A new DSTM scheme based on Weyl group for MIMO systems with 2, 4 and 8 transmit antennas

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Abstract—In this paper, we propose a differential space-time modulation (DSTM) scheme for multiple input multiple output (MIMO) systems with 2, 4, and 8 transmit antennas. This scheme is used for MIMO systems where the channel coefficients are not available at both the transmitter and the receiver. For 2 transmit antennas, the transmission matrices are selected from Weyl group. We use the Kronecker product of matrices of the Weyl group to design new groups for the schemes using 4 and 8 transmit antennas. Simulation results show the performance of these schemes.

Keywords—MIMO, Differential Space-Time Modulation, Non-coherent, Weyl group, Kronecker product.

I. INTRODUCTION

The last decade of the 20th century witnessed the great demand for data communications in the wireless communication domain. Researchers have to search new methods to enlarge the capacity and robustness of wireless communication systems. Among these methods, multiple-input-multiple-output (MIMO) technics using multiple antennas at both the transmitter and receiver are considered as a competitive method and have been widely analyzed.

Generally, MIMO systems can be divided into two types according to whether the receiver needs the precise channel state information (CSI). The first one is represented by the coherent MIMO systems which need to estimate the CSI at the receive side. References [1], [2] analyzed the capacity and the error exponents of such systems with Gaussian noise. Lots of coding schemes have been proposed such as space-time block codes (STBC) [3], space-time trellis codes (STTC) [4], Bell Labs layered space-time codes (BLAST) [5], etc.

Actually, the CSI is often obtained by training. Known signals are periodically transmitted to the receiver in order to estimate the channel coefficients. However, when many antennas are used or when the propagation channel changes rapidly, the training based scheme doesn’t work effectively. For MIMO systems, the number of channel coefficients to be estimated is equal to the product of the number of transmit antennas by the number of receive antennas. In addition, the length of the training sequences is proportional to the length of the training sequences is proportional to the number of transmit antennas by the number of receive antennas. At time slot $t$, the antenna $n$ receives the signal:

$$y_{nt} = \sum_{m=1}^{M} h_{nm} x_{mt} + w_{nt}, n = 1, \ldots, N$$

In [7], Marzetta and Hochwald analyzed the capacity of the MIMO systems without CSI. They found that the transmission matrices have a special structure to achieve capacity, called unitary space-time modulation (USTM) [8]. In succession, Hochwald and Sweldens proposed the differential unitary space-time modulation (DUSTM) scheme [9]. These two schemes are difficult to design. Moreover, there is not general design criteria for these two schemes. Meanwhile, based on Alamouti’s transmit diversity scheme [10], Tarokh and Jafarkhani proposed a differential space-time block coding (DSTBC) scheme [11] for MIMO systems with 2 transmit antennas and expanded this scheme to systems with 4 transmit antennas in [12]. The spectral efficiency of this scheme for 4 transmit antennas is limited to 1 bps/Hz. In [13], [14], Arab et al invented a new kind of non-coherent space-time modulation scheme—matrix coded modulation (MCM) which is suitable for $2 \times 2$ MIMO systems. However, the above schemes are just suitable for MIMO systems with 2 or 4 transmit antennas.

In the study of the MCM scheme, we found that the Weyl group used in this scheme can also be used for differential MIMO schemes. Hence, we proposed a new differential space-time modulation scheme in [15] and used in [16] the distance spectrum to improve the error performance for MIMO systems with 2 and 4 transmit antennas by selecting transmit matrices separated by the largest distances. In this paper, we expand the scheme to MIMO systems with 8 transmit antennas. The simulation results demonstrate the advantage of this new scheme.

The following notations will be used through the paper: $\text{Tr}\{A\}$ denotes the trace of the matrix $A$ and $A^H$ means the conjugate transpose of $A$. $\|A\|$ is the Frobenius norm of $A$, i.e., $\|A\| = \sqrt{\sum_{i,j} |a_{ij}|^2} = \sqrt{\text{Tr}\{A^H A\}}$. $\text{Re}\{z\}$ is the real part of the complex number $z$. The sign $\lfloor x \rfloor$ denotes the largest integer number not exceeding $x$, and $\otimes$ means the Kronecker product. The zero-mean, unit-variance, circularly symmetric, complex Gaussian distribution is denoted as $CN(0,1)$.

