What is Quantum Field Theory?

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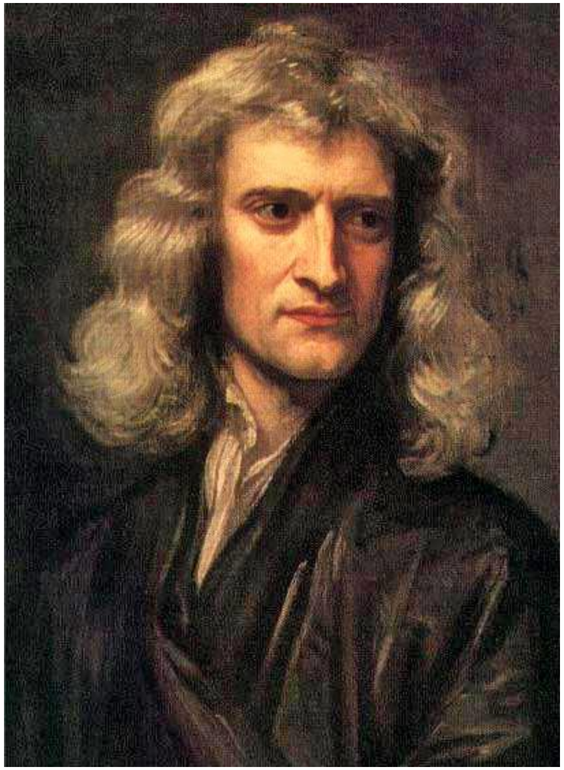
Main points

- Quantum field theory is the natural language of physics:
  - Particle physics
  - Condensed matter
  - Cosmology
  - String theory/quantum gravity
- Applications in mathematics especially in geometry and topology
- Quantum field theory is the modern calculus
  - Natural language for describing diverse phenomena
- Enormous progress over the past decades, still continuing
- Indications that it should be reformulated

Calculus vs. Quantum Field Theory

- New mathematics
- Motivated by physics (motion of bodies)
- Many applications in mathematics, physics and other branches of science and engineering
- Sign that this is a deep idea
- Calculus is a mature field. Streamlined – most books and courses are more or less the same



Calculus vs. Quantum Field Theory

- New mathematics (in fact, not yet rigorous)
- Motivated by physics (particle physics, condensed matter)
- Many applications in mathematics and physics
- Sign that this is a deep idea
- QFT is not yet mature – books and courses are very different (different perspective, order of presentation)
- Indications that we are still missing big things – perhaps QFT should be reformulated

Should quantum field theory be reformulated?

- Should we base the theory on a Lagrangian?
  - Examples with no semi-classical limit – no Lagrangian
  - Examples with several semi-classical limits – several Lagrangians
  - Many exact solutions of QFT do not rely on a Lagrangian formulation (integrability, bootstrap in CFT)
  - Magic in amplitudes – beyond Feynman diagrams
- Not mathematically rigorous
- Extensions of traditional local QFT

Generalized Global Symmetries

D. Gaiotto, A. Kapustin, NS, B. Willett, arXiv:1412.5148

- Ordinary global symmetries
  - Associated with operators of co-dimension one manifolds (e.g. space)
  - The charged operators are point-like
  - The charged states are particles (0-branes)
- *q*-form global symmetries
  - Associated with operators of co-dimension $q+1$-manifold
  - The charged operators are of dimension *q*, e.g. Wilson and 't Hooft lines)
  - The charged states are *q*-dimensional (*q*-branes), e.g. strings

q-form global symmetries

As with ordinary symmetries:

- Selection rules on amplitudes
- Couple to a background classical gauge field (twisted boundary conditions)
  - Interpret ‘t Hooft twisted boundary conditions as an observable in the untwisted theory
- Gauging the symmetry by summing over twisted sectors (like orbifolds)
  - New parameters in gauge theories – discrete θ-parameters (like discrete torsion)
- Dual theories often have different gauge symmetries. But the global symmetries must be the same
  - non-trivial tests of duality including non-BPS operators

q-form global symmetries

As with ordinary symmetries:

- The symmetry could be spontaneously broken
  - Continuous: the photon is a Goldstone boson
  - Discrete: a topological theory in the IR. Long range topological order
- Anomalies and anomaly inflow on boundaries or defects
  - ‘t Hooft matching – Symmetry Protected Topological phases
- Characterize phases – unified description, extending Landau’s
  - Confinement = one-form global symmetry unbroken
    - Strings, loops with area law
  - Higgs or Coulomb = one-form global symmetry broken
  - Various other phases (mixed, oblique, partial breaking)

Clearly, a lot more can be said about these generalized symmetries (see the paper and future work).

More generally we should study Quantum Field Theory from many points of view

- because of its many applications
- in order to understand it better

Hopefully, we can learn how to improve its presentation – reformulate it.

Thank you for your attention