Joint Power and Bandwidth Allocation for Discrete-Rate Multi-User Link Adaptation with Imperfect Channel State Information

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Abstract—In practical wireless networks, the available transmission power and bandwidth are limited resources. Therefore, joint bandwidth and power allocation for wireless multi-user networks is essential in order to improve the network performance. Most of the research has focused on continuous rate, power, and bandwidth allocations in the presence of perfect channel knowledge. However, this is not the case with practical systems. In this paper, we therefore consider the issue of discrete power and bandwidth allocation for discrete-rate multi-user link adaptation with imperfect channel state information. To be more specific, we discuss how the system can be designed in such a scenario for i) sum rate maximization and ii) average power minimization in a multi-user setting. The results show that with only a few codes, we can approach the performance of systems that employ continuous (infinite) rates. We have also found that imperfect channel information at the base station affects the performance such that the sum rate is decreased and the average power consumption is increased.

I. INTRODUCTION

In practical wireless networks, the available power and bandwidth are limited. Equal allocation of bandwidth and power to all users may not be efficient for multi-user networks. Therefore, joint bandwidth and power allocation is essential in order to improve the network performance. In literature [1]–[4], joint bandwidth and power allocation has been studied for i) maximizing the sum rate of all users, ii) maximizing the rate of the worst user, and iii) minimizing the average power consumption of all users. However, most of the research has focused on continuous rate, power, and bandwidth allocations, i.e., it is assumed that there are infinite number of rates, power and bandwidth levels available. However, this is not the case with practical systems. In this paper, we therefore consider the issue of joint discrete power and bandwidth allocation for discrete-rate multi-user link adaptation.

Link adaptation or adaptive coded modulation (ACM) has been shown to improve the throughput of wireless networks even in the presence of fading [5]–[7]. For a total of $C$ codes and (close-to-)capacity-achieving codes for AWGN channels, single-user ACM systems are designed in [6], such that the chosen signal-to-noise ratio (SNR) thresholds and corresponding rates (codes) are optimal with respect to maximal average spectral efficiency. In [7] continuous and discrete power adaptation for discrete-rate single-user link adaptation with perfect channel knowledge is investigated. The resulting schemes yield high transmission rates using only a few codes. In real systems, the channel prediction is not always perfect. However, robustness against channel prediction errors can be achieved by increasing the thresholds and being more cautious in the selection of codes [8].

In this work, we assume imperfect channel knowledge at the base station, and analyze the issue of joint power and bandwidth allocation for discrete rates by extending the idea of [7] to multi-user systems, e.g., OFDM or CDMA systems. To be more specific, we discuss how the ACM can be designed for i) sum rate maximization and ii) average power minimization in a multi-user system, with imperfect channel knowledge at the base station.

The rest of the paper is organized as follows. We present the system model and the ACM in Section II. In Section III, we discuss the joint power and bandwidth problem for the two scenarios mentioned above. Numerical results are presented in Section IV, and we list our conclusions in Section V.

II. SYSTEM MODEL

We consider a wireless downlink system, where in each time-slot the base station resources, i.e., bandwidth and power, are divided among $M$ users, based on the channel feedback from them. It is assumed that the total number of users in the system remains constant and equal to $M$. We further assume that users cannot share the same bandwidth, and thus they do not interfere with each other. The wireless system between the base station and user $i$ is shown in Fig. 1. The discrete-time channel between the base station and user $i$ is a wide-sense stationary fading channel, with time-varying gain $g_i[n]$ and AWGN noise $n_i[n]$. The fading is assumed to be slow and flat. Let $P$ denotes the average transmit power without power adaptation, and $W$ [Hz] is the transmission bandwidth allocated to user $i$ without adaptation, such that the total

\footnote{Otherwise the system needs to run the optimization problem again for each new value of $M$.}
available bandwidth is $W_i = MW$ [Hz]$^2$. The instantaneous pre-adaptation received SNR is then $\gamma_i[n] = P_g \gamma_i[n]/(N_0 W)$, where $N_0$ is the noise power spectral density. The average pre-adaptation received SNR is denoted by $\tilde{\gamma}_i[n]$. Assuming that the transmit power and bandwidth are adapted instantaneously based on a predicted received SNR, $\hat{\gamma}_i[n]$, we denote the transmit power and bandwidth as $P_l(\hat{\gamma}_i[n])$ and $W_l(\hat{\gamma}_i[n])$ respectively. The post-adaptation received SNR is then given as follows:

$$P_l(\hat{\gamma}_i[n]) W_l(\hat{\gamma}_i[n])^{-1} \gamma_i[n], \quad (1)$$

It should be noted that the users will have different noise powers after adaptation, since they are allocated different bandwidths.

