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Alternative methodology for Scott-Knott test

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ABSTRACT - *The test proposed by Scott Knott (1974), a procedure of means grouping, is an effective alternative to perform procedures of multiple comparisons without ambiguity. This study aimed to propose a modification related to the partitioning and means grouping in the said procedure, to obtain results without ambiguity among treatments, organized in more homogeneous groups. In the proposed methodology, treatments that did not participate in the initial group are joined for a new analysis, which allows for a better group distribution. In a comparative study, four experiments were simulated in a randomized complete block design. The first consisted of 10 and the other 3 of 100 treatments. All experiments were performed in three replications at a significance level of 0.05 for the means grouping test. Only in the third experiment of those of 100 treatments the groups formed by Scott-Knott did not differ from the methodology proposed here. The proposed methodology is considered effective, aiming at the identification of elite cultivar groups for recommendation.*

Key words: multiple comparison procedures, plant breeding, simulation.

INTRODUCTION

The analysis of variance (ANOVA) is a widely used methodology to prove the statistical hypothesis test. It covers important subjects in several areas that involve experimentation. When a fixed group of treatments are evaluated, the point of interest is mostly the existence of statistical significance between treatment pairs or groups of means. In ANOVA, this statistical significance among means is evaluated by the F-test. If the null hypothesis (H0) is rejected and several treatments are tested, it is important to know which pair of means differ from each other. In this case, multiple comparison methods are used.

Numerous procedures of multiple comparisons are proposed in the literature. However, breeders encounter difficulties of interpretation, arising from the ambiguity of results. An efficient alternative, mainly when a large number of treatments is evaluated, is the use of the Scott-Knott test (1974). This test it a method of grouping means, which distinguishes results without ambiguity.

Silva et al. (1999) studied the power and rates of type error I in the Scott-Knott test and almost always verified high power and type error I in agreement with the nominal levels. The authors further stated an increase in the test power as the number of treatments increases. When smaller differences among the treatment levels (2 standard deviations) were tested, the test power was almost twice as high as in the statistical tests Duncan, t and SNK. The largest discrepancies were verified in a comparison of the Scott-Knott with the Tukey test. In some circumstances, the power of the test was eight or more times higher than

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the Tukey test. The only test with similar power as the Scott-Knott was the Bayesian t-test. When the differences between the treatment levels were largest, with magnitudes equivalent to six standard deviations or more, results were similar to those obtained by Perecin and Barbosa (1988).

The grouping method proposed by Scott-Knott consists in partitioning the original group of treatments. The partitioning aims at a maximum differentiation between groups. Each group formed can be partitioned again if the new groups are significantly different. This partitioning is stopped when the groups obtained are not significantly different in the constituent treatments. This process is quite interesting when the number of treatments is large and it has been widely used in the literature.

This study aimed to propose a modification of the procedure described by Scott-Knott in relation to partitioning and means grouping, while ensuring results without ambiguity among treatments, forming groups that can be more interesting in certain areas of research.

MATERIAL AND METHODS

To compare the two methodologies an experiment was simulated arranged in a randomized block design with 10 treatments and 3 replications. The treatment means were: 255.5; 259.3; 271.6; 290.6; 298.8; 334.9; 341.0; 348.7; 384.3; 495.5, and the residual variance was estimated at 1254.327 associated to 18 degrees of freedom. The grouped means were tested at a significance level of 5 %.

a- The original Scott-Knott Methodology

The procedure begins by partitioning the groups to maximize the sum of squares between groups. The process is facilitated when the means are ordered, since the number of possible partitions (g-1 partitions) is reduced.

The sum of squares is defined as B_0, according to the expression:

$$B_0 = \frac{T_1^2}{K_1} + \frac{T_2^2}{K_2} - \frac{(T_1 + T_2)^2}{K_1 + K_2}$$

Where T_1 and T_2 are the totals of the two groups with K_1 and K_2 treatments in each.