II. DIFFERENTIAL MIMO SYSTEM MODEL

Consider a MIMO system with $M$ transmit antennas and $N$ receive antennas. At time slot $t$, the antenna $n$ receives the signal:

$$y_{nt} = \sum_{m=1}^{M} h_{nm} x_{mt} + w_{nt}, n = 1, \ldots, N$$

(1)
The SNR is defined as follows:

$$\text{SNR} = \frac{E[|y_{nt} - w_{nt}|^2]}{E[|w_{nt}|^2]} = \frac{E \left[ \sum_{m=1}^{M} h_{nm} x_{mt} \right]^2}{\sigma^2} = \frac{E \left[ \sum_{m=1}^{M} |x_{mt}|^2 \right]}{\sigma^2} = \frac{1}{\sigma^2}$$

(4)

where $E[\cdot]$ denotes the mathematical expectation.

It is proved in [7] that for non-coherent MIMO systems with block length $T$ and $M$ transmit antennas, the capacities obtained with $M > T$ and $M = T$ are equal, no matter how much the SNR (signal to noise ratio) and the number of receive antennas are. Therefore, we choose $M = T$ in our study.

A. Differential space-time modulation

As for the differential space-time modulation systems, the information matrix is used to multiply the previous transmitted matrix. In general, the information matrix is selected from a group $P$ according to the incoming information bits. For example, at time $\tau$, $X_{\tau}$ is transmitted. At the next time $\tau + 1$, a block of information bits is mapped onto the matrix $V_{i_{\tau+1}}$ from the group $P$, and then the matrix

$$X_{\tau+1} = X_{\tau} V_{i_{\tau+1}}$$

(5)

is transmitted. This relation is the fundamental differential transmission equation.

Therefore, the sequence of transmitted matrices is:

$$X_0 = V_0$$
$$X_1 = X_0 V_{i_1} = V_0 V_{i_1}$$
$$X_2 = X_1 V_{i_2} = V_0 V_{i_1} V_{i_2}$$
$$\ldots$$
$$X_{\tau} = X_{\tau-1} V_{i_\tau} = V_0 V_{i_1} \ldots V_{i_\tau}$$
$$\ldots$$

In general, the reference matrix $V_0$ is the identity matrix. To satisfy the constraint (3) imposed on the total transmit power, all the matrices of the group $P$ should be unitary matrices, otherwise the power of the transmit signal maybe some extremely high or low values.

Assume that the synchronization is perfect, subsequently, a matrix stream $Y_0, Y_1, Y_{\tau+1}, \ldots$ is detected by the receive antennas. We have the relations

$$Y_{\tau} = H X_{\tau} + W_{\tau}$$

(6)

and

$$Y_{\tau+1} = H X_{\tau+1} + W_{\tau+1}$$

(7)

With the differential transmission equation (5), we get

$$Y_{\tau+1} = H X_{\tau+1} + W_{\tau+1} = Y_{\tau} V_{i_{\tau+1}} + W_{\tau+1}$$

(8)

where $W'_{\tau+1} = W_{\tau+1} - W_{\tau} V_{i_{\tau+1}}$. 

where $h_{nm}$ is the path gain of the quasi-static channel from the transmit antenna $m$ to the receive antenna $n$. It corresponds to a narrow band (hence frequency non-selective) channel, and its envelope is Rayleigh distributed. The channel coefficients are independent and identically distributed (iid), $h_{nm} \sim C N(0,1)$. $x_{mt}$ is the symbol transmitted from antenna $m$ at time slot $t$. $w_{nt}$ is the additive white Gaussian noise, $w_{nt} \sim C N(0, \sigma^2)$ and $\sigma^2$ is also the power of the noise.

For convenience, we use the matrix form to analyze a MIMO system. Therefore, the matrix $Y_{\tau}$ of the received signals $y_{nt}$ can be expressed as:

$$Y_{\tau} = H_{\tau} X_{\tau} + W_{\tau}$$

(2)

where $\tau$ is the time index for the matrices, while $t$ is the time index for the transmitted symbols. $Y_{\tau}$ is the $N \times T$ received matrix, where $T$ denotes the number of symbols of each matrix. $H_{\tau}$ is the channel coefficients matrix at time $\tau$ and its size is $N \times M$. $X_{\tau}$ is the $M \times T$ transmission matrix and $W_{\tau}$ is the $N \times T$ additive white Gaussian noise matrix.

