International Journal of Engineering Research (ISSN:2319-6890)(online),2347-5013(print)
Volume No.3, Issue No.1, pp : 13-17 01 Jan. 2014

Transient Magnetohydrodynamic flow of two immiscible Fluids through a horizontal channel

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Abstract* - An analysis is made for transient Magnetohydrodynamic (MHD) flow of two immiscible fluids thorough a horizontal channel. The flow above the interface is assumed to be electrically conducting while the other fluid and the channel walls are assumed to be electrically insulating. The partial differential equations governing the flow are solved analytically using two-term periodic and non-periodic functions in both the regions of the channels. Velocity profiles are analyzed for various physical parameters such as viscosity ratio, frequency parameter and Hartmann number.

Keywords– Magnetohydrodynamic, immiscible fluids, Unsteady flow, horizontal channel

I. INTRODUCTION

Unsteady flow of immiscible fluids through a channel is of special importance in the petroleum extraction and transport, the reservoir rock of an oil field always contains several immiscible fluids in its pores. Lighthil [1] was the first to have studied the unsteady forced flow of a viscous incompressible fluid past a flat plate and a circular cylinder with small amplitude oscillation in free stream. The corresponding problem of unsteady free convection flow along a vertical plate with oscillating surface temperature was studied by Nanda and Sharma [2]. Later Muhuri and Maiti [3] and Verma [4] have analyzed the effect of oscillation of the surface temperature on the unsteady free convection from a horizontal pipe. Hossain et al [5] used the linearised theory to study the free convection boundary layer flow of electrically conducting fluid along a vertical plate to surface temperature oscillations.

All these studies pertain to one fluid model. Most of the problems relating to petroleum industry, geophysics, plasma physics, magneto-fluid dynamics etc, involve multi-fluid flow model. Both theoretical and experimental work is found in literature on stratified laminar flow of two immiscible fluids in a horizontal pipe. In the petroleum industry, as well as in other engineering fields, stratified two-phase flow often occurs. There have been some experimental and analytical studies on hydrodynamic aspects of two-phase flow reported in the recent literature. The interest in these types of problems stems from the possibility of reducing the power required to pump oil in a pipeline by suitable addition of water. The first investigations were associated with the LM-MFM generator project at the Argone National Laboratory. Rudraiah et al [6] studied the Hartmann flow past a permeable bed in the presence of a transverse magnetic field with an interface at the surface of the permeable bed. Packham and Shail [7] analyzed stratified laminar flow of two immiscible liquids in a horizontal pipe. Hartmann flow of conducting fluid in a channel with a layer of non-conducting fluid between upper channel wall and the conducting fluid has been studied by Shail [8]. He found that increase of order 30% could be achieved in the flow rate for suitable ratios of depths and viscosities of two fluids. Loharsbi and Sahai [9] dealt with two-phase MHD flow and heat transfer in a parallel-plate channel. Both phases are incompressible and the flow was assumed to be steady, one-dimensional and fully developed. The study was expected to be useful in the understanding of the effect of the presence of slag layers on the heat transfer characteristics of a coal-fired MHD generator. Alireza and Sahai [10] studied the effect of temperature-dependent transport properties on the developing MHD flow and heat transfer in a parallel plate channel whose walls are held at constant and equal temperatures. Following the work of Alireaz and Sahai [10], Malashetty and Umavathi [11] and Malashetty et al [12-14] studied two-phase MHD flow and heat transfer in an inclined channel. Stamenković,M. Ž., et. al [15] investigated the MHD flow of two immiscible, electrically conducting fluids between isothermal and insulated moving plates in the pressure of an applied electric and inclined magnetic field with the effect of induced magnetic field.

The above investigations are related to that the flow is steady. However the study closed to practical problems is to consider unsteadiness. Chamkha [16] studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Recently Umavathi et al [17-19] have presented analytical solutions for unsteady/oscillatory two-fluid flow and heat transfer in a horizontal channel. Keeping in view the wide range of applications in geophysics and MHD generators, an attempt is made to analyze flow nature and heat transfer of unsteady Hartmann flow of two immiscible fluids in a horizontal channel.

