Fast algorithms for antenna selection in MIMO systems

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Invited Paper

***Abstract*— We consider the capacity of multiple-input – multiple-output (MIMO) systems with reduced complexity. One link end uses all available antennas, while the other chooses the "best" L out of N antennas. The selection of the optimum antenna subset requires an exhaustive search of all possible combinations, involving $\binom{N}{L}$ computations of determinants of size $L \times L$, which can become prohibitively complex. In this paper, we suggest a class of fast antenna selection algorithm that are based on the correlation or mutual information between the signals at the different antenna elements. It requires less than N^2 vector multiplications and thus leads to dramatic savings on the computation time. Its performance is very close to the optimum selection procedure: the capacity penalty is less than $1 bit/s/Hz$ for the analyzed examples. The algorithm thus offers the possibility of almost-optimum selection even in fast-changing environments.**

I. Introduction

MIMO (multiple-input - multiple output) wireless systems are those that have antenna arrays at both transmitter and receiver. First simulation studies that reveal the potentially large capacities of those systems were already done in the 1980s [1], and later papers explored the capacity analytically [2], [3]. Since that time, interest in MIMO systems has exploded. Layered space-time (ST) receiver structures [4], [5], [6] and space-time codes [7] allow to approach the capacity limits revealed by [2]; such systems have become known as "spatial multiplexing" or "BLAST" systems [8]. The standard for third-generation cellular phones (3GPP) foresees the use of a space-time coding for voice communications, and true MIMO for packet data [9].

One major drawback of conventional MIMO systems is the high implementation cost caused by the requirement for multiple RF chains (one for each antenna element). This fact led to increased interest in antenna selection schemes that optimally choose a subset of the available antennas [10], [11], [12], [13], [14], [15], [16], [17], [18], [19]; for an overview see [20]. Antenna subset selection maximally benefits from the multiple antennas within the RF cost constraint; if used together with spatial multiplexing, it is also known as "hybrid selection/MIMO" (H-S/MIMO) [11]. In this paper, we are considering a system that uses all available antennas at one link end, while employing H-S/MIMO at the other link end.

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One challenge in the use of H-S/MIMO is the selection of the optimum set of antennas. Maximization of the capacity requires first the computation of the capacity value of any possible combination of L processed data streams out of the possible N received signals. This requires the evaluation of $\binom{N}{L}$ determinants, which is computationally prohibitive, especially if the channel changes rapidly and the antenna selection has to be re-evaluated frequently. The investigation of fast antenna selection algorithms is thus of great practical as well as theoretical interest.

Ref. [11] showed that simply choosing the antennas that instantaneously receive the most energy does not give good performance. The algorithm in [21] requires on the order of N^2 matrix inversions, and thus requires considerable computational complexity. In this paper, we are proposing a novel family of fast antenna selection algorithms that require less than N^2 *vector multiplications*, while providing excellent performance. In all considered examples, the penalty in terms of outage capacity is small.

The remainder of the paper is organized as follows: in Sec. II, we describe the system model and the basic principle of H-S/MIMO. Next, we describe the fast selection algorithm, and in Sec. IV, we give simulation results, demonstrating the good performance of our algorithm. A summary and conclusions wrap up this paper.

II. System model

Figure 1 shows the block diagram of the considered system. We consider the case where the transmitter does not have channel state information (CSI); thus no waterfilling can be used, and the available transmitter power is equally distributed among the N_t transmit antennas. The data stream originating at the (possibly source-encoded) data source is first sent to a space-time processor/encoder, whose outputs are forwarded to the N_t transmit antennas, resulting (at each symbol time) in a $N_t \times 1$ transmit vector $\overrightarrow{s}$. The signals are subsequently upconverted to passband, amplified by a power amplifier, and filtered. For our model, we omit these stages, as well as their equivalents at the receiver, which allows us to treat the whole problem in equivalent baseband. Note, however, that it is exactly these parts that are most expensive and make the use of reduced-complexity systems desirable.

