Multi-item integrated supply chain model for deteriorating items with stock dependent demand under fuzzy random and bifuzzy environments

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A R T I C L E   I N F O

Article history:
Received 14 November 2014
Received in revised form 24 June 2015
Accepted 28 June 2015
Available online 3 July 2015

Keywords:
Multi-item supply chain
Credit period
Deterioration
Genetic algorithm
Fuzzy random
Bifuzzy

A B S T R A C T

In this paper, we have investigated multi-item integrated production-inventory models of supplier and retailer with a constant rate of deterioration under stock dependent demand. Here we have considered supplier’s production cost as nonlinear function depending on production rate, retailers procurement cost exponentially depends on the credit period and suppliers transportation cost as a non-linear function of the amount of quantity purchased by the retailer. The models are optimized to get the value of the credit periods and total time of the supply chain cycle under the space and budget constraints. The models are also formulated under fuzzy random and bifuzzy environments. The ordering cost, procurement cost, selling price of retailer’s and holding costs, production cost, transportation cost, setup cost of the supplier’s and the total storage area and budget are taken in imprecise environments. To show the validity of the proposed models, few sensitivity analyses are also presented under the different rate of deterioration. The models are also discussed in non deteriorating items as a special case of the deteriorating items. The deterministic optimization models are formulated for minimizing the entire monetary value of the supply chain and solved using genetic algorithm (GA). A case study has been performed to illustrate those models numerically.

1. Introduction

In the past few years, the research in deteriorating inventory is becoming important under various circumstances (Chung, Cárdenas-Barrón, & Ting, 2014; Guchhait, Maiti, & Maiti, 2014; Lee & Dye, 2012; Ouyang, Wu, & Yang, 2006; Widyadana, Cárdenas-Barrón, & Wee, 2011; Wu, Ouyang, Cárdenas-Barrón, & Goyal, 2014). This is because in the real life, decay and deterioration occur in almost all products, such as fruit, medicine and vegetables. Ghare and Schrader (1963) were the first to consider the effect of decaying inventory when the demand is constant. Heng, Labban, and Linn (1991) have presented an order-level lot-size inventory model for deteriorating items with finite replenishment rate. Chung (2000) researched on the inventory replacement policy for deteriorating items under permissible delay in payments. Benkherouf, Bournenir, and Aggoun (2003) worked on a diffusion inventory model for deteriorating items. Yang and Wee (2003) discussed integrated multi-lot-size production inventory model for deteriorating item. Sarkar, Ghosh, and Chaudhuri (2012) developed deteriorating inventory with time-quadratic demand and time-dependent partial backlogging with shortage. Ahmed, Al-Khamis, and Benkherouf (2013) proposed inventory models with ramp type demand rate, partial backlogging and general deterioration rate. Yang, Teng, and Chern (2001) discussed deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuating demand. Saadany and Jaber (2010) introduces production/remanufacturing inventory model with price and quality dependant return rate. Taleizadeh, Stojkovska, and Penticoc (2015) developed economic order quantity model with partial backordering and incremental discount.

As the commercial market has become more competitive, supplier and retailer are comfortable to go with long term cycles with each other in order to reduce costs and improve efficiency. That is why supply chain coordination is essential. The basic model is based on the implicit assumption that the retailer must pay for the product as soon as he receives them from a supplier. However, in practical situation, the supplier will allow a certain
fixed period (credit period) for reconciling the amount that the supplier be indebted to retailer for the items supplied. Many researchers have expressed their interest in optimizing the supply chain integrated systems in various environments. Goyal (1977) first introduce the idea of optimize the supplier and retailer total cost jointly. Das, Das, and Mondal (2013) developed integrated supply chain cost model for a deteriorating item with procurement cost dependent credit period. Wong proposed (Wong, 2010) supply chain with the distribution processing and replenishment policy under asymmetric information and deterioration. Sarkar (2013) presented a production-inventory model with probabilistic deterioration in two-echelon supply chain management. Sarkar, Jamal, and Wang (2000) developed supply chain models for perishable products under inflation and permissible delay in payment. Uthayakumar and Parvathi (2011) worked on two-stage supply chain with order cost reduction and credit period incentives for deteriorating items. Teng and Lou (2012) discussed sellers optimal credit period and replenishment time in a supply chain with up-stream and down-stream trade credits. Huang (2001) worked on integrated supply chain model for supplier and retailer with defective items. Recently Wang, Teng, and Lou (2014) studied on supply chain for deteriorating items with maximum lifetime.


In many real-life cases, several factors in the decision making procedure, such as incomplete data, additional qualitative criteria and imprecision preferences. Such determination can be reached using the parameter under fuzzy, fuzzy random, etc. environments. Several researchers has worked on those imprecise environments to solve EOQ or EPQ model. Maiti and Maiti (2006) discussed fuzzy inventory model with two warehouses under possibility constraints. Pedro, Mula, Poler, and Verdegay (2009) developed fuzzy supply chain planning under supply, demand and process uncertainties. Chen, Lin, and Huang (2006) introduced fuzzy approach for supplier evaluation and selection in supply chain management. Xu and Zhai (2010) discussed two-stage supply chain under fuzzy demand. Jana, Das, and Maiti (2014) developed multi-item inventory models over random planning horizon in random fuzzy environment. In spite of the above mentioned developments, coming after, following additions can also be made in the formulation and solution of supply chain models for deteriorating items.

- Till now, none has formulated multi-item integrated supply chain models (MISCM) allowing credit period, procurement cost and stock dependent demand for deteriorating items. This vacuum has been removed by this investigation.
- In a real-life supply chain system, limitations on available budget and storage space are very often faced by the suppliers and retailers. These resources are sometimes fuzzy, random fuzzy or bifuzzy in nature. The MISCM under fuzzy, random fuzzy or bifuzzy environments has been introduced first time.
- The objective and constraints under fuzzy random and bifuzzy environments has been successively introduced for the first time in MISCM and transformed to a corresponding deterministic model using probability-possibility and possibility-possibility chance constraint techniques.
- A practical real life example has been provided to validate the proposed model and some sensitivity analysis has been done and changes are depicted via graphs.
- MISCM has been solved by using contractive mapping genetic algorithm.

The rest of this paper is organized as follows. In Section 2, we recall some preliminary knowledge about fuzzy, fuzzy random and bifuzzy. Section 3 provides the notations and assumption used throughout in this paper. In Sections 4 and 5, multi-item supply chain models are proposed for deteriorating and non-deteriorating items respectively. In Sections 6 and 7, deterministic models have been formulated and also the models under imprecise environments have been consider. In those section we have also discussed the solution procedure to those models. Section 8 provides the general information about contracting mapping genetic algorithm and how it is used to solve the proposed models. A real life example is solved, results are compared and explained graphically in Section 9. Section 10 summarizes the paper and also discusses about the scope of future work.

### 2. Preliminaries

**Definition 2.1** (LR fuzzy variable Dubois and Prade, 1980). Let $L(·)$ and $R(·)$ be two reference functions. If the membership function of fuzzy variable $\xi$ has the following form

$$
\mu_\xi(x) = \begin{cases} 
L\left(\frac{m-x}{\alpha}\right), & x \leq m, \alpha > 0; \\
R\left(\frac{x-m}{\beta}\right), & x \geq m, \beta > 0. 
\end{cases}
$$

then $\xi$ is called LR fuzzy variable, $L$, $R$ are called left and right branch of $\xi$ respectively, the reference function $L, R : [0, 1] \rightarrow [0, 1]$

### Table 1

Summary of related literature for multi-item supply chain models.

<table>
<thead>
<tr>
<th>Author(s) and year</th>
<th>Constraints</th>
<th>Demand rate</th>
<th>Credit period</th>
<th>Imprecise environment</th>
<th>Multi-item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wu and Golbasi (2004)</td>
<td>Yes</td>
<td>Uniform</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Chen and Chen (2005)</td>
<td>No</td>
<td>Uniform</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Chen et al. (2006)</td>
<td>No</td>
<td>Uniform</td>
<td>No</td>
<td>Only Fuzzy</td>
<td>No</td>
</tr>
<tr>
<td>Pedro et al. (2009)</td>
<td>Yes</td>
<td>Uniform</td>
<td>No</td>
<td>Only Fuzzy</td>
<td>Yes</td>
</tr>
<tr>
<td>Wong (2010)</td>
<td>No</td>
<td>Uniform</td>
<td>No</td>
<td>Only Fuzzy</td>
<td>No</td>
</tr>
<tr>
<td>Xu and Zhai (2010)</td>
<td>No</td>
<td>Uniform</td>
<td>No</td>
<td>Only Fuzzy</td>
<td>No</td>
</tr>
<tr>
<td>Taleizadeh et al. (2011)</td>
<td>No</td>
<td>Uniform</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Das et al. (2013)</td>
<td>No</td>
<td>Uniform</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Sarkar (2013)</td>
<td>No</td>
<td>Uniform</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Wang et al. (2014)</td>
<td>No</td>
<td>Credit Period Dependent</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Present paper</td>
<td>Yes</td>
<td>Stock-dependent</td>
<td>Yes</td>
<td>Fuzzy, Random and Bifuzzy</td>
<td>Yes</td>
</tr>
</tbody>
</table>
satisfies that $L(1) = R(0) = 0$, $L(0) = R(0) = 1$, and it is monotone function. $\alpha$, $\beta$ are called left and right spread of $\xi$ respectively, $m$ is called the main value of $\xi$. Denote $\xi$ by $(m, \alpha, \beta)_{LR}$.