II.MATHEMATICAL FORMULATION

The geometry under consideration consists of two infinite parallel plates extending in the Z and X directions. The regions $0 \le y \le h$ and $-h \le y \le 0$ are denoted as Region-I and Region-II respectively. The fluid in Region-I is a conducting fluid having density ρ_1 and viscosity μ_1. A constant magnetic field of strength B_0 is applied transverse to the flow field. The Region-II is filled with a non-conducting fluid having density ρ_2 and viscosity μ_2. It is assumed that the flow is unsteady, fully developed and that all fluid properties are constants. The flow is assumed to be driven by a pressure gradient $\left(-\frac{\partial p_1}{\partial x}\right)$ for region-I and $\left(-\frac{\partial p_2}{\partial x}\right)$ for region-II.

Under these assumptions and taking $\rho_1 = \rho_2 = \rho_0$ the governing equations of motion and energy are given by:

Region-I

$$\rho_0 \frac{\partial u_1}{\partial t} = -\frac{\partial p_1}{\partial x} + \mu_1 \frac{\partial^2 u_1}{\partial y^2} - \sigma_1 B_0^2 u_1 \qquad (1)$$

Region-II

$$\rho_0 \frac{\partial u_2}{\partial t} = -\frac{\partial p_2}{\partial x} + \mu_2 \frac{\partial^2 u_2}{\partial y^2} \qquad (2)$$

where *u* is the x-component of fluid velocity. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions which require that the x-component of velocity must vanish at the wall. We also assume continuity of velocity and shear stress at the interface between the two fluid layers at y=0.

The hydrodynamic boundary and interface conditions for the two fluids can then be written as

$$\begin{aligned} &u_1(h) = 0 \\ &u_2(-h) = 0 \\ &u_1(0) = u_2(0) \\ &\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \quad at\ y = 0 \end{aligned} \qquad (3)$$

By using the following non-dimensional quantities:

$$u_i = U_o u_i^*,\ y = h y^*,\ t = \frac{h^2}{\nu} t^*,$$
$$P_i = \frac{h_1^2}{\mu_1 U_0}\left(-\frac{\partial P_i}{\partial x}\right),\ M = B_0 h \sqrt{\frac{\sigma_1}{\mu_1}} \qquad (4)$$

and for simplicity dropping the asterisks, equations (1) and (2) become

Region-I

$$\frac{\partial u_1}{\partial t} = P_1 + \frac{\partial^2 u_1}{\partial y^2} - M^2 u_1 \qquad (5)$$

Region-II

$$\frac{\partial u_2}{\partial t} = P_2 + \alpha \frac{\partial^2 u_2}{\partial y^2} \qquad (6)$$

$\alpha = \frac{\mu_2}{\mu_1}$ is the ratio of viscosity.

The hydrodynamic boundary and interface conditions for both fluids in non-dimensional form become

$$\begin{aligned} &u_1(h) = 0 \\ &u_2(-h) = 0 \\ &u_1(0) = u_2(0) \\ &\frac{\partial u_1}{\partial y} = \alpha \frac{\partial u_2}{\partial y} \quad at\ y = 0 \end{aligned} \qquad (7)$$

III. SOLUTIONS

The governing momentum equations (5) and (6) are solved subject to the boundary and interface conditions (7) for the velocity distributions in both regions. These equations are partial differential equations that can not be solved in closed form. However, it can be reduced to ordinary differential equations by assuming

$$u_i(y,t) = u_{i0} + e^{i\omega t} u_{i1(y)} \qquad (8)$$
$$P_i(y,t) = P_{i0} + e^{i\omega t} P_{i1(y)} \qquad (9)$$
$$for\ i = 1,2$$

By substitution of equation (8) and (9) into equations (5) and (6), one obtains the following pairs of equations:

Region-I

Non-periodic coefficients

$$\frac{d^2 u_{10}}{dy^2} - M^2\ u_{10} + P_{10} = 0 \qquad (10)$$

Periodic coefficients

$$\frac{d^2 u_{11}}{dy^2} - \left(M^2 + i\,\omega\right) u_{11} + P_{11} = 0 \qquad (11)$$