We define $L$ equal to the normalized coherence interval $[T_c/T_s]$ during which the channel matrix $H_{\tau}$ is approximately constant, where $T_c$ is the coherence interval and $T_s$ is the symbol duration. A popular definition of $T_c$ is: $T_c = \frac{\lambda}{4\pi f_d} = \frac{0.423}{f_d}$ [17], where $f_d = \frac{V}{\lambda}$ is the Doppler spread, $V$ is the relative velocity between the transmitter and receiver, and $\lambda$ is the signal wavelength. In practice, for simplicity, people usually use it as $T_c \approx 0.5/f_d$. For example, with velocity $V = 120$ km/h, and carrier frequency $f = 900$ MHz, the Doppler spread is approximately 100 Hz and the coherence interval is approximately 5 ms. During this period, for a symbol rate of 30 kHz, $L = 150$ symbols are transmitted. For high speed vehicular $V = 350$ km/h channels [18], and carrier frequency $f = 2.5$ GHz, the Doppler spread is approximately 810 Hz and the coherence interval is approximately 0.6 ms. For a symbol rate of 500 kHz, $L = 300$ symbols are transmitted.

We can see that during $L(L >> T)$ transmitted symbols, the channel coefficients can be considered approximately constant. Therefore, we ignore the index $\tau$ of the matrix $H_{\tau}$ for a length of $L$ symbols or $[L/T]$ transmitted matrices and the channel matrix is written to be $H$. As in [11], the channel matrix varies from a block of $L$ transmitted symbols to the next one.

Furthermore, we set the total power over $M$ transmit antennas at each transmit time to be 1:

$$\sum_{m=1}^{M} |x_{mt}|^2 = 1, \quad t = 1, \ldots, T.$$  

(3)
As $Y_\tau$ and $Y_{\tau+1}$ are known by the receiver, we can use the maximum likelihood detector to estimate the information matrix:

$$\hat{V}_{\tau+1} = \arg \min_{V} \| Y_{\tau+1} - Y_\tau V \|$$

$$= \arg \min_{V} \Tr \{(Y_{\tau+1} - Y_\tau V)^H (Y_{\tau+1} - Y_\tau V)\}$$

$$= \arg \max_{V} \Tr \{ \Re (Y_{\tau+1}^H Y_\tau V) \}$$

(9)

III. NEW DIFFERENTIAL SPACE-TIME MODULATION SCHEME

A. The constellation of the new scheme

As mentioned before, each transmission matrix should be unitary. In the study of MCM schemes [13], [14], we found that the Weyl group is a unitary matrices group. Therefore, the matrices of this group can be used to design a new differential space-time modulation scheme.

The multiplicative Weyl group $G_w$ [19] is generated by two matrices $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$. As these two matrices are unitary, all the matrices generated by them are also unitary. For convenience, we divide the group into 12 cosets $(C_0, C_1, \ldots, C_{11})$. Each coset contains 16 invertible matrices. The first coset which is also a subgroup of the Weyl group is defined as:

$$C_0 = \alpha \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

(10)

with $\alpha \in \{1, -1, i, -i\}$. The 12 cosets of $G_w$ are derived from $C_0$ as follows:

$$C_k = A_k C_0, \forall k = 0, 1, \ldots, 11$$

(11)

where the matrices $A_k, k = 0, 1, \ldots, 5$ are respectively:

$$A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, A_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, A_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}, A_4 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix}, A_5 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ 1 & 1 \end{bmatrix},$$

and the matrices $A_k, k = 6, 7, \ldots, 11$ are given by:

$$A_{k+6} = \eta A_k, with \ \ \eta = (1 + i)/\sqrt{2}, \forall k = 0, 1, \ldots, 5$$

(12)

There are 192 matrices in this group, and we number the matrices as $M_0, M_1, \ldots, M_{191}$. As they are all unitary matrices, they satisfy the constraint (3) imposed on the total transmit power.

We define the distance between two matrices $M_a$ and $M_b$ as:

$$D_{a,b} = \| M_a - M_b \|.$$  

(13)

This distance can be used to improve the error performance of the new differential space-time modulation scheme by selecting the subset of transmitted matrices separated by the largest distances.

