The IMAP Hybrid Method for Learning Gaussian Bayes Nets

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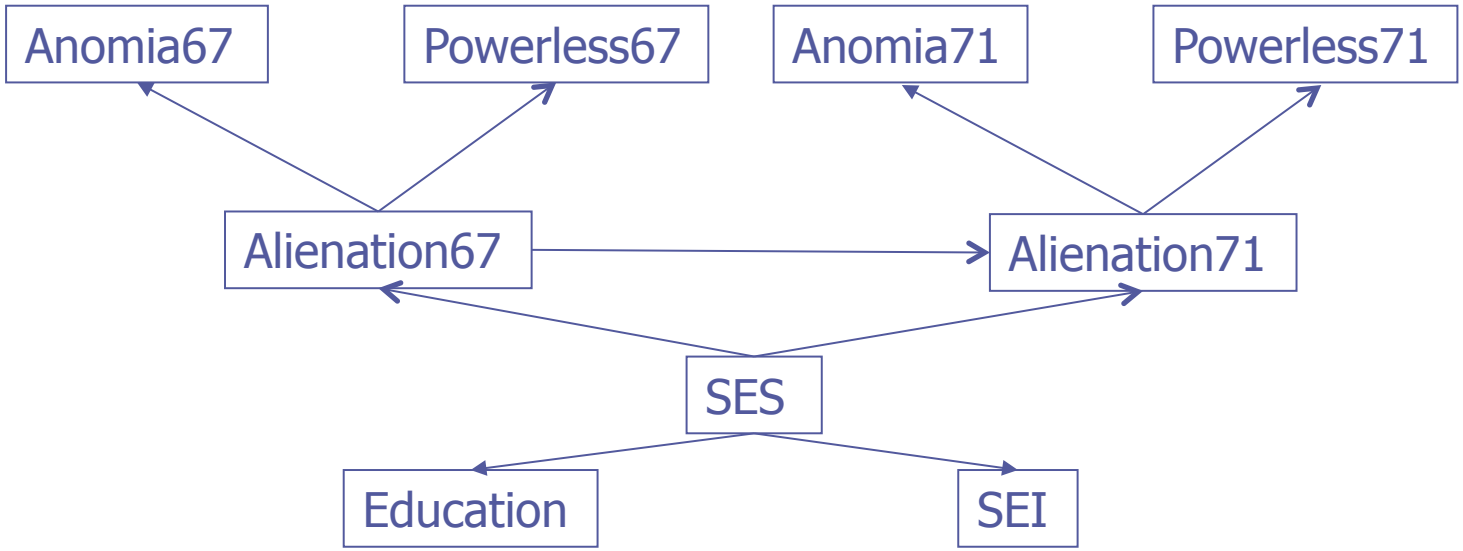
Outline

- Brief Intro to Bayes Nets (BN)
- BN structure learning: Score-based vs. Constraint-based.
- Hybrid Design: Combining Dependency (Correlation) Information with Model Selection.
- Evaluation by Simulations

Bayes Nets: Overview

- Bayes Net Structure = Directed Acyclic Graph.
- Nodes = Variables of Interest.
- Arcs = direct "influence", "association".
- Parameters = CP Tables
  = Prob of Child given Parents.
- Structure represents (in)dependencies.
- (Structure + parameters) represents joint probability distribution over variables.
- Many applications: Gene expression, finance, medicine, …

Example: Alienation Model



- Wheaton et al. 1977. Structure entails (in)dependencies:
  - Anomia67 dependent on SES.
  - Education independent of any node given SES
- Can predict any node, e.g. "Probability of Powerless67 = x, given SES = y?"

Gaussian Bayes Nets

- aka (recursive) structural equation models.
- Continous variables.
- Each child is linear combination of parents, plus normally distributed error ϵ (mean 0).
- E.g. Alienation71 = 0.61*Alienation67 -0.23 SES + ϵ

Two Approaches to Learning Bayes Net Structure

Input: random sample

A	B
3.2	4.5
10	10
-2.1	-3.3
-5.4	-4.3
…	…

Search and Score

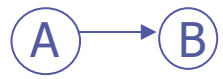
- Select graph G as model with parameters to be estimated
- score BIC, AIC
- balances data fit with model complexity

Constraint-Based (CB)

- use statistical correlation test (e.g., Fisher's *z*) to find (in)dependencies.
- choose G that entails (in)dependencies in sample.

