Mathematical and Computational Applications, Vol. 15, No. 3, pp. 461-471, 2010.

THE EMERGENCE OF SPHERICAL MAGNETO-GAS DYNAMIC STRONG SHOCK WITH RADIATION NEAR THE SURFACE OF A STAR WITH A ROTATING , GRAVITATING, NON-UNIFORM ATMOSPHERE

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Abstract - The propagation of a strong magneto-gas dynamic, rotating and gravitating shock wave originating in a stellar interior is considered , when it approaches the surface of the star . The flow behind the shock wave is assumed to be spatially isothermal rather than adiabatic to simulate the conditions of large radiative transfer near the stellar surface. It has been observed that gravitation and rotation have important impact upon the emergence of shock at the surface of the star.
Keywords – Spherical Magneto-gas dynamic shock wave, Gravitation, Rotation.

1. INTRODUCTION

In the present paper, we discuss the propagation of a strong magneto-gas dynamic shock wave as it approaches the surface of a star. Such shocks are formed in the interior of stars due to the steepening of internal disturbances and then propagate outward to the periphery of the star .We consider the stage when the shock is so close to the surface of the star that we can treat the motion as spherical. Differing from Srivastava [1], Sachdev and Ashraf [2] , we take into consideration the gravitational effect . We consider that the initial conditions of the solution have been “forgotten”. In fact , we are referring to self-similar solution of the second kind as considered by Zel’dovich and Raizer([3], p.812-817) while discussing the emergence of a strong shock near the edge of a star. Further we also consider a rotating stellar atmosphere .Recently, it has been observed that the outer atmosphere of the planets rotates due to the rotation of the planets .Macroscopic motion with supersonic speed occurs in the interplanetary atmosphere with rotation and shock waves are generated (Jana and Ganguly[4]).

As the shock propagates in the outer layers of the star, it accelerates and the temperature behind it increases. Besides, the mean free path of radiation which is inversely proportional to density becomes extremely large so that there is intense radiative transfer leading to the leveling down of the temperature gradient .Thus, the flow behind the shock is rendered approximately isothermal. This picture is different from the one normally envisaged by Rogers [5] where the shock front is assumed isothermal .We have closely followed Zel’dovich and Raizer[3] adopting Eulerian coordinates rather than the Lagrangian coordinates , employed by Laumbach and Probstein[6] .

The shock position is assumed to be given by the similarity relation $X = A(-t)^{\alpha}$ where X is the shock distance measured from the surface of a star and t is the time which is negative before the shock reaches the surface of the star .We take t = 0 to be the instant at which the shock wave emerges at the surface. A and α (<1) are constants.

The variations of flow variables have been depicted graphically and the relevant comparison between the cases of presence and absence of gravitation have been shown.

2. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

We take the origin of co-ordinates on the surface of the star and the positive x axis points into the interior of the star. We assume that the undisturbed density ahead of the shock is given by

$$\rho_0 = br^{\delta} \quad , \quad (-3 \le \delta \le 0) \tag{1}$$

where b and δ are constants so that $\rho_0 = 0$ on the surface .

The magnetic field is taken after slight modification of Rosenau and Frankenthal [7],

$$h_0 = h_c X^{\mu} \quad (-1 < \mu < 0) \tag{2}$$

directed tangentially to the advancing shock front and h_c is a constant .

The basic equations governing the isothermal flow in spherically symmetric Eulerian form are : (Shrivastava[1] , Jana and Ganguly[4])

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0 \tag{3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{h}{\rho r}\frac{\partial}{\partial r}(hr) + \frac{Gm}{r^2} - \frac{v^2}{r} = 0 \tag{4}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{hu}{r} = 0 \tag{5}$$

$$\frac{d}{dt}(v\,r) = 0 \tag{6}$$

$$\frac{\partial m}{\partial r} = 4\pi\rho\, r^2 \tag{7}$$

$$\frac{\partial T}{\partial r} = 0 \tag{8}$$

where u , p , ρ , T , h ,G , m , v , r , t are radial component of velocity , pressure , density, temperature , magnetic field , gravitational constant , the mass contained between a fixed surface and the surface under consideration , azimuthal component of velocity , radial distance , time respectively behind the shock wave .

