Equivalent Circuit Model for Double Split Ring Resonators

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Abstract – This article focuses on the electric modeling of split ring resonators with a new approach based on an equivalent circuit. The elements of circuit are determined from geometric parameters forming the three dimensional model of this type of resonators. In the first part, an analytic study is proposed about the determination of the components of the circuit according to a function of physical parameters and a review about comparison between obtained results when the simulation of two different models is presented. In the second part, a study is devoted to demonstrate the rate influence, according to the variation of relative permittivity and the variation of physical parameters, on the frequency characteristics of our electric equivalent model. The results of simulations are demonstrated with using the two commercials software HFSS and ADS.

Index Terms-Metamaterials, Split Ring Resonator, Equivalent Circuit, High Frequency Structure Simulator (HFSS), Advanced Design System (ADS)

I. INTRODUCTION

In electromagnetism, the term metamaterial shows an artificial composite material which has electromagnetic properties not found in nature. They are generally periodic dielectric or metallic structures, which act as a homogeneous material. This important discovery had been proposed by physicist V. Veselago in 1968 [1]. Thirty years later, thesis properties were verified experimentally by J. Pendry [2-3]. One year later, Dr. Smith and al. [4-5] got validation to demonstrate experimentally the existence of a material by the left hand through the combination of two periodic and homogeneous respectively contain split ring resonators, which can produce a negative effective permeability, and metallic files produce a negative effective permittivity and consequently obtaining a negative refractive index metamaterials [6-7]. Some years later, C. Caloz and Itoh [8-9] proposed method makes left hand metamaterials use the lines transmission based on microstrip technology. There are many types of metamaterials in electromagnetism, the most know are based on split ring resonator. These resonators or resonators with magnetic response have many important properties and that let them be presented in the microwave field applications as waveguide, filters and antennas [10-12]. Generally, electric modeling takes a great interest in the microwave filed [13-15]. The most interesting electric modeling properties, the facilitation of integration and manufacturing smaller devices. In this sector of researching, there are a lot of works interested by split ring resonator modeling [16-20].

Taking into account the three-dimensional modeling, we propose in this article and for the first time an equivalent circuit of SRR more adapted to preferred frequency. This circuit having electrical components LC is correlated to geometric parameters of this type of resonators in 3D model. The second part is dedicated to guide our studies to extract effect of variation of relative permittivity and physical quantities on frequency characteristics of our proposed electric model.
II. EQUIVALENT CIRCUIT

The SRR may be modeled by an equivalent electrical circuit. This circuit behaves as a resonant cavity modeled by an LC circuit. The presence of a magnetic field \( H \) perpendicular the plane supporting the two splits rings, allows the creation of \( C_{\text{gap}} \) capacity at the opening of each ring. The interaction between these two metal rings is through a mutual capacity. In addition, each ring resembles a solenoid that can be represented by an inductance \( L_m \). The space between the two rings will be modeled by a capacity \( C_m \). The electrical circuit is show in the following figure.

![Equivalent circuit model of SRR](image)

Thus the capacity "\( C_{\text{gap}} \)" of the slot ring is determined by the following "Equation".

\[
C_{\text{gap}} = \frac{\varepsilon_0 \varepsilon_r I_c}{D}
\]

(2)

Therefore the capacity "\( C_m \)" created between the two rings may be expressed as follows:

\[
C_m = \frac{A \varepsilon_0 \varepsilon_r W(2l_{\text{out}} + 2l_{\text{in}} - D)}{2S}
\]

(3)

With "\( l_{\text{out}} \)" and "\( l_{\text{in}} \)" as successively the lengths of the outer and inner ring and \( W \) is the width of the strip of metal ring. "\( S \)" is shown the space between the two rings and "\( D \)" is the opening rings or the gap. And the letter «\( A \)» is a balance constant. With

\( \mu_0 \): The permeability of vacuum equal to \( 1.256 \times 10^{-6} [m Kg s^{-2} A^{-2}] \)

\( \varepsilon_0 \): The permittivity of vacuum equal to \( 8.854 \times 10^{-12} [A s V^{-1} m^{-1}] \)

\( C_0 \): The speed-load equal to \( 3 \times 10^8 [Km/s] \)

