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Liquidity Shocks, Systemic Risk, and Market Collapse: Theory and Application to the Market for Perps

Chitru S. Fernando
University of Michigan Business School

Richard J. Herring
The Wharton School, University of Pennsylvania

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* Please address all correspondence to Richard J. Herring, The Wharton School, University of Pennsylvania, Lauder-Fischer Hall, 2nd Floor, 256 South 37th Street, Philadelphia, PA 19104-6330. Tel. (215) 898-5613; Fax: (215) 898-2067; Email: herring@wharton.upenn.edu.

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Liquidity Shocks, Systemic Risk and Market Collapse: Theory and Application to the Market for Perps

ABSTRACT

Traditional explanations of market crashes rely on the collapse of an asset price bubble or the exacerbation of an information asymmetry sufficient to cause less-informed participants to withdraw from the market. We show that markets can crash even though asset prices have not deviated from fundamental values and information is shared symmetrically among all market participants. We present a model in which markets crash when investors shift their beliefs about the liquidity of the secondary market. While such shifts in liquidity may be a factor in explaining many market crashes, the collapse of the market for perpetual floating-rate notes (perps) provides an especially clear illustration of the theory because a shift in liquidity beliefs appears to have been the sole determinant of the market crash. Such a shift can be precipitated by a systemic liquidity shock that is transitory or permanent. The latter proved to be the case with perps because perceptions of the liquidity of the secondary market were permanently altered. In addition to providing new insights into why markets crash, our findings are particularly relevant for unseasoned financial products that are often priced and marketed on the assumption that liquid secondary markets will develop. The perp episode also highlights the importance of broad placement of securities. Since market liquidity arises endogenously from the diversity of liquidity needs across the investor base, the broader the investor base, the lower the probability of a systemic liquidity shock. We also show how simple modifications in security design can mitigate the impact of such a shock should it occur.
Liquidity, according to Keynes, offers a classic example of the fallacy of composition: what is true for a part is not necessarily true for the whole. The ability to reverse positions and get out quickly vanishes when everyone tries to do it at once. – Merton Miller (1991).

I. Introduction

The literature on asset bubbles and asymmetric information applies most readily to markets for equity, junk bonds, or more generally, markets where rational investors can disagree about future cash flows or can be prevented by various market imperfections from exploiting large price deviations from fundamental value. On the other hand the literature on irrational bubbles may apply when assets are owned primarily by unsophisticated investors who may exhibit irrational behavior such as the case of Swedish lottery bonds studied by Green and Rydqvist (1997). But, these explanations for market crashes seem less plausible in the case of markets for high-quality, fixed-income securities such as government and corporate bonds where the determination of fundamental value is usually straightforward, deviations from this value are easy to exploit and trading tends to be dominated by sophisticated, institutional investors. Perpetual floating-rate notes (perps) fall into this latter category. Issuers of perps generally had very high credit ratings and perps were traded in well-organized markets by sophisticated investors, primarily banks and other institutions. Our model, which emphasizes beliefs about the liquidity of the secondary market, explains market crashes in such cases where future cash flows are not in doubt and information is shared symmetrically across all market participants. While

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1 Well known episodes in this category are the collapse of the junk bond market, triggered by a series of defaults culminating in the default of the Campeau Group in September 1989, and the collapses of the LDC debt market, first in August 1982 triggered by the Mexican default and more recently in August 1998 triggered by the Russian default.

2 Perps are floating rate notes (FRNs) of infinite maturity, bearing a coupon indexed to a benchmark rate (usually the London Inter-Bank Offered Rate, LIBOR) and re-set at fixed intervals (usually every three or six months).
both information effects (pertaining to fundamental value) and liquidity effects (pertaining to investor clientele) may contribute to market crashes in general, the perp market crash provides a particularly clear illustration of the role of liquidity effects in causing a market to collapse.

The rise and collapse of the perp market is illustrated in Figure 1, which shows the prices of three obligations—a perpetual floating rate note, preference shares (which were subordinate to the perp), and a long-dated floating rate note (FRN)—of the National Westminster Bank (NatWest), a major U.K. clearing bank. After trading steadily at its par value, the price of the NatWest perp shows a sudden decline, which coincided with the collapse of the perp market. Clearly, this collapse cannot be attributed to a decline in the creditworthiness of NatWest since the prices of its long-dated FRNs and preference shares held steady while the prices of its perps dropped.

[Insert Figure 1 about here]

The proximate cause of the perp collapse was a rumor that began circulating in December 1986, of an impending change in international banking regulations. This change, if implemented, was expected to affect holders of the vast majority of the outstanding stock of perps. The rumor appears to have caused a large number of perp investors to attempt to reduce their holdings, creating liquidity demands that were highly correlated across the investor base. This liquidity shock did not affect the anticipated cash flow from the perps in any way, yet caused a dramatic downturn in market liquidity and asset values that turned out to be permanent. This episode provides a particularly striking illustration of the importance of beliefs regarding the breadth of

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3 Primarily banks issued perps, and 80% of the outstanding perps were thought to be held by Japanese banks at the time of the collapse. The rumor was that the Basel Committee on Banking Regulation and Supervisory Practices would require that banks deduct holdings of perps issued by other banks in computing their capital for regulatory purposes. See Herring and Litan (1995) or Wagster (1996) for further discussion of the negotiation of the 1988 Basel Accord.
the secondary market in determining its liquidity and asset prices. We argue that the drop in price experienced by perps was the result of investors losing confidence in the liquidity of the secondary market.

As Kindleberger (1978) has noted, market crashes are often viewed as the bursting of a bubble in which asset prices have diverged significantly above their fundamental values. Allen and Gale (2000) review a variety of models that can explain how bubbles can develop, and present their own theory, which relies on imperfect information and an agency problem that leads investors to bid up asset prices far above what they would be willing to pay if they were fully exposed to all potential losses. Our model does not depend on imperfect information regarding the fundamentals that determine asset prices, nor does it rely on agency problems.

Of course, markets can collapse even in the absence of a pronounced bubble in asset prices. Such market crashes can be explained by a worsening of information asymmetry about asset price fundamentals across market participants as in Glosten and Milgrom (1985), Glosten (1989), or Bhattacharya and Spiegel (1991). In these models, uninformed investors withdraw from the market for fear of being taken advantage of by better-informed market participants. The withdrawal of uninformed investors reduces demand for the asset and causes prices to fall. We achieve a comparable result in our model without assuming that information regarding asset-pricing fundamentals is asymmetrically distributed across investors. This difference has important implications about how a crisis can be resolved. If the crash is caused by an exacerbation of asymmetric information, it can be resolved by alleviating the information

4 However, the implications of our model are similar to Glosten (1989) in that reducing competition will reduce the susceptibility of market makers to systemic liquidity shocks.
asymmetry. If the crash is caused by a liquidity shock, however, prices will rebound only if market participants believe that the liquidity shock was transitory.