**Definition 2.2** Dubois and Prade. 1980. Let $\xi = (m_1, \alpha_1, \beta_1)_{LR}$ and $\xi_2 = (m_2, \alpha_2, \beta_2)_{LR}$, $k \neq 0$ a real number, then

1. $\xi_1 + \xi_2 = (m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$;
2. $\xi_1 - \xi_2 = (m_1 - m_2, \alpha_1 - \alpha_2, \beta_1 - \beta_2)_{LR}$;
3. $k\xi_1 = \begin{cases} (km_1, \alpha_1, \beta_1)_{LR}, & k > 0; \\ (km_1, -\alpha_1, \beta_1)_{LR}, & k < 0. \end{cases}$

**Definition 2.3** (Possibility space (Liu, 2004)). Let $\Theta$ be a nonempty set, and $P(\Theta)$ be the power set of $\Theta$. For each $A \in P(\Theta)$, there is a nonnegative number $Pos(A)$, called its possibility, such that

1. $Pos(\emptyset) = 0$, $Pos(\Theta) = 1$; and
2. $Pos(\cup_i A_k) = \sup_i Pos(A_k)$ for any arbitrary collection $A_k$ in $P(\Theta)$.

The triplet $(\Theta, P(\Theta), Pos)$ is called a possibility space, and the function $Pos$ is referred to as a possibility measure. Then, a random fuzzy variable is firstly defined by Liu (2004) as a function from a possibility space to a collection of random variables.

**Definition 2.4** (Random fuzzy variable (Liu, 2004)). A random fuzzy variable is defined as a function from the possibility space $(\Theta, P(\Theta), Pos)$ to the set of random variables.

**Definition 2.5** (Membership function of a random fuzzy variable (Liu, 2004)). Let $\tilde{\xi}$ be a random fuzzy variable on the possibility space $(\Theta, P(\Theta), Pos)$. Then its membership function is derived from the possibility measure $Pos$ by

$$\mu(\bar{\xi}) = Pos(\theta \in \Theta | \tilde{\xi}(\theta)), \bar{\xi} \in \Gamma$$

**Theorem 1** (Li, Xu, and Gen, 2006). Assume that $\tilde{e}_i$, $\tilde{b}_i$ are LR fuzzy random variables, for any $w \in \Omega$ the membership function of $\tilde{e}_i(w)$ and $\tilde{b}_i(w)$ are

$$\mu_{\tilde{e}_i}(t) = \begin{cases} L\left(\frac{e_i(w)(t)}{\bar{\xi}_i(w)}\right), & t \leq e_i(w), \bar{\xi}_i(w) > 0; \\ R\left(\frac{e_i(w)}{e_i(w)}\right), & t \geq e_i(w), \bar{\xi}_i(w) > 0. \end{cases}$$

$$\mu_{\tilde{b}_i}(t) = \begin{cases} L\left(\frac{b_i(w)-1}{\bar{\xi}_i(w)}\right), & t \leq b_i(w), \bar{\xi}_i(w) > 0; \\ R\left(\frac{b_i(w)+1}{b_i(w)}\right), & t \geq b_i(w), \bar{\xi}_i(w) > 0. \end{cases}$$

where $(e_i(w))_{n+1} = (e_1(w), e_2(w), \ldots, e_n(w))^T \sim N(d_i, V_i)$, $b_i(w) \sim N(d_i, \sigma_i^2)$. $\bar{\xi}_i$ and $\bar{\xi}_i$ are the left and right spread of $\tilde{e}_i(w)$ and $\tilde{b}_i(w)$, respectively.

**Definition 2.6** (Bifuzzy variable (Liu, 2002)). A bifuzzy $\xi$ is a variable with fuzzy parameters.

**Theorem 2** (Pramanik, Jana, and Maity, 2014). Assume that $\tilde{e}_{i1}$, $\tilde{b}_i$ are LR bi-fuzzy variables for $\theta \in \Theta$, the membership function of $\tilde{e}_{i1}(\theta)$ and $\tilde{b}_i(\theta)$ are

$$\mu_{\tilde{e}_{i1}(\theta)}(t) = \begin{cases} L\left(\frac{e_{i1}(\theta)-1}{\bar{\xi}_{i1}(\theta)}\right), & t \leq e_{i1}(\theta), \bar{\xi}_{i1}(\theta) > 0; \\ R\left(\frac{e_{i1}(\theta)}{\bar{\xi}_{i1}(\theta)}\right), & t \geq e_{i1}(\theta), \bar{\xi}_{i1}(\theta) > 0. \end{cases}$$

$$\mu_{\tilde{b}_i(\theta)}(t) = \begin{cases} L\left(\frac{b_i(\theta)-1}{\bar{\xi}_{i1}(\theta)}\right), & t \leq b_i(\theta), \bar{\xi}_{i1}(\theta) > 0; \\ R\left(\frac{b_i(\theta)+1}{\bar{\xi}_{i1}(\theta)}\right), & t \geq b_i(\theta), \bar{\xi}_{i1}(\theta) > 0. \end{cases}$$

where the vector $(e_{i1}(\theta))_{n+1} = (e_{i11}, e_{i12}, \ldots, e_{i1n})^T$, $b_i(w)$ are also fuzzy variables which membership function are

$$\mu_{\tilde{e}_{i1}(\theta)}(t) = \begin{cases} L\left(\frac{e_{i1}(\theta)-1}{\bar{\xi}_{i1}(\theta)}\right), & t \leq e_{i1}(\theta), \bar{\xi}_{i1}(\theta) > 0; \\ R\left(\frac{e_{i1}(\theta)}{\bar{\xi}_{i1}(\theta)}\right), & t \geq e_{i1}(\theta), \bar{\xi}_{i1}(\theta) > 0. \end{cases}$$

$$\mu_{\tilde{b}_i(\theta)}(t) = \begin{cases} L\left(\frac{b_i(\theta)-1}{\bar{\xi}_{i1}(\theta)}\right), & t \leq b_i(\theta), \bar{\xi}_{i1}(\theta) > 0; \\ R\left(\frac{b_i(\theta)+1}{\bar{\xi}_{i1}(\theta)}\right), & t \geq b_i(\theta), \bar{\xi}_{i1}(\theta) > 0. \end{cases}$$

and $\bar{\xi}_{i1}$, $\bar{\xi}_{i1}$ are the left and right spread of $\tilde{e}_i$ and $e_i$ for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \bar{\xi}_i, \bar{\xi}_i, \bar{\xi}_i$ and $\bar{\xi}_i$ are the left and right spread of $\tilde{b}_i$ and $b_i$ for $r = 1, 2, \ldots, p, j = 1, 2, \ldots, n$. Then

$$\text{Pos}\left\{\phi \mid \text{Pos}\left(\tilde{e}_{i1}(\theta) \leq b_i(\theta)\right) \geq \theta_i\right\}$$

where $\theta_i, \eta_i \in [0, 1]$ are predetermined confidence level.

### 3. Notations and assumptions

#### 3.1. Notations

For convenience, the following notations are used throughout the entire paper:

- $m$: number of items
- $A_i$: retailer ordering cost per order for $i$th item
- $T_{C_i}$: supplier transportation cost per item
- $S_i$: supplier setup cost per production run for $i$th item
- $P_i$: production rate of the supplier for $i$th item
- $c_i$: retailer procurement cost for $i$th item
- $h_i^T$: inventory holding cost rate, excluding interest charge for $i$th item
- $h_i^S$: supplier’s inventory holding cost rate for $i$th item
- $p_i^T$: retailer’s selling price per unit item for $i$th item
- $d_i$: deterioration rate of the $i$th item for the retailer only
- $T_i$: replenishment time interval of the retailer in a year unit for $i$th item
- $M_i^C$: credit period offered by the supplier to the retailer for $i$th item
- $q_i^C$: interest rate of revenue deposited by the retailer for $i$th item
interest rate to be paid to the supplier for the remaining stock from $M_i$ to $T_i$ for $i$th item

interest rate to calculating supplier’s opportunity interest loss due to delay payment for $i$th item

the number of replenishment of retailer for $i$th item

initial quantity which is taken by the retailer for a cycle for the supplier for $i$th item

demand rate = $\frac{\alpha_i}{a+(\beta_i q_i(t))}$ for all $i$

retailer storage area for $i$th item (per unit)

supplier storage area for $i$th item (per unit)

total storage area of the supply chain total budget of the supply chain

3.2. Assumptions

The mathematical model of the inventory replenishment problem is based on the following assumptions:

(i) The model deals with single supplier single retailer and multi product.

(ii) The supplier produces the items and then fulfill the retailer demands simultaneously.