It is also assumed that a maximum a posteriori (MAP) optimal predictor is used to predict $\hat{\gamma}_i[n]$ [9]. A brief introduction of the MAP optimal predictor is given in the next section.

A. Pilot-Symbol assisted MAP-Optimal Prediction

Channel prediction is done using $\mathcal{K}$ maximum-likelihood (ML) estimates at time instant $n$ and its elements represent the correlation between the fading to be predicted at time $n + \Delta$ and the fading at the pilot-symbol instants. $\mathbf{R}_\mathcal{K}$ is the autocorrelation matrix of the fading at the pilot-symbol instants. With the assumption of Jakes spectrum in the fading process, the elements of $\mathbf{r}_{\Delta \mathcal{K}}$ and $\mathbf{R}_\mathcal{K}$ can be calculated from the following equations [10]:

$$[\mathbf{r}_{\Delta \mathcal{K}}]_l = R((\Delta + l \mathcal{L})T_s), \quad (5)$$

and

$$[\mathbf{R}_\mathcal{K}]_{lm} = R(l - m \mathcal{L}T_s), \quad (6)$$

where $T_s$ is the symbol duration, $l$ and $m$ are integers, and $R(\tau)$ is the autocorrelation function given as

$$R(\tau) = J_0(2\pi f_D \tau), \quad (7)$$

where $J_0(x)$ is the zeroth-order Bessel function of the first kind, and $f_D = v/c$ is the Doppler frequency shift due to carrier frequency $f_c$ and a user speed $v$, and $c$ is the speed of light.

With a MAP-optimal predictor, the predicted SNR follows an exponential distribution with average SNR $\tilde{\gamma}_i = \rho_i \gamma_i$ [10], where $\rho_i$ is the normalized correlation between the actual and predicted SNRs, and $\gamma_i$ is the actual average SNR of user $i$. The probability density function (PDF) of the predicted SNR of user $i$, denoted by $f_{\tilde{\gamma}_i}(\tilde{\gamma})$, is then given as

$$f_{\tilde{\gamma}_i}(\tilde{\gamma}) = \frac{1}{\rho_i \gamma_i} e^{-\frac{\tilde{\gamma}}{\gamma_i}}, \quad (8)$$

and the normalized correlation, $\rho_i$, is given as [10]

$$\rho_i = \mathbf{r}_{\Delta \mathcal{K}}^T \left( \mathbf{R}_\mathcal{K} + \frac{1}{\gamma_i} \mathbf{I}_{\mathcal{K} \times \mathcal{K}} \right)^{-1} \mathbf{r}_{\Delta \mathcal{K}}. \quad (9)$$

We observe from (9) that $\rho_i$ is a function of average SNR $\gamma_i$, the Doppler spread $f_D$, the delay (prediction lag) $\Delta T_s$, the filter length $\mathcal{K}$, and the pilot-spacing $\mathcal{L}$.

B. Adaptive Coding and Modulation (ACM)

We assume that the base station has $C$ codes, with $L$ power levels for each code. Following [7], the range of SNRs is thus divided into $CL + 1$ intervals, the boundaries of which are given by the SNR thresholds $\gamma_c, l$ for $c = 1, \cdots, C$ and $l = 1, \cdots, L$, as illustrated in Fig. 2. We further assume that $\gamma_{0, 1} = 0$ and $\gamma_{C+1, 1} = \infty$.