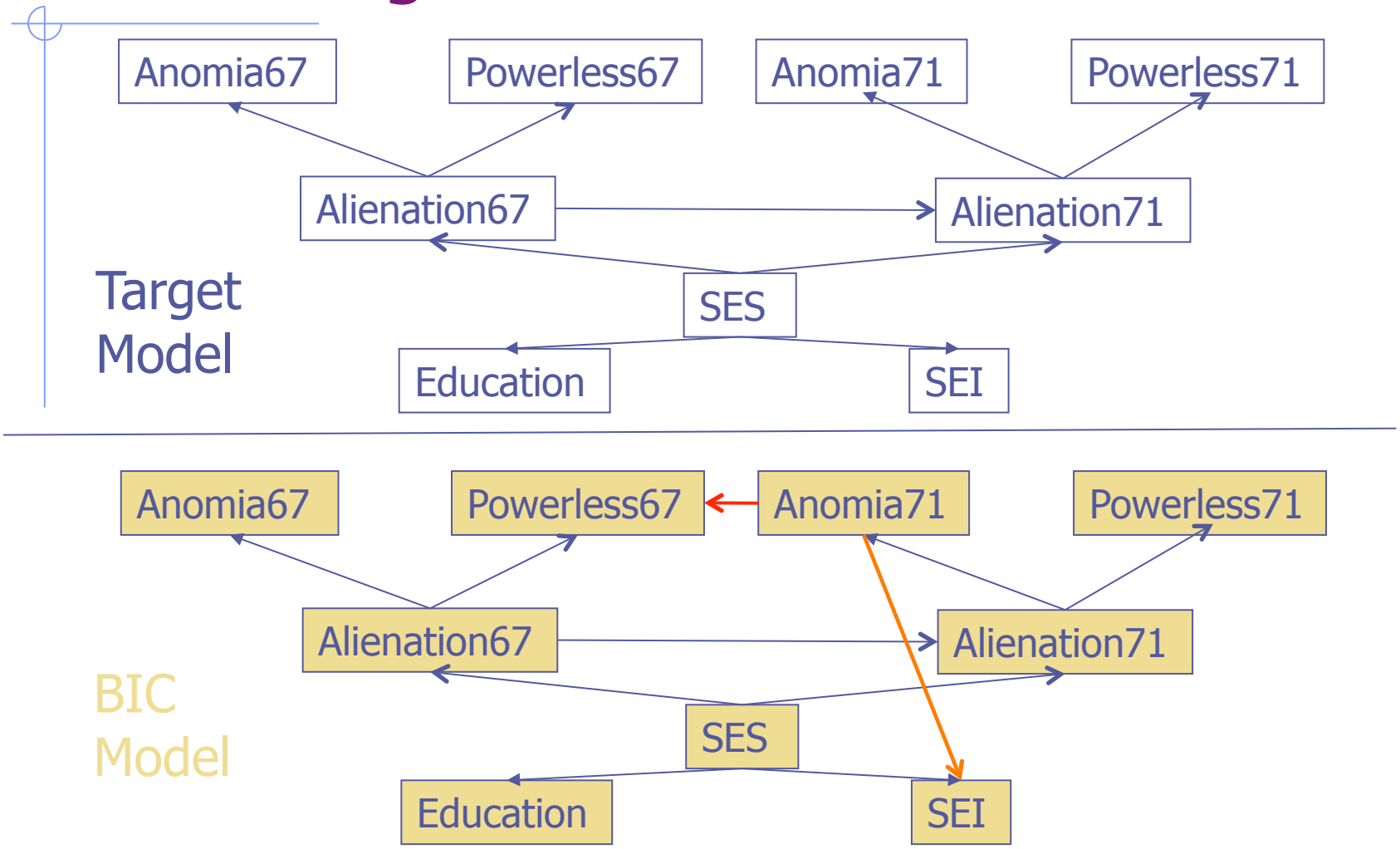
Score = 5

Score = 10

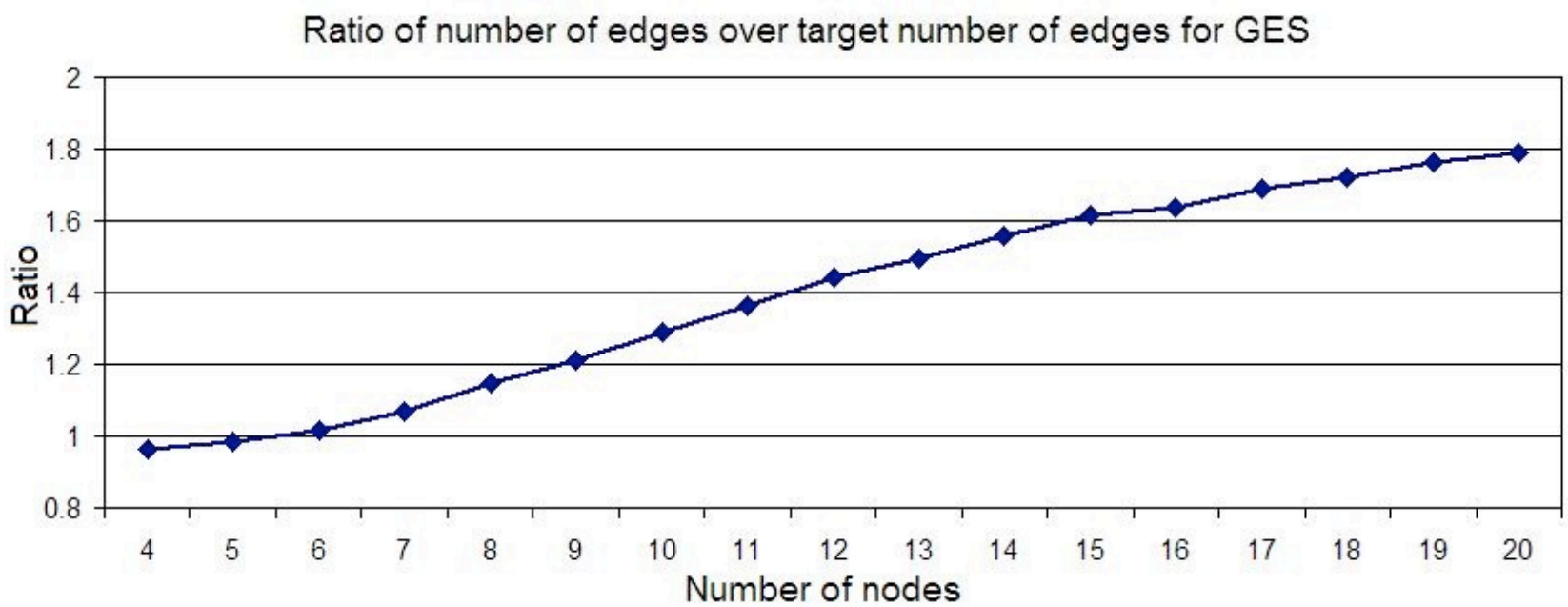


Covers correlation between A and B

Overfitting with Score-based Search



Overfitting with Score-based Search



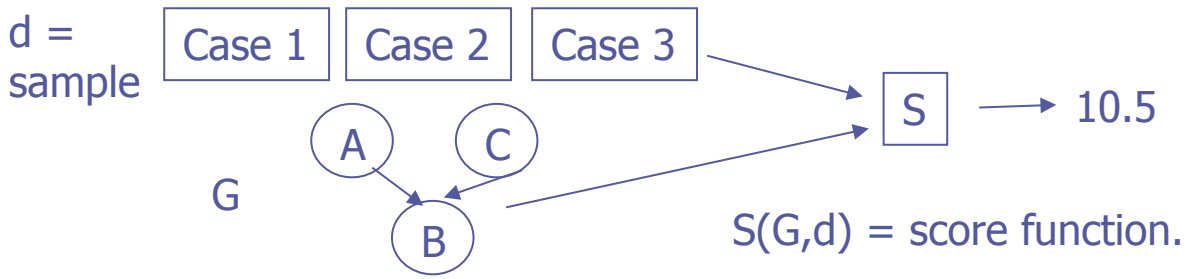
- Generate random parametrized graphs with different sizes, 30 graphs for each size.
- Generate random samples of size 100-20,000.
- Use Bayes Information Criterion (BIC) to learn.