(iii) Demand rate is assumed to be depend on the existing stock level.

(iv) In this production-inventory system the whole business period is assumed to be one year i.e. time horizon is finite.

(v) It is assumed that the credit period ($M_i$) offered by supplier must be within each replenishment period ($T_i$) (i.e. $M_i \leq T_i$) for $i$th item. Because in this paper production-inventory model is considered for multi-periods i.e. for finite time horizon with multiple replenishment, for the convenience of mathematical representation only $M_i \leq T_i$ for $i$th item is considered. It has a physical/practical consideration also. Normally in developing countries like India, Bangladesh, etc., suppliers do not take much risk in giving credits. The suppliers insist on the retailers that you (retailers) clear your dues before taking the next installment of goods i.e. in that case $M_i \leq T_i$ is always valid.

(vi) Shortages are not allowed.

(vii) Here the deterioration is consider only for retailer.

(viii) Supplier charges an interest at the rate of $\frac{i}{s}$ on the remaining amount of stock after the credit period $M_i$ for each $i$th item.

(ix) Retailer procurement cost ($c_i$) depends on the credit period ($M_i$) by the relation $c_i = c_0^i + c_1^i e^{t_i}$, where $c_0^i$ is the procurement cost in the absence of credit period and $c_1^i > 0$ for each $i$th item.

(x) Supplier transportation cost ($TC_i$) depends on the quantity purchased by the retailer by the relation $TC_i = TC_0^i + TC_1^i$ for each item.

(xi) $F_i(P_i) = R_i + \frac{P_i}{P_0} + B_i P_i^\gamma$ = unit production cost is related to production rate $P_i$, where $R_i$, $G_i$, $B_i$ and $\gamma$ are non negative real numbers to be chosen to provide the best fit for the estimated unit production cost function. $R_i$ = cost component independent of production rate. This cost component includes raw material. $\frac{P_i}{P_0}$ = cost component per unit that decreases with increase of production rate. $K_i P_i^\gamma$ = cost component per unit that increases with increases of production rate.

(xii) Retailer and supplier desire to settle the credit period $M_i$ and replenishment period $T_i$ for each $i$th item in such a way that the system cost is minimum.

4. MISCM formulation for deteriorating item

The production of supplier begins at time $t = 0$ at the rate of $P_i$ for $i$th item. At first, after the replenishment period of the retailer $T_i$ (for $i$th item), the retailer receives the amount $Q_i$ (for $i$th item) from the supplier. Then retailer fulfills the demands of the customers from his/her stock during the time period $T_i$ and then the retailer again demand the amount to the supplier and supplied the same amount $Q_i$ from the supplier after the time period $T_i$. This procedure continues up to time $n_i T_i$, where $n_i$ is the total number of replenishment of retailer for $i$th item and at the time $(n_i + 1)T_i$, the stock of the retailer vanishes. The behavior of the integrated inventory system in one year business period is shown in Fig. 1.

4.1. Retailer inventory

The differential equation describing the inventory level $q_i(t)$ for $i$th item for each replenishment cycle is given by

$$\frac{dq_i(t)}{dt} + \theta_i q_i(t) = - (\alpha_i + \beta_i q_i(t)), 0 \leq t \leq T_i$$

with boundary conditions

$$q_i(0) = Q_i$$ and $q_i(T_i) = 0$

Solving the differential Eq. (1) using the boundary condition (2), we have

$$q_i(t) = \frac{\alpha_i}{(\beta_i + \theta_i)} (e^{(\beta_i + \theta_i)T_i - t} - 1)$$

and

$$Q_i = \frac{\alpha_i}{(\beta_i + \theta_i)} e^{(\beta_i + \theta_i)T_i} - 1.$$  (4)

Retailer average ordering cost ($ROC_i$) for $i$th items is given by

$$ROC_i = \frac{A_i}{T_i}$$  (5)

Retailer average holding cost ($RHC_i$) for $i$th items is given by

$$RHC_i = c_i h_i^R \int_0^{T_i} q_i(t) dt = c_i h_i^R \frac{\alpha_i}{(\beta_i + \theta_i)} \int_0^{T_i} (e^{(\beta_i + \theta_i)T_i - t} - 1) dt$$

$$= \frac{c_i h_i^R A_i}{T_i (\beta_i + \theta_i)} \left[ \frac{1}{(\beta_i + \theta_i)} (e^{(\beta_i + \theta_i)T_i} - 1) - T_i \right].$$  (6)

Retailer average deterioration cost ($RDC_i$) for $i$th items is given by

![Fig. 1. Supply chain model.](image-url)
\[ RDC_i = \frac{C_i}{P_i} \left[ Q_i - \int_0^{T_i} D_i(t) \, dt \right] = \frac{C_i}{P_i} \left[ Q_i - \int_0^{T_i} (x_i + \beta_i q_i(t)) \, dt \right] = \frac{C_i}{P_i} \left[ Q_i - \int_0^{T_i} \left( x_i + \beta_i \left( \frac{x_i}{(\beta_i + 0)} \left( e^{(\beta_i + 0)(T_i - t)} - 1 \right) \right) \right) \, dt \right] = \frac{C_i}{P_i} \left[ Q_i - \alpha_i T_i + \frac{\beta_i x_i}{(\beta_i + 0)^2} \left( 1 - e^{(\beta_i + 0)T_i} \right) + \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right]. \] 

Retailer average interest charged \( (RIC_i) \) for ith items given by
\[ RIC_i = \frac{C_i q_i^2}{P_i} \int_0^{T_i} D_i(t) \, dt = \frac{C_i q_i^2}{P_i} \int_0^{T_i} \left( x_i + \beta_i q_i(t) \right) \, dt = \frac{C_i q_i^2}{P_i} \left[ \frac{1}{(\beta_i + 0)} (e^{(\beta_i + 0)(T_i - t)} - 1) - (T_i - M_i) \right] \]

Retailer average interest earned \( (RIE_i) \) for ith items given by
\[ RIE_i = \frac{P_i q_i^2}{P_i} \int_0^{T_i} D_i(t) \, dt - \frac{P_i q_i^2}{P_i} \int_0^{T_i} (x_i + \beta_i q_i(t)) \, dt = \frac{P_i q_i^2}{P_i} \left[ \frac{1}{(\beta_i + 0)^2} (1 - e^{(\beta_i + 0)T_i}) + \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right] \times \frac{C_i q_i^2}{P_i} \left( \frac{1}{(\beta_i + 0)^2} (1 - e^{(\beta_i + 0)T_i}) + \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right) \]

Therefore, the retailer’s average total cost for ith item \( (TR_i) \) is given by
\[ TR_i = ROC_i + RHC_i + RDC_i + RIC_i - RIE_i \]

Therefore, the retailer’s average total cost \( (TR) \) is given by
\[ TR = \sum_{i=1}^{m} TR_i = \sum_{i=1}^{m} (ROC_i + RHC_i + RDC_i + RIC_i - RIE_i) = \sum_{i=1}^{m} \left( A_i + \frac{C_i q_i^2}{P_i} \left( \frac{1}{(\beta_i + 0)^2} (1 - e^{(\beta_i + 0)T_i}) - (T_i - M_i) \right) \right) \]

\[ \sum_{i=1}^{m} \left[ \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right] \times \frac{C_i q_i^2}{P_i} \left( \frac{1}{(\beta_i + 0)^2} (1 - e^{(\beta_i + 0)T_i}) + \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right) \]

4.2 Supplier inventory

The supplier’s inventory starts initially from time \( t = 0 \) and will vanished at time \( n_iT_i \) for ith item, at the moment when retailer receives last replenishment of the ith item. The supplier’s production has continues up to a certain time of the cycle for the ith item. Therefore the inventory level of supplier increases up-to this time and after every cycle time \( T_i \), the inventory level of supplier decreases of amount \( Q_i \) due to demand \( (D_i) \) of the retailer for the ith item, as shown in Fig. 1.