But the resonant frequency ‘\( f_0 \)’ can be determined based on geometric parameters such as the SRR

\[
f_0 = \frac{1}{2\pi \sqrt{L_m(C_m + C_{\text{gap}})}} \approx \frac{1}{2\pi \sqrt{L_m C_0}}
\]

(4)

If we want to work at an operating frequency equal to 2.4GHz with a dielectric substrate such as FR04-epoxy having a relative permittivity \( \varepsilon_r \) equal to 4.4, and if we fix the value of mutual inductance “\( L_m \)” at 60 nH, we can determine by using “Equation (4)” the mutual capacity “\( C_m \)” created between the two rings such as:

\[
C_m = \frac{1}{4\pi^2 f_0^2 I_m}
\]

(5)

After a numerical application we find \( C_m \approx 75 fF \).

According to our knowledge on the design and implementation of this type of resonator, we can determine the geometric parameters "\( W \)" and "\( S \)" meaning successively the track width of the ring and the space between the two rings 1mm to
facilitate the calculation. So, we can now determine the sum of two lengths rings $L_{\text{out}}$ and $L_{\text{in}}$ with the aid of “Equation (1)” for the determination of the inductance.

After a numerical application we find that:

$$L_{\text{in}} + L_{\text{out}} = 47.77 \text{mm}$$  \hspace{1cm} (6)

According the "Fig 3" for the SRR and especially the microstrip model present in three-dimensional structure we can determine the expression of the length of the outer ring by the following “Equation”.

$$L_{\text{out}} = L_{\text{in}} + 2S + 2W$$  \hspace{1cm} (7)

We note that both “Equations (6)” and “Equation (7)” are equal when:

$$L_{\text{in}} = \frac{47.77 \text{mm} - 2(S + W)}{2}$$  \hspace{1cm} (8)

Then according to a numerical application, we find an internal length ‘$l_{\text{in}}$’ equal to 21.885mm and an external length ‘$l_{\text{out}}$’ equal to 25.885mm for a working frequency chosen to 2.4GHz.

Also, if we replace “Equations (1)” and “Equation (3)” in “Equation (4)” one can determine the equilibrium constant ‘$A$’.

After simplification we find:

$$A = \frac{C^2}{4\pi^2(l_{\text{out}} + l_{\text{in}})^2 f_0^2 \varepsilon_r}$$  \hspace{1cm} (9)

With the resonance frequency has the following form:

$$f_0 = \frac{C}{2\pi(l_{\text{out}} + l_{\text{in}}) \sqrt{A \varepsilon_r}}$$  \hspace{1cm} (10)

C: the speed in a vacuum electromagnetic waves equal to $3 \times 10^8 \text{ m/s}$.

From “Equation (3)”, we can deduce the expression of the physic parameter ‘D’ which index the opining gap.

$$D = 2 \left( \frac{l_{\text{out}} + l_{\text{in}} - C_m}{A \varepsilon_0 \varepsilon_r} \right)$$  \hspace{1cm} (11)

So the numerical application gives us "D" equal to 1.14mm.

IV. MEASURED RESULTS AND COMPARAISON

From the results of obtained simulations supported by two commercial software the High Frequency Structure Simulator (HFSS) for three dimensional model of SRR offered in the “Fig. 2” and the Advanced Design System (ADS) for electrical modeling illustrated in the “Fig. 1”, we distinguish from the curve of the return loss $S_{11}$ according to a function of frequency “Fig. 3” that there is a low lag of 0.03GHz at the resonance frequency of the two patterns.
Thus the equivalent circuit having a good adaptation compared with which is in the 3D model where we find an adaptation equal to -21dB around the resonance's frequency 2.4 GHz which is in the electric model and -12dB around the resonance's frequency 2.37GHz about the three-dimensional structure. As we note that the electric model has a frequencies band less wide than which is obtained in the three-dimension.

These light different at the resonant frequency of the two models operating and at the adaptation S11 and frequencies band, these are logical and reasonable because these are due to the effect of the geometric parameters and the tangential loss of dielectric substratum which are not intercalated in the components \( L_m, C_m \) and \( C_{\text{gap}} \).

V. EFFECT OF VARIATION OF THE RELATIVE PERMITTIVITY

In this case, we will verify the effect of variation of relative permittivity on our equivalent circuit SRR square. We first take 4.4 as a nominal relative value of permittivity “\( \varepsilon_r \)” and then this value is varied while keeping the other physical parameters \( w, D \) and \( S \).

