Transmit Power Optimization for Multiantenna Decode-and-Forward Relays with Loopback Self-Interference from Full-Duplex Operation

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Session TP1a “Resource Allocation in Multi-Antenna Systems,” Nov. 8, 2011
45th Asilomar Conference on Signals, Systems and Computers
The information theory of full-duplex MIMO relaying has been investigated extensively (2005–2007)

- Implementation aspects are out of the scope of those studies
  - The limitations of transceiver electronics?
  - How to isolate the two hops?
  - Relay with a single antenna array?
- In this work: A step to more practical direction

First prototypes of full-duplex MIMO repeaters (2009–2010)
- But the background theory still needs further development
The existence of self-interference was recognized only recently
  - Ahead of their time: Bliss, Parker, and Margetts (Aug. 2007)
  - The relay must be equipped with separated Rx and Tx arrays

Interference mitigation schemes for full-duplex MIMO relays
  - Our contributions: [ACSSC’09], [ACSSC’10], [CISS’11], [IEEE TSP 12/2011]

Achievable transmission rates of full-duplex amplify-and-forward MIMO relay links with self-interference
  - In this work: Decode-and-forward relaying
Our earlier studies on full-duplex SISO relays
  ▶ Transmit power adaptation (i.e., gain control in AF relaying) taking into account the self-interference
    - See [IEEE TWC 6/2009], [SPAWC’09], [PIMRC’10], [IEEE TWC 9/2011]
  ▶ Comparison of full-duplex and half-duplex relaying in the presence of (residual) self-interference
    - See [WCNC’09], [SPAWC’09], [PIMRC’10], [IEEE TWC 9/2011]

In this work: The above aspects generalized for the MIMO case
Two-hop transmission through a full-duplex MIMO relay

- The source (S) and the relay (R) transmit simultaneously

\[ x_S \in \mathbb{C}^{N_S \times 1} \quad \text{and} \quad x_R \in \mathbb{C}^{N_{tx} \times 1} \]

- and the relay and the destination (D) receive

\[ y_R = H_{SR} x_S + H_{RR} x_R + n_R \in \mathbb{C}^{N_{tx} \times 1} \]
\[ y_D = H_{RD} x_R + H_{SD} x_S + n_D \in \mathbb{C}^{N_D \times 1} \]

- \( H_{RR} \) represents the residual channel if mitigation is used
- The direct link is assumed to be blocked, i.e., \( H_{SD} \approx 0 \)
The Reference System: Half-Duplex Relay Link

- Two-hop transmission through a half-duplex MIMO relay
  - 1st time slot (duration $\tau_S$): The source transmits $x_S \in \mathbb{C}^{N_S \times 1}$ and the relay receives
    \[ y_R = H_{SR} x_S + n_R \in \mathbb{C}^{N_{rx} \times 1} \]
  - 2nd time slot (duration $\tau_R$): The relay transmits $x_R \in \mathbb{C}^{N_{tx} \times 1}$ and the destination receives
    \[ y_D = H_{RD} x_R + n_D \in \mathbb{C}^{N_D \times 1} \]
- No self-interference ($H_{RR} = 0$) at the cost of using two time slots
Regenerative (DF) MIMO Relaying Protocol

- Pre-whitening: \( W = (H_{RR}R_xR_x^H + I)^{-\frac{1}{2}} \) and \( \tilde{H}_{SR} = WH_{SR} \)
- Spatial-division multiplexing transforms the signal model to
  \[ \tilde{y}_R = \tilde{\Sigma}_{SR}\tilde{x}_S + \tilde{n}_R \]
  \[ \tilde{y}_D = \Sigma_{RD}\tilde{x}_R + \tilde{n}_D \]

  ▶ SVDs: \( \tilde{H}_{SR} = \tilde{U}_{SR}\tilde{\Sigma}_{SR}\tilde{V}_{SR}^H \) and \( H_{RD} = U_{RD}\Sigma_{RD}V_{RD}^H \)

- The half-duplex link operates in the same way but with \( H_{RR} = 0 \)
Spatial-division multiplexing diagonalizes $\tilde{H}_{SR}$ and $H_{RD}$, but not the self-interference channel: $\tilde{H}_{RR} = \tilde{U}^H_{SR} WH_{RR} V_{RD}$

- Transmission of independent spatial streams with
  \[
  P_S = \mathcal{E}\{\tilde{x}_S\tilde{x}^H_S\} = \text{diag} \{p_S[1], \ldots, p_S[N_S]\} \\
  P_R = \mathcal{E}\{\tilde{x}_R\tilde{x}^H_R\} = \text{diag} \{p_R[1], \ldots, p_R[N_{tx}]\}
  \]