Our Hybrid Approach

CB: weakness is type II error, false acceptance of independencies.
⇒ use **only dependencies**
Schulte et al. 2009 IEEE CIDM

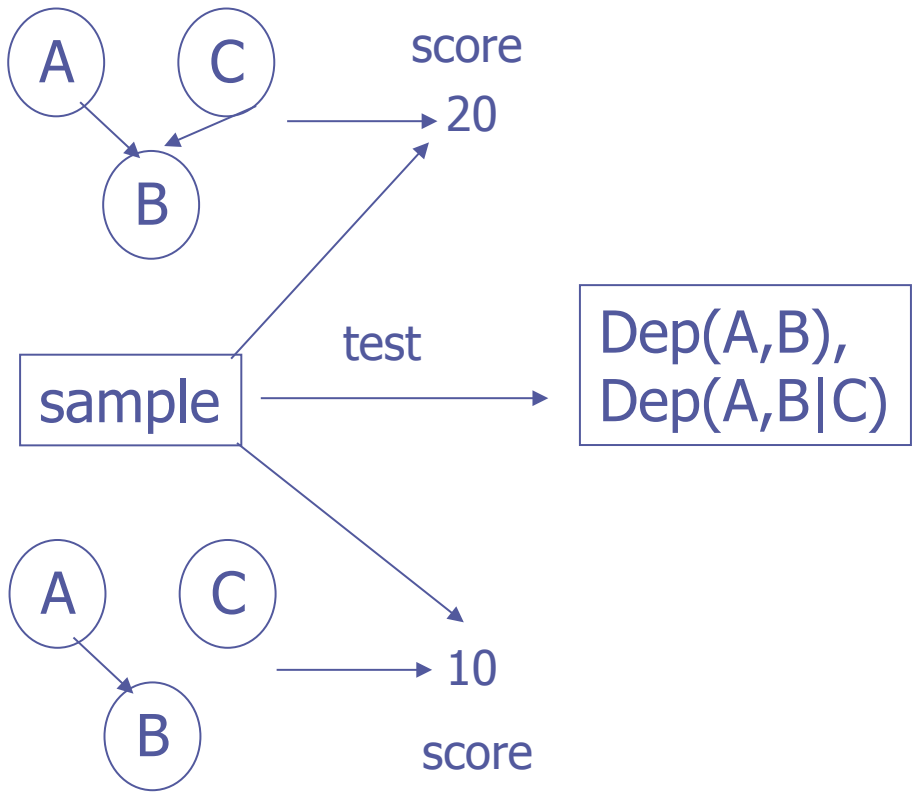
Score-based: produces overly dense graphs
⇒ cross-check score with **dependencies**

New idea



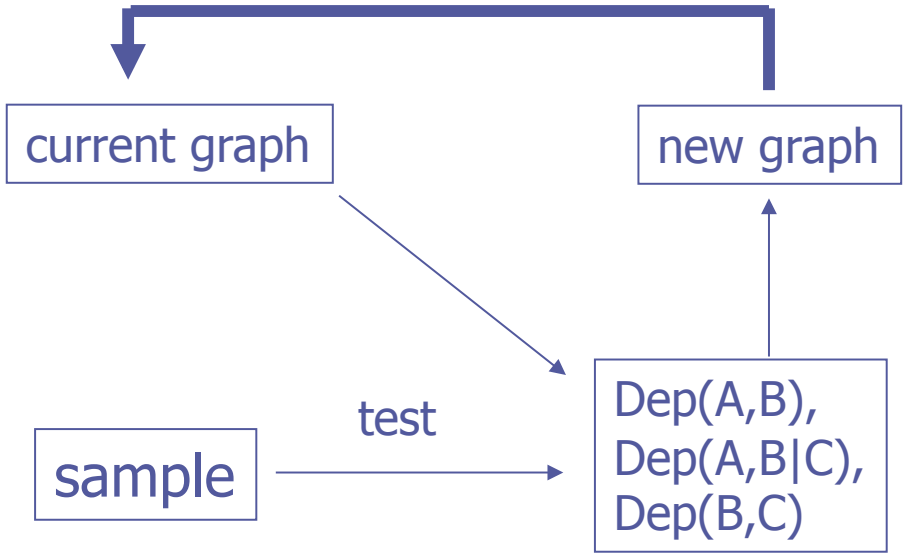
- Let Dep be a set of conditional dependencies extracted from sample d.
- Move from graph G to G' only if
  1. score increases: S(G',d) > score(G,d) **and**
  2. G' entails strictly more dependencies from Dep than G.