We define a “liquid market” as a market where participants can execute large transactions at short notice with minimal impact on the price. An asset will be liquid if it is traded in a liquid market. Generally, the liquidity of a secondary market depends on its depth, breadth and resiliency as well as its organizational structure and the reliability of clearing and settlement arrangements. Liquidity in our model, as in Diamond and Dybvig (1983), is a characteristic determined by the diversity of liquidity needs among investors who hold the asset. In our model, market participants form beliefs about future liquidity by observing past trading, and these liquidity beliefs in turn determine their valuation of the asset.

“Liquidity shocks” are unanticipated changes in the demand for liquidity. We consider two kinds of liquidity shocks: “idiosyncratic liquidity shocks” that are independently distributed across holders of an asset; and “systemic liquidity shocks” that are identically distributed, causing all investors to want to trade identically at the same time. Clearly, a systemic liquidity shock that causes investors to herd can render a market illiquid. In addition, we consider two durations of market illiquidity caused by systemic liquidity shocks: “transitory illiquidity” that is quickly reversed and “permanent illiquidity” which arises from the withdrawal of market participants due to their belief that the risk of systemic liquidity shocks exceeds the threshold for a liquid market to exist. Such liquidity shocks need not be related to asset fundamentals, although in many historical cases, herding behavior may have been triggered by news about asset fundamentals.

Since liquid assets are easily marketable, they will be priced at a premium to illiquid

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5 Of course, even in the absence of a liquid market, high-quality short-term securities are regarded as liquid due to their imminent cash settlement.
assets. We define the price differential between liquid and illiquid assets as the “marketability premium”.

Like Romer (1993) we assume that trading can reveal information that affects asset prices, but the information in Romer’s model (and other papers in this genre) is about asset price fundamentals. In contrast, we focus on the information generated by the trading process relating to potential liquidity demands of market participants, especially the extent to which these demands are correlated.

We explain the availability of markets, the cost of transacting in them, asset prices, and trading volume as the outcome of the demand for liquidity by individual market participants and their beliefs about the future availability of a liquid secondary market. This extends the work of Amihud and Mendelson (1986) and Brennan and Subrahmanyam (1996), who relate asset prices to transactions costs and other liquidity measures. While the behavior of market participants that precipitates market collapse in our model can be characterized as herding (Scharfstein and Stein (1990), Bikhchandani, Hirshleifer and Welch (1992) and Banerjee (1992)), in our model market participants collectively update their beliefs about future market liquidity by observing past market behavior, in contrast to the existing literature where herding occurs when one set of individuals learns sequentially from another.

The rest of the paper is organized as follows. In Section II, we develop our basic theoretical model, which relates asset prices to the liquidity of the secondary market. In Section III, we examine how investor beliefs about future market liquidity can be formed by observing past states of the market, and how changes in these beliefs can lead to shifts in market liquidity. In Section IV, we discuss the rise and fall of the market for perps in the context of our theoretical
framework, and provide a brief postscript on attempts to restore liquidity to the market. Section V provides some concluding observations.

II. Liquidity and Asset Prices

In this section, we develop our basic theoretical model and relate liquidity to asset prices by considering three cases that are differentiated by assumptions about the liquidity of the secondary market. In the first case, investors experience idiosyncratic liquidity shocks and the secondary market permits investors to mutualize their liquidity shocks by trading. The second case is identical to the first in all respects except that no secondary market is available. Comparing these two cases permits us to show how the liquidity of the secondary market affects asset prices. In the third case, investors experience a systemic liquidity shock. Since a systemic liquidity shock cannot be mutualized through trading, the secondary market will collapse resulting in a price that is identical to the second case in which there is no secondary market.

II.A. Model

We model a two-period economy with a group of $N$ risk averse investors who are identical at time 0. Each investor is endowed at time 0 with 1 unit of the single risky asset and 1 unit of the riskless asset. The risky asset pays off a random quantity of the numeraire riskless asset, $\tilde{v}$, at time 2, where $E(\tilde{v}) > 1$. All investors know that the return, $\tilde{v}$, is distributed normally with mean $\bar{v}$ and variance $\sigma_v^2$. The risk-free return is assumed to be zero. Investors maximize negative exponential utility functions of their wealth at time 2, $W_2$: $U(W_2) = -\exp(-aW_2)$, where $a \geq 0$ is the coefficient of risk aversion.
All investors experience identically distributed liquidity shocks at time 1, with the
distribution of these shocks being known *ex ante* at time 0. In general, such shocks can arise due
to a broad range of events that give rise to a change in the investor’s valuation of the risky asset
without new information about its fundamental payoff. In the literature, liquidity shocks have
been most frequently motivated as arising from shocks to preferences as in Diamond and Dybvig
(1983) or to endowments as in Glosten (1989) or Bhattacharya and Spiegel (1991). While such
shocks will also give rise to changes in the investor’s marginal valuation of the security, for ease
of exposition, we shall follow the formulation of Michaely and Vila (1995) and Michaely, Vila
and Wang (1996), and model such shocks as idiosyncratic tax or regulatory effects that change
the way in which individual investors value a security even in the absence of new information
regarding the cash flows associated with the security. This approach fits the example of perps
particularly well since the proximate cause of the collapse was a rumored regulatory change that
would have effectively imposed a regulatory tax on some investors who held perps. We
characterize this shock as a random additive increment, $\tilde{\theta}_i$, to the payoff $\tilde{v}$ of the risky asset to
investor $i$. $\tilde{\theta}_i$ is also distributed normally with mean 0 and variance $\sigma^2_{\theta}$, and is independent of $\tilde{v}$.

As in Karpoff (1986), differences in personal valuation caused by these shocks induce trading
when it is possible.

The correlation of liquidity shocks across investors is determined by the realization of one
of two possible states, “idiosyncratic” or “systemic” which will be revealed at time 1. In the
idiosyncratic state, liquidity shocks are independently distributed across investors. In the
systemic state, liquidity shocks are perfectly correlated across all investors. The implications are
quite straightforward. If the idiosyncratic state occurs, there is mutual benefit to trading at time 1
since shocks are uncorrelated. By trading, investors can mutualize the risk of the idiosyncratic
liquidity shocks. But if shocks are perfectly correlated, investors are unable to mutualize their liquidity shocks by trading at time 1 and prices will simply adjust to reflect the shock, just as in a Milgrom-Stokey (1982) no-trade equilibrium. The secondary market will collapse.

We assume that trading at time 1 occurs in a simple batch market in which all trades clear at the same price subject to a bid-ask spread. Trading is facilitated by $M$ identical, competing, risk-neutral market makers each of whom incurs a fixed cost of $C$ in setting up the market for each round of trade. These costs are recovered by the bid-ask spread. In a no-trade equilibrium that accompanies a systemic liquidity shock, market makers lose the cost sunk into setting up the market.