Supplier average inventory cost for ith item \( (SIC_i) \) is given by
\[ SIC_i = \frac{h_i}{n_i T_i} \left[ n_i Q_i \left( \frac{1}{2} - \frac{n_i Q_i}{P_i} \right) - Q_i (T_i + 2T_i + \ldots + (n_i - 1)T_i) \right] \]

\[ = \frac{h_i}{2} \left( n_i + 1 \right) - \frac{n_i Q_i}{P_i} \]

Supplier average setup cost \( (SSC_i) \) for ith items is given by
\[ SSC_i = \frac{S_i}{n_i T_i} \]

Supplier average production cost \( (SPC_i) \) for ith items is given by
\[ SPC_i = \frac{F(P_i) Q_i}{T_i} \]

Supplier average opportunity interest loss \( (SOIL_i) \) for ith item is given by
\[ SOIL_i = \frac{q_i c_i M_i Q_i}{T_i} \]

Supplier average transportation cost \( (STC_i) \) for ith item is given by
\[ STC_i = \frac{T_i}{T_i} \]

Therefore, the supplier’s average total cost \( (TS_i) \) for ith item is given by
\[ TS_i = SIC_i + SPC_i + SOIC_i + SSC_i + STC_i \]

Therefore, the supplier’s average total cost \( (TS) \) is given by
\[ TS = \sum_{i=1}^{m} TS_i = \sum_{i=1}^{m} \left( \frac{h_i Q_i}{2} \left( \frac{1}{2} - \frac{n_i Q_i}{P_i} \right) + \frac{P_i Q_i}{T_i} + \frac{q_i c_i M_i Q_i}{T_i} + \frac{S_i}{n_i T_i} + \frac{T_i}{T_i} \right) \]

Therefore, for the integrated model in the multi-item stock dependent consumption production supply chain model with deterioration, the total cost \( (TSC) \) of the system is given by
\[ TSC(T_i, M_i) = \sum_{i=1}^{m} (TR_i + TS_i) = \sum_{i=1}^{m} \left( A_i + \frac{C_i q_i^2}{P_i} \left( \frac{1}{(\beta_i + 0)^2} (1 - e^{(\beta_i + 0)T_i}) - (T_i - M_i) \right) \right) \]

\[ + \sum_{i=1}^{m} \left[ \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right] \times \frac{C_i q_i^2}{P_i} \left( \frac{1}{(\beta_i + 0)^2} (1 - e^{(\beta_i + 0)T_i}) + \frac{\beta_i x_i}{(\beta_i + 0)} T_i \right) \]

4.2 Supplier inventory

The supplier’s inventory starts initially from time \( t = 0 \) and will vanished at time \( n_iT_i \) for ith item, at the moment when retailer receives last replenishment of the ith item. The supplier’s production has continues up to a certain time of the cycle for the ith item. Therefore the inventory level of supplier increases up-to this time and after every cycle time \( T_i \), the inventory level of supplier decreases of amount \( Q_i \) due to demand \( (D_i) \) of the retailer for the ith item, as shown in Fig. 1.

Supplier average inventory cost for ith item \( (SIC_i) \) is given by
\[ SIC_i = \frac{h_i}{n_i T_i} \left[ n_i Q_i \left( \frac{1}{2} - \frac{n_i Q_i}{P_i} \right) - Q_i (T_i + 2T_i + \ldots + (n_i - 1)T_i) \right] \]

5. MISCM for non-deteriorating item

The multi-item supply chain model for non-deteriorating items will be formed by making \( b_i \to 0 \) for ith item.
5.1. Retailer inventory

Retailer average ordering cost for non-deteriorating item (NROC) is given by
\[
NROC_i = \frac{A_i}{T_i} \tag{22}
\]

Retailer average holding cost for non-deteriorating item (NRHC) is given by
\[
NRHC_i = \lim_{n_i \to 0} RHC_i \]
\[
= \lim_{n_i \to 0} \frac{c_i h_i}{T_i} \left(1 - e^{-\frac{T_i}{\beta_i}}\right) = \frac{c_i h_i}{T_i} \left(1 - e^{-\frac{T_i}{\beta_i}}\right) \tag{23}
\]

Retailer average deterioration cost for non-deteriorating item (NRDC) is given by
\[
NRDC_i = \lim_{n_i \to 0} RDC_i \]
\[
= \lim_{n_i \to 0} \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) = \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) \tag{24}
\]

Retailer average interest charged for non-deteriorating item (NRIC) is given by
\[
NRIC_i = \lim_{n_i \to 0} RIC_i \]
\[
= \lim_{n_i \to 0} \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) = \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) \tag{25}
\]

Retailer average interest earned for non-deteriorating item (NRIE) is given by
\[
NRIE_i = \lim_{n_i \to 0} RIE_i \]
\[
= \lim_{n_i \to 0} \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) = \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) \tag{26}
\]

Therefore, the retailer’s average total cost for non-deteriorating item (NTR) is given by
\[
NTR_i = NRDC_i + NRHC_i + NRIC_i - NRIE_i \tag{27}
\]

Therefore, the retailer’s average total cost for non-deteriorating item (NTR) is given by
\[
NTR = \sum_{i=1}^{m} NTR_i = \sum_{i=1}^{m} (NRDC_i + NRHC_i + NRIC_i - NRIE_i) \]
\[
= \sum_{i=1}^{m} \frac{A_i}{T_i} + \frac{c_i h_i}{T_i} \left(1 - e^{-\frac{T_i}{\beta_i}}\right) + \frac{C_i}{T_i} \left(\frac{Q_i}{N_i} - \frac{\alpha_i}{\beta_i} (1 - e^{-\frac{T_i}{\beta_i}})\right) \tag{28}
\]

5.2. Supplier inventory

Supplier average inventory cost for non-deteriorating item (NSIC) is given by
\[
NSIC_i = \lim_{n_i \to 0} SIC_i = \lim_{n_i \to 0} \frac{h_i Q_i}{2 T_i} \left(\frac{n_i + 1}{\beta_i} \right) = \frac{h_i Q_i}{2 T_i} \left(\frac{n_i + 1}{\beta_i} \right) \tag{29}
\]

Supplier average setup cost for non-deteriorating item (NSSC) is given by
\[
NSSC_i = \frac{S_i}{n_i T_i} \tag{30}
\]

Supplier average production cost for non-deteriorating item (NSPC) is given by
\[
NSPC_i = \lim_{n_i \to 0} SPC_i = \lim_{n_i \to 0} \frac{F(P_i) Q_i}{T_i} = \frac{F(P_i) Q_i}{T_i} \tag{31}
\]

Supplier average opportunity interest loss for non-deteriorating item (NSOIL) is given by
\[
NSOIL_i = \lim_{n_i \to 0} SOIL_i = \lim_{n_i \to 0} \frac{q_i c_i M_i}{T_i} = \frac{q_i c_i M_i}{T_i} \tag{32}
\]

Supplier average transportation cost for non-deteriorating item (NSTC) is given by
\[
NSTC_i = \lim_{n_i \to 0} STC_i = \lim_{n_i \to 0} \frac{T_i C_i}{T_i} = T_i C_i \tag{33}
\]

Therefore, the supplier’s average total cost for non-deteriorating item (NTS) is given by
\[
NTS = NSIC_i + NSPC_i + NSOIL_i + NSSC_i + NSTC_i \tag{34}
\]

Therefore, the supplier’s average total cost for non-deteriorating item (NTS) is given by
\[
NTS = \sum_{i=1}^{m} NTS_i = \sum_{i=1}^{m} (NSIC_i + NSPC_i + NSOIL_i + NSSC_i + NSTC_i) \]
\[
= \sum_{i=1}^{m} \left(\frac{h_i Q_i}{2 T_i} \left(\frac{n_i + 1}{\beta_i} \right) \right) + \frac{F(P_i) Q_i}{T_i} + \frac{q_i c_i M_i}{T_i} (e^{\frac{Ti}{\beta_i}} - 1) \tag{35}
\]

Therefore, for the integrated model in the multi-item stock dependent consumption production supply chain model with non-deterioration, the total cost (NTS) of the system is given by
\[
NTS(T_i, M_i) = NTR + NTS \tag{36}
\]

where NTR and NTS are given by (28) and (35) respectively.

6. Deterministic MISCM model

Our objective is to minimize TSC and NTSC under the space constraints and budget constraints. So the supply chain deterministic model is given by.

6.1. Deterministic model for deteriorating items (Model-1)

Minimize \[ TSC \]
subject to \[ \sum_{i=1}^{m} (W_i + W_i^*) Q_i \leq W \]
\[ \sum_{i=1}^{m} (F(P_i) P_i + c_i Q_i) \leq B \]
where TSC is given by (21).
6.2. Deterministic model for non-deteriorating items (Model-2)

Minimize \( \text{NTSC} \)

subject to \[
\sum_{i=1}^{m} (w_i + w'_i)Q_i \leq W \\
\sum_{i=1}^{m} (F(P_i)P_i + c_iQ_i) \leq B 
\]

where \( \text{NTSC} \) is given by (36).

7. MISCM under imprecise environment

In real life situation various parameter like ordering cost, procurement cost, selling price of retailer and holding cost, production cost, transportation cost, setup cost of supplier and total storage area and total budget of supply chain uncertain in nature. In this section MISCM models are considered by using those parameters in various imprecise environments. The models are solved using the chance constraints methods.

7.1. MISCM for deteriorating items under imprecise environment

Minimize \( \overline{TSC} \)

subject to \[
\sum_{i=1}^{m} (w_i + w'_i)Q_i \leq \bar{W} \\
\sum_{i=1}^{m} (\bar{F}(P_i)P_i + \bar{c}_iQ_i) \leq \bar{B} 
\]

where \( \overline{TSC} \) is given by (37).