Example for Hybrid Criterion



- The score prefers the denser graph.
- The correlation between B and C is *not* statistically significant.
- So the hybrid criterion prefers the simpler graph.

Adapting Local Search



apply local search operator to maximize score while covering additional dependencies

test n^2 Markov blanket dependencies:

is A independent of B given the Markov blanket of A?
n= number of nodes

- any local search can be used
- inherits convergence properties of local search if test is correct in the sample size limit

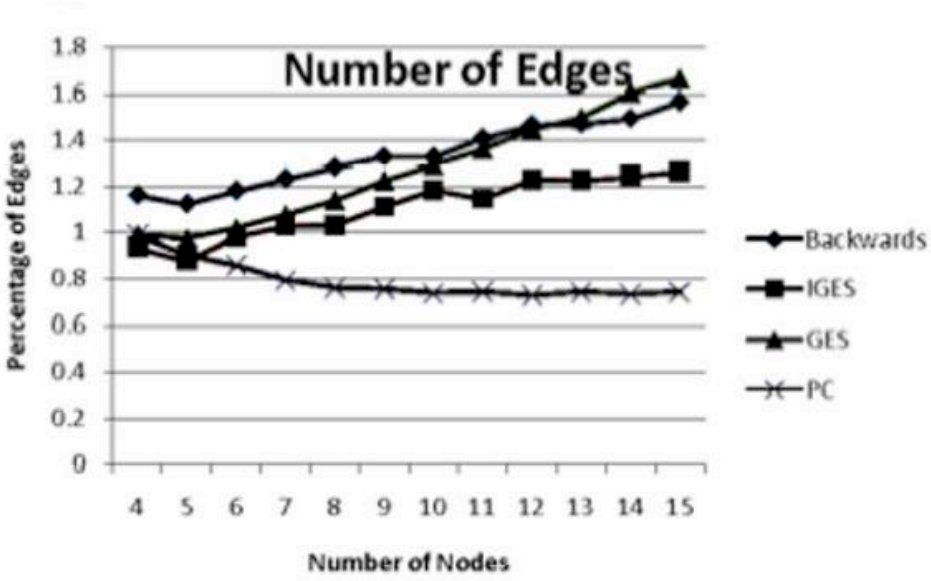
Simulation Setup (key results)

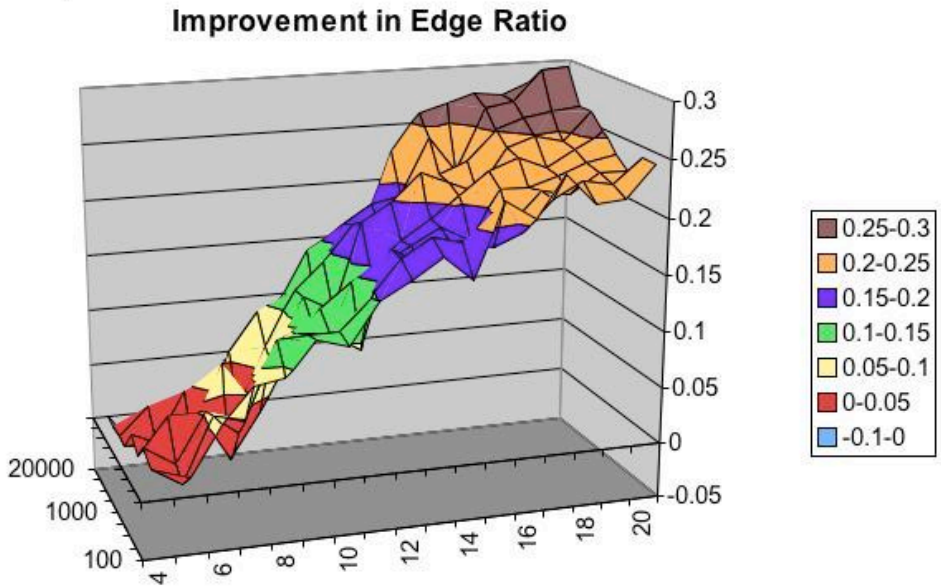
- Statistical Test: Fisher z-statistic, α = 5%
- Score S: BIC (Bayes information score).
- Search Method: GES [Meek, Chickering 2003].