All market participants use a Bayesian updating framework (developed in Section III) to update their $ex\ ante$ probability beliefs about the state of the market by observing past states. The market makers offer a market only if, $ex\ ante$, the (subjective) probability that an idiosyncratic state will prevail exceeds a cut-off level beyond which their expected profit is non-negative. If investors expect that the secondary market will be liquid at time 1, they will attach a value to being able to rebalance their portfolios optimally at time 1, based on what they learn at time 1 about their liquidity shocks. This value (marketability premium) will be reflected in the time 0 equilibrium price of the risky asset. For tractability, we assume that market makers set bid-ask spreads as follows:

$$P_{i} = P_{i} + \lambda A X_{i},$$

(1)
where \( \lambda > 0 \) is determined by competition among market makers. \( P_i \) is the market-clearing price in the absence of transactions costs, \( \Delta X_{i,t} \) is the trade size of individual \( i \) and \( P_{i,t} \) is the actual price paid or received by individual \( i \).

**II.B. Equilibrium at \( t = 1 \) and \( t = 0 \)**

With the transactions costs described in the preceding section, the investor’s time 1 problem can be expressed as:

\[
\begin{align*}
\text{Max } & E_i \left[ -\exp \left\{ -a \left[ W_{i,t} + X_{i,t} (\tilde{\theta} + \theta_i - P_{i,t}) - \lambda (X_{i,t} - X_{0,i})^2 \right] \right\} \right] \\
\end{align*}
\]

where \( \theta_i \) is the liquidity shock realized by investor \( i \). We consider three cases in turn.

**Case 1: Idiosyncratic Liquidity Shocks and a Liquid Secondary Market at \( t = 1 \).**

This would be the case of perfect investor heterogeneity (maximum market breadth for given \( N \)), where each investor’s liquidity need is uncorrelated with the liquidity needs of the other investors. In this case, the equilibrium price at time 1, \( P_{i,t} \) becomes:

\[
P_{i,t} = \bar{v} + \hat{\theta}_A - a_i^2
\]

where

\[
\hat{\theta}_A = \frac{\sum_{j=1}^{N} \hat{\theta}_j}{N}
\]

and the optimal portfolio adjustment of individual \( i \), \( \Delta X_{i,t} = X_{i,t} - X_{0,i} \) will be:

\[\text{...}\]

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6 The transactions cost structure that we are assuming here is equivalent to the structure in Kyle (1985), where \( \lambda \) is the inverse measure of market depth. However, the rationale is different in Kyle’s model since \( \lambda \) is derived from his assumptions about information asymmetry.
\[ \Delta X_{it} = \frac{\hat{\theta} - \hat{\vartheta} + a\sigma^2_v(1 - X_{0t})}{a\sigma^2_v + 2\lambda} \]  

(5)

and the total volume of trade at time 1, \( Q_1 \), will be:

\[ Q_1 = \frac{\sigma_\theta}{a\sigma^2_v + 2\lambda} \sqrt{\frac{N(N-1)}{2\pi}} \]  

(6)

While the cost of transacting does not affect \( P^1_0 \), it depresses the volume of trade. When \( \lambda \) becomes very large, \( Q_1 \) shrinks and the market effectively shuts down.

Noting that \( \hat{\theta} \to 0 \) as \( N \to \infty \), we observe that a liquid secondary market enables investors to adjust to liquidity shocks at an equilibrium price which is not affected by the liquidity shocks. Such a market provides a valuable option to investors. Even if an investor does not plan to sell the asset before maturity, the investor’s future portfolio allocation preferences are inevitably subject to uncertainty and so the opportunity to sell the claim in a liquid secondary market enhances the investor’s willingness to buy the claim in the primary market. We examine next how this is reflected in the time 0 price.

The investor’s time 0 problem reduces to:

\[ \text{Max}_{X_{0i}} E_0 \left[ -a \left( W_{0i} + X_{0i}(\tilde{P}_1 - P_0) + \tilde{X}_{0i}(\tilde{v} + \tilde{\vartheta} - \tilde{\varsigma}) - \lambda(\tilde{X}_{0i} - X_{0i})^2 \right) \right] \]  

(7)

yielding the time 0 equilibrium price, \( P^1_0 \):

\[ P^1_0 = \bar{v} - a\sigma^2_v - a\sigma^2_\theta \left( \frac{2\lambda}{N} \right) - \frac{2\lambda}{a\sigma^2_v + 2\lambda + a\sigma^2_\theta \left( \frac{N - 1}{N} \right)} \]  

(8)

As \( \lambda \to 0 \) and \( N \to \infty \), \( P^1_0 \to \bar{v} - a\sigma^2_v \). Given idiosyncratic liquidity shocks, \( N \) becomes a measure of market breadth. Hence, with frictionless trading and an infinitely broad market,
investors will no longer price the risk of their idiosyncratic liquidity shocks, since this risk can be perfectly mutualized by trading.

Total expected market maker revenue, $R^1$, will be given by:

$$R^1 = E \sum_{i=1}^{N} \lambda (\Delta \tilde{X}_{i1})^2 = \frac{\lambda \sigma^2_{\theta}(N-1)}{(a\sigma^2_v + 2\lambda)^2}$$  \hspace{1cm} (9)

**Case 2: Idiosyncratic Liquidity Shocks with No Secondary Market at $t = 1$.**

In contrast to the previous case, investors do not expect to be able to satisfy their liquidity needs at time 1 because there is no secondary market. Since there is no portfolio rebalancing at time 1, the investor’s time 0 problem reduces to:

$$\max_{\lambda_{W_0}} E_0 \left[ -\exp \left\{ -a \left[ W_{0i} + X_{0i}(\tilde{v} + \tilde{\theta}_i - P_0) \right] \right\} \right]$$  \hspace{1cm} (10)

yielding the time 0 equilibrium price, $P_0^2$:

$$P_0^2 = \tilde{v} - a\sigma^2_v - a\sigma^2_{\theta}. \hspace{1cm} (11)$$

In contrast to the case when the market is liquid at time 1, the risk of a liquidity shock at time 1 is fully discounted in $P_0^2$ because there is no possibility for investors to mutualize these shocks by trading.\[7\]

Since there is no difference in the fundamental determinants of the price of the risky asset between Cases 1 and 2, the price differential between the two cases is entirely determined by the absence of the secondary market in Case 2. The price differential $\Phi$, the marketability premium, is:

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\[7\] Note that even in the previous case, it follows from (8) that the same result obtains in the limit when $\lambda \to \infty$. When transactions costs become very high, the secondary market shuts down.
Given idiosyncratic liquidity shocks, we can infer that the liquidity-driven price differential $\Phi$ is higher, the greater the demand for liquidity (as measured by the volatility of liquidity shocks $\sigma_{\theta}^2$), the greater the market breadth (as measured by the number of investors $N$) and the lower the transactions costs. Note that the latter, which is parameterized by $\lambda$, will be minimized by the competition among market makers. Given a level of liquidity demand, $\lambda$ provides a measure of the degree of liquidity supplied to the market by the market makers. In the limiting case of $\lambda \to 0$ and $N \to \infty$ that we considered previously, $\Phi$ will converge to $a\sigma_{\theta}^2$. At the opposite end of the liquidity spectrum, the case of $\lambda \to \infty$, $\Phi$ will converge to 0. Thus, the marketability premium, $\Phi$, will be bounded by $0 \leq \Phi \leq a\sigma_{\theta}^2$, and the value of $\Phi$ will reflect the degree of marketability.