7.1.1. Fuzzy random model for deteriorating items (Model-3)

To deal with real life problem, MISCM model in Eq. (37), parameters \( \bar{A}_i, \bar{c}_i, \bar{P}_i, \bar{R}_i, \bar{K}, \bar{C}_i, \bar{T}_C, \bar{T}_C^c, \bar{S}_i, \bar{B} \) are taken as fuzzy random in nature and \( W \) take as fuzzy in nature. Then due to our assumption objective function \( TSC \) becomes \( \overline{TSC} \). Since optimization under imprecise environment is not well defined, so we first convert the problem into equivalent deterministic or pessimistic problem. We use chance constraints method to convert the problem (37) into equivalent deterministic problem to determine \( T \) and \( M \) so as to

Minimize \( F \)

subject to \[
\begin{align*}
\text{Pr}\left\{w|\text{Pos}\left\{\overline{TSC}x \leq F\right\} \geq \mu_1\right\} & \geq \phi \\
\text{Pr}\left\{\sum_{i=1}^{m} (w_i + w'_i)Q_i \leq W\right\} & \geq \mu_2 \\
\text{Pr}\left\{w|\text{Pos}\left\{\sum_{i=1}^{m} (\bar{F}(P_i)P_i + \bar{c}_iQ_i) \leq \bar{B}\right\} \geq \mu_3\right\} & \geq \psi 
\end{align*}
\]

Following Li et al. (2006), Liu and Iwamura (1998) and Theorem 1, expression in (39) can be written as

Minimize \( F \)

subject to \[
\begin{align*}
\text{Pr}\left\{\Phi^{-1}(\phi) \left[ x^2 \left( \frac{1}{\pi} \right)^2 - L^{-1}(\phi) \right] \text{NSC}x + d \text{NSC}x \leq F \right\} \\
\sum_{i=1}^{m} (w_i + w'_i)Q_i \leq W_1 + (1 - \mu_2)(W_2 - W_1) \\
L^{-1}(\mu_3) \sum_{i=1}^{m} (\bar{F}(P_i)P_i + \bar{c}_iQ_i) - \sum_{i=1}^{m} (d_{\bar{F}(P_i)P_i} + \bar{d}_{\bar{c}_iQ_i}) \\
- \Phi^{-1}(\psi) \sum_{i=1}^{m} \left( \left( \frac{w_i}{\pi} \right)^2 \frac{1}{\pi} + \left( \frac{w'_i}{\pi} \right)^2 \frac{1}{\pi} \right) + \left( \frac{w_i}{\pi} \right)^2 \frac{1}{\pi} + \left( \frac{w'_i}{\pi} \right)^2 \frac{1}{\pi} \\
+ R^{-1}(\mu_3)\bar{b}^2 + \bar{d}^2 & \geq 0 
\end{align*}
\]

the above problem has been optimized via contracting mapping genetic algorithm Section 8.1.

7.1.2. Bifuzzy model for deteriorating items (Model-4)

In MISCM model (37), parameters \( \bar{A}_i, \bar{c}_i, \bar{P}_i, \bar{R}_i, \bar{K}, \bar{C}_i, \bar{T}_C, \bar{T}_C^c, \bar{S}_i, \bar{B} \) are taken in bi-fuzzy in nature and \( W \) take as fuzzy in nature. Since optimization under imprecise environment is not well defined, so we first convert the problem into equivalent optimistic or pessimistic problem. We use chance constraints method to convert the problem (37) into equivalent deterministic problem to determine \( T \) and \( M \) so as to

Minimize \( F \)

subject to \[
\begin{align*}
\text{Pos}\left\{w|\text{Pos}\left\{\overline{TSC}x \leq F\right\} \geq \mu_1\right\} & \geq \phi \\
\text{Pos}\left\{\sum_{i=1}^{m} (w_i + w'_i)Q_i \leq W\right\} & \geq \mu_2 \\
\text{Pos}\left\{w|\text{Pos}\left\{\sum_{i=1}^{m} (\bar{F}(P_i)P_i + \bar{c}_iQ_i) \leq \bar{B}\right\} \geq \mu_3\right\} & \geq \psi 
\end{align*}
\]

Following Pramanik et al. (2014), Liu and Iwamura (1998), Das et al. (2007) and Theorem 2 the expression in (42) can be written as

Minimize \( F \)

subject to \[
\begin{align*}
\text{NSC} - L^{-1}(\phi)\frac{\text{NSC}}{\pi}x - L^{-1}(\mu_2)\frac{\text{NSC}}{\pi}x \leq F \\
\sum_{i=1}^{m} (w_i + w'_i)Q_i \leq W_1 + (1 - \mu_2)(W_2 - W_1) \\
R^{-1}(\mu_3)\bar{b}^2 + L^{-1}(\mu_3)\sum_{i=1}^{m} (\bar{F}(P_i)P_i + \bar{c}_iQ_i) - F(P_i)P_i + c_iQ_i \\
+ B + L^{-1}(\phi)\sum_{i=1}^{m} (\bar{F}(P_i)P_i + \bar{c}_iQ_i) - F(P_i)P_i + c_iQ_i & \geq 0 
\end{align*}
\]
7.2. MISCM for non-deteriorating items under imprecise environment

Minimize \( \text{NTSC} \)

subject to \( \sum_{i=1}^{m} (\tilde{W}_i + \tilde{w}_i) Q_i \leq \tilde{W} \)

\[
\sum_{i=1}^{m} \left( \tilde{F}(P_i) P_i + \tilde{e}_i Q_i \right) \leq \tilde{B}
\]

where \( \text{NTSC} = \sum_{i=1}^{m} \left( \tilde{A}_i \tilde{h}_i \tilde{b}_i \left( \frac{1}{\beta_i} \left( e^{\tilde{h}_i T_i} - 1 \right) - T_i \right) \right. + \tilde{c}_i \tilde{q}_i \tilde{r}_i \left( \frac{1}{\beta_i} \left( \frac{1}{\beta_i} \left( e^{\tilde{h}_i T_i} - 1 \right) - (T_i - M_i) \right) \right) - \tilde{P}(\tilde{T}) \left( \frac{1}{\beta_i} \left( \frac{1}{\beta_i} \left( e^{\tilde{h}_i T_i} - 1 \right) - (T_i - M_i) \right) \right) + \left. \sum_{i=1}^{m} \tilde{c}_i \tilde{q}_i \left( e^{\tilde{h}_i T_i} - 1 \right) \left( (n_i + 1) - \frac{n_i \tilde{a}_i \tilde{P}_i \tilde{T}_i}{\beta_i} (e^{\tilde{h}_i T_i} - 1) \right) \right) \]

\[+ \tilde{S}_i \frac{1}{\beta_i} \tilde{h}_i + \tilde{T}_i \frac{1}{\beta_i} \tilde{h}_i \left( \frac{1}{\beta_i} \left( e^{\tilde{h}_i T_i} - 1 \right) \right) \]

and \( \tilde{F}(P_i) = \tilde{R}_i + \tilde{c}_i + \tilde{P}_i \tilde{r}_i \). \( \tilde{c}_i = \tilde{c}_i^0 + \tilde{c}_i^1 M_i \), where \( \tilde{a} \) represents that, the parameter \( \tilde{a} \) is in imprecise environments.

7.2.2. Bifuzzy model for non-deteriorating items (Model-6)

For MISCM model in Eq. (45), parameters \( \tilde{A}_i, \tilde{c}_i, \tilde{P}_i, \tilde{R}_i, \tilde{T}_i, \tilde{C}_i, \tilde{S}_i, \tilde{B} \) are taken in fuzzy random environment and \( \tilde{W} \) in fuzzy environment. Using chance constraints method (45) can be written as

Minimize \( F \)

subject to \( \Phi^{-1}(\phi) \sqrt{\hat{x}^2 \left( \frac{\sigma_{\text{NTSC}}}{\mu_{\text{NTSC}}} \right)^2} + L^{-1}(v_1) \frac{\theta_{\text{NTSC}}}{\mu_{\text{NTSC}}} x + d_{\text{NTSC}} x \leq F \)

and (40)–(41)

where \( Q_i = \frac{1}{\tilde{h}_i} (e^{\tilde{h}_i T} - 1) \).

8. Genetic Algorithm

Genetic Algorithms (GA) are heuristic search process for optimization that resembles natural selection. Because of its generality and other advantages over conventional optimization methods it has been successfully applied to different decision making problems. Genetic algorithms find application in bioinformatics, phylogenetics, computational science, engineering, economics, chemistry, manufacturing, mathematics, physics, pharmacometrics and other fields. Normally a GA starts with a set of potential (feasible) solutions (called initial population) of the decision making problem under consideration. Stochastic operations crossover and mutation are made on the solutions to get improve set of solutions and it continues until termination conditions are satisfied. Here a special type of GA is used to solve our model where movement to new population from old population takes place only when average fitness of solutions of new population is better than the average fitness of old population. Such GA is named as contractive mapping genetic algorithm (CMGA). Michaelewicz (1992) proposed the algorithm and proved the asymptotic convergence of the algorithm by Banach fixed point theorem. A CMGA in general form is given below.