From equations (4), (8) and the gas law $p = R\rho T$ we get

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{p}{\rho^2}\frac{\partial \rho}{\partial r} + \frac{h}{\rho r}\frac{\partial}{\partial r}(hr) + \frac{Gm}{r^2} - \frac{v^2}{r} = 0 \tag{9}$$

We assume that the heat flux across the optically thin shock front is continuous so that the classical shock conditions hold . So for a strong shock we have the boundary conditions at the shock as(see, Zel'dovich and Raizer([3], p.814), Nath et.al.[8])

$$u_s = (1-\beta)\dot{X} \quad , \quad v_s = (1-\beta)\dot{X}$$

$$\rho_s = \frac{\rho_0}{\beta} \quad , \quad p_s = (1-\beta)\rho_0\dot{X}^2 \quad ,$$

$$h_s = \frac{h_0}{\beta} = \sqrt{(1-\beta)}\sqrt{\rho_0}\,\dot{X} \quad , \quad m_s = m_0 = \frac{4\pi b r^{\delta+3}}{(\delta+3)} \tag{10}$$

where $\beta = \frac{(\gamma-1)}{(\gamma+1)}$ is the density ratio across the shock and $\dot{X}$ is the shock velocity.

3. SIMILARITY SOLUTIONS

We introduce a similarity variable $\xi = \frac{r}{X}$ and seek a solution of the form

$$u = (1-\beta)\dot{X}\,U(\xi) \tag{11}$$

$$\rho = \frac{\rho_0(X)}{\beta} g(\xi) \tag{12}$$

$$p = (1-\beta)\rho_0(X)\dot{X}^2 P(\xi) \tag{13}$$

$$v = (1-\beta)\dot{X}\,V(\xi) \tag{14}$$

$$h = \sqrt{(1-\beta)}\sqrt{\rho_0(X)}\,\dot{X}\,H(\xi) \tag{15}$$

$$m = \frac{4\pi b r^{\delta+3}}{(\delta+3)} Y(\xi) \tag{16}$$

At the shock , where $\xi = 1$, the boundary conditions (10) for the reduced functions $U(\xi)$, $g(\xi), P(\xi), V(\xi), H(\xi)$ and $Y(\xi)$ become

$$U(1) = g(1) = P(1) = V(1) = H(1) = Y(1) = 1 \tag{17}$$

The Alf 'ven Mach number and the usual Mach number are defined as

$$M_A{}^2 = \frac{\rho_0\dot{X}^2}{h_0{}^2} \quad \text{and} \quad M^2 = \frac{\rho_0\dot{X}^2}{\gamma p_0} \quad \text{respectively.}$$

Under the equilibrium conditions, we have from (4)

$$G = -\frac{(3+\delta)(1+\delta)\dot{X}^2}{2\pi b r^{\delta+2}}\left[\frac{(\mu+1)}{2\mu {M_A}^2} + \frac{1}{\gamma M^2}\right].$$

Equation (8) provides the following integral in terms of the reduced functions:

$$g(\xi) = P(\xi) \tag{18}$$

Substituting equations (11)-(16) into equations (3) and (9) and making use of equations (4)-(7) we obtain

$$\delta + [(1-\beta)U - \xi]\frac{1}{g}\frac{dg}{d\xi} + (1-\beta)\frac{dU}{d\xi} + \frac{2(1-\beta)U}{\xi} = 0 \tag{19}$$

$$[(1-\beta)U - \xi]\frac{dU}{d\xi} + \alpha^{-1}(\alpha-1)U + \frac{\beta}{g}\left[\frac{dg}{d\xi} + H\frac{dH}{d\xi} + \frac{H^2}{\xi}\right] - \frac{2(1+\delta)}{(1-\beta)\xi}\left[\frac{(\mu+1)}{2\mu {M_A}^2} + \frac{1}{\gamma M^2}\right]Y - \frac{(1-\beta)V^2}{\xi} = 0 \tag{20}$$

$$\frac{\delta}{2} + \alpha^{-1}(\alpha-1) + [(1-\beta)U-\xi]\frac{1}{H}\frac{dH}{d\xi} + (1-\beta)\frac{dU}{d\xi} + \frac{(1-\beta)U}{\xi} = 0 \tag{21}$$

$$[(1-\beta)U - \xi]\frac{dV}{d\xi} + \left[\alpha^{-1}(\alpha-1) + \frac{(1-\beta)U}{\xi}\right]V = 0 \tag{22}$$

$$\frac{dY}{d\xi} = \frac{4\pi b g \xi^2 A^{\delta}(-t)^{\alpha\delta+2}}{\beta\alpha^2} \tag{23}$$