Case 3: Systemic Liquidity Shocks Causing an Illiquid Secondary Market at $t = 1$.

In this case, the investor’s situation will be identical to the previous case in which there was no secondary market. Since investors expect to experience the same liquidity shock at time 1, they will have no opportunity to mutualize these shocks by trading. Thus, the secondary market will break down and the price will adjust without trade as in Milgrom and Stokey (1982). The risk of liquidity shocks will again be fully discounted in the time 0 equilibrium price. Hence, the time 0 price in this case, $P^3_0$, will be equal to the time 0 price in Case 2, $P^2_0$. Obviously, both trading volume and market maker revenue will be zero in Cases 2 and 3.
II.C. Transition from a Liquid to an Illiquid Secondary Market

It is clear from the above analysis that if the secondary market collapses, the security will experience a price decline of $\Phi \geq 0$ reflecting the elimination of the marketability premium. This is consistent with the price drop observed in the perp market. Moreover, as we will discuss later, the price drop experienced by perps is consistent with the price differentials between liquid and illiquid securities in other markets.

The analysis of the three cases in this section proceeded on the assumption that market participants had perfect foresight about the liquidity of the secondary market, and rationally incorporated their beliefs in asset prices. Furthermore, the decision by market makers to open or close the secondary market was assumed to be exogenous to the model. Next, we focus on the question of how market participants form beliefs about future market liquidity, and how this process can lead to an endogenous shift in the liquidity of the secondary market that affects asset prices.

III. Beliefs about Liquidity

In this section we extend our theoretical framework to examine how beliefs about liquidity in the secondary market evolve and change, drawing on the literature on herding and informational cascades (Scharfstein and Stein (1990), Bikhchandani, Hirshleifer and Welch (1992) and Banerjee (1992)) as well as the literature on market breakdowns in the presence of asymmetric information (Glosten and Milgrom (1985) and Bhattacharya and Spiegel (1991)).

Informational cascades occur when individuals deduce the information of preceding market participants sequentially by observing their behavior. Similarly, in our model individuals deduce the degree of liquidity of the secondary market by observing past states of the secondary
market. As we demonstrate, when the true probability of a systemic liquidity shock is low, a
continued sequence of experiences with a liquid secondary market can cause market participants
to underestimate the probability of a systemic liquidity shock. In our model, market participants
update their beliefs in Bayesian fashion, so that their subjective probability of a systemic liquidity
shock progressively diminishes as the sequence of periods without a systemic liquidity shock
continues. This can cause security prices to deviate from the level that reflects the true
probability of a systemic liquidity shock. A reevaluation occurs only when market participants
experience a liquidity shock that turns out to be systemic.

III.A. Bayesian Updating Framework

Market makers offer markets at time 1 and investors value the risky asset at time 0 based
on their subjective (uniform) probability estimate of the occurrence of the state in which liquidity
shocks are idiosyncratic (liquid state). At the outset, we assume that the subjective probability
estimate of the state in which a systemic liquidity shock occurs (illiquid state) is low enough for
market makers to open the secondary market. The indicator \( \chi = 1 \) if the illiquid state occurs and
0 otherwise.

We assume that the unconditional probability of the illiquid state is \( \pi \). Market participants
form a subjective probability estimate of the occurrence of the two types of events. The prior
probability density function of \( \pi \) is assumed to be a Beta distribution with parameters \( \alpha > 0 \) and \( \beta > 0 \):\(^8\)

\[
f(\pi) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \pi^{\alpha-1}(1-\pi)^{\beta-1}, \quad 0 \leq \pi \leq 1
\]  

(13)

and has an unconditional expected value of:

$$E(\pi) = \frac{\alpha}{\alpha + \beta}$$  (14)

As shown by DeGroot (1970), the posterior probability distribution is also a Beta distribution with parameters $\alpha' = \alpha + y$; and $\beta' = \beta + n - y$:

$$g(\pi | \chi_{t-1}, \ldots, \chi_{t-n}) = \frac{\Gamma(\alpha' + \beta')}{\Gamma(\alpha')\Gamma(\beta')} \pi^{\alpha'-1} (1-\pi)^{\beta'-1}$$  (15)

where $n + 1$ is the number of observations in the sequence of events and $y$ is the total number of illiquid states observed in this sequence.

The market participants form their subjective probability beliefs as follows:

1. If the preceding $n + 1$ events are liquid states, the subjective probability assessment after observing $n + 1$ liquid states will be:

$$E\left[\pi \left| \sum_{t=-n+1}^{-1} \chi_t = 0 \right. \right] = \frac{\alpha}{\alpha + \beta + n}$$  (16)

As $n$ increases, the subjective probability of an illiquid state goes to zero.

2. If the immediately preceding event is an illiquid state, then a new sequence of observations begins and the market participants form their subjective probability assessment using the posterior probability distribution, as follows:

$$E(\pi | \chi_{t-1} = 1) = \frac{\alpha + 1}{\alpha + \beta}$$  (17)

Note that:

$$\frac{\alpha}{\alpha + \beta + n} < E(\pi) = \frac{\alpha}{\alpha + \beta} < \frac{\alpha + 1}{\alpha + \beta}$$  (18)

This means that as the number of periods ($n$) without a systemic shock increases, the subjective probability of a systemic liquidity shock will continue to decline relative to the unconditional
probability of such a shock. The occurrence of a systemic liquidity shock will cause market participants to overestimate initially the probability that a systemic liquidity shock will reoccur. But, if the secondary market reopens, as the number of periods without an additional systemic liquidity shock increases, the subjective probability of a systemic liquidity will again decline, ultimately falling below the unconditional probability. This shift in subjective probability of a systemic liquidity shock is the critical determinant of whether market makers will open a secondary market.

**III.B. Liquidity Shifts**

Given the market set-up costs \( C \), the number of market makers \( M \), the total expected market maker revenue derived in (9), and the assumption that bid-ask spreads are set competitively such that the market makers’ expected profits will be zero, we show in the appendix that the necessary condition for a market to be offered is:

\[
\frac{(1 - \xi) \sigma_y^2 (N - 1)}{8 a \sigma_y^2 MC} \geq 1
\]

(19)

where \( \xi \) is the *ex ante* subjective probability attached by the market makers to a systemic liquidity shock. For a given value of \( \lambda \), dealers are more likely to make a secondary market (the left-hand-side of (19) is more likely to exceed one), the lower the subjective probability of a systemic liquidity shock \( \xi \), the lower the fixed costs of making a market \( C \), and the lower the number of market-makers \( M \). As in Glosten (1989), increases in the number of competitive market makers may render the secondary market more vulnerable to collapse. In Glosten’s model the result is driven by the reduction in the ability of market makers to withstand losses to informed traders when competition reduces their profits. In our model, the result is drive by the reduction in the ability of market makers to withstand a systemic liquidity shock.
Equation (19) defines a threshold level of the subjective probability of a systemic liquidity shock above which dealers will not be willing to make a secondary market because they do not expect it to be profitable. The threshold level is the value of $\xi$ that equates the left-hand-side of (19) to one. Whether the secondary market is reopened after a systemic liquidity shock depends on the unconditional probability ($\pi$) of a systemic liquidity shock. It follows from (18) that prior to the first occurrence of a systemic liquidity shock, the \textit{ex ante} subjective probability of a systemic liquidity shock ($\xi$) will fall below $\pi$. But after the occurrence of a systemic liquidity shock $\xi$ will rise above $\pi$. If $\pi$ is sufficiently high, $\xi$ may rise above the threshold level at which market makers can expect to earn non-negative profits. As a result, they will not reopen the secondary market. In contrast, when the unconditional probability of a systemic liquidity shock is sufficiently low, even though $\xi$ rises above $\pi$, market makers will expect to earn non-negative profits and so they will reopen the secondary market.