From the antennas, the signal is sent through the mobile radio channel. The channel is assumed to be flat-fading, so that it can

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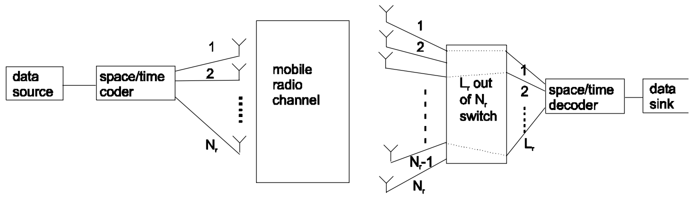

Fig. 1. Block diagram of the system

be characterised by the $N_\mathrm{r} \times N_\mathrm{t}$ matrix

$$\underline{H} = \begin{pmatrix} h_{11} & h_{12} & . & . & . & h_{1N_\mathrm{t}} \\ h_{21} & h_{22} & . & . & . & h_{2N_\mathrm{t}} \\ . & . & . & . & . & . \\ h_{N_\mathrm{r}1} & h_{N_\mathrm{r}2} & & & & h_{N_\mathrm{r}N_\mathrm{t}} \end{pmatrix}. \tag{1}$$

so that the $N_\mathrm{r} \times 1$ vector $\overrightarrow{y}$ of the receive signal can be written as

$$\overrightarrow{y} = \underline{H}\,\overrightarrow{s} + \overrightarrow{n} = \overrightarrow{x} + \overrightarrow{n}$$

where $\overrightarrow{n}$ is the vector containing the (complex) noise samples at the receive antenna elements.

We furthermore assume that the channel has block fading with "very long" block duration T_block. This implies that the data within one channel realization (i.e., within T_block) can be encoded with an almost ideal code that approaches the Shannon limit. Such a code could be, e.g., the combination of space-time-processing [6] with a low-density parity check code LDPCC [22]. It can be shown that LDPCC codes with a block-length of 10000 approach the Shannon limit within less than 1dB [23]. Thus, each channel realization is associated with a (Shannon - AWGN) capacity value. The capacity thus becomes a random variable, rendering the concept of a "capacity cumulative distribution function" and "outage capacity" a meaningful measure [2]. In physical terms, this requires that the ratio of symbol rate to Doppler frequency of the channel must be large. In cellular systems and wireless LANs, maximum Doppler frequencies are typically on the order of 300Hz and 30Hz, respectively.

We furthermore assume for the computations that the channel is Rayleigh fading, the h_{ij} are i.i.d. zero-mean, circularly symmetric complex Gaussian random variables with unit variance, i.e., the real and imaginary part each have variance $1/2$. Consequently, the power carried by each transmission channel (h_{ij}) is chi-square distributed with 2 degrees of freedom. Following [2], we consider the case in which the h_{ij} are independently fading. More involved channel models are discussed, e.g., in [24], [25], [26].

At the receiver, a control algorithm (to be discussed in Sec. III) selects the best L_r of the available N_r antenna elements and downconverts their signals for further processing (note that only L_r receiver chains are required). The space-time processing and encoding/decoding are assumed to be ideal so that the capacity can be achieved. We assume ideal knowledge of the channel at the receiver, so that it is always possible to select the best antennas. However, we do *not* assume knowledge of the channel at the transmitter.

III. Antenna selection algorithms

A. Optimum antenna selection

The capacity of a MIMO system using all antenna elements is given by [2]

$$C_\mathrm{full} = \log_2 \left[\det \left(\underline{I}_{N_\mathrm{r}} + \frac{\overline{\Gamma}}{N_\mathrm{t}} \underline{H}\underline{H}^\dagger \right) \right], \tag{2}$$

where $\underline{I}_{N_\mathrm{r}}$ is the $N_\mathrm{r} \times N_\mathrm{r}$ identity matrix, $\overline{\Gamma}$ is the mean signal-to-noise ratio (SNR) per receiver branch, and superscript † denotes the Hermitian transpose. The receiver now selects those antennas that allow a maximization of the capacity, so that

$$C_\mathrm{select} = \max_{S(\underline{\widetilde{H}})} \left(\log_2 \left[\det \left(\underline{I}_{L_\mathrm{r}} + \frac{\overline{\Gamma}}{N_\mathrm{t}} \underline{\widetilde{H}}\underline{\widetilde{H}}^\dagger \right) \right] \right), \tag{3}$$

where $\underline{\widetilde{H}}$ is created by deleting $N_\mathrm{r} - L_\mathrm{r}$ rows from $\underline{H}$, and $S(\underline{\widetilde{H}})$ denotes the set of all possible $\underline{\widetilde{H}}$, whose cardinality is $\binom{N_\mathrm{r}}{L_\mathrm{r}}$.