Region -II

Non-periodic coefficients

$$\alpha \frac{d^2 u_{20}}{dy^2} + P_{20} = 0 \qquad (12)$$

Periodic coefficients

$$\alpha \frac{d^2 u_{21}}{dy^2} - i\,\omega\ u_{21} + P_{21} = 0 \qquad (13)$$

The corresponding boundary and interface conditions become as follows:

Non-periodic coefficients

$$\begin{aligned} &u_{10}(h) = 0 \\ &u_{20}(-h) = 0 \\ &u_{10}(0) = u_{20}(0) \\ &\frac{du_{10}}{dy} = \alpha \frac{du_{20}}{dy} \quad at\ y = 0 \end{aligned} \qquad (14)$$

Periodic coefficients

$$
\begin{aligned}
&u_{11}(h) = 0 \\
&u_{21}(-h) = 0 \\
&u_{11}(0) = u_{21}(0) \\
&\frac{du_{11}}{dy} = \alpha \frac{du_{21}}{dy} \qquad at\ y = 0
\end{aligned} \tag{15}
$$

Equations (10) to (13) along with boundary and interface conditions (14) and (15) represents a system of ordinary differential equations and conditions that can be solved in closed form. Since the solutions can be obtained directly and depicted graphically.

III. RESULTS AND DISCUSSINS

Unsteady MHD flow through a horizontal channel consisting of two immiscible fluids between to non-conducting plate is studied. Separate solutions for each fluid are obtained and these solutions are matched at the interface using suitable matching conditions. The partial differential equations governing the flow are transformed into ordinary differential equations and solved analytically considering cosine, sinusoidal and exponential function of ωt. The effect of governing parameter such as viscosity ratio, periodic frequency parameter ωt, Hartmann number presented graphically for cosine, sinusoidal and exponential function of ωt. The parameters are fixed except the varying one, $\omega t = 45^o$, α = h = P_{i0} = P_{i1}= 1, and M=2.

The effect of viscosity ratio α on velocity is shown in Fig.1 for cosine function of ωt. As the viscosity ratio increases the flow is suppress in both the regions, the magnitude of suppression is large in region-I compare to region-II. Similar effects are observed for sinusoidal and exponential function of ωt in Fig. 4 and Fig.7 respectively.

The effect of frequency parameter ωt on velocity profile is shown in Fig.2 for cosine, Fig.5 for sinusoidal and Fig.8 for exponential. As ωt increases the flow decreases in Fig.2 and Fig.8. For ωt = 0^0 and 180^0 display the velocity profiles for steady case as seen in Fig. 5.

Fig.3, Fig. 6 and Fig. 9 are shown the effect of Hartmann numbers on velocity profile for cosine, sinusoidal and exponential function of ωt respectively. The magnetic field will falttern the velocity distribution profile and reduced the flow which is the usual Hartmann flow problem.

ACKNOWLEDGMENT

The author would like to thank UGC-New Delhi for the financial support under UGC-Minor Research Project.[MRP(S)-1075/11-12/KAGUG009/UGC-SWRO]

NOMENCLATURE

M	Hartmann number
P	pressure
t	time
U_0	average velocity

Greek letters

ρ	fluid density
μ	viscosity of fluid
α	ratio of viscosity
ω	frequency parameter
ωt	periodic frequency parameter

Subscripts

1,2 quantities for region-I and region-II respectively.