Random Gaussian DAGs.

- #Nodes: 4-20.
- Sample Sizes 100-20,000.
- 30 random graphs for each combination of node number with sample size, average results.
- Graphs generated with Tetrad's random DAG utility (CMU).

Simulation Results: Number of Edges

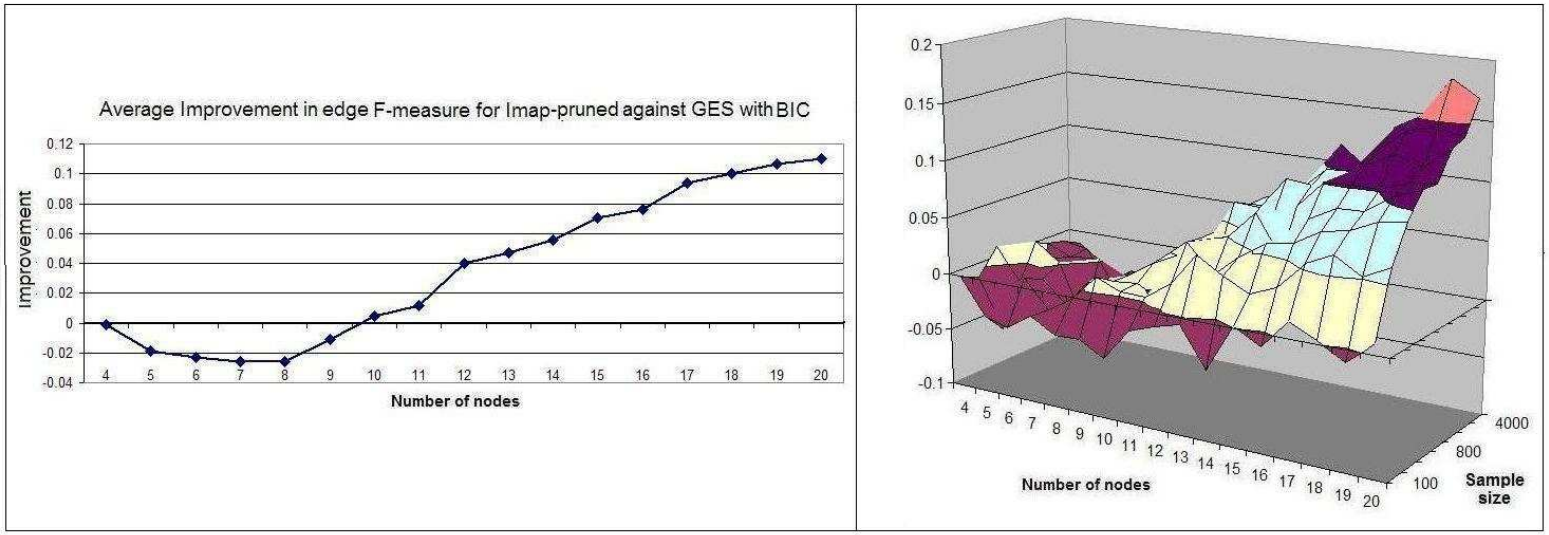




- Edge ratio of 1 is ideal.
  GES = standard score search. IGES = hybrid search.
- Improvement = ratio(GES) – ratio(IGES).