The optimum choice of antennas requires knowledge of the complete channel matrix. This may seem to necessitate the use of N_r RF chains, which would not agree with the goal of reducing the number of RF chains in a low-complexity system. However, in a sufficiently slowly-changing environment, the antennas can be multiplexed to the L_r RF chains during the training bits. This principle, which is similar in spirit to multiplexed channel sounders [27], means that a receiver chain is connected to the first antenna during the first part of the training sequence, then to the second antenna during the next part, and so on. This allows the determination of the channel state information at all N_r antenna elements. At the end of the training sequence, we pick the best L_r antennas. Thus, we only need a few more training bits instead of additional RF chains. Especially in high-data-rate systems, those additional training bits decrease the spectral efficiency in a negligible way.

B. Fast antenna selection

The optimum selection of the antennas requires $\binom{N}{L}$ computations of determinants, and is thus rather computationally intensive. It seems thus worthwhile to investigate suboptimum algorithms with lower computational complexity. In this section, we present a family of such algorithms that results in a small SNR penalty while drastically reducing computation time.

The determinant in (3) can be written as

$$\det \left(\underline{I}_{L_\mathrm{r}} + \frac{\overline{\Gamma}}{N_\mathrm{t}} \underline{\widetilde{H}}\underline{\widetilde{H}}^\dagger \right) = \prod_{k=1}^{r} \left(1 + \frac{\overline{\Gamma}}{N_\mathrm{t}} \lambda_k^2 \right) \tag{4}$$

where r is the rank of the channel matrix and λ_k is the singular value of $\underline{\widetilde{H}}$. The rank and the singular values should be maximized for the maximum capacity. Suppose there are two rows of $\underline{H}$ which are identical. Clearly only one of these rows should be selected in $\underline{\tilde{H}}$. Since these two rows carry the same information, we can delete any of these two rows without losing any information regarding the transmitted vector. In addition if they have different powers (i.e. square of the norm of the row), we select the row with the higher power. When there are no identical rows, we choose those two rows for the possible deletion whose correlation is the highest. In this manner we can have the channel matrix $\underline{\tilde{H}}$ whose rows are maximally uncorrelated and have maximum powers. This intuition leads to the following algorithm, which in the remainder of the paper will be called "Correlation Based Method (CBM)":

1) The channel vector h_k is defined as the k-th row of H, with k being an element of the set $X = \{1, ...N_r\}$
2) for all k and l, $k > l$ compute the correlation $\Xi(k,l)$. The correlation $\Xi(k,l)$ is defined as $\Xi(k,l) = |\langle h_k, h_l\rangle|$ where $\langle a, b\rangle$ represents an inner product between vector a and b.
3) Loop
   a) choose the k and l (with $k, l \in X$, $k > l$) that give the largest $\Xi(k,l)$. If $\|h_k\|^2 > \|h_l\|^2$, eliminate h_l, otherwise eliminate h_k.
   b) delete l (or k) from X
   c) goto Loop until $N_r - L_r$ rows are eliminated.

The CBM does not require knowledge of the SNR value and is based on the correlation of the rows of the channel matrix $< h_k h_l^* >$, which is approximated by the correlation of the noisy estimates $E\{y_k y_l^*\}$.

As an alternative method when the SNR is available, we suggest to use the mutual information between y_k and y_l. The zero valued mutual information means the k-th receive antenna output y_k and the l-th output y_l carry totally different information. This occurs when the corresponding channel vectors h_k and h_l are orthogonal. On the other hand when the mutual information between y_k and y_l has maximum value, y_k and y_l carry the same information so that we can delete one of them. The mutual information is defined as [28]

$$I(y_k; y_l) = G(y_k) + G(y_l) - G(y_k, y_l). \tag{5}$$

where G denotes the entropy (we deviate from the usual entropy notation H to avoid confusion with the channel matrix H).