![Fig. 2. The SNR thresholds $\gamma_{c,l}$ for the ACM scheme](image)

When $\gamma_i$ lies in the interval $[\gamma_{c, 1}, \gamma_{c+1, 1})$, we select code $c$ with spectral efficiency $SE_c$. We have constant transmission rate inside a single interval, but the transmitted power and bandwidth can be adapted to achieve the system objective.

\[\text{outage} \quad R_1 \cdots \quad R_2 \cdots \quad R_3 \cdots \quad R_C \cdots\]

$\gamma_{1, 1} \quad \gamma_{1, 2} \quad \gamma_{1, 3} \quad \gamma_{2, 1} \quad \gamma_{2, 2} \quad \gamma_{2, 3} \quad \gamma_{3, 1} \quad \gamma_{3, 2} \quad \gamma_{3, 3} \quad \gamma_{C, 1} \quad \gamma_{C, 2} \quad \gamma_{C, 3} \quad \gamma_{C+1, 1}$
C. Code Selection

The actual SNR is not always known at the base station. Therefore based on the predicted SNR, a new set of thresholds is defined for code selection [8]. We denote the switching thresholds as \( \hat{\gamma}_{c,l} \) with \( \hat{\gamma}_{0,1} = 0 \) and \( \hat{\gamma}_{C+1,1} = \infty \). Code \( c \) is now selected when \( \hat{\gamma}_{i} \) lies within \( [\hat{\gamma}_{c,1}, \hat{\gamma}_{c+1,1}) \). The probability of selecting code \( c \) for user \( i \) is given as

\[
Pr_{c,i} = \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_{i}}(\hat{\gamma})d\hat{\gamma},
\]

where \( f_{\hat{\gamma}_{i}}(\hat{\gamma}) \) is given in (8). For perfect channel prediction (\( \rho_{i} = 1 \)), the SNR thresholds in Fig. 2 will be the switching thresholds, i.e. \( \hat{\gamma}_{c,l} = \gamma_{c,l} \). However, the normalized correlation \( \rho_{i} \) can be lower due to, for example, lower \( \hat{\gamma}_{i} \), larger delay, or higher user speed. This may cause the actual SNR to fall in a lower interval than the predicted SNR, resulting in a higher error probability. We cannot avoid this completely, but we can reduce the probability of this event by demanding [8]

\[
Pr[\gamma_{i} < \gamma_{c,l}] | \hat{\gamma}_{i} \geq \hat{\gamma}_{c,l} \leq \epsilon,
\]

where \( \epsilon \) is a small constant selected by the system designer. The probability of the complementary event in this case is given as [8]

\[
1 - \epsilon = Pr[\gamma_{i} > \gamma_{c,l} | \hat{\gamma}_{i} = \hat{\gamma}_{c,l}] = Q \left( \frac{\sqrt{\gamma_{c,l} - \rho_{i}}}{\sqrt{\gamma_{i} - \rho_{i}}} / 2, \frac{\sqrt{\gamma_{c,l} - \rho_{i}}}{\sqrt{\gamma_{i} - \rho_{i}}} / 2 \right)
\]

where

\[
Q(a, b) = \int_{b}^{\infty} x I_0(ax)e^{-\frac{1}{2}(a^2+x^2)}dx
\]

is the Marcum-Q function [11].

D. Average Rate for ACM

It is assumed that the fading is slow enough so that (close-to)-capacity-achieving codes for AWGN channels can be employed [7]. Furthermore, the \( C \) different codes are selected so that capacity is achieved at the lower end of the corresponding SNR intervals. The spectral efficiency of code \( c \) is given as

\[
SE_c = \log_2 \left( 1 + \frac{P_i(\hat{\gamma}_{c,1})}{P_{W_i}(\hat{\gamma}_{c,1})} \gamma_{c,1} \right) [\text{bits/s/Hz}],
\]

and the average rate \( \bar{R}_i \) is then

\[
\bar{R}_i = \sum_{c=1}^{C} W_i(\hat{\gamma}_{c,1}) \cdot SE_c \cdot Pr_{c,i} [\text{bits/s}].
\]