Noting the fact that motion is self-similar , we have from (23)

$$\alpha\delta + 2 = 0 \quad \Rightarrow \quad \alpha = \frac{-2}{\delta} \tag{24}$$

From equations (19) , (21) and (22) we obtain

$$\frac{dg}{d\xi} = -\frac{\left[(1-\beta)\frac{dU}{d\xi} + \frac{2(1-\beta)U}{\xi} - \delta\right]g}{[(1-\beta)U - \xi]} \tag{25}$$

$$\frac{dH}{d\xi} = -\frac{\left[(1-\beta)\frac{dU}{d\xi} + \frac{(1-\beta)U}{\xi} + \frac{\delta}{2} + \left(1-\alpha^{-1}\right)\right]H}{[(1-\beta)U - \xi]} \tag{26}$$

$$\frac{dV}{d\xi} = -\frac{\left[\left(1-\alpha^{-1}\right)+\frac{(1-\beta)U}{\xi}\right]V}{\left[(1-\beta)U-\xi\right]} \quad (27)$$

Substituting the values of $\frac{dg}{d\xi}$ and $\frac{dH}{d\xi}$ in equation (20), we obtain

$$\frac{dU}{d\xi} = [-(1-(1/\alpha))U((1-\beta)U-\xi)+2\beta(1-\beta)(U/\xi)-\beta\delta+(\beta H^2/g)((\delta/2)+(2-(1/\alpha)))$$
$$+(2(1+\delta)/(1-\beta)\xi)((((\mu+1)/2\mu M_A^2)+(1/\gamma M^2))((1-\beta)U-\xi)Y$$
$$+(1-\beta)(V^2/\xi)((1-\beta)U-\xi)] \div ((1-\beta)U-\xi)^2-\beta(1-\beta)(1+(H^2/g))$$

(28)

From equations (23) and (24), we obtain

$$\frac{dY}{d\xi} = \frac{4\pi bg\xi^2 A^{\delta}}{\beta\alpha^2} \quad (29)$$

ξ varies from 1 at the shock to ∞ at a large distance behind the shock . To reduce the range of integration we change the variable ξ to η by the transformation $\eta = \frac{1}{\xi}$, (30)

So that η varies from 1 at the shock to 0 far behind the shock .
Making use of the transformation (30), we obtain

$$\frac{dg}{d\eta} = -\frac{\left[(1-\beta)\eta\frac{dU}{d\eta}-2(1-\beta)U+\frac{\delta}{\eta}\right]g}{\left[(1-\beta)U\eta-1\right]} \quad (31)$$

$$\frac{dH}{d\eta} = -\frac{\left[(1-\beta)\eta\frac{dU}{d\eta}-(1-\beta)U-\frac{\delta}{2\eta}-\frac{\left(1-\alpha^{-1}\right)}{\eta}\right]H}{\left[(1-\beta)U\eta-1\right]} \quad (32)$$

$$\frac{dV}{d\eta} = \frac{\left[\frac{\left(1-\alpha^{-1}\right)}{\eta}+(1-\beta)U\right]V}{\left[(1-\beta)U\eta-1\right]} \quad (33)$$

$$\frac{dU}{d\eta} = [-(1-(1/\alpha))(U/\eta)((1-\beta)U\eta-1)+2\beta(1-\beta)U\eta-\beta\delta+(\beta H^2/g)((\delta/2)+(2-(1/\alpha)))$$
$$+(2(1+\delta)/(1-\beta))((((\mu+1)/2\mu M_A^2)+(1/\gamma M^2))((1-\beta)U\eta-1)Y$$

$$+(1-\beta)V^2((1-\beta)U\eta-1)\div[-((1-\beta)U\eta-1)^2+\beta(1-\beta)\eta^2(1+(H^2/g))]$$