Our theoretical framework thus shows how the availability of a secondary market can give rise to a marketability premium and how this marketability premium can grow as investors gain confidence in the liquidity of the secondary market. We also show how this confidence can collapse in the event of a systemic liquidity shock, leading to either transitory or permanent illiquidity in the secondary market depending on how the expectations of market participants are changed by the systemic liquidity shock. We next turn to the details of the perp market to illustrate our theoretical framework.

\textbf{IV. The Rise and Fall of the Perp Market}

The perp was introduced in the early eighties and was quickly hailed as a successful financial innovation. In this section, we examine the rise and fall of the perp market, and link its
initial success to the growth of investor confidence about the liquidity of the secondary market for perps, and its collapse to a systemic liquidity shock that substantially increased expectations of the reoccurrence of a systemic liquidity shock and led market makers to abandon the market permanently.

**IV.A. The Rise**

The perp was a simple variation on the Floating Rate Note (FRN). The standard FRN is a bond bearing a coupon that changes at a set interval (usually every three or six months) over the life of the bond. The coupon is a fixed margin or spread over some benchmark rate (usually the London Inter-Bank Offered Rate (LIBOR)).

The FRN has particular investor appeal when interest rates are expected to be volatile because the principal value of the FRN is likely to be much more stable than that of a conventional, fixed-rate bond of identical maturity. Indeed if the borrower’s relative credit standing (as reflected in the spread over the benchmark rate) has not changed since issued, the FRN will normally be repriced at par on the day on which the coupon is reset.

The first perp was issued on behalf of National Westminster Bank in April 1984. The original NatWest issue was junior, subordinated debt, ranking in payment priority after other outstanding debt, but before common or preferred stock. Banks experienced pressure to increase their capital during the mid-eighties and sought permission to count issuance of perps as capital for regulatory purposes. The Bank of England did not permit this first issue to qualify as

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9 Citicorp issued a quasi-perp in 1980. Since this issue gave investors the right to put the perp to Citicorp at each coupon reset date, the instrument effectively had a fixed, if indefinite, maturity and traded accordingly.
regulatory capital. In rejecting this request, however, the Bank of England set conditions under which a perp could be counted as capital.\textsuperscript{10}

Regulators in several other countries followed the Bank of England’s lead so that issuance of perps became a feasible solution to the perceived need to increase regulatory capital.\textsuperscript{11} From the perspective of bank issuers, the perp was an especially attractive capital instrument because interest payments on perps, unlike dividends on preferred stock, could be deducted in computing taxable income.\textsuperscript{12}

The challenge in marketing perps was to convince prospective investors that perps were close substitutes for fixed-maturity, floating-rate notes and money market instruments. Underwriters argued that the floating-rate feature made the interest-rate risk on perps equivalent to that on any other floating rate instrument. Moreover, they addressed concerns about the infinite maturity of perps by arguing that they could be sold any time at a price close to par in a broad, deep secondary market. Investor confidence in the liquidity of the secondary market was thus key to pricing perps in line with money market and other finite-maturity instruments of

\textsuperscript{10} The critical requirement was that perps must be automatically converted into preferred stock in the event of default with the number of preferred stock shares equal to the principal amount of the perp plus all arrears of interest and all interest accrued. Most perps were dollar-denominated and so the Bank of England also required that, in the event of a default by a British issuer, the dollar amount should be converted into pounds at the prevailing exchange rate. In effect, investors who had dollar-denominated debt would end up with pound-sterling-denominated preferred shares.

\textsuperscript{11} Following the Bank of England, several other central banks—including those in Australia, Canada, France and the United States—established conditions under which perps could be counted as capital for regulatory purposes. Many of these conditions were less favorable to the investor than those established by the Bank of England. The Japanese authorities were about to authorize the use of perps to meet capital requirements when the market collapsed.

\textsuperscript{12} All issues of perps include a provision that allows the borrower to automatically call the perp if the tax authorities disallow the deductibility of the interest paid on perps as a business expense. US banks were very late in entering the market because there was a presumption that the Internal Revenue Service (IRS) would construe the interest paid on perps as the equivalent of dividends and therefore not tax-deductible. Just before the market collapsed, Goldman Sachs introduced a perp structure for Citibank that seemed likely to gain a favorable ruling from the IRS. This involved giving investors the right to put the security back to Citibank after a specified period so that the perp was, for all practical purposes, equivalent to a long-dated FRN that had received a favorable ruling from the IRS.
comparable quality. The evidence suggests that underwriters were increasingly successful in this regard.

Banks, especially Japanese banks, who were eager to invest in floating-rate, dollar-denominated instruments during the eighties, became the main buyers of perps. They found perps an attractive way to increase returns over interbank placements (which yielded LIBOR or less) at what appeared to be little additional risk.

From 1984 to the end of 1986 the spread over LIBOR steadily declined, indicating an increasing marketability premium as investors gained confidence in the liquidity of the secondary market. The first perps were priced at margins over LIBOR that were as much as 20-25 basis points higher than comparable FRNs; but, by mid-1986, the margin had declined to around 10-15 basis points. Figure 2 illustrates this favorable trend in the spread over LIBOR for Barclays Overseas Investment BV, one of the few perps for which price data is available going back to Fall 1984. As investors grew more confident in the liquidity of the secondary market, their subjective probability of a systemic liquidity shock declined and the marketability premium increased thus reducing the yield spreads attached to perps. The spread relative to the LIBOR benchmark fell so low that even some governments, such as the Kingdoms of Belgium, Denmark and Sweden, issued perps, although they had no tax or regulatory incentive to do so. Other quasi-government issuers included the World Bank and Hydro-Quebec.

The market grew rapidly following the inaugural NatWest issue in April 1994. The volume of perps outstanding stood at $3.5 billion by the end of 1984, $16 billion by the end of

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13 Some market observers estimate that as much as 80% of the perps outstanding were placed with Japanese financial institutions. Overall, banks held an estimated 90% of outstanding perps (IFR, December 6, 1986, p.3633).

14 Figure 2 captures the declining yield spread in terms of the corresponding increase in price.
1985, and $22 billion by the end of 1986, accounting for 29% and 46%, respectively, of total FRN issuance in the latter two years (Meerschwa (1987)). Through 1986, nearly 60 perps were issued.