The switching thresholds \( \hat{\gamma}_{c,l} \) are related to the SNR thresholds \( \gamma_{c,l} \) via the Marcum-Q function given in (12). To our best knowledge, the inverse Marcum-Q function does not have a closed form expression. But as suggested in [8], we can obtain the inverse of one of the arguments by Ridders’ method [12]. Thus, the inverse of \( Q(\cdot, \cdot) \) with respect to \( \hat{\gamma}_{c,l} \) is given as

\[
\gamma_{c,l}(\epsilon) = \left\{ \begin{array}{cc}
\frac{1}{2} \left( \sqrt{\frac{\gamma_{c,l}(1-\rho_{i})}{\epsilon}} + \frac{\sqrt{\gamma_{c,l}(1-\rho_{i})}}{2} \right), & \rho_{i} < 1 \\
\hat{\gamma}_{c,l}, & \rho_{i} = 1
\end{array} \right.
\]

where \( b = q_{b}(a, \epsilon) \) is the inverse of \( 1 - Q(a, b) \), w.r.t. its second argument. The average rate \( \bar{R}_i \) in (15) can now be written as

\[
\bar{R}_i(\epsilon) = \sum_{c=1}^{C} \log_2 \left( 1 + \left( \frac{P_i(\hat{\gamma}_{c,1})}{P_{W_i}(\hat{\gamma}_{c,1})} \gamma_{c,1} \right) \right) \times W_i(\hat{\gamma}_{c,1}) \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_{i}}(\hat{\gamma})d\hat{\gamma}.
\]

III. JOINT BANDWIDTH AND POWER ADAPTATION

Different optimization objectives can be considered while allocating resources in wireless multi-user networks. We shall consider i) sum rate maximization and ii) average power minimization in our work. We now formulate and solve the problems of joint bandwidth and power allocation for each of these two different objectives.

A. Sum Rate Maximization

In applications without delay constraints, a high data rate from any user in the network is favorable. Thus, it is desirable to allocate the resources to maximize the overall network performance, e.g., the sum rate of all users. The joint bandwidth and power allocation problem aiming at maximizing the sum rate for the network can be formulated as

\[
\bar{R}_{max} = \max \sum_{i=1}^{M} \bar{R}_i(\epsilon),
\]

subject to the following average power and total bandwidth constraints

\[
\sum_{c=0}^{C} P_i(\hat{\gamma}_{c,1}) f_{\hat{\gamma}_{i}}(\hat{\gamma})d\hat{\gamma} \leq \bar{P},
\]

\[
\sum_{i=1}^{M} W_i(\hat{\gamma}_{i}) \leq \bar{W}.
\]

We now consider different adaptation strategies for allocating power and bandwidth.

1) Constant Bandwidth Constant Power Adaptation: Under this strategy, the transmission bandwidth allocated to user \( i \) is restricted to be constant in the interval \([\hat{\gamma}_{c,1}, \hat{\gamma}_{c+1,1})\). I.e.,

\[
W_i(\hat{\gamma}_{i}) = \left\{ \begin{array}{cc}
\omega_i \hat{\gamma}_{1,1}, & \text{if } \hat{\gamma}_{c,1} \leq \hat{\gamma}_{i} \leq \hat{\gamma}_{c+1,1}, \\
1, & \text{if } \hat{\gamma}_{i} < \hat{\gamma}_{c,1}
\end{array} \right.,
\]

for all \( i = 1, \cdots, M \). The constraint in (20) then becomes

\[
\sum_{i=1}^{M} \omega_i \hat{\gamma}_{1,1} = M, \text{ for all } c = 1, \ldots, C.
\]

Furthermore, we employ a single power level for all codes for user \( i \). From (19) the optimal constant power policy is (similar to the one in [7])

\[
\frac{P_i(\hat{\gamma}_{i})}{P} = \left\{ \begin{array}{cc}
\frac{1}{1-P_i(\hat{\gamma}_{c,1})}, & \text{if } \hat{\gamma}_{c,1} \leq \hat{\gamma}_{i} \leq \hat{\gamma}_{c+1,1}, \\
1, & \text{if } \hat{\gamma}_{i} < \hat{\gamma}_{c,1}
\end{array} \right.,
\]

where \( b = q_{b}(a, \epsilon) \) is the inverse of \( 1 - Q(a, b) \), w.r.t. its second argument. The average rate \( \bar{R}_i \) in (15) can now be written as

\[
\bar{R}_i(\epsilon) = \sum_{c=1}^{C} \log_2 \left( 1 + \left( \frac{P_i(\hat{\gamma}_{c,1})}{P_{W_i}(\hat{\gamma}_{c,1})} \gamma_{c,1} \right) \right) \times W_i(\hat{\gamma}_{c,1}) \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_{i}}(\hat{\gamma})d\hat{\gamma}.
\]
where $F_{\hat{\gamma}_i}(\cdot)$ is the cumulative density distribution function (CDF) of $\hat{\gamma}_i$. We obtain $\bar{R}_{\text{max},C}$:

$$\bar{R}_{\text{max},C} = \max \sum_{i=1}^{M} \sum_{c=1}^{C} \log_2 \left(1 + \frac{\gamma_{c,1}(\epsilon)}{\omega_i \hat{\gamma}_{i,1} (1 - F_{\hat{\gamma}_i}(\hat{\gamma}_{i,1}))} \right)$$

$$\times \omega_i \hat{\gamma}_{i,1} \int_{\hat{\gamma}_{i,1}}^{\hat{\gamma}_{c,1}+1} f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma}$$

subject to constraint (22). The variables $\{\omega_i\}_{i=1}^{M}$ and $\{\hat{\gamma}_{c,1}\}_{c=1}^{C}$ are then obtained by numerical optimization. This can for example be achieved in MATLAB by using the function fmincon.

2) Constant Bandwidth Discrete Power adaptation: The transmission bandwidth allocated to user $i$ is again restricted to be constant in the interval $[\hat{\gamma}_{i,1}, \hat{\gamma}_{C+1,1}]$, given by (21). However, there are now $L \geq 1$ power levels within each interval [7]. For each interval $[\hat{\gamma}_{c,1}, \hat{\gamma}_{c+1,1}]$ we again employ a capacity-achieving code for an AWGN channel to ensure an arbitrarily low BER with received post-adaptation SNR greater than or equal to $\frac{P_i(\hat{\gamma}_{c,1})}{W_i(\hat{\gamma}_{c,1})} \gamma_{c,1}(\epsilon)$, applying the following restriction (similar to the one in [7]):

$$\frac{P_i(\hat{\gamma}_{c,1})}{W_i(\hat{\gamma}_{c,1})} \gamma_{c,1}(\epsilon) \geq \frac{P_i(\hat{\gamma}_{c+1,1})}{W_i(\hat{\gamma}_{c+1,1})} \gamma_{c+1,1}(\epsilon)$$

In each interval, the rate and the bandwidth are constant. However, the transmitted power can be reduced when the channel is better. We define $\frac{P_i(\hat{\gamma}_{c,1})}{W_i(\hat{\gamma}_{c,1})} \gamma_{c,1}(\epsilon) = \beta_c$. The minimum received SNR within the interval $c$ (for $1 \leq c \leq C$) is then given by $\beta_c$ (due to (25)). The jointly optimal power and bandwidth adaptation scheme is then given as (following [7])

$$P_i(\hat{\gamma}_{c,1}) = \left\{ \begin{array}{ll} \frac{\beta_{c}}{\gamma_{c,1}(\epsilon)}, & \text{if } \hat{\gamma}_{c,l} \leq \hat{\gamma}_{i} \leq \hat{\gamma}_{c+1,l}, \\ 0, & \text{if } \hat{\gamma}_{c} < \hat{\gamma}_{i,1} \end{array} \right.$$ (25)

Or

$$P_i(\hat{\gamma}_{c,1}) = \left\{ \begin{array}{ll} \frac{\omega_i \beta_{c} \gamma_{c,1}(\epsilon)}{\gamma_{c,1}(\epsilon)}, & \text{if } \hat{\gamma}_{c,l} \leq \hat{\gamma}_{i} \leq \hat{\gamma}_{c+1,l}, \\ 1, & \text{if } 1 \leq c \leq C, 1 \leq l \leq L, \\ 0, & \text{if } \hat{\gamma}_{c} < \hat{\gamma}_{i,1} \end{array} \right.$$ (26)

We thus have the following problem with respect to the variables $\{\omega_i\}_{i=1}^{M}$, $\{\beta_c\}_{c=1}^{C}$ and $\{\hat{\gamma}_{c,1}\}_{c=1}^{C,L}$: Maximize

$$\bar{R}_{\text{max},C \times L} = \sum_{i=1}^{M} \sum_{c=1}^{C} \omega_i \hat{\gamma}_{i,1,1} \log_2 \left(1 + \beta_c \right) \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma}$$

such that for all $i = 1, \ldots, M$,

$$\sum_{c=1}^{C} \omega_i \beta_c \hat{\gamma}_{i,1,1} \sum_{l=1}^{L} \frac{1}{\gamma_{c,1}(\epsilon)} \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma} = 1, \quad (27)$$

and for all $c = 1, \ldots, C$,

$$\sum_{i=1}^{M} \omega_i \hat{\gamma}_{i,1,1} = M. \quad (28)$$

This problem can again be solved in MATLAB by using the function fmincon.