8.1. CMGA Algorithm

1. Set iteration counter \( T = 0 \).
2. Initialize probability of crossover \( p_c \) and probability of mutation \( p_m \).
3. Initialize \( P(T) \).
4. Evaluate \( P(T) \).
5. Repeat
   a. Select \( N \) solutions from \( P(T) \), for mating pool using Roulette-wheel selection process. Let this set be \( P(T)^1 \).
   b. Select solutions from \( P(T)^1 \), for crossover depending on \( p_c \).
   c. Made crossover on selected solutions for crossover to get population \( P(T)^2 \).
   d. Select solutions from \( P(T)^2 \), for mutation depending on \( p_m \).
   e. Made mutation on selected solutions for mutation to get population \( P(T + 1) \).
   f. Evaluate \( P(T + 1) \).
   g. If average fitness of \( P(T + 1) \) > average fitness of \( P(T) \)
   h. \( T \rightarrow T + 1 \).
6. Until (Termination condition does not hold).
7. Output: Fittest solution (chromosome) of \( P(T) \).

The stepwise procedure of this CMGA is shown as below:

8.2. CMGA procedures for the proposed model

(a) Representation: A \( n \)-dimensional real vector \( Q = (q_1, q_2, \ldots, q_n) \) is used to represent a solution, where \( q_1, q_2, \ldots, q_n \) represent different decision variable of the problem.

(b) Initialization: Initially many individual solutions are (usually) randomly generated to form an initial population. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions. Traditionally, the population is generated randomly, allowing the entire range of possible solutions (the search space). Occasionally, the solutions may be “seeded” in areas where optimal solutions are likely to be found. \( N \) such solutions \( Q_1, Q_2, \ldots, Q_N \) are randomly generated by random number generator such that each solution satisfies the resource constraints of the problem.

(c) Fitness value: The value of the objective function \( Z(Q) \) due to the potential solution \( Q = (q_1, q_2, \ldots, q_n) \) is taken as fitness value.

(d) Selection process for mating pool: The following steps are followed for this purpose:

(i) Find total fitness of the population

\[ F = \sum_{i=1}^{N} Z(Q_i) \]
Table 2
Input data in deterministic environments.

<table>
<thead>
<tr>
<th>$A_1$, $A_2$, $A_3$</th>
<th>$P_1$, $P_2$, $P_3$</th>
<th>$q_1^t$, $q_2^t$, $q_3^t$</th>
<th>$c_1^t$, $c_2^t$, $c_3^t$</th>
<th>$c_1$, $c_2$, $c_3$</th>
<th>$q_0^t$, $q_1^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[130, 140, 160]</td>
<td>[30, 20, 25]</td>
<td>[0.05, 0.03, 0.03]</td>
<td>[25, 35, 40]</td>
<td>[0.8, 0.7, 0.9]</td>
<td>[0.07, 0.06, 0.05]</td>
</tr>
<tr>
<td>$G_0 = 0.65$</td>
<td>$G_1 = 0.85$</td>
<td>[0.04, 0.04, 0.03]</td>
<td>[1500, 1200, 150]</td>
<td>[550, 500, 450]</td>
<td>[0.6, 0.5, 0.4]</td>
</tr>
<tr>
<td>$G_i$, $G_j$, $G_k$</td>
<td>$G_i$, $G_j$, $G_k$</td>
<td>[0.0006, 0.0005, 0.0007]</td>
<td>[150, 160, 155]</td>
<td>[0.05, 0.10, 0.15]</td>
<td>[15, 18, 20]</td>
</tr>
<tr>
<td>$[TC_1^t, TC_2^t, TC_3^t]$</td>
<td>$[w_1^t, w_2^t, w_3^t]$</td>
<td>[5, 6, 7]</td>
<td>[15, 16, 17]</td>
<td>[3000, 3500, 3300]</td>
<td>98000</td>
</tr>
</tbody>
</table>

Table 3
Optimal result of Model-1 and Model-2.

<table>
<thead>
<tr>
<th>Item</th>
<th>$n_i$</th>
<th>$c_i$</th>
<th>$M_i$</th>
<th>$T_i$</th>
<th>$Q_i$</th>
<th>$TR$</th>
<th>$TS$</th>
<th>$TSC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic model with deterioration (Model-1)</td>
<td>1</td>
<td>6</td>
<td>25.85145</td>
<td>0.062330</td>
<td>0.1666</td>
<td>96.816</td>
<td>5311.980</td>
<td>50482.862</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>35.73707</td>
<td>0.051611</td>
<td>0.1666</td>
<td>87.642</td>
<td>5112.954</td>
<td>50795.037</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>40.95524</td>
<td>0.059574</td>
<td>0.1666</td>
<td>78.544</td>
<td>5161.928</td>
<td>50892.981</td>
</tr>
<tr>
<td>Alternative solution of Model-1</td>
<td>1</td>
<td>6</td>
<td>25.85354</td>
<td>0.051345</td>
<td>0.1666</td>
<td>96.816</td>
<td>5317.675</td>
<td>50477.403</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>35.73495</td>
<td>0.048728</td>
<td>0.1666</td>
<td>87.642</td>
<td>5122.954</td>
<td>50795.037</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>40.94744</td>
<td>0.051345</td>
<td>0.1666</td>
<td>78.544</td>
<td>5161.928</td>
<td>50892.981</td>
</tr>
<tr>
<td>Deterministic model without deterioration (Model-2)</td>
<td>1</td>
<td>6</td>
<td>25.85387</td>
<td>0.065175</td>
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<td>2</td>
<td>6</td>
<td>35.73697</td>
<td>0.051466</td>
<td>0.1666</td>
<td>86.904</td>
<td>4842.485</td>
<td>50183.675</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>40.95759</td>
<td>0.062033</td>
<td>0.1666</td>
<td>77.556</td>
<td>4842.485</td>
<td>50183.675</td>
</tr>
<tr>
<td>Alternative solution of Model-2</td>
<td>1</td>
<td>6</td>
<td>25.85277</td>
<td>0.063885</td>
<td>0.1666</td>
<td>96.406</td>
<td>4844.042</td>
<td>50182.134</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>35.73592</td>
<td>0.050053</td>
<td>0.1666</td>
<td>86.904</td>
<td>4844.042</td>
<td>50182.134</td>
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<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>40.95807</td>
<td>0.062531</td>
<td>0.1666</td>
<td>77.556</td>
<td>4844.042</td>
<td>50182.134</td>
</tr>
</tbody>
</table>

Fig. 2. Objective value and iteration number by GA for Model-1.

Fig. 3. Objective value and iteration number by GA for Model-2.

(ii) Calculate the probability of selection $f_i$ of each solution $Q_i$ by the formula
$$f_i = P(Q_i) / F.$$ 

(iii) Calculate the cumulative probability $c_{p_i}$ for each solution $Q_i$ by the formula
$$c_{p_i} = \sum_{j=1}^{i} f_j.$$ 

(iv) Generate a random number ‘r’ from the range [0, 1].

(v) If $r < c_{p_1}$ then select $Q_1$ otherwise select $Q_i$ $(2 \leq i \leq N)$ where $c_{p_{i-1}} < r < c_{p_i}$.

(vi) Repeat step (iv) and (v) $N$ times to select $N$ solutions for mating pool. Clearly one solution may be selected more than once.

(vii) Selected solution set is denoted by $P(T)$ in the proposed GA algorithm.

(e) Crossover:

(i) Selection for crossover: For each solution of $P(T)$ generate a random number $r$ from the range [0, 1]. If $r < p_c$, then the solution is taken for crossover, where $p_c$ is the probability of crossover.
(ii) Crossover process: Crossover taken place on the selected solutions. For each pair of coupled solutions \(Y_1, Y_2\) a random number \(c\) is generated from the range \([0,1]\) and \(Y_1, Y_2\) are replaced by their offspring's \(Y_{11}\) and \(Y_{21}\) respectively where \(Y_{11} = cy_1 + (1-c)y_2\), \(Y_{21} = cy_2 + (1-c)y_1\).

(f) Mutation:
(i) Selection for mutation: For each solution of \(P^i\) generate a random number \(r\) from the range \([0,1]\). If \(r < p_m\) then the solution is taken for mutation, where \(p_m\) is the probability of mutation.
(ii) Mutation process: To mutate a solution \(Q = (q_1, q_2, \ldots, q_n)\) select a random integer \(r\) in the range \([1, n]\). Then replace \(q_r\) by randomly generated value within the boundary of \(r\)th component of \(Q\).

(g) Implementation and usage: The algorithm is implemented to optimize the objective function of the proposed models. Each set of numerical data the program is run about five times and the best result found are taken as near optimum solution.

Table 4
Optimal result of MISCM (Model-1) for different value of \(h\).