Transactions in the secondary market for perps were cleared and settled through Euroclear and CEDEL, the two principal systems for clearing Eurobonds, both of which are designed to accomplish simultaneous delivery of assets against payment (Kamata (1990)). This forestalled any potential investor concerns about “settlement risk”—the risk that a counterparty would not fulfill its settlement obligation or that the settlement mechanism would break down—and undoubtedly helped to sustain the confidence of investors in the liquidity of the secondary market.

Investor confidence in the liquidity of a secondary market also depends on the cost of finding a counterparty and executing a transaction, which varies with the structure of the market. Some secondary markets are primitive, direct-search markets where the transactions costs are borne by the initiator of the transaction and are likely to be quite heavy. Assets that are traded in such markets are not very liquid because, if obliged to sell on short notice, investors must often accept less than full market value. At the other end of the spectrum, dealer markets are usually regarded as especially liquid because, in addition to providing information and matching buyers and sellers, dealers also provide immediacy by buying and selling from their own inventories of securities. The perp market was a dealer market, whose liquidity increased steadily as reflected by transactions costs and the size of the standard lot for which dealers would quote a price. By November 1986, more than fifty dealers stood ready to quote two-way prices for standard lots of $5 million at a 10 basis point spread (Williams and Hole (1987)). Although volume data is not available, an indirect indication of volume can be inferred from the number of perp issues in the
Euroclear listing of the twenty most actively traded money market issues each month. As Figure 3 indicates, perps were among the most actively traded money market instruments through the first quarter in 1987, with average daily volumes of as much as $1 billion.

[Insert Figure 3 about here]

IV.B. The Collapse

The success of the perp market was short-lived. The proximate cause of the collapse in the market for perps appears to have been a rumor that the Basel Committee on Banking Regulation and Supervisory Practices (Basel Committee) would require that banks deduct holdings of perps issued by other banks in computing their capital for regulatory purposes.

Although the proposed regulations pertained only to British and American banks, the potential implications for banks in Japan were clear (Wagster (1996)). As one market participant observed, “The Ministry of Finance in Tokyo must eventually insist on similar provisioning, particularly if Japanese banks are allowed to offer perpetual debt themselves. That will just about kill off the market,” (IFR, December 6, 1986, p.3633).

Even the possibility of this kind of change in regulations provided a powerful incentive for banks holding perps to sell. That’s precisely what happened on Wednesday, December 3, 1986, when almost all of the core fifty dealers in the perp market were overwhelmed with sell

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15 The Committee was expected to take the view that the banking system would be more resilient, and the danger of contagion would be less, if nonbanks held capital claims on banks. The basis for this rumor was the proposed Anglo-American Accord on the assessment of capital adequacy. The proposed regulations were officially released in the United States on January 8, 1987, but major banks were well aware of the general outlines of the approach. The part of the proposal that was of particular relevance to the market for perps was the decision regarding bank holdings of other banks’ capital instruments. The official release (Comptroller of the Currency, Federal Deposit Insurance Corporation and Federal Reserve Board, 1987, p.7) noted that the Bank of England already deducts such holdings from capital: “…except for limited concessions to allow some banks to play an active role in market-making in the primary (new issues) and/or secondary markets. This policy will be maintained. The U.S. authorities accept the principle underlying this policy and will monitor bank holdings of capital instruments issued by other banks and may, as appropriate, deduct these items on a case-by-case basis.”
orders and suspended normal trading. Thus the proposed regulatory change produced a systemic liquidity shock

As participants noted at the time, “We have seen the door slammed shut on the only way in which investors can really leave this market—trading liquidity.... The whole psychology of this market has now changed—it’s never going to be the same again,” (IFR, December 6, 1986, p.3632). Another participant observed, “The liquidity myth has been exploded with perpetuals. We all now know that liquidity is only there when nobody wants to use it -- if everybody wants to pile out, then the market can’t accommodate it,” (IFR, December 6, 1986, p.3632). Along the same lines, another market participant (IFR, January 10, 1987, p. 3) concluded, “The crisis is basically one of confidence, and perpetuals are undergoing a general re-evaluation of worth separate from any underlying change in the quality and credit of the debt involved and external influences such as interest rates.”

Ironically, the Basel Committee ultimately rejected this unfavorable treatment of perps.\textsuperscript{16} By then, however, the damage to investor confidence in market liquidity was irreparable.

From Wednesday, December 3, 1986, the secondary market began to collapse. Market makers withdrew from the market in anticipation of continuing losses. Typical dealer-to-dealer price spreads increased from 10 basis points to 50 basis points while at the same time standard lot sizes declined from $5 million to $1 million (Williams and Hole (1987)). The number of

\textsuperscript{16} The Committee’s Consultative Paper of December 1987 (p.9) stated: “The Committee carefully considered the possibility of requiring deduction of banks’ holdings of capital issued by other banks.... Several G-10 supervisory authorities currently require such a deduction to be made in order to discourage the banking system as a whole from creating cross-holdings of capital, rather than drawing capital from outside investors. The Committee is very conscious that such double-gearing (or “double-leveraging”) can have systemic dangers for the banking system by making it more vulnerable to the rapid transmission of problems from one institution to another and some members consider these dangers justify a policy of full deduction of such holdings... Despite these concerns, however, the Committee as a whole is not presently in favor of a general policy of deducting all holdings of other banks’ capital, on the grounds that to do so could impede certain significant and desirable changes taking place in the structure of domestic banking systems.”
active market makers plummeted from 50 to fewer than 10 by March 1987, and these functioned mainly as brokers—trying to match buyers with sellers—rather than standing ready to buy or sell at a stated spread. After December 1986, only perps issued by the British clearing banks made the list of the twenty most active issues maintained by Euroclear. As illustrated in Figure 3, by May 1987, perps had dropped from the list altogether. In the absence of a liquid secondary market, the new issue market completely disappeared.

Secondary market prices fell sharply during this period. Figure 4 plots the value from November 1984 through June 1991 of a price index of eleven perps issued by British Clearing Banks. The sudden collapse and lack of recovery of the secondary market is clearly evident.

As further evidence that concerns over the liquidity of the secondary market, rather than default risk, were at the heart of the difficulty, consider the behavior of the four perps issued by Standard Chartered Bank (Figure 5). The perp with a put option continued to trade at par, while the other three issues that could not be put back to the issuer declined sharply in price, losing as much as half their value by December 1990.

The events of December 1986 had exposed the narrow investor base of the secondary market for perps. The episode made clear that the liquidity of the perp secondary market depended critically on confidence in the breadth of the market—on the belief that other investors would not change their portfolio preferences in the same way at the same time. When a liquidity shock affected all the bank holders of perps, the negative implications of the overwhelming

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17 With the exception of issues that gave holders the option of putting the notes back to the issuer at par value, such as the issue by Standard Chartered Bank discussed below, and by the World Bank and the Kingdom of Belgium.
concentration of perps in the hands of Japanese banks for the prospect of additional systemic
liquidity shocks became all too evident.