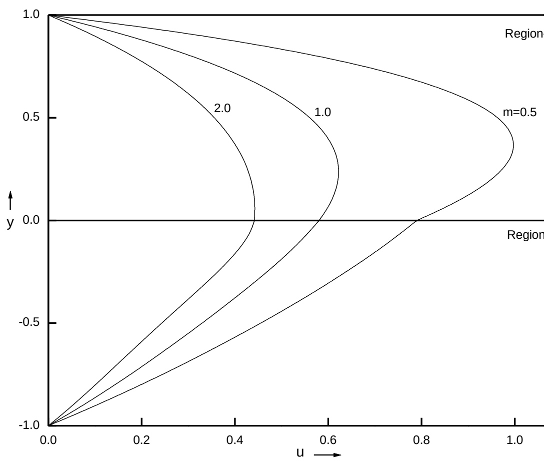

Fig.1 Velocity profiles for different values of viscosity ratio α

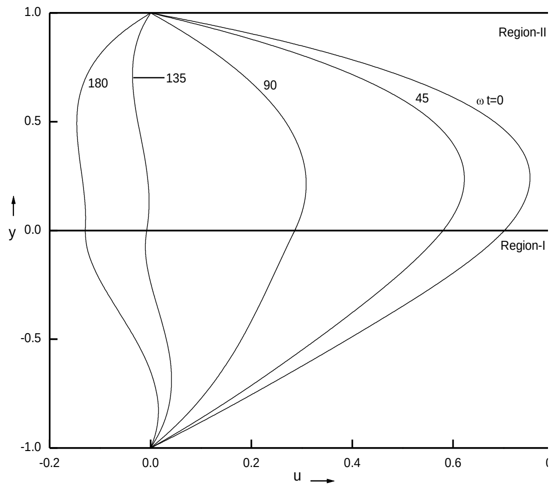

Fig.2 Velocity profiles for different values of periodic frequency parameter ωt

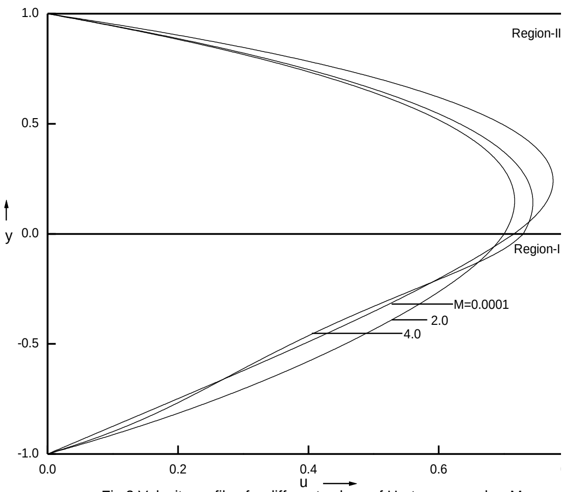

Fig.3 Velocity profiles for different values of Hartmann number M

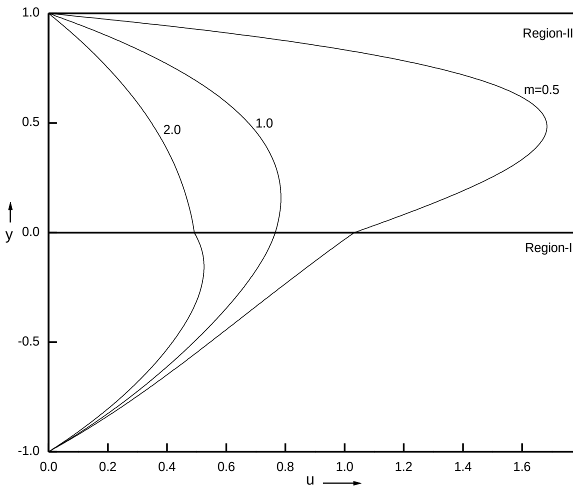

Fig.4 Velocity profiles for different values of viscosity ratio α

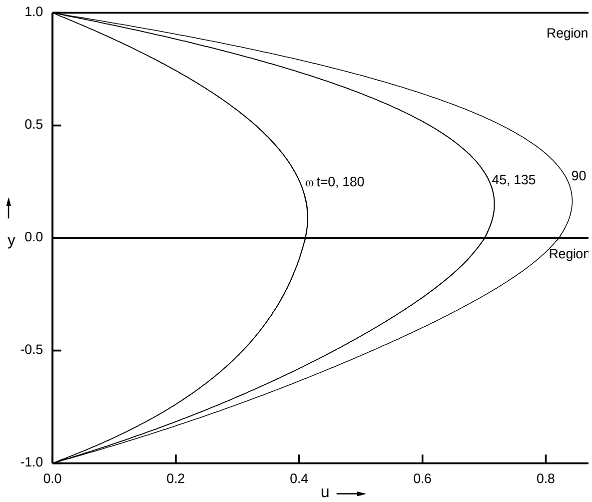

Fig.5 Velocity profiles for different values of periodic frequency parameter ωt