In the MIMO system the mutual information can be written as

$$\begin{aligned} &I(y_k; y_l) \\ &= \log \frac{\left(\|h_k\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right)\left(\|h_l\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right)}{\left(\|h_k\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right)\left(\|h_l\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right) - |\langle h_k, h_l\rangle|^2 \frac{\overline{\Gamma}^2}{N_t^2}}. \end{aligned} \tag{6}$$

Since the mutual information is bounded as follows

$$0 \le I(y_k; y_l) \le \min(G(y_k), G(y_l)), \tag{7}$$

we define the normalized mutual information

$$I_0(y_k; y_l) = \frac{I(y_k; y_l)}{\min(G(y_k), G(y_l))} \tag{8}$$

as a measure of how close the two random variables are. The entropy calculation of y_k requires both the signal and noise power whereas the mutual information needs the SNR only. This can be overcome as follows. The scaling of y_k to $c \cdot y_k$, where the non-zero real number c is chosen in a way that the noise variance is equal to one, will not change the order of the normalized mutual information. Clearly, the scaling does not change the mutual information while the entropy of $c \cdot y_k$ becomes

$$\begin{aligned} G(c \cdot y_k) &= \log\left(c^2 \pi e \left(\|h_k\|^2 \frac{P}{N_t} + \sigma^2\right)\right) \\ &= \log\left(\|h_k\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right) \end{aligned} \tag{9}$$

where $c = 1/\sqrt{\pi e \sigma^2}$ and $P = \overline{\Gamma}\sigma^2$ represents signal power.

Now redefine the normalized mutual information as

$$I_0(y_k; y_l) = \frac{I(c \cdot y_k; c \cdot y_l)}{\min(G(c \cdot y_k), G(c \cdot y_l))}. \tag{10}$$

Then, the normalized mutual information becomes

$$I_0(y_k; y_l) = \frac{I(y_k; y_l)}{\min\left(\log\left(\|h_k\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right), \log\left(\|h_l\|^2 \frac{\overline{\Gamma}}{N_t} + 1\right)\right)}. \tag{11}$$

We can also apply the mutual information based technique to x_k, which is the signal component of y_k, in order to avoid requiring the SNR value. Then, the mutual information between the data component x_k and x_l is

$$I(x_k; x_l) = \log \frac{\|h_k\|^2 \|h_l\|^2}{\|h_k\|^2 \|h_l\|^2 - |\langle h_k, h_l\rangle|^2}. \tag{12}$$

Similarly, we can define the normalized mutual information as

$$\begin{aligned} I_0(x_k; x_l) &= \frac{I(c \cdot x_k; c \cdot x_l)}{\min(G(c \cdot x_k), G(c \cdot x_l))} \\ &= \frac{I(x_k; x_l)}{\min\left(\log\|h_k\|^2, \log\|h_l\|^2\right)}. \end{aligned} \tag{13}$$

The antenna selection algorithms based on mutual information then have a similar program structure as the one based on correlation (CBM); we just replace Ξ by I_0 as defined in (11) (henceforth referred to as MIBM) or (13) (MIBM2).

C. *The frequency-selective case*

We next consider the case where the channel is frequency-selective. In that case, the use of OFDM (orthogonal frequency division multiplexing) is optimum. In MIMO OFDM with N_s subcarriers, the channel matrix can be modeled as a block diagonal matrix

$$\underline{H} = \begin{bmatrix} \underline{H}(1) & 0 & \cdots & 0 \\ 0 & \underline{H}(2) & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \underline{H}(N_s) \end{bmatrix} \tag{14}$$

where $\underline{H}(n)$ represents the channel matrix at subcarrier n. The normalized capacity (capacity per tone) becomes

$$C = \frac{1}{N_s}\sum_{n=1}^{N_s} \log_2\left[\det\left(\underline{I}_{N_r} + \frac{\overline{\Gamma}}{N_t}\underline{H}(n)\underline{H}(n)^\dagger\right)\right] \tag{15}$$

In the correlation based methods, the correlation is averaged over the subcarriers

$$\Xi(k,l) = \frac{1}{N_s}\left|\sum_{n=1}^{N_s}\langle h_k(n), h_l(n)\rangle\right| \tag{16}$$

where $h_k(n)$ is the k-th row vector of the matrix $H(n)$.