<table>
<thead>
<tr>
<th>Rate of deterioration</th>
<th>Item</th>
<th>(n_i)</th>
<th>(c_i)</th>
<th>(M_i)</th>
<th>(T_i)</th>
<th>(Q_i)</th>
<th>TR</th>
<th>TS</th>
<th>TSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.05</td>
<td>1</td>
<td>6</td>
<td>25.8531</td>
<td>0.064285</td>
<td>0.1666</td>
<td>97.228</td>
<td>5537.173</td>
<td>50637.035</td>
<td>56174.209</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>35.7362</td>
<td>0.050541</td>
<td>0.1666</td>
<td>88.014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>40.9562</td>
<td>0.060659</td>
<td>0.1666</td>
<td>78.878</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.10</td>
<td>1</td>
<td>7</td>
<td>25.8457</td>
<td>0.055641</td>
<td>0.1428</td>
<td>82.935</td>
<td>5747.311</td>
<td>50795.900</td>
<td>56543.211</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>35.7358</td>
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<td>88.389</td>
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</tr>
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<td>7</td>
<td>40.9479</td>
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<td></td>
<td></td>
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<tr>
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<td>7</td>
<td>25.8454</td>
<td>0.055258</td>
<td>0.1428</td>
<td>83.237</td>
<td>5918.245</td>
<td>50951.173</td>
<td>56869.419</td>
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<td>7</td>
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<td>40.9476</td>
<td>0.051590</td>
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<td>67.609</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>+0.30</td>
<td>1</td>
<td>7</td>
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<td>0.054230</td>
<td>0.1428</td>
<td>84.152</td>
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<td>51345.170</td>
<td>57852.003</td>
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<tr>
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<td>76.223</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>40.9466</td>
<td>0.050565</td>
<td>0.1428</td>
<td>68.351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.35</td>
<td>1</td>
<td>8</td>
<td>25.8392</td>
<td>0.047944</td>
<td>0.1250</td>
<td>73.231</td>
<td>6699.982</td>
<td>51478.602</td>
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<tr>
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<td></td>
</tr>
<tr>
<td></td>
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<td>7</td>
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<td>0.050221</td>
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<td>68.601</td>
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<td>0.047697</td>
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<td>73.465</td>
<td>6824.936</td>
<td>51647.002</td>
<td>58471.938</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>35.7265</td>
<td>0.037273</td>
<td>0.1250</td>
<td>66.574</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td>40.9408</td>
<td>0.044379</td>
<td>0.1250</td>
<td>59.726</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.45</td>
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<td>8</td>
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<td>0.047421</td>
<td>0.1250</td>
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<td>6997.530</td>
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</tr>
<tr>
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<td>0.044255</td>
<td>0.1250</td>
<td>59.916</td>
<td></td>
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</tr>
</tbody>
</table>
9. A case study

A fruit-vegetable products factory of Kolkata, West Bengal, India manufactured three types of fruit-vegetable products and distributed it to retailer. Depending on the demand of market owner of fruit-vegetable factory offer credit period to the retailer. Data during one year production and retailer commercializing are collected and given in Table 2. Recently many researchers (Guchhait, Maiti, & Maiti, 2013; Maiti, 2011) have developed the genetic algorithm (GA) to solve optimization problem. Here GA (Section 8) is used to solve the deterministic models (Model-1 and Model-2), fuzzy random models (Model-3, Model-5) and bifuzzy models (Model-4 and Model-6).

9.1. Optimum results for Model-1 and Model-2

In Model-1 and Model-2 we consider the stock dependent supply chain for deteriorating and non deteriorating items respectively. Using GA optimum result for deteriorating and non-deteriorating item under deterministic environments are given in Table 3. The graphical changes in the objective values with generation number by GA techniques for model-1 and model-2 are given in Figs. 2 and 3. A sensitivity analysis is performed for the minimum total cost of the supply chain with respect to rate of deterioration for Model-1 and display in Table 4. It is observed that as rate of deterioration is increases total cost of the supply chain also increases, this is as per our expected. The result in Table 7 is also reflected in Figs. 4–7.

9.2. Optimum result for Model-3 and Model-5

For Model-3 and Model-5, we consider the storage area as a triangular fuzzy number, i.e. \( \tilde{W} = (5500, 6000, 6500) \). The parameters \( \tilde{A}_1, \tilde{T}_1, \tilde{P}_1, \tilde{R}_1, \tilde{Q}_1, \tilde{C}_1, \tilde{R}_1, \tilde{\Delta}_1, \tilde{\Delta}_1, \tilde{R}_1 \) as stated in Sections 7.1.1 and 7.2.1 are considered under fuzzy random environments, given in Table 5 and all other remaining data same as deterministic model (Model-1). We transform the Model-3 and Model-5.

---

### Table 5

Input data in fuzzy random environments.

<table>
<thead>
<tr>
<th>( \tilde{A}_1 )</th>
<th>( \tilde{A}_2 )</th>
<th>( \tilde{A}_3 )</th>
<th>( \tilde{P}_1 )</th>
<th>( \tilde{R}_1 )</th>
<th>( \tilde{R}_2 )</th>
<th>( \tilde{R}_3 )</th>
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<tbody>
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<td>(( \rho_1, 7, 7 ))</td>
<td>(( \rho_1, 7, 7 ))</td>
<td>(( \rho_1, 7, 7 ))</td>
<td>(( \rho_4, 3, 3 ))</td>
<td>(( \rho_4, 3, 3 ))</td>
<td>(( \rho_4, 3, 3 ))</td>
<td>(( \rho_4, 3, 3 ))</td>
</tr>
<tr>
<td>( \rho_1 \sim N(103, 2) )</td>
<td>( \rho_1 \sim N(104, 2) )</td>
<td>( \rho_1 \sim N(160, 2) )</td>
<td>( \rho_4 \sim N(30, 1) )</td>
<td>( \rho_4 \sim N(20, 1) )</td>
<td>( \rho_4 \sim N(25, 1) )</td>
<td></td>
</tr>
<tr>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
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</tr>
<tr>
<td>(( \rho_1, 1, 1 ))</td>
<td>(( \rho_1, 1, 1 ))</td>
<td>(( \rho_1, 1, 1 ))</td>
<td>(( \rho_1, 1, 1 ))</td>
<td>(( \rho_1, 1, 1 ))</td>
<td>(( \rho_1, 1, 1 ))</td>
<td></td>
</tr>
<tr>
<td>( \rho_1 \sim N(25, 1) )</td>
<td>( \rho_1 \sim N(35, 1) )</td>
<td>( \rho_1 \sim N(40, 1) )</td>
<td>( \rho_1 \sim N(8, 1) )</td>
<td>( \rho_1 \sim N(0, 7, 1) )</td>
<td>( \rho_1 \sim N(0, 9, 1) )</td>
<td></td>
</tr>
<tr>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td></td>
</tr>
<tr>
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<td>(( \rho_1, 2, 2 ))</td>
<td>(( \rho_1, 2, 2 ))</td>
<td>(( \rho_1, 2, 2 ))</td>
<td>(( \rho_1, 2, 2 ))</td>
<td>(( \rho_1, 2, 2 ))</td>
<td></td>
</tr>
<tr>
<td>( \rho_1 \sim N(15, 1) )</td>
<td>( \rho_1 \sim N(18, 1) )</td>
<td>( \rho_1 \sim N(20, 1) )</td>
<td>( \rho_1 \sim N(60, 2) )</td>
<td>( \rho_1 \sim N(70, 2) )</td>
<td>( \rho_1 \sim N(55, 2) )</td>
<td></td>
</tr>
<tr>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td></td>
</tr>
<tr>
<td>(( \rho_1, 0.005, 0.005 ))</td>
<td>(( \rho_1, 0.005, 0.005 ))</td>
<td>(( \rho_1, 0.005, 0.005 ))</td>
<td>(( \rho_2, 0.005, 0.005 ))</td>
<td>(( \rho_2, 0.005, 0.005 ))</td>
<td>(( \rho_2, 0.005, 0.005 ))</td>
<td></td>
</tr>
<tr>
<td>( \rho_1 \sim N(0.04, 1) )</td>
<td>( \rho_2 \sim N(0.02, 1) )</td>
<td>( \rho_2 \sim N(0.03, 1) )</td>
<td>( \rho_2 \sim N(0.03, 1) )</td>
<td>( \rho_2 \sim N(0.03, 1) )</td>
<td>( \rho_2 \sim N(0.03, 1) )</td>
<td></td>
</tr>
<tr>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td>( \tilde{R}_1 )</td>
<td>( \tilde{R}_2 )</td>
<td>( \tilde{R}_3 )</td>
<td></td>
</tr>
<tr>
<td>(( \rho_1, 5, 5 ))</td>
<td>(( \rho_1, 5, 5 ))</td>
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<td>(( \rho_1, 5, 5 ))</td>
<td>(( \rho_1, 5, 5 ))</td>
<td>(( \rho_1, 5, 5 ))</td>
<td></td>
</tr>
<tr>
<td>( \rho_1 \sim N(150, 2) )</td>
<td>( \rho_1 \sim N(160, 2) )</td>
<td>( \rho_1 \sim N(155, 2) )</td>
<td>( \rho_1 \sim N(98000, 5) )</td>
<td>( \rho_1 \sim N(98000, 5) )</td>
<td>( \rho_1 \sim N(98000, 5) )</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6

Optimal result of Model-3 and Model-5.