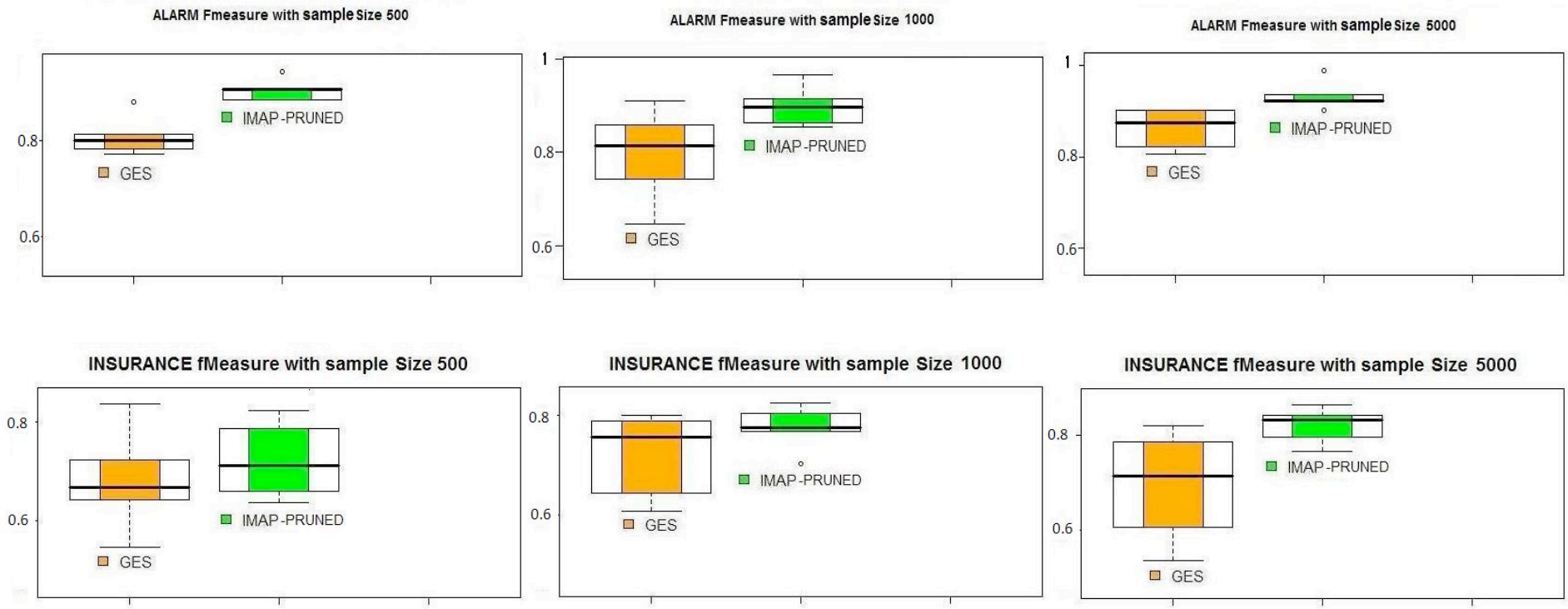
Simulation Results: f-measure

f-measure on correctly placed links combines false positives and negatives:
2 correctly placed/ (2 correctly placed + incorrectly placed + missed edges.)



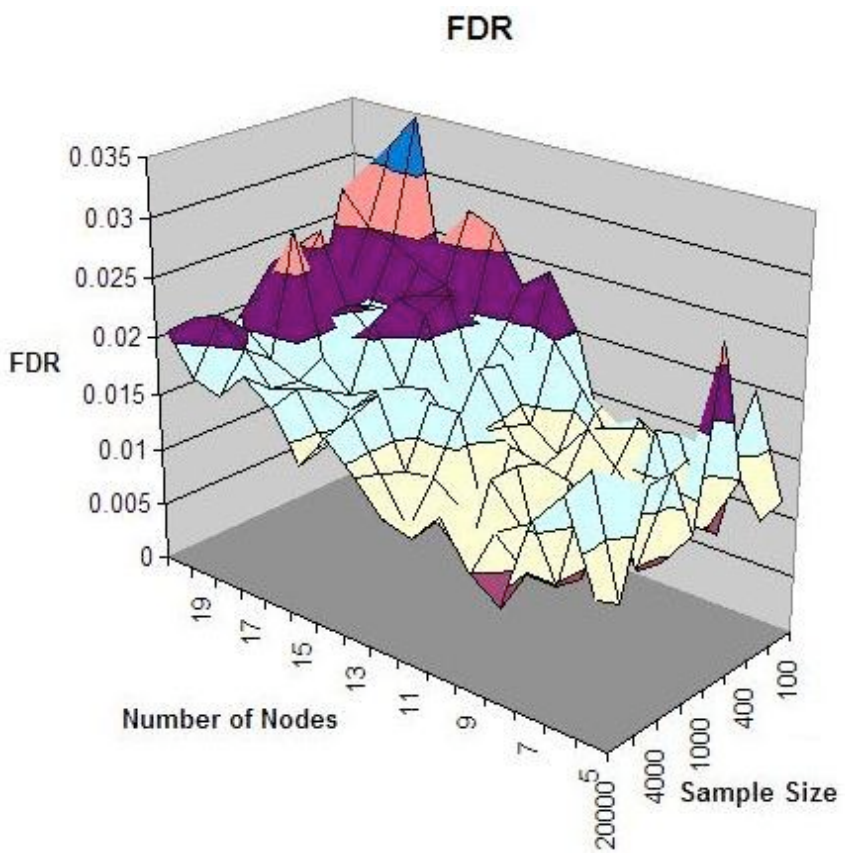
Structure fit improves most for >10 nodes.