Defining the received vector at k-th receive antenna as $y_k = \begin{bmatrix} y_k(1) & y_k(2) & \cdots & y_k(N_s) \end{bmatrix}^T$ where $y_k(n)$ is k-th receive antenna output at n-th subcarrier, the mutual information in MIMO OFDM becomes

$$I(y_k; y_l) = G(y_k) + G(y_l) - G(y_k, y_l) \tag{17}$$

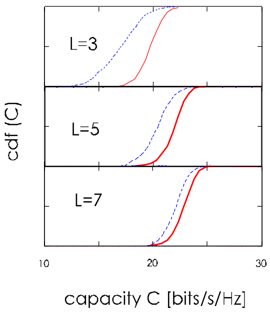

Fig. 2. Cdf of the capacity of a system with $N_{\rm r} = 8$, $N_{\rm t} = 3$. Selection of antenna by capacity criterion (solid) and by power criterion (dashed).

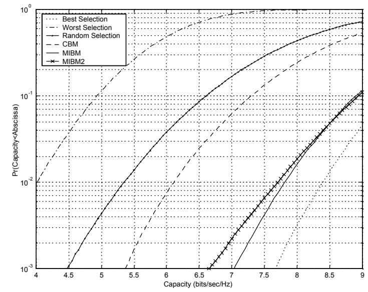

Fig. 3. Outage Probabilities of fast algorithms, $N_{\rm r} = 8$, $N_{\rm t} = L_{\rm r} = 3$, SNR=10 dB.

The block diagonal property of the channel matrix leads to the following expression for the mutual information

$$I\left(y_k; y_l\right) = \frac{1}{N_s} \sum_{n=1}^{N_s} G(y_k(n)) + G(y_l(n)) - G(y_k(n), y_l(n)). \quad (18)$$

Hence, the mutual information based techniques are modified to use the following normalized mutual information

$$I_0\left(y_k; y_l\right) = \frac{\sum_n I\left(y_k(n); y_l(n)\right)}{\min\left(\sum_n G(c \cdot y_k(n)), \sum_n G(c \cdot y_l(n))\right)}. \quad (19)$$

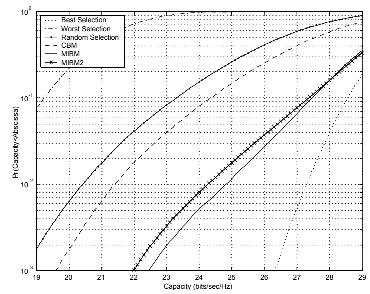

Fig. 4. Outage Probabilities of fast algorithms, $N_{\rm r} = 8$, $N_{\rm t} = L_{\rm r} = 3$, SNR=30 dB.

IV. Results

In this section, we evaluate the capacity as obtained with our selection algorithm, as well as that obtained by exhaustive search, for practical system parameters. For the computer experiments (Monte Carlo simulations), we created random realizations of mobile radio channels. For the exhaustive search, we create a complete set $S(\underline{\widetilde{H}})$ of all possible matrices $\underline{\widetilde{H}}$ by eliminating all possible permutations of $N_{\rm r} - L_{\rm r}$ rows from the matrix realization $\underline{H}$. For each of the $\underline{\widetilde{H}}$, we compute the capacity by (3), and select the largest capacity from the set.

Figure 2 shows the performance of a power-based selection algorithm [11]. The receive antennas with the largest received powers (for a given channel realization) are selected. The figure shows the results for $N_{\rm r} = 8$, $N_{\rm t} = 3$. $\overline{\Gamma} = 20dB$. We see that this algorithm leads to a considerable loss in performance for small values of L_r. Especially, the outage capacity for low outage values is small.

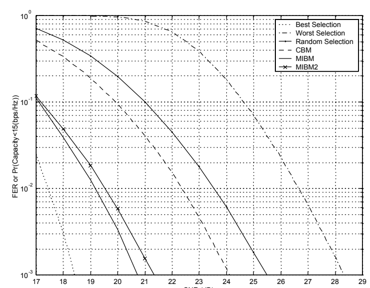

Fig. 5. FER comparison, $N_{\rm r} = 8$, $N_{\rm t} = L_{\rm r} = 3$.