B. Average Power Minimization

Another relevant design objective is the minimization of the average power consumption of all users. This minimization is performed under the constraint that the minimum rate requirements of all users must be satisfied. The corresponding joint bandwidth and power allocation problem can be written as

$$\min \sum_{i=1}^{M} \sum_{c=1}^{C} \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} P_i(\hat{\gamma}_i) f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma}$$

subject to $\sum_{i=1}^{M} W_i(\hat{\gamma}_i) \leq W_t$ (31)

and

$$\bar{R}_i(\epsilon) \geq r_i, \quad (32)$$

Two different adaptation strategies for allocating power and bandwidth are discussed below.

1) Constant Bandwidth Constant Power Adaptation: For constant bandwidth constant power adaptation, the problem in (30)-(32) becomes

$$\min \sum_{i=1}^{M} P_i(\hat{\gamma}_i) (1 - F_{\gamma_i}(\hat{\gamma}_{i,1}))$$

such that

$$\sum_{i=1}^{M} \omega_i \hat{\gamma}_{i,1,1} = M, \quad \text{for all } c = 1, \ldots, C, \quad (34)$$

and for all $i = 1, \ldots, M$,

$$\sum_{c=1}^{C} \omega_i \hat{\gamma}_{i,1,1} \log_2 \left(1 + \frac{P_i(\hat{\gamma}_{i,1})}{W_i(\hat{\gamma}_{i,1})} \gamma_{i,1}(\epsilon) \right) \int_{\hat{\gamma}_{i,1}}^{\hat{\gamma}_{i+1,1}} f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma} \geq r_i. \quad (35)$$

The variables $\{\omega_i\}_{i=1}^{M}$ and $\{\hat{\gamma}_{i,1}\}_{c=1}^{C}$ are then found by using the function fmincon in MATLAB.

2) Constant Bandwidth Discrete Power adaptation: The corresponding problem for constant bandwidth discrete power adaptation becomes

$$\min \sum_{i=1}^{M} \sum_{c=1}^{C} \omega_i \beta_c \hat{\gamma}_{i,1,1} \sum_{l=1}^{L} \frac{1}{\gamma_{c,1}(\epsilon)} \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma}, \quad (36)$$

such that for all $c = 1, \ldots, C$,

$$\sum_{i=1}^{M} \omega_i \hat{\gamma}_{i,1,1} = M, \quad (37)$$

and for all $i = 1, \ldots, M$,

$$\sum_{c=1}^{C} \omega_i \hat{\gamma}_{i,1,1} \log_2 \left(1 + \beta_c \right) \int_{\hat{\gamma}_{c,1}}^{\hat{\gamma}_{c+1,1}} f_{\hat{\gamma}_i}(\hat{\gamma}) d\hat{\gamma} \geq r_i. \quad (38)$$

The variables $\{\omega_i\}_{i=1}^{M}$, $\{\beta_c\}_{c=1}^{C}$ and $\{\hat{\gamma}_{c,1}\}_{c=1}^{C,L}$ can then be found by using the function fmincon in MATLAB.

The ACM scheme for multi-user scenario can now be designed as follows: For the given system objective, the
number of users $M$, the average SNRs $\bar{\gamma}_i$s for all users, the number of codes $C$, and a bandwidth/power adaptation scheme in mind, find the set of switching thresholds, corresponding SNR thresholds, the bandwidth allocations, and the maximal spectral efficiencies $\bar{SE}_i$s. Then design optimal codes for these spectral efficiencies for each $M$ and $\hat{\gamma}_i$s of interest.