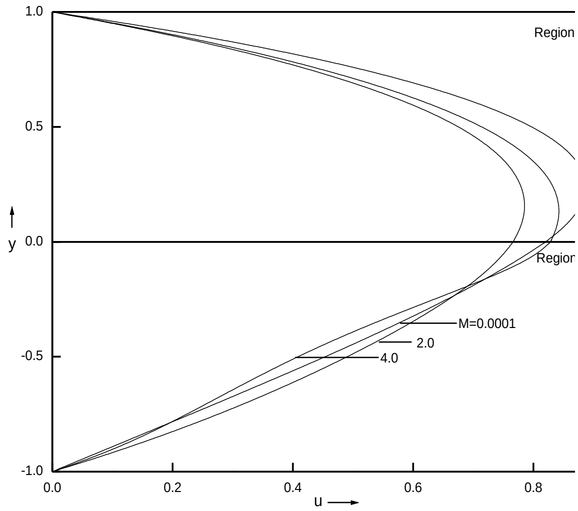

Fig.6 Velocity profiles for different values of Hartmann number M

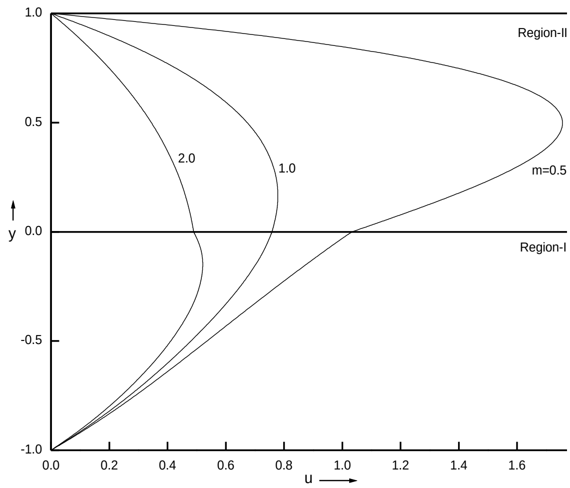

Fig.7 Velocity profiles for different values of viscosity ratio α

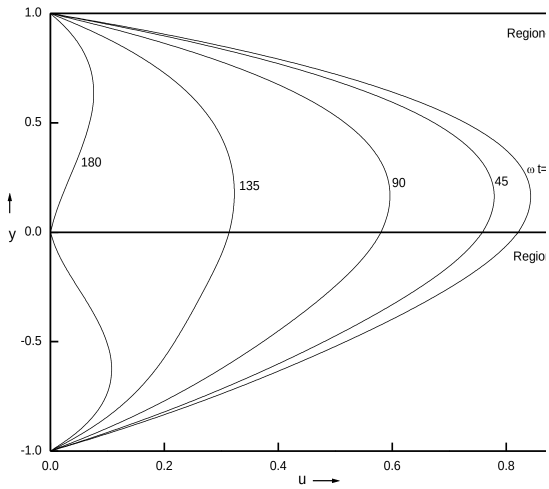

Fig.8 Velocity profiles for different values of periodic frequency parameter ωt

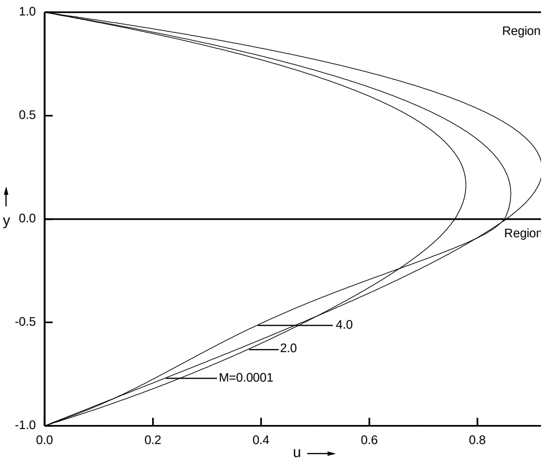

Fig.9 Velocity profiles for different values of Hartmann number M

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