The performance of our new fast antenna selection algorithms is detailed in Figures 3 and 4. Again, the number of transmit and receive antennas is $N_{\rm t}$ =3 and $N_{\rm r}$ =8, respectively; each algorithm uses $L_r = 3$. Among the fast algorithms, the mutual information based methods outperform the correlation-based technique. Assuming that ideal coding is employed, the FER (frame error rate) is shown in Figure 5 when the bandwidth efficiency is 15 bits/s/Hz, i.e., this figure plots the probability that the capacity is smaller than 15bit/s/Hz. The worst selection has 10 dB loss at 10^{-3} FER. The MIBM has about 2 dB loss while the correlation based method exhibits around 6 dB loss. The performance of the fast algorithm MIBM2 is comparable to that of the MIBM at high FER. The

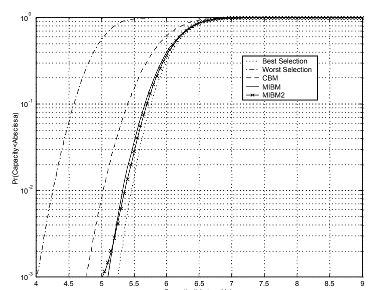

Fig. 6. Outage Probability in MIMO OFDM ($N_s = 64$, $T_d/T = 1/4$, $\tau_d/T_d = 1/4$, exponential distribution, 6 receive, 2 transmit antennas, select 2 antennas out of 6); SNR=10 dB.

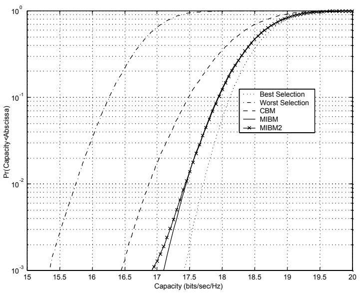

Fig. 7. Outage Probability in MIMO OFDM ($N_s = 64$, $T_d/T = 1/4$, $\tau_d/T_d = 1/4$, exponential distribution, 6 receive, 2 transmit antennas, select 2 antennas out of 6); SNR=30 dB.

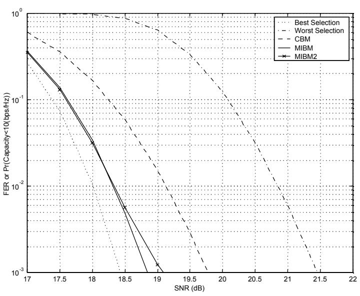

Fig. 8. FER comparison, $N_{\mathrm{t}} = 2$, $N_{\mathrm{r}} = 6$, $L_{\mathrm{r}} = 2$.

MIBM2 has a good performance overall while it does not require the SNR value.

Figures 6 and 7 show the outage probability of each fast algorithm in the MIMO OFDM system under a frequency selective Rayleigh fading channel with 10 and 30 dB SNR, respectively. The number of subcarriers is 64. The maximum delay spread T_d is $\frac{1}{4}$ of the symbol duration and the r.m.s. delay spread τ_d is assumed to be $\frac{1}{4}$ of the maximum delay spread with an exponential power distribution. The number of transmit and receive antennas is 2 and 6, respectively. Each algorithm selects 2 receive antennas out of 6 receive antennas. Figure 8 shows the FER for the same antenna configuration. Among fast algorithms the mutual information based method outperforms the correlation-based technique.

V. Summary and Conclusions

We have investigated the behavior of MIMO systems that select a subset of available antennas at the receiver. Important applications for such systems are cellular systems with MIMO capability, and future wireless LANs. The necessity of selecting antennas at one link end (instead of using all of them) stems either from complexity or cost considerations. The main advantage is the savings in hardware costs: instead of a full N_{r} receiver chains, only L_{r} receiver chains, plus an RF switch, are required. Antenna selection can also be especially beneficial in low-rank and interference-limited systems.

We have derived and compared several algorithms that allow the selection of the antennas without an exhaustive search over all possible antenna combinations. By minimizing the correlation, or the mutual information, between the signals at the receive antennas, we can find an antenna subset that achieves high capacity (within 1 bit/s/Hz) of theoretical capacity, while reducing the computiational effort for the search from $\binom{N}{L}$ computations of determinants to N^2 vector multiplications. The schemes thus allow the fast determination of good subsets even for a large number of available antennas.

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