Table 1: Effect of variation the relative permittivity on frequency characteristic of equivalent circuit of SRR

<table>
<thead>
<tr>
<th>Nature of Dielectric</th>
<th>Relative permittivity</th>
<th>Frequencies [GHz]</th>
<th>Return loss [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rogers RO5880</td>
<td>2.2</td>
<td>3.427</td>
<td>-23.6</td>
</tr>
<tr>
<td>Rogers RO4232</td>
<td>3.2</td>
<td>2.839</td>
<td>-33.4</td>
</tr>
<tr>
<td>FR404-epoxy</td>
<td>4.4</td>
<td>2.4</td>
<td>-21</td>
</tr>
<tr>
<td>Mica</td>
<td>5.7</td>
<td>2.127</td>
<td>-24.3</td>
</tr>
<tr>
<td>Rogers TMM6</td>
<td>6.0</td>
<td>2.06</td>
<td>31</td>
</tr>
</tbody>
</table>

According the “table.1” of results aided from software ADS, we find, in first time, that the resonant frequency is increases at 0.439GHz for \( \varepsilon_r = 3.2 \) and at 1.027GHz for \( \varepsilon_r=2.2 \). On second time, we note that the resonance frequency of equivalent circuit is decreases at 0.273GHz for \( \varepsilon_r= 5.7 \) and at 0.33GHz for \( \varepsilon_r= 6.0 \).

VI. EFFECT OF VARIATION THE PHYSICAL PARAMETERS

The objective of these studies is to check the effect of variation of the physical parameters \( D \), \( W \) and \( S \) on the frequency characteristics of the equivalent circuit. Where references will be set.
by the electrical model works 2.4GHz where geometric variables are all equal to 1mm nominal value.

VI.1 VARIATION OF THE GAP

In this study and on the one hand, if we decrease the nominal value of the gap, we find using the ADS simulation that the resonance frequency decreases compared to the reference case at 0.12GHz for D = 0.1 mm and at 0.722GHz for D = 0.01. On the other hand, if we proceed to increase the nominal value of the gap, we note that the resonance frequency increases in this case at 0.13GHz for D=10mm and at 0.28GHz for D=20mm.

Subsequently, the variation of the physic parameter D which index the opening rings demonstrate that the resonant frequency is proportional to the value of the gap D, as shown in the following figure.

Fig. 6. Curve of the Return Loss as a function of frequency then variation the track width

From this study we can show that the value of the track width proportionally affects the values of the resonant frequency, the return loss and the band width of frequencies

VI.2 VARIATION OF THE TRACK WIDTH OF THE RING

In the first part, if we exercise a reduction on the value of the track width of the ring, we conclude a low degradation at level the resonance frequency. By against this degradation is clearly noteworthy at the wide band of frequencies and the return loss with respect to the reference case. In the second part, if we increase the value of this parameter, we observe in this case a low increase appear on the value of the resonant frequency and the return loss of electric model. The simulation results are illustrated in the following figure.

Fig. 5. Curve of the Return Loss as a function of frequency then variation the opening ring

VI.3 VARIATION OF SPACE «S»

Using the same principle to fix the physical parameters D and W at nominal value and varying the parameter S. Therefore if we drive a diminution in value of the space between the two rings, we observe an increase in the value of the resonant frequency compared to the case of reference by 0.032GHz for S equal to 0.1mm. On the contrary, if we increase this parameter we note a degradation of 0.036GHz and 0.09GHz for successively S as having 10mm and 20mm value. The simulation results are illustrated in the following figure.
Fig. 7. Curve of the Return Loss as a function of frequency then variation the space "S"

According to variation the value of the distance between the two rings “S”, we can show that the resonance frequency of the equivalent circuit is inversely proportional to this geometric parameter. These results are the consequences of appearance of the capacitive effect in this space and we verified our well of results.

VII. CONCLUSION

The electric modeling of split ring resonators is elaborated and proposed in the present article. The using of equivalent circuit having electric components which are correlated with geometric parameters of SRR in three-dimensional structure, allowing it to be effective, very useful and help us to do good analytic studies. Moreover, the time of simulation was reduced and that means a fast tool of simulation.

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REFERENCES