- DF: The relay decodes $\tilde{y}_R$ and re-encodes the data into $\tilde{x}_R$

- Separate transmit power constraints:
  \[
  p_S = \text{tr}\{P_S\} = \sum_{n=1}^{N_S} p_S[n] \leq 1, \quad p_R = \text{tr}\{P_R\} = \sum_{n=1}^{N_{tx}} p_R[n] \leq 1
  \]
Transmission Rates
The rates of the two hops are given by

\[ R_{SR} = \log_2 \det \{ \mathbf{I} + \tilde{\Sigma}_{SR} \mathbf{P}_S \tilde{\Sigma}_{SR}^H \} = \min\{N_S, N_{rx}\} \sum_{n=1}^{\min\{N_S, N_{rx}\}} \log_2 \left( 1 + p_S[n] \tilde{\sigma}_{SR}^2[n] \right) \]

\[ R_{RD} = \log_2 \det \{ \mathbf{I} + \Sigma_{RD} \mathbf{P}_R \Sigma_{RD}^H \} = \min\{N_{tx}, N_D\} \sum_{n=1}^{\min\{N_{tx}, N_D\}} \log_2 \left( 1 + p_R[n] \sigma_{RD}^2[n] \right) \]

The end-to-end rate is given by

\[ R_{FD} = \min\{R_{SR}, R_{RD}\} \]

since data should not accumulate in the relay
• By setting $H_{RR} = 0$, the rates of the two hops are given by $R_{SR}$ and $R_{RD}$ as shown in the previous slide.

• The end-to-end rate is given by

$$R_{HD} = \min\{\tau_S R_{SR}, \tau_R R_{RD}\}$$

- The typical reference case is $\tau_S = \tau_R = \frac{1}{2}$
- Optimal time shares are $\tau_S = R_{RD}/(R_{SR} + R_{RD})$, $\tau_R = 1 - \tau_S$:

$$R_{HD} = \max_{\tau_S + \tau_R \leq 1} \min\{\tau_S R_{SR}, \tau_R R_{RD}\} = \frac{R_{SR} R_{RD}}{R_{SR} + R_{RD}}$$
Separately Optimal Transmit Powers

- The reference case: Two-step approach for power allocation
  1. Maximize the second-hop rate:

\[
C_{RD} = \max_{P_R} R_{RD} = \min\{N_{tx}, N_D\} \sum_{n=1} \max\left\{0, \log_2 \left( \mu_R \sigma^2_{RD}[n] \right) \right\}
\]

2. And then, given \( P_R \), maximize the first-hop rate:

\[
C_{SR} = \max_{P_S} R_{SR} = \min\{N_S, N_{rx}\} \sum_{n=1} \max\left\{0, \log_2 \left( \mu_S \tilde{\sigma}^2_{SR}[n] \right) \right\}
\]

Water-filling: \( p_S[n] = \max\{0, \mu_S - \frac{1}{\sigma^2_{SR}[n]}\} \), \( p_R[n] = \max\{0, \mu_R - \frac{1}{\sigma^2_{RD}[n]}\} \)

- This approach is optimal for half-duplex relaying:

\[
R_{HD} = \frac{1}{2} \min\{C_{SR}, C_{RD}\}, \quad C_{HD} = \frac{C_{SR}C_{RD}}{C_{SR} + C_{RD}}
\]
Jointly Optimal Transmit Powers

- Separate power adaptation is suboptimal for full duplex relaying, because the hops are coupled:
  \[ R_{FD} = \min\{C_{SR}, C_{RD}\} \]

- Single-step approach for jointly optimal power allocation:
  \[ C_{FD} = \max_{P_S, P_R} \min\{R_{SR}, R_{RD}\} = \max_{P_R} \min\{C_{SR}, R_{RD}\} \]
  - Only numerical solution available except for the SISO case
  - The solution lies in the subspace for which \( C_{SR} = R_{RD} \)

- Two-fold benefit from jointly optimal power allocation
  1. Interference is directed to the least harmful spatial dimensions
  2. End-to-end rate improvement by decreasing relay’s Tx power
Example System Setup