The values obtained are tested by the statistics l according to the expression:

$$\lambda = \frac{\pi}{2(\pi - 2)} x \frac{B_0}{\hat{\sigma}_0^2}$$

where $\hat{\sigma}_0^2$ is the estimator of maximum likelihood σ_y^2 obtained by:

$$\hat{\sigma}_0^2 = \frac{1}{g+v}\left[\sum_{i=1}^{g}\left(\bar{Y}_i - \bar{Y}\right)^2 + vs_y^2\right]$$

Where

$\bar{Y}_i$: mean of treatment i (i=1, 2,.... g);

$\bar{Y}$: overall mean of treatments to be separated;

g: number of means to be separated;

v: number of residual degrees of freedom;

s_y^2: QMR/r being r the number of observations that created the means to be grouped.

The statistics l is tested by the chi square statistic (χ^2), where l $\lambda < \chi^2(\alpha, v_0)$ implies that all means are considered homogeneous and, further partitioning is therefore unnecessary. The condition l $\lambda > \chi^2(\alpha , v_0)$ indicates that the two groups are statistically different and should be tested separately for new possible divisions.

In the considered example, the first group formed consisted of the treatments 1, 2, 3, 4, 5, 6, 7 and 8 and the second of the treatments 9 and 10. The next step consists in the attempt to partition the formed groups again. The first group was once more divided into two new subgroups (one with treatments 1, 2, 3, 4 and 5 and the other with 6, 7 and 8). In these newly formed groups, new possible partitions were sought. The statistics showed that the two groups could not be partitioned. The group composed of treatments 9 and 10 was also divided into two subgroups with one treatment each. This way, all statistically possible partitions were performed, forming homogeneous subgroups. The final result obtained by the test Scott-Knott is shown in Figure 1.

The treatment means can be presented as follows:

255.5D; 259.3D; 271.6D; 290.6D; 298.8D; 334.9C; 341.0C; 348.7C; 384.3B; 495.5A

(treatments followed by the same letter belong to the same group).

Treatments with means within a same subgroup are statistically equal, while the subgroups formed differ from each other.

b- Description and Illustration of the Proposed Methodology

This methodology proposes an alteration in the way of partitioning groups. The process begins with

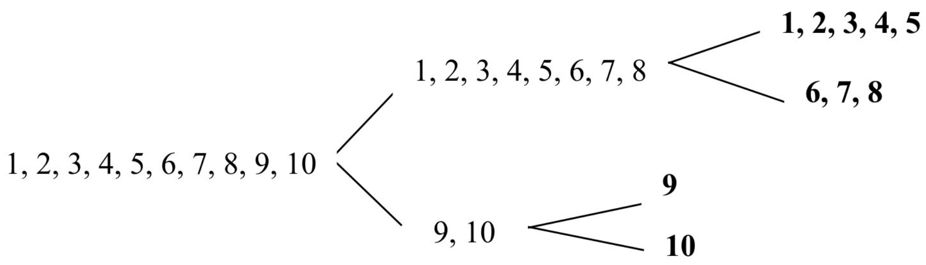

Figure 1. Partitions performed by the Scott-Knott test at 5 % probability

the formation of groups that maximize the sum of squares, based on the same concept as the Scott-Knott test. Two groups were formed first (one with treatments 9 and 10 and the second with 1, 2, 3, 4, 5, 6, 7 and 8). Upon the formation of these groups, the second group was discarded and the possible partitions in the first group performed, resulting in two new subgroups (one with treatment 10 and the other with 9). This second subgroup was also discarded. Consequently, treatment 10 represented a group, the first formed group.

A new grouping analysis was performed with all previously discarded treatments (1, 2, 3, 4, 5, 6, 7, 8, and 9), which divided the treatments in two groups (group one 6, 7, 8 and 9 and group two 1, 2, 3, 4 and 5). Once again, the treatments of the second group were discarded and new possible partitions sought in the first group. No possibility of forming new subgroups was verified. Consequently, the treatments 6, 7, 8 and 9 represent the second group.

As the procedure continues, new analyses are carried out with the previously discarded treatments (1, 2, 3, 4 and 5), until all treatments are grouped. Summing up, the new procedure consists in the removal of the treatments that form a new group and in the performance of new analyses with the remaining treatments, so that at each step a new group is formed while the number of remaining treatments decreases. The illustration of the divisions separated in the example is shown in Figure 2.

The treatment means can be presented as follows: 255.5C; 259.3C; 271.6C; 290.6C; 298.8C; 334.9B; 341.0B; 348.7B; 384.3B; 495.5A (treatments followed by the same letter belong to the same group).