The gradual increase in the marketability premium as the secondary market became more
liquid is consistent with our model as was the dramatic collapse of the secondary market after the
experience of a systemic liquidity shock. More broadly, Davis (1989) and Guttentag and Herring
(1984) have shown that innovative financial instruments may be particularly subject to disaster
myopia because the empirical record for judging how they will perform over a variety of
conditions is very limited and those who market a new instrument have an incentive to
emphasize the robustness of its features. The consequence is that buyers often have a very
imperfect a priori understanding of the attributes of a new instrument. It is not unusual for
buyers of new instruments to extrapolate favorable performance into the future. Because the
secondary markets in perps were well organized, with low transaction costs and more that fifty
dealers providing immediacy by standing willing to buy or sell perps at stated prices, it must have
been tempting to dismiss the probability of a systemic liquidity shock as inconsequential.

After the collapse, why didn’t nonbanks (who, even if the rumor had proved to be true,
would not be penalized by the regulators for holding perps) buy the bank holdings of perps?
Why didn’t the market quickly equilibrate at a new lower price that would compensate investors
for higher perceived risk with higher anticipated capital gains? Prices did fall, but the volume of
trading activity never recovered. Ironically, the fundamental problem standing in the way of a
recovery was also the lack of breadth in the market. The Japanese regulatory authorities
permitted Japanese banks (which held the vast majority of perps) to defer recognition of the

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18 This normal tendency toward disaster myopia was undoubtedly exacerbated by the general inexperience of many
market participants. One senior banker admitted that, “When I look around the trading areas of my bank, and realize
that most of the dealers are well under 30 years of age, and have never been exposed to a consistent bear market, I
realize that this is probably the greatest single area of exposure I have,” (IFR, December 6, 1986, p.3653).
capital loss until the perps were sold. Market makers and other potential market participants believed that any rise in the prices of perps would be swiftly met by sales of perps by Japanese banks and so they were unwilling to buy. But, so long as perps traded below par, Japanese banks, which were under international pressure to increase regulatory capital, were not eager to sell below par and realize a capital loss, particularly since the stream of cash flows from perps was never in doubt. If Japanese banks had been obliged to mark their holdings to market, it is likely that the volume of trading would have increased, albeit at lower initial prices, and perps would have been redistributed to a broader range of investors, thereby paving the way for a restoration of liquidity.

**IV.C. Postscript**

The collapse of the secondary market for perps caused a sharp decline in prices of 12% to 25% due to the collapse of the marketability premium. These discounts for illiquidity are consistent with other measures of the discount for illiquidity reported by the SEC (1971) and Pratt (1989) with regard to letter stocks. Using the midpoints of the discount range for letter stocks relative to their freely traded counterparts, Pratt found that the discount was 25.8%. The SEC Study (1971) found that most letter stock transactions were at a discount of 10% to 30% of the analogous securities traded freely on the public exchanges.

Several attempts have been made to restore liquidity to perps by repackaging the promised cash flows as instruments with fixed maturities (Meerschwam (1987)). The basic idea

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19 Pratt (1989) notes that “A letter stock is identical in all respects to the freely traded stock of a public company except that it is restricted from trading on the stock exchanges for a certain period” and reports comparisons of the value of such letter stocks to their freely traded counterparts. Publicly traded corporations issue letter stock frequently in making acquisitions or raising capital when the time and cost of registering the new stock with the SEC would make the transaction impractical. Even though such stock cannot be sold to the public on the open market, it may be sold in private transactions under certain circumstances. Such transactions must be reported to the SEC where they become a matter of public record (Pratt, 1989, p.240).
was to add a high quality zero-coupon bond (an instrument that was all principal repayment with no interest payments) to the perp (an instrument with only interest payments and no principal repayment) to create a synthetic, dated FRN that would appeal to a broader range of investors. These efforts have met with limited success, but they did succeed in producing a lower bound for the price of perps. This transformation could restore some of the marketability premium because the secondary market for FRNs remained liquid. In addition, some issuers are known to have called substantially all their outstanding perps at prices close to face value, apparently to preserve or strengthen their reputations in the credit markets.

The perp episode emphasizes the importance of market breadth in establishing and maintaining liquid secondary markets. Market breadth can be achieved by placing the security with a broad range of investors during the initial public offering. We have presented a case in which a secondary market became very liquid despite its narrow investor base. But the liquidity of the secondary market proved to be temporary. Once a systemic liquidity shock revealed the lack of breadth, the secondary market could not recover. Although it is still possible to have systemic liquidity shocks with a heterogeneous investor base, they are likely to be transitory since investors’ idiosyncratic liquidity shocks are likely to predominate most of the time and investors can expect to mutualize their idiosyncratic shocks by trading in secondary market. The marketability premium may fall briefly, but it is likely to be restored relatively rapidly.

V. Concluding Remarks

We have provided new insights into why markets crash. As we have shown, markets can crash even in the absence of the two conditions that are thought in the literature to give rise to a crash: the bursting of a bubble concerning fundamental value or an exacerbation of information
asymmetry about the value of the fundamental determinants of asset prices. Our theoretical framework and the perp episode demonstrate that market collapse can be an endogenous phenomenon, having nothing to do with the fundamental value of assets, and everything to do with the liquidity needs of the investor clientele that holds the assets and their beliefs about whether these needs can be met in the future. Of course, the collapse of the marketability premium can also accompany the collapse of a fundamental value bubble or an exacerbation of information asymmetry, deepening the crisis.

Our framework readily extends to the case considered by Glauber (1997) in which everyone adopts the same trading model, leading to potentially harmful herding behavior. And as we have demonstrated, the triggers for such behavior need not arise from asset fundamentals. Recent empirical evidence of commonality in liquidity (Chordia, Roll and Subrahmanyam (2000)) lends support to this view.

The perp episode emphasizes the importance of market breadth in establishing and maintaining liquidity in financial markets. Market breadth can be assured through broad-based placement of securities. But the perp episode also provides an example of how the effects of systemic shocks can be mitigated through security design. As illustrated in Figure 5, the Standard Chartered Bank perp with the attached put option continued to trade at par throughout the crisis, while the other three Standard Chartered perps, which had no puts attached, followed the other perps down when the market crashed. The effect here is remarkably similar to deposit insurance in the Diamond-Dybvig (1983) framework, especially since there was no change in Standard Chartered Bank’s credit quality that could potentially have undermined the value of the put option that provided a de facto maturity for this issue.
Appendix

The $t = 1$ equilibrium in case 1:

Individual $i$’s holding at time $1$ is given by the first order condition to (2):

$$\nabla + \hat{\theta}_i - P_i - a\sigma_i^2 X_{ii} - 2\lambda(X_{ii} - X_{0i}) = 0$$  \hspace{1cm} (A.1)

Noting that:

$$\sum_{i=1}^{N} X_{0i} = \sum_{i=1}^{N} X_{ii} = N$$  \hspace{1cm} (A.2)

we can aggregate over the $N$ agents to obtain the expression for the equilibrium price, $P^1_1$, in (3), from which (5) and (6) follow directly.