(34)

$$\frac{dY}{d\eta}=-\frac{4\pi b\, g\, A^{\delta}}{\eta^4\beta\alpha^2} \tag{35}$$

4. RESULTS AND DISCUSSIONS

The set of differential equations (31)- (35) have been integrated numerically with the help of the boundary conditions (17) by the well known Runga-Kutta method. The variations of the flow variables with the distance are illustrated in the figures (1) – (5) for $\gamma=1.66$, $M^2=5$ and $M_A^2=10$, $\alpha=0.8$, $b=2$, $\delta=-2.5$ and the respective influences of gravitation, magnetic field and rotation have been studied by their presence and absence.

The line patterns used in the figures (1) –(5) stand for the following cases:

Case I : All present

Case II: gravitation absent

Case III: magnetic field absent

Case IV: rotation absent

and we have considered the effects of these factors like rotation ,gravitation , magnetic field and also the role of γ in the variations of flow parameters.

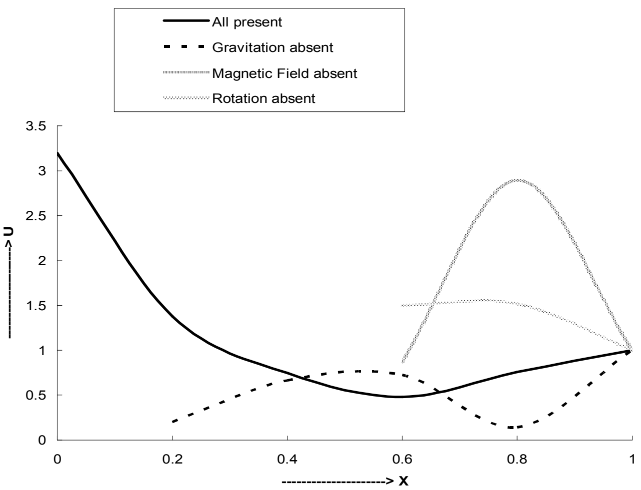

Fig. 1 - Variation of Radial Velocity with Distance

As we move towards the centre of shock , in the case I , there is a remarkable variation in radial velocity . As we go towards the centre of shock , it first decreases and then increases abnormally .So also in the absence of magnetic field ,the radial velocity tries to increase but after some time drops down towards zero much before we reach the centre of the shock.

In the absence of gravitation, radial velocity behaves like a sinusoidal wave and tries to arrive at zero before we reach the centre of the shock. When the rotation is absent radial velocity, like in the case when the magnetic field is absent, tries to increase and then gradually tries to attain some constant value as we approach towards the centre of the shock.

In fact, presence of rotational effect helps gravitational and magnetic presence to take the radial velocity to upper bound as we approach the centre of the shock .Thus we conclude the gravitational presence creates a conducive condition for rotational velocity to increase radial velocity as we approach towards the centre of the shock.

In Fig. 2 , we observe that , in absence of gravitational field , variation of rotational velocity is similar to that of radial velocity with distance as we move towards the centre of the shock . This shows presence of magnetic field dampens rotational velocity as well as radial velocity to decrease as we go towards the centre of the shock, which is why we notice in Fig.2 that in the absence of magnetic field gravitational presence tries to take rotational velocity to higher values as we move towards the centre of the shock. This was noticeable evidently in Fig.1 when magnetic field was absent. Thus we find in Fig.2 gravitational presence here also has as an important role in the increase of radial velocity as we move towards the centre of the shock.

In Fig. 3, we observe that absence of gravitational field puts a dampening effect on the variation of density and pressure as we move towards the centre of the shock. In fact it decreases and almost attains zero value before we reach at the centre of the shock. Even in case I, density and pressure tend to zero as we move towards the centre of the shock.

Thus we conclude, in spite of presence of rotation, the role of magnetic field is significant in reduction of density and pressure of the shock wave as we move towards the centre of the shock.