The estimators used to determine the possible partitions are the same as determined originally by the test of Scott-Knott (1974), briefly described in this study.

c) Application

To exemplify and compare the proposed methodology three experiments were simulated, arranged in a randomized block design. The experiments consisted of 100 treatments with three replications. The simulations and all statistical analyses were performed using software GENES (Cruz, 2006). The Scott-Knott test was carried out at a significance level of 5%.

RESULTS AND DISCUSSION

The simulated data of three experiments were considered for a broader comparison of the differences obtained by the application of the two methodologies. For the first experiment the treatment means varied from 300 to 542. Both the Scott-Knott test and the proposed methodology separated the treatments into eight groups. The groups formed by the novel methodology were however more homogeneous and grouped the means more satisfactorily (Table 1). To verify the superiority of the proposed method, the variances among the elements of each group were calculated in both methodologies. In both cases the groups A, E and H contained the same treatments, which is why their variances were not used. In the methodology proposed by Scott-Knott the variances of each group were: B (35.56); C (57.40); D (132.27); F (46.43); G (25.01); obtaining an average of the variances of 59.33. In the proposed methodology the variances of the groups were: B (103.86); C (81.77); D (0); F (48.95); G (22.88); obtaining a medium estimate of the variances of 51.49, which is lower than by the traditional Scott-Knott methodology. We emphasize that the goal of our proposal is not the formation of groups with an inferior variance than by the Scott-Knott methodology in all

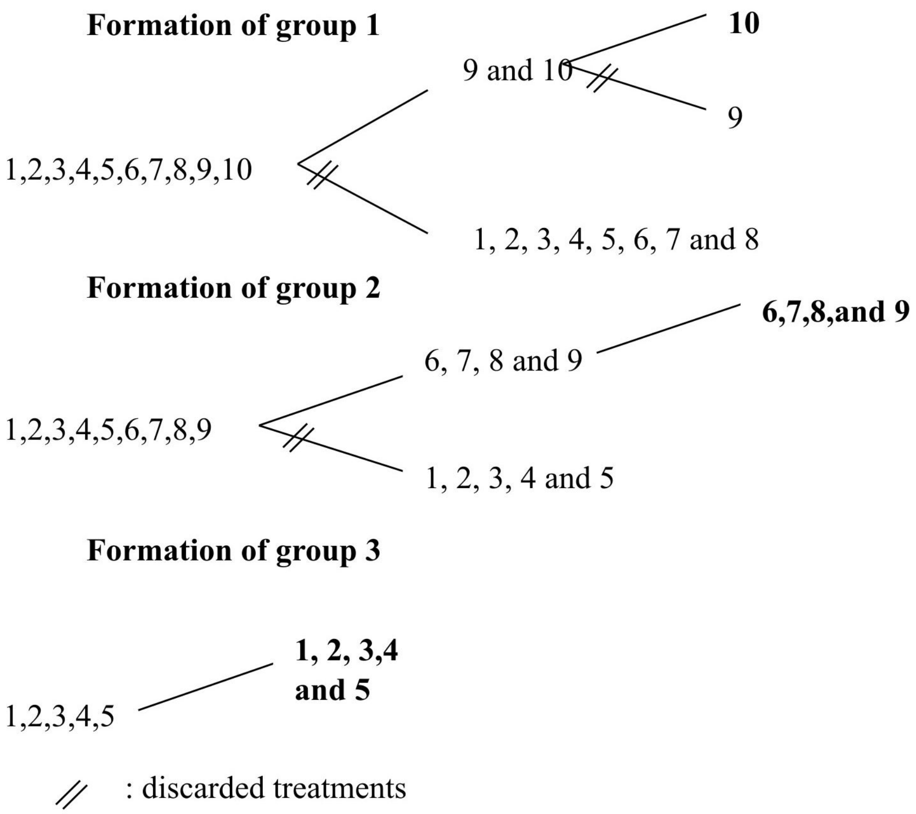

Figure 2. Partitioning by the methodology proposed, at 5% probability

Table 1. Results of the experiments, with 100 treatments, used to compare the Scott-Knott test (SK) with the new proposed methodology (P)