- Let us illustrate next power allocation in the simplest MIMO case, i.e., $N_S = N_{rx} = N_{tx} = N_D = 2$
- and choose randomized example channels as

$$
H_{SR} = \sqrt{\gamma_{SR}} \left( \begin{bmatrix} 0.5036 & 0.4348 \\ -0.5794 & 0.8751 \end{bmatrix} + j \begin{bmatrix} 0.7546 & 0.8125 \\ 1.1061 & 0.0528 \end{bmatrix} \right)
$$

$$
H_{RR} = \sqrt{\gamma_{RR}} \left( \begin{bmatrix} 0.5387 & 0.3153 \\ 0.7987 & -0.7633 \end{bmatrix} + j \begin{bmatrix} -1.3410 & 0.2403 \\ -0.6481 & 0.3370 \end{bmatrix} \right)
$$

$$
H_{RD} = \sqrt{\gamma_{RD}} \left( \begin{bmatrix} 0.0281 & 0.7647 \\ -0.2892 & -0.5713 \end{bmatrix} + j \begin{bmatrix} 0.4119 & -0.1978 \\ -1.2992 & 1.0524 \end{bmatrix} \right)
$$

where $\gamma_{SR}$, $\gamma_{RR}$, and $\gamma_{RD}$ represent the channel SNRs

- Next slides: $\gamma_{SR} = 15$dB, $\gamma_{RR} = 5$dB, and $\gamma_{RD} = 20$dB
**Effect of Self-Interference at First Hop**

(a) $R_{SR}$ when $p_{R}[1] = p_{R}[2] = 0$

(b) $R_{SR}$ when $p_{R}[1] = p_{R}[2] = \frac{1}{2}$

- The rate of the first hop drops significantly when the relay transmits.
- The source should use maximum transmit power: $\sum_{n=1}^{N_S} p_S[n] = 1$
First Hop with Self-Interference vs. Second Hop

- Increasing relay transmit power benefits the second hop but harms the first hop. Equilibrium is optimal: $C_{SR} = R_{RD}$
Benefit of Transmit Power Optimization

(a) Full-duplex relay: $R_{FD}$

(b) Half-duplex relay

- Equal time shares:
  $R_{HD} = 4.8$ bit/s/Hz

- Optimal time shares:
  $C_{HD} = 5.0$ bit/s/Hz

- Full transmit power is used:
  $p_{R[1]} + p_{R[2]} = 1$

- Transmit power adaptation increases the transmission rate by 25% while decreasing the total relay transmit power by 56%
Transmission Rates vs. Self-Interference Level

- On the right: Varying $\gamma_{RR}$ when
  $\gamma_{SR} = 15\,\text{dB}$
  $\gamma_{RD} = 20\,\text{dB}$

- Next slides: Varying $\gamma_{SR}$ and $\gamma_{RD}$ when
  $\gamma_{RR} = 10\,\text{dB}$

- Spatial-division duplexing (SDD) vs. time-division duplexing (TDD)
  ▶ If transmit power adaptation is used, full-duplex relay can achieve non-zero rate with any level of self-interference
Power Optimization vs. Time Optimization

(a) $C_{FD}/R_{FD}$

- FD: Up to *five* times larger rate using optimal transmit powers

(b) $C_{HD}/R_{HD}$

- HD: Up to *two* times larger rate using optimal time shares
Full-Duplex Relaying vs. Half-Duplex Relaying (1)

- Prefer FD when $\gamma_{SR} \gg \gamma_{RD}$
- Prefer HD when $\gamma_{SR} \ll \gamma_{RD}$

FD is better or equal to HD within the whole SNR range
Full-Duplex Relaying vs. Half-Duplex Relaying (2)

(a) $R_{FD}/C_{HD}$

- FD: Up to 60% higher rate
- HD: Up to 440% higher rate

(b) $C_{FD}/C_{HD}$

- Up to 60% (FD) or 40% (HD) higher rate with mode selection
Conclusion
Conclusion

- Full-duplex MIMO relaying can offer significantly improved spectral efficiency w.r.t. half-duplex MIMO relaying
- Main technical problem: self-interference in the relay
  - Separated Rx and Tx antenna arrays for natural isolation
  - Mitigation schemes for additional isolation
- Optimal transmit power allocation for alleviating the effect of potential residual interference
  - Spatial-division duplexing instead of time-division half-duplex
  - Interference is directed to the least harmful dimensions
  - Simultaneous power savings and rate improvement
- Full-duplex vs. half-duplex relaying in an example case
  - Fair comparison: Half-duplex time slots can be optimized


• T. Riihonen, S. Werner, and R. Wichman, “Hybrid full-duplex/half-duplex relaying with transmit power adaptation,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 9, pp. 3074–3085, September 2011.
References (MIMO)