The $t = 0$ equilibrium in case 1:

At $t = 0$, the individual’s problem becomes:

$$\begin{aligned}
\max_{x_{0i}} E_{0} \left[ \max_{x_{1i}} - \exp \left\{ -a \left[ W_{0i} + X_{0i} (P_{0} - P_{1}) + X_{1i} (\bar{v} + \theta_i - P_{1}) - \lambda (X_{ii} - X_{0i})^2 \right] \right\} \right] \\
\end{aligned}$$ \hspace{1cm} (A.3)

which is equivalent to:

$$\begin{aligned}
\max_{x_{0i}} E_{0} \left[ -\exp \left\{ -a \left[ W_{0i} + X_{0i} (P_{1}^i - P_{0}) + X_{1i}^* (\bar{v} + \theta_i - P_{1}^i) - \lambda (X_{ii}^* - X_{0i})^2 - \frac{1}{2} a (X_{ii}^*)^2 \sigma_i^2 \right] \right\} \right] \\
\end{aligned}$$ \hspace{1cm} (A.4)

where $X_{1i}^*$ and $P_{1}^i$ are the optimal time 1 holding and equilibrium price, respectively. This is of the form:
Substituting from the time 1 first order condition, \( \bar{Z} \) reduces to:

\[
\bar{Z} = X_{0i}(P^1 - P_0) + \lambda(X_{1i}^2 - X_{0i}^2) + \frac{1}{2}aX_{1i}^2\sigma_v^2
\]  

(A.6)

and substituting for \( X_{1i}^* \) and \( P^1 \) yields:

\[
\bar{Z} = X_{0i}(\bar{v} - a\sigma_v^2 - P_0) - \lambda X_{0i}^2 + \frac{(a\sigma_v^2 + 2\lambda X_{0i})^2}{2(a\sigma_v^2 + 2\lambda)} \\
+ \frac{1}{2(a\sigma_v^2 + 2\lambda)}\Delta\tilde{\theta}_i^2 + \frac{(a\sigma_v^2 + 2\lambda X_{0i})}{(a\sigma_v^2 + 2\lambda)}\Delta\tilde{\theta}_A + X_{0i}\tilde{\theta}_A
\]

(A.7)

where \( \Delta\tilde{\theta}_i = \tilde{\theta}_i - \tilde{\theta}_A \). This expression is of the form:

\[
\tilde{Z} = A + B(\Delta\tilde{\theta}_i^2) + C(\tilde{\theta}_A\Delta\tilde{\theta}_i) + D(\tilde{\theta}_A^2) + E(\Delta\tilde{\theta}_i) + F(\tilde{\theta}_A)
\]

(A.8)

where \( C = D = 0 \), and \( A, B, E \) and \( F \) are non-random. Krishnan (1987) derives the moment generating function of a non-homogenous quadratic in a correlated bivariate normal.

Disregarding terms uncorrelated with \( X_{0i} \), we apply Krishnan’s result directly to obtain:

\[
Max_{X_{0i}}E_0\left[-\exp\left\{-a[\bar{Z}]ight\}\right] \equiv Max_{X_{0i}}\left[aA - \frac{1}{2L_B}\left[M_1^2 + \frac{(L_B M_2)^2}{L_B}\right]\right]
\]

(A.9)

where:

\[
L_B = 1 + 2aB\sigma_{\Delta\theta}^2 \\
M_1 = (-a)a\sigma_v^2 + 2\lambda X_{0i}\sigma_v\sigma_{\Delta\theta} \\
M_2 = (-a)X_{0i}\sigma_{\theta_i}
\]

(A.10)
\( \sigma_{\Delta \theta}^2 \) and \( \sigma_{\theta_i}^2 \) are the variances of \( \Delta \theta \) and \( \theta_i \), respectively. Taking the first order condition with respect to \( X_{\theta i} \) and noting that in equilibrium, \( X_{\theta i} = 1 \), we obtain the market clearing price at \( t = 0 \):

\[
P_0^l = \nu - a\sigma_v^2 - a\sigma_{\theta_i}^2 - \frac{2\lambda a\sigma_{\Delta \theta}^2}{a\sigma_v^2 + 2\lambda + a\sigma_{\theta}^2}
\]  
(A.11)

Noting that

\[
\sigma_{\theta_i}^2 = \frac{\sigma_{\theta}^2}{N} \tag{A.12}
\]

and

\[
\sigma_{\Delta \theta}^2 = \frac{\sigma_{\theta}^2 (N-1)}{N} \tag{A.13}
\]

yields the expression for \( P_0^l \) in (8).

The necessary condition (19) for offering a market:

In the case where the probability of a systemic liquidity shock, \( \xi \), is non-zero, the total expected market maker revenue in (9) modifies to:

\[
R^l = \frac{(1 - \xi)\lambda \sigma_{\theta}^2 (N-1)}{(a\sigma_v^2 + 2\lambda)^2}
\]  
(A.14)

Since market makers have zero expected profits,

\[
MC = \frac{(1 - \xi)\lambda \sigma_{\theta}^2 (N-1)}{(a\sigma_v^2 + 2\lambda)^2}
\]  
(A.15)

which is quadratic in \( \lambda \). It is straightforward to show that the condition in (19) must be satisfied in order for this quadratic equation to yield a real non-negative root for \( \lambda \). If (19) is satisfied, market makers will offer a market. If (19) is not satisfied, there is no real non-negative value for \( \lambda \) at which market makers can expect to generate sufficient revenue to recover their set up costs.


Figure 1: The price behavior of 3 NatWest financial securities. This figure shows the prices of three obligations—a perpetual floating rate note, preference shares (which were subordinate to the perp), and a long-dated floating rate note (FRN)—of the National Westminster Bank (NatWest), a major U.K. clearing bank. After trading steadily at its par value, the price of the NatWest perp shows a sudden decline, which coincided with the collapse of the perp market. Clearly, this collapse cannot be attributed to a decline in the creditworthiness of NatWest since the prices of its long-dated FRNs and preference shares held steady while the prices of its perps dropped.
Figure 2: The rising trend in the marketability premium prior to the crash. This figure captures the declining yield spread over LIBOR (in terms of the corresponding increase in price) for the Barclays Overseas Investment BV perp. As investors grew more confident in the liquidity of the secondary market, their subjective probability of a systemic liquidity shock declined and the marketability premium increased, thus reducing the yield spreads attached to perps.
Figure 3: Percentage of perp issues in twenty most actively traded money market issues. This figure provides the number of perp issues in the Euroclear listing of the twenty most actively traded money market issues each month, an indirect indication of trading activity in perps. As shown, perps were among the most actively traded money market instruments through the first quarter in 1987. Average daily volumes of as much as $1 billion were recorded during this period. A dramatic decline in trading activity accompanied the perp crash.
Figure 4: Behavior of perp price index. This figure plots the value from November 1984 through June 1991 of a price index of eleven perps issued by British Clearing Banks. The sudden collapse and lack of recovery of the secondary market is clearly evident.
Figure 5: Effect of an attached put option. This figure plots the price behavior of the four perps issued by Standard Chartered Bank. The perp with an attached put option continued to trade at par, while the other three issues that could not be put back to the issuer declined sharply in price, losing as much as half their value by December 1990. This evidence supports the view that concerns over the liquidity of the secondary market, rather than default risk, were at the heart of the crisis in the perp market.