Experiment 1				Experiment 2				Experiment 3			
Genotype	Mean	SK	P	Genotype	Mean	SK	P	Genotype	Mean	SK	P
5	542.75	a	a	93	650.6261	a	a	90	631.447	a	a
1	542.5	a	a	95	649.5351	a	a	93	630.6261	a	a
3	540.4421	a	a	80	647.8417	a	a	95	629.5351	a	a
4	540	a	a	81	646.0592	a	a	82	629.2549	a	a
2	539.0307	a	a	94	644.542	a	a	97	627.755	a	a
25	533.2933	a	a	90	636.447	b	b	81	626.0592	a	a
6	532.4005	a	a	92	636.1917	b	b	83	625.7013	a	a
8	531.3028	a	a	91	630.8681	c	b	98	625.6156	a	a
9	524.3248	b	b	82	629.2549	c	c	85	625.5948	a	a
10	521.2165	b	b	97	627.755	c	c	87	625.2544	a	a
12	520.1503	b	b	83	625.7013	c	c	88	624.6018	a	a
11	517.7394	b	b	98	625.6156	c	c	100	624.0402	a	a
13	516.6014	b	b	96	623.7211	c	c	96	623.7211	a	a
18	515.4928	b	b	84	623.5586	c	c	84	623.5586	a	a
15	511.0155	b	b	89	621.7379	c	c	86	623.5438	a	a
14	510.2093	b	b	88	619.6018	c	c	80	622.8417	a	a

Table 1. *Cont...*

Experiment 1				Experiment 2				Experiment 3			
Genotype	Mean	SK	P	Genotype	Mean	SK	P	Genotype	Mean	SK	P
17	508.0385	b	b	87	615.2544	c	c	89	621.7379	a	a
16	506.5988	b	b	100	609.0402	d	d	92	621.1917	a	a
19	500.1404	c	b	86	608.5438	d	d	91	620.8681	a	a
20	498.5249	c	b	85	605.5948	d	d	94	619.542	a	a
22	497.9659	c	b	99	596.6134	e	e	99	616.6134	a	a
21	497.9213	c	b	67	550.6261	f	f	67	530.6261	b	b
23	497.7177	c	b	79	543.8428	g	f	77	530.2967	b	b
24	494.9031	c	b	77	540.2967	g	f	69	529.5351	b	b
26	493.794	c	b	78	538.1307	g	f	71	527.755	b	b
28	491.236	c	c	69	529.5351	h	g	72	525.6156	b	b
27	489.997	c	c	76	527.9685	h	g	75	524.6742	b	b
29	485.7167	c	c	75	524.6742	i	g	74	524.0402	b	b
30	483.572	c	c	70	523.7211	i	g	79	523.8428	b	b
33	482.2531	c	c	68	519.542	i	g	70	523.7211	b	b
31	482.2327	c	c	74	519.0402	i	g	78	523.1307	b	b
32	479.7844	c	c	72	510.6156	j	h	76	522.9685	b	b
34	479.0627	c	c	71	507.755	j	h	68	519.542	b	b
35	472.1322	d	c	73	506.6134	j	h	73	516.6134	b	b
36	468.1733	d	c	66	452.2964	k	i	59	430.6822	c	c
37	461.6978	d	c	64	440.6711	l	j	56	429.4006	c	c
7	446.0016	d	d	65	439.5751	l	j	66	427.2964	c	c
38	412.0879	e	e	63	433.1433	l	k	60	427.0185	c	c
39	399.6853	e	e	56	429.4006	m	k	61	426.6052	c	c
40	391.4712	e	e	61	426.6052	m	k	55	426.0961	c	c
41	381.848	e	e	55	426.0961	m	k	64	425.6711	c	c
42	379.8977	e	e	62	425.9128	m	k	57	424.922	c	c
43	371.7489	f	f	60	422.0185	m	k	58	424.2197	c	c
44	370.3968	f	f	59	420.6822	m	k	63	423.1433	c	c
45	369.641	f	f	54	419.6937	m	k	62	420.9128	c	c
46	368.319	f	f	58	409.2197	n	l	54	419.6937	c	c
47	366.9272	f	f	57	404.922	n	l	65	419.5751	c	c
48	364.6831	f	f	52	347.8461	o	m	48	333.6519	d	d
49	364.3974	f	f	51	342.5962	o	m	46	328.0692	d	d
51	363.0924	f	f	50	342.511	o	m	44	327.5692	d	d
50	360.6469	f	f	53	340.6414	o	m	50	327.511	d	d
52	358.9296	f	f	48	338.6519	o	m	41	327.1049	d	d
54	358.3174	f	f	49	334.1029	o	m	45	325.3603	d	d
53	358.312	f	f	41	327.1049	p	n	49	324.1029	d	d
55	357.6615	f	f	46	323.0692	p	n	47	323.0458	d	d
56	357.0284	f	f	47	323.0458	p	n	42	322.9765	d	d
59	356.8743	f	f	42	322.9765	p	n	52	322.8461	d	d
57	356.673	f	f	45	315.3603	q	n	51	322.5962	d	d
58	356.0337	f	f	44	312.5692	q	n	53	320.6414	d	d
60	354.4826	f	f	43	300.2625	r	o	43	320.2625	d	d
61	353.9297	f	f	39	248.9897	s	p	40	230.104	e	e
66	353.4657	f	f	38	245.5709	s	p	39	228.9897	e	e