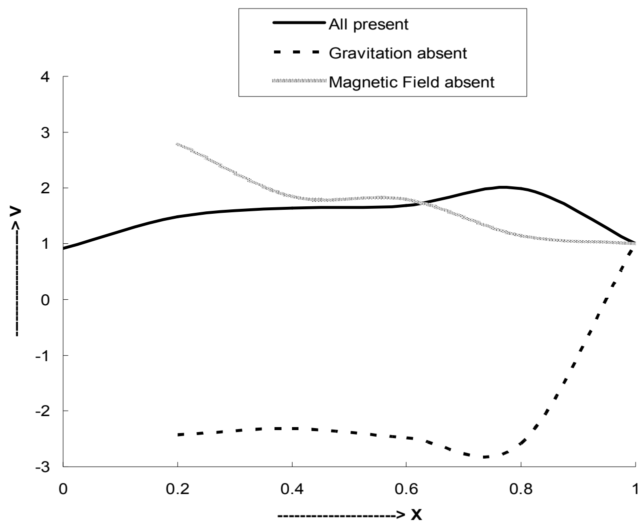

Fig.2 - Variation of Rotational Velocity with Distance

Fig. 4 shows the presence of magnetic field tries to reduce intensity of shock, as its presence leads to immediate decrease as we move towards the centre of the shock.

Lastly, Fig.5 also justifies that absence of magnetic field gives a leap to mass as we go towards the centre of the shock.

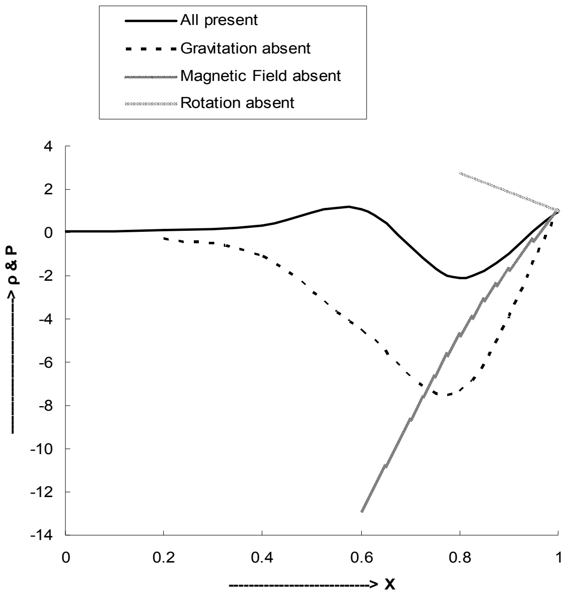

Fig. 3 - variation of Density and Pressure with Distance

We finally conclude that magnetic field has a disaster effect. Srivastava[1] discussed the problem of the emergence of a spherical magneto gas dynamic shock wave at the surface of a star with the help of Mc Vittie[9] solution technique . He concluded that radial velocity , density , pressure and magnetic field increase as one moves towards the nucleus of the star . But in our case the presence of magnetic field and the absence of gravitation leads to sharp declined in radial velocity, rotational velocity, density and pressure, and magnetic field. In fact, presence of gravitation only helps along with rotational factor to keep the values of velocity, density , pressure to have nonzero positive values as one moves towards the centre of the shock or nucleus of star .

So our conclusion is that gravitation and rotation have important impact upon the
emergence of shock at the surface of the star.

In the Table 1-Table 5, when the variables are absent as we approach zero, values become very high for comparative study of presence and absence. It is therefore not possible to display it in the same figure. Otherwise graphs for variations can not be displayed properly .There will be overlapping of the graphs.

It should be noted that due to the possibility of overlapping of the graphs for the values of distance going towards the centre of the shock (which are coming very high),we are presenting the tabulated values in appendix 1 for the interest of the readers.