B. Differential scheme for MIMO systems with 2 transmit antennas

For MIMO system with $M = 2$ transmit antennas, the transmit matrix should be sent during $T = M = 2$ symbol durations. The number of receive antennas is arbitrary, i.e. it can be $N = 1, 2, 3, \ldots$. For our first experiment, the coset $C_0$ is selected as the set of matrices used for mapping the blocks of information bits. There are 16 matrices $(M_0, M_1, \ldots, M_{15})$ in the coset $C_0$, so 4 bits are mapped to a given matrix of the coset $C_0$. As 4 bits are sent during 2 symbol durations, the spectral efficiency $R$ is 2 bps/Hz.

We use a general binary mapping rule from the information bits to the information matrices, as shown in Table I.

<table>
<thead>
<tr>
<th>Information bits</th>
<th>Matrix in coset $C_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>$M_0 = \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>0001</td>
<td>$M_1 = \begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
</tr>
<tr>
<td>0010</td>
<td>$M_2 = \begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>0011</td>
<td>$M_3 = \begin{bmatrix} 0 &amp; 1 \ 1 &amp; -1 \end{bmatrix}$</td>
</tr>
<tr>
<td>0100</td>
<td>$M_4 = \begin{bmatrix} 1 &amp; 0 \ -1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>0101</td>
<td>$M_5 = \begin{bmatrix} -1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>0110</td>
<td>$M_6 = \begin{bmatrix} 0 &amp; -1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>0111</td>
<td>$M_7 = \begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1000</td>
<td>$M_8 = \begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1001</td>
<td>$M_9 = \begin{bmatrix} 1 &amp; 0 \ i &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1010</td>
<td>$M_{10} = \begin{bmatrix} 0 &amp; 1 \ i &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1011</td>
<td>$M_{11} = \begin{bmatrix} 0 &amp; 1 \ i &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1100</td>
<td>$M_{12} = \begin{bmatrix} 0 &amp; 1 \ -i &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1101</td>
<td>$M_{13} = \begin{bmatrix} 1 &amp; 0 \ 0 &amp; i \end{bmatrix}$</td>
</tr>
<tr>
<td>1110</td>
<td>$M_{14} = \begin{bmatrix} 0 &amp; -1 \ -i &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>1111</td>
<td>$M_{15} = \begin{bmatrix} 0 &amp; -1 \ i &amp; 0 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

| TABLE I |
|------------------|-----------------------|
| THE GENERAL MAPPING RULE FROM THE INFORMATION BITS TO MATRICES OF THE COSET $C_0$ |

At time $\tau = 0$, the reference matrix $X_0 = M_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is transmitted.

Suppose that at time $\tau$, $X_\tau$ is transmitted. At the next instant $\tau + 1$, a new block of 4 information bits must be transmitted. These bits are mapped onto one of the matrices $M_{\tau+1}(\tau+1 \in \ldots, 11)$.
The matrices in this set among which only 4608 matrices are transmitted. The demodulation procedure was shown in Section II-A. For this scheme, the maximum spectral efficiency is: \( R = \frac{1}{3M} \lfloor \log_2 K \rfloor = \frac{1}{3} \lfloor \log_2 192 \rfloor = 3.5 \text{ bps/Hz}. \)

### C. Differential MIMO systems with 4 transmit antennas

For the differential MIMO system with 4 transmit antennas, each transmission matrix should be a 4 \( \times \) 4 matrix, according to the assumption \( M = T \) indicated in Section II. The Kronecker product is a feasible method to expand the 2 \( \times \) 2 Weyl group to a 4 \( \times \) 4 matrices group.

The Kronecker product of two arbitrary matrices A and B is defined as:

\[
A \otimes B = \begin{bmatrix}
a_{11}B & \cdots & a_{1n}B \\
\vdots & \ddots & \vdots \\
a_{m1}B & \cdots & a_{mn}B
\end{bmatrix}
\]

where A is an \( m \times n \) matrix, B is a \( p \times q \) matrix and the resulting matrix is an \( mp \times nq \) matrix. The Kronecker product has the properties:

1) The Kronecker product is not commutative:
\( A \otimes B \neq B \otimes A. \)

2) \( A \otimes B \) is invertible if and only if A and B are invertible:
\( (A \otimes B)^{-1} = A^{-1} \otimes B^{-1}. \)