Table 1. *Cont...*

Experiment 1				Experiment 2				Experiment 3			
Genotype	Mean	SK	P	Genotype	Mean	SK	P	Genotype	Mean	SK	P
62	353.4031	f	f	37	243.5466	s	p	34	228.4187	e	e
67	353.2014	f	f	36	240.1369	s	p	29	226.6473	e	e
64	353.1425	f	f	34	233.4187	t	q	28	226.4739	e	e
63	352.8043	f	f	35	231.616	t	q	32	225.7315	e	e
68	352.5672	f	f	40	230.104	t	q	36	225.1369	e	e
65	351.6484	f	f	28	226.4739	u	q	33	223.979	e	e
69	349.2532	f	f	33	223.979	u	q	37	223.5466	e	e
71	349.0631	f	f	32	220.7315	u	q	31	223.4759	e	e
70	348.5293	f	f	31	213.4759	v	r	30	222.58	e	e
73	348.1981	f	f	30	207.58	v	r	35	221.616	e	e
72	346.56	g	f	29	206.6473	v	r	38	220.5709	e	e
74	343.983	g	g	24	152.5386	x	s	27	130.811	f	f
75	343.5468	g	g	25	144.9072	z	t	15	129.1063	f	f
76	343.3235	g	g	23	142.0541	z	t	20	128.2245	f	f
77	343.1007	g	g	22	140.6058	z	t	18	127.9417	f	f
78	342.7038	g	g	21	134.3638	A	u	24	127.5386	f	f
79	341.9649	g	g	20	133.2245	A	u	22	125.6058	f	f
82	341.096	g	g	27	130.811	A	u	25	124.9072	f	f
80	340.961	g	g	26	124.6838	B	u	26	124.6838	f	f
83	340.4884	g	g	18	122.9417	B	u	21	124.3638	f	f
81	340.328	g	g	19	120.6464	B	v	23	122.0541	f	f
84	340.0172	g	g	17	109.6036	C	x	16	121.1945	f	f
85	339.2385	g	g	15	109.1063	C	x	19	120.6464	f	f
86	335.3879	g	g	16	106.1945	C	x	17	119.6036	f	f
87	333.8369	g	g	11	46.8994	D	z	7	30.6027	g	g
88	333.7094	g	g	9	46.76	D	z	1	29.8366	g	g
91	332.8715	g	g	10	46.1222	D	z	2	28.0035	g	g
89	331.7901	g	g	7	40.6027	D	z	11	26.8994	g	g
90	331.5215	g	g	8	35.6676	D	z	9	26.76	g	g
92	329.5654	g	g	12	26.4971	E	A	12	26.4971	g	g
93	326.7311	h	h	14	26.4629	E	A	14	26.4629	g	g
94	326.0146	h	h	13	25.9764	E	A	4	26.1963	g	g
95	325.3018	h	h	5	22.9545	E	A	13	25.9764	g	g
96	322.6017	h	h	4	21.1963	E	A	10	22.9807	g	g
97	318.8178	h	h	6	19.0901	E	A	5	22.9545	g	g
98	314.0447	h	h	2	17.6208	F	B	8	20.6676	h	h
99	305.4996	h	h	1	13.9781	F	B	3	17.816	h	h
100	300.0906	h	h	3	7.816	F	B	6	14.0901	h	h

cases. The objective is simply to present a new methodology of group formation.