Real-World Structures: Insurance and Alarm



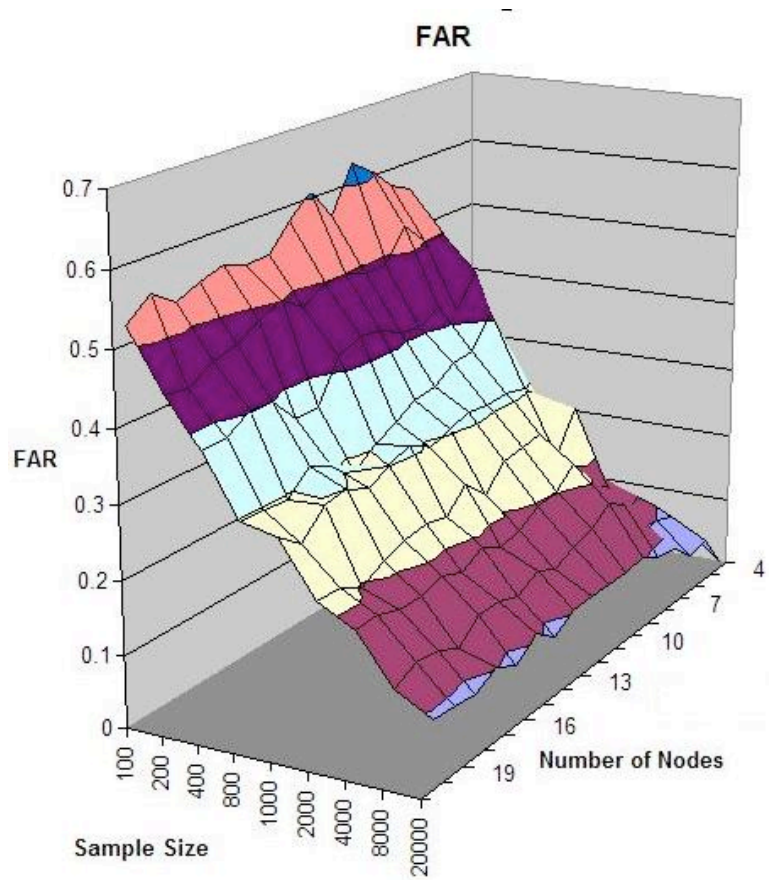
- Alarm (Beinlich et al. 1989)/Insurance (Binder et al.) 37/25 nodes.
- Average over 5 samples per sample size.
- F-measure, higher is better.

False Dependency Rate



type I error (false correlations) rare, frequency less than 3%.

False Independency/Acceptance Rate



- type II error (false independencies) are quite frequent.
- e.g., 20% for sample size 1,000.

Conclusion

- In Gaussian BNs, score-based methods tend to produce overly dense graphs for >10 variables.
- Key new idea: constrain search s.t. adding edges is justified by fitting more statistically significant correlations.
- Also: use only dependency information, not independencies (rejections of null hypothesis, not acceptances).
- For synthetic and real-world target graphs finds less complex graphs, better fit to target graph.

The End

Thank you!

full paper