IV. Numerical Results

We consider a system with 5 users. All the user channels are Rayleigh distributed with constant average SNRs that are distributed with an average of $\bar{\gamma}_i$, and the user with the worst channel has an average SNR of $\frac{1}{2}\bar{\gamma}_i$. For example, for $\bar{\gamma}_i = 10$ dB, the worst and the best users have average SNRs of 7.5 dB and 11.58 dB respectively. The value of $\epsilon$ is set to $2 \times 10^{-3}$. For simplicity, we also assume that all the users have the same value of the normalized correlation $\rho_t$.

A. Sum Rate Maximization

In this section, we analyze the effect of imperfect channel knowledge on the sum rate of the system. Fig. 3 a) gives the optimal switching thresholds for $R_{max,2 \times 2}$ as a function of $\bar{\gamma}_i$. We observe that the thresholds for the imperfect case ($\rho_t = \rho = 0.99$) are higher than the thresholds for $\rho = 1$ (perfect prediction). Since the outage probability depends on $\bar{\gamma}_i$, imperfect prediction will result in a higher outage probability. Therefore an outage probability constraint can be introduced if the considered application has strict real-time requirements. The corresponding minimum received SNR values $\beta_i$ are depicted in Fig. 3 b). For $\rho < 1$, the SNR values are lower and therefore the values of designed spectral efficiencies $\bar{SE}_c$ will also be lower, causing a reduction in the sum rate.

Under the average power and bandwidth constraints and with perfect channel knowledge, the average sum rates corresponding to $R_{max,C \times L}$ and $R_{max,C}$ are plotted in Fig. 4 a). We see that the discrete rate schemes approach the performance of the continuous rate scheme using only a few codes. The effect of imperfect channel knowledge is shown in Fig. 4 b) for the $R_{max,2 \times 2}$ scheme. We observe (as expected) that the sum rate is reduced due to imperfect channel prediction. To approach the sum rate of the perfect case, $\epsilon$ should be as large as possible. But then the probability that the actual SNR is in a lower interval than the predicted SNR will be increased, resulting in high BER. We have chosen a very small value for $\epsilon$ in this section, which results in reduced spectral efficiencies and hence the reduced sum rate. Insertion of equally spaced pilot-symbols also reduces the sum rate by a factor of $1 - (\ell - 1)/L$, where $L$ is the pilot-spacing.

An example of the optimized power and bandwidth allocation for the $R_{max,2 \times 3}$ scheme is shown in Fig. 5, for $\bar{\gamma}_i = 10$ dB. The plot is shown for user 1 only, whose average SNR is 7.5 dB. The figure also shows the spectral efficiencies ($SE_c$) in the two intervals. Fig. 5 can be interpreted as follows: The base station will first determine the interval in which the instantaneous pre-adaptation SNR of user 1, given by $\bar{\gamma}_i$, exists. It will then allocate the corresponding power and bandwidth to the user 1, and send the data at the given rate for that interval using the corresponding $SE_c$. Within the interval, the rate and the bandwidth is constant, therefore the power is reduced as shown in the figure.

B. Average Power Minimization

For simplification, we have assumed identical rate requirements for all the users, i.e., $r_t = r = 1$ bit/s. Fig. 6 a) shows average power consumed for constant bandwidth constant power ($P_{min,C}$) and constant bandwidth discrete power
allocation for discrete-rate multi-user link adaptation with needs to consume more power to fulfill the rate requirements.

SE efficiencies (\(SE_{\text{eff}}\)) in the two intervals are also shown. The effect of imperfect channel prediction on \(\rho \leq 1\), the base station needs to consume more power to fulfill the rate requirements of all the users in the system.

V. Conclusion

In this paper, we consider discrete power and bandwidth allocation for discrete-rate multi-user link adaptation with imperfect channel state information. We have shown that with only a few rates (codes) and joint power and bandwidth optimization, the system can approach the performance of continuous rate system. We have also observed that imperfect channel knowledge affects the system such that the sum rate is decreased and the average power is increased. The outage probability also increases. Therefore for applications with low delay requirements, a constraint on the outage probability may also be applied. Furthermore, since there was no peak power constraint, the system uses higher instantaneous power for imperfect channel knowledge at the base station.

This work makes a lot of idealistic assumptions. For example, we assumed that the number of users and the average SNRs remain constant for the time-window of interest. A good topic for future research can be to find a heuristic solution to deal with the variation of the number of users and the channel conditions.

References