In the second experiment the treatment means varied from 7 to 650. The Scott-Knott test grouped the treatments in 30 and the alternative methodology in 27 groups and the variations within the groups were considered less conflicting Table 1).

In the third experiment the treatment means varied from 14 to 631. The treatments were however organized in 8 groups with discrepant mean values in each group. For example, one group was formed with means varying from 616 to 631 and the subsequent group varying from 516 to 530, with a great difference (gap) between group means (approximately 85 units). In this case it was the two grouping methods proved to be similar, separating the same treatments in each group.

The proposed methodology is an alternative of grouping treatments, in view of the greater homogeneity of partitions than by the Scott-Knott test, obtained by a new analysis with the treatments that did not participate in the initial group. This allows for a better distribution of the groups. Moreover, the methodology preserves one of the main features of the Scott-Knott - the unambiguous results; it is therefore recommended for situations where the number of treatments is high.

It is important to point out that new analyses of variance at every step are not necessary because the original data are the same. This way, only one analysis of variance is performed to obtain the residual mean square and the degree of freedom. These values will be used during the performance of all analyses for groupings means.

The proposed strategy makes a differentiated partitioning of the treatments possible, because the groups are formed step-by-step and not, as proposed originally, simultaneously. Both methodologies maintain the concept that the first established group is formed by those with higher means, considered, therefore, the elite group. This elite group often involves a smaller number of treatments than desirable, and the researcher might want to use the second or other of the remaining groups. From this point onwards, the two methodologies differ substantially in the partition strategies.

The search for a second group (or other groups) within the yet ungrouped treatments for a new partition process seems most interesting for agrarian purposes, where, in spite of the high number of treatments evaluated, the main interest focuses on the comparatively best treatments. Intermediate or inferior groups are quickly discarded and no inference is made. In this case, the researcher is not very interested in the global structure of partitioning, but rather in taking the statistically superior treatments from the original group.

Finally, it is important to emphasize that the use of either the new or the Scott-Knott methodology should be in agreement with the researcher's needs and objectives. The new proposal is intended as one more auxiliary technique in research, with no intention of replacing the traditional Scott-Knott methodology.

CONCLUSION

1. The proposed methodology makes a differentiated partitioning of the available treatments possible while ensuring the principle of absence of ambiguity or superposition of treatment groups.

2. The proposed methodology is a more effective option when the objective is to identify one or few elite groups and discard inferior and intermediate groups.

3. There is a loss of the global partitioning structure, while the identification of a specific subgroup with better performance is facilitated.

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Metodologia alternativa para o teste Scott-Knott

RESUMO - *Com intuito de realizar procedimentos de comparações múltiplas com ausência de ambigüidade o teste proposto por Scott-Knott (1974) torna-se boa alternativa por tratar de procedimento eficaz de agrupamento de médias. O objetivo deste trabalho foi propor alteração no procedimento descrito por Scott-Knott (1974) relativo à forma de partição e agrupamento de médias, proporcionando resultados com ausência de ambigüidade entre tratamentos, porém com formação de grupos mais homogêneos. Na metodologia proposta, os tratamentos que não participaram do grupo inicial são novamente reunidos e nova análise é realizada, permitindo melhor distribuição dos grupos. Para estudo comparativo, foram simulados quatro experimentos no delineamento em blocos ao acaso. O primeiro constituído de 10 tratamentos e os demais 100 tratamentos. Todos experimentos possuíam 3 repetições e utilizou-se nível de significância de 5 % para o teste de agrupamento entre médias. Apenas no terceiro experimento daqueles de 100 tratamentos não houve alteração nos agrupamentos formados pela metodologia de Scott-Knott e a metodologia aqui proposta. Considera-se que a metodologia proposta é eficaz, tendo em vista a identificação de grupos elites de cultivares, para fins de recomendação.*

Palavras-chave: Melhoramento vegetal, procedimentos de comparações múltiplas, simulação.

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