3) The operation of transposition is distributive over the Kronecker product:
\( (A \otimes B)^T = A^T \otimes B^T. \)

4) The Kronecker product is associative:
\( (A \otimes B) \otimes C = A \otimes (B \otimes C). \)

Computing the Kronecker product between each couple of 2 \( \times \) 2 matrices of the Weyl group, a 4 \( \times \) 4 matrices set \( \{G_w \otimes G_{w4}\} \) is obtained. There are 192 \( \times \) 192 = 36864 matrices in this set among which only 4608 matrices are distinct. They are denoted \( N_0, N_1, \ldots, N_{4607}. \) The set of these matrices is also a group denoted \( G_{w4}. \)

The maximum spectral efficiency we can get is \( R = \frac{1}{4M} \lfloor \log_2 K \rfloor = \frac{1}{4} \lfloor \log_2 4608 \rfloor = 3 \text{ bps/Hz}. \)

We presented the detailed information of this new differential space-time modulation scheme in [15]. Based on the distance spectrum of the group \( G_{w4}, \) as shown in [16], it is possible to select a set of information matrices separated by the largest distances to reduce the bit error rate (BER) and hence to improve the system performance.

Fig. 1 presents the simulation results obtained for 4 transmit antennas. For spectral efficiency \( R = 1, \) our scheme is about 1.5 dB better than the DSTBC scheme [12]. For spectral efficiency \( R = 2, \) our scheme with an improved set of matrices operates much better than the DUSTM scheme [9]. Indeed, for BER values lower than \( 10^{-5}, \) the SNR is reduced with about 2.5 dB.

### D. Differential MIMO systems with 8 transmit antennas

As for the scheme using 4 transmit antennas, we can expand the new scheme for MIMO systems with 8 transmit antennas. Using the Kronecker product between the matrices of the Weyl group and the matrices of \( G_{w4}, \) 8 \( \times \) 8 matrices are obtained.

In fact, there are 192 \( \times \) 4608 = 884736 matrices in the set \( \{G_w \otimes G_{w4}\}. \) However, only 110592 matrices are distinct. We denote this set of distinct 8 \( \times \) 8 matrices \( G_{w8}. \) The maximum spectral efficiency we can get is \( R = \frac{1}{8M} \lfloor \log_2 K \rfloor = \frac{1}{8} \lfloor \log_2 110592 \rfloor = 2 \text{ bps/Hz}. \)

The simulation results obtained for this new scheme with 8 transmit antennas are presented in Fig. 2. In our simulation, we assume that the channel coefficients are constant during the transmission of \( L = 200 \) symbols and change randomly to new values for the next block of symbols. As shown in this figure, we select the first 16, 256, 4096, 65536 matrices from the set \( G_{w8} \) for spectral efficiencies 0.5, 1, 1.5, 2 bps/Hz respectively. We can remark that for the last two values of the spectral efficiency, the results are quite similar.

As mentioned before, we can use the distance spectrum to improve the error performance. We select a new set for \( R = 1 \) scheme.

The new set is selected as follows: first, we generate a set of 16 matrices of \( C_{44} \) by using the Kronecker product between the first 4 matrices of \( G_w (M_0, M_1, M_2, \text{ and } M_3). \) Second, the Kronecker product between \( C_0 \) (16 matrices of the size 2 \( \times \) 2) and \( C_{44} \) produces a set \( C_{88} \subset G_{w8} \) with 256 matrices. For \( R = 1, \) this new set is used for the mapping between blocks of 8 information bits and matrices of \( C_{88}. \) This new set has a better distance spectrum than the original one. Simulation result in Fig. 2 shows the great advance of this new selected set. For example, for BER=\( 10^{-5}, \) the SNR is reduced with 3 dB.
In this paper, based on the initial scheme proposed in [15], we propose new differential space-time modulation schemes. For $2 \times 2$ MIMO systems, the transmitted matrices belong to the Weyl group. For $4 \times 4$ and $8 \times 8$ MIMO systems, we used the Kronecker product to expand the Weyl group and design new differential MIMO schemes. This method can be further used for MIMO systems using $2^n$ transmit antennas, $n$ being any positive integer. Simulation results show that these schemes perform quite well.

Furthermore, the performance of these schemes can be improved using for each value of the spectral efficiency the set of matrices separated by the largest distance, as in [16], [20]-[22].

**REFERENCES**