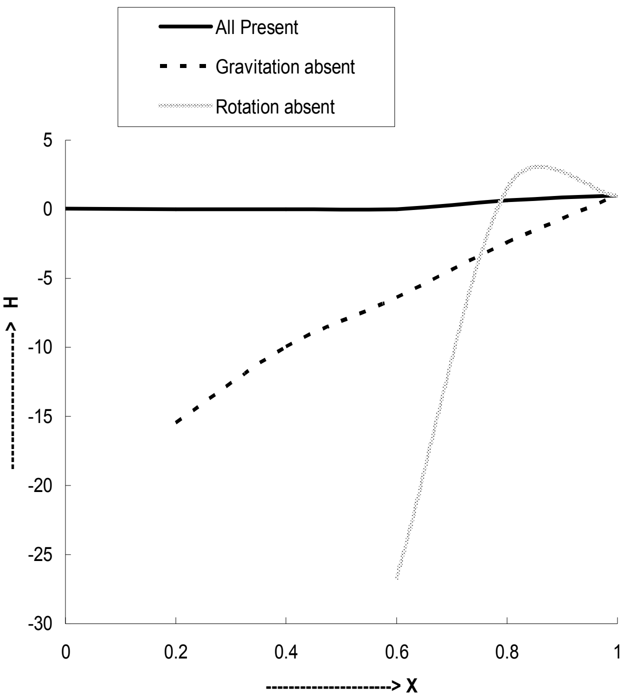

Fig. 4 - Variation o f Magnetic Field with Distance

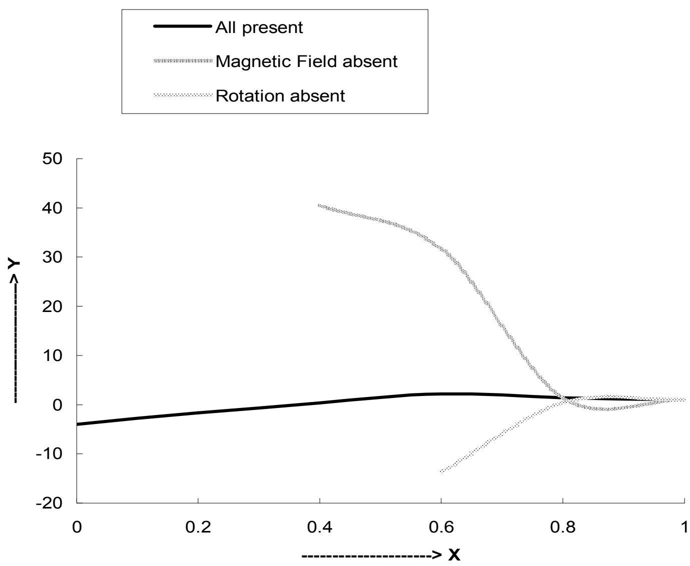

Fig. 5 - Variation of Mass with Distance

Appendix 1

Table 1-Variation of Radial velocity with Distance.

X	All Present	Gravitation absent	Magnetic Field absent	Rotation absent
1	1	1	1	1
0.8	0.757	0.142	2.895	1.518
0.6	0.481	0.727	0.871	1.5
0.4	0.746	0.664	99.735	115.007
0.2	1.381	0.199	116.643	-4218.448
0	3.2	-	-	-

Table 2-Variation of Rotational velocity with Distance.

X	All Present	Gravitation absent	Magnetic Field absent	Rotation absent
1	1	1	1	0
0.8	1.989	-2.583	1.146	0
0.6	1.683	-2.479	1.793	0
0.4	1.638	-2.319	1.846	0
0.2	1.483	-2.425	2.791	0
0	0.918	-	-	-

Table 3- Variation of Density and Pressure with Distance.

X	All Present	Gravitation absent	Magnetic Field absent	Rotation absent
1	1	1	1	1
0.8	-2.116	-7.33	-4.778	2.755
0.6	1.056	-4.526	-12.896	110.128
0.4	0.343	-1.056	-889.627	128864.47
0.2	0.089	-0.286	-1774.839	36075880
0	0.035	-	-	-

Table 4- Variation of Magnetic Field with Distance

X	All Present	Gravitation absent	Magnetic Field absent	Rotation absent
1	1	1	0	1
0.8	0.647	-2.41	0	1.461
0.6	0.001	-6.386	0	-26.658
0.4	0.002	-9.958	0	5768.997
0.2	0.005	-15.466	0	795336.38
0	0.045	-	-	-

Table 5- Variation of Mass with Distance.

X	All Present	Gravitation absent	Magnetic Field absent	Rotation absent
1	1	0	1	1
0.8	1.384	0	1.402	0.396
0.6	2.173	0	31.697	-13.787
0.4	0.419	0	40.504	3476.713
0.2	-1.636	0	12674.19	-11296578
0	-3.941	-	-	-

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