<table>
<thead>
<tr>
<th>Item</th>
<th>( n_i )</th>
<th>( c_i )</th>
<th>( M_i )</th>
<th>( T_i )</th>
<th>( Q_i )</th>
<th>( TR )</th>
<th>( TS )</th>
<th>( TSC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy random model with deterioration (Model-3)</td>
<td>1</td>
<td>7</td>
<td>25.84681</td>
<td>0.056866</td>
<td>0.1428</td>
<td>82.334</td>
<td>5350.765</td>
<td>49837.352</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>35.73700</td>
<td>0.051520</td>
<td>0.1666</td>
<td>87.642</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>40.94993</td>
<td>0.054003</td>
<td>0.1666</td>
<td>86.904</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuzzy random model without deterioration (Model-5)</td>
<td>1</td>
<td>7</td>
<td>25.84706</td>
<td>0.057165</td>
<td>0.1428</td>
<td>82.037</td>
<td>4932.022</td>
<td>49572.435</td>
</tr>
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<td>6</td>
<td>35.73734</td>
<td>0.051978</td>
<td>0.1666</td>
<td>86.904</td>
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<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>40.94993</td>
<td>0.054003</td>
<td>0.1666</td>
<td>86.904</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Model-5 into deterministic problem, then use GA under the optimistic level $\mu_1 = \xi = 0.75$, $\mu_2 = \phi = 0.85$ and $\mu_3 = 0.5$ to solve this problem. The optimal result of fuzzy random model with deteriorating (Model-3) and non deteriorating items (Model-5) are given in Table 6. To validate the Model-3 a sensitivity analysis is performed under rate of deterioration and the result are given in Table 9 and it is also reflected in Figs. 8, 10 and 12. The results are also compared with the results of the Model-4 (Fig. 14).

9.3. Optimum result of Model-4 and Model-6

To get the optimum result of Model-4 and Model-6, we assume the storage area as a triangular fuzzy number, i.e. $\tilde{W} = (5500, 6000, 6500)$. The parameters $\tilde{A}_i$, $\tilde{C}_i$, $\tilde{P}_i$, $\tilde{H}_i$, $\tilde{G}_i$, $\tilde{K}_i$, $\tilde{c}_0$, $\tilde{c}_1$, $\tilde{H}_c$, $\tilde{I}_c$, $\tilde{R}_i$, $\tilde{B}_i$, $\tilde{S}_i$, $\tilde{B}$ as stated in Sections 7.1.2 and 7.2.2 are consider under bifuzzy environments, given in Table 7 and all other remaining data same as deterministic model (Model-1). We transform the Model-4 and Model-6 into deterministic problem, then use GA under the optimistic level $\mu_1 = \xi = 0.75$, $\mu_2 = \phi = 0.85$ and $\mu_3 = 0.5$ to solve this problem. The optimal result of bifuzzy model with deteriorating (Model-4) and non deteriorating (Model-6) are given in Table 8. To validate the Model-3 a sensitivity analysis is performed under rate of deterioration and the result are given in Table 9 and it is also reflected in Figs. 9, 10 and 12. The result are also compared with the result of the Model-3 (Fig. 14).

10. Discussion

From Table 4 following decision can be made, which are also reflected in Figs. 4–7.
Table 7
Input data in bifuzzy environments.

<table>
<thead>
<tr>
<th>$\tilde{A}_1$</th>
<th>$\tilde{A}_2$</th>
<th>$\tilde{A}_3$</th>
<th>$\tilde{F}_1$</th>
<th>$\tilde{F}_2$</th>
<th>$\tilde{F}_3$</th>
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</thead>
<tbody>
<tr>
<td>$A_1 \times 10^{10} i_{12}$</td>
<td>$A_2 \times 10^{10} i_{12}$</td>
<td>$A_3 \times 10^{10} i_{12}$</td>
<td>$A_4 \times 3 i_{12}$</td>
<td>$A_5 \times (25.30) i_{12}$</td>
<td>$A_6 \times (15.20) i_{12}$</td>
</tr>
<tr>
<td>$\theta_1 = (115, 130, 145)$</td>
<td>$\theta_2 = (125, 140, 155)$</td>
<td>$\theta_3 = (145, 160, 175)$</td>
<td>$\theta_4 = (25.30, 35)$</td>
<td>$\theta_5 = (15.20, 25)$</td>
<td>$\theta_6 = (20.25, 30)$</td>
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</table>

<table>
<thead>
<tr>
<th>$\tilde{B}_1$</th>
<th>$\tilde{B}_2$</th>
<th>$\tilde{B}_3$</th>
<th>$\tilde{B}_4$</th>
<th>$\tilde{B}_5$</th>
<th>$\tilde{B}_6$</th>
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</thead>
<tbody>
<tr>
<td>$B_1 \times 10^{10} i_{12}$</td>
<td>$B_2 \times 10^{10} i_{12}$</td>
<td>$B_3 \times 10^{10} i_{12}$</td>
<td>$B_4 \times 10^{10} i_{12}$</td>
<td>$B_5 \times (25.30) i_{12}$</td>
<td>$B_6 \times (15.20) i_{12}$</td>
</tr>
<tr>
<td>$\psi_1 = (24.25, 26)$</td>
<td>$\psi_2 = (34, 35, 36)$</td>
<td>$\psi_3 = (39, 40, 41)$</td>
<td>$\psi_4 = (0.7, 0.8, 0.9)$</td>
<td>$\psi_5 = (0.6, 0.7, 0.8)$</td>
<td>$\psi_6 = (0.8, 0.9, 1.0)$</td>
</tr>
</tbody>
</table>

Table 8
Optimal result of Model-4 and Model-6.

<table>
<thead>
<tr>
<th>Item</th>
<th>$n_i$</th>
<th>$c_i$</th>
<th>$M_i$</th>
<th>$T_i$</th>
<th>$Q_i$</th>
<th>$TR$</th>
<th>$TS$</th>
<th>$TSC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bifuzzy model with deterioration (Model-4)</td>
<td>1</td>
<td>7</td>
<td>25.8475</td>
<td>0.057516</td>
<td>0.1428</td>
<td>82.334</td>
<td>5231.419</td>
<td>4520.429</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>35.7322</td>
<td>0.044966</td>
<td>0.1428</td>
<td>74.579</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>40.9574</td>
<td>0.061901</td>
<td>0.1428</td>
<td>78.544</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bifuzzy model without deterioration (Model-6)</td>
<td>1</td>
<td>7</td>
<td>25.8475</td>
<td>0.057716</td>
<td>0.1428</td>
<td>82.037</td>
<td>4801.122</td>
<td>44962.813</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>35.7324</td>
<td>0.045381</td>
<td>0.1428</td>
<td>74.041</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>40.9592</td>
<td>0.063764</td>
<td>0.1428</td>
<td>77.556</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9
Optimal result of Model-3 and Model-4 for different rate of deterioration.

<table>
<thead>
<tr>
<th>Rate of deterioration</th>
<th>Item</th>
<th>Fuzzy random model (Model-3)</th>
<th>Fuzzy model (Model-4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_i$</td>
<td>$M_i$</td>
<td>$T_i$</td>
</tr>
<tr>
<td>+0.05</td>
<td>1</td>
<td>7</td>
<td>0.050766</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>0.051383</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>0.052850</td>
</tr>
<tr>
<td>+0.10</td>
<td>1</td>
<td>7</td>
<td>0.056184</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>0.056565</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>0.052492</td>
</tr>
<tr>
<td>+0.15</td>
<td>1</td>
<td>7</td>
<td>0.059966</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>0.043719</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>0.052103</td>
</tr>
<tr>
<td>+0.30</td>
<td>1</td>
<td>7</td>
<td>0.055212</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>0.043093</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>0.051291</td>
</tr>
<tr>
<td>+0.35</td>
<td>1</td>
<td>8</td>
<td>0.048597</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>0.042820</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>0.050953</td>
</tr>
<tr>
<td>+0.40</td>
<td>1</td>
<td>8</td>
<td>0.048372</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>0.037799</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td>0.044933</td>
</tr>
<tr>
<td>+0.45</td>
<td>1</td>
<td>8</td>
<td>0.048110</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>0.037602</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td>0.044808</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>8</td>
<td>0.045460</td>
</tr>
</tbody>
</table>
In this paper, we introduced realistic stock dependent integrated multi-item supply chain models for deteriorating and non-deteriorating items. Until now, no multi-item integrated supply chain models has been formulated with such considerations i.e. multi-item, stock dependent demand, deterioration, credit period, procurement, space constraints, budget constraints, etc. in fuzzy, fuzzy random and bifuzzy environments. The proposed models are optimized via soft computing methods GA. The imprecise models—fuzzy-random and bifuzzy models are normally completely different in their inputs and representations. So these results cannot be compared. But in our models as the inputs are presented about the some mid-values, for these reason we can say that bifuzzy model has given the minimum value than the rest of the models. Thus, this model incorporates some practical features that are likely to relate with some kinds of inventory. The model be used for electronic components, domestic goods, gas, alcohol and other products which are more likely with the characteristics above. The present idea can be extended to multi-objective, multi-supplier, multi-retailer supply chain models. The presents model can also be consider under inflation and variable deterioration for future research work.

### Acknowledgements

The authors sincerely thank the anonymous reviewers and editor-in-chief for their careful reading, constructive comments and fruitful suggestions. The authors are also thankful to Prof. Manoranjan Maiti, Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapur-721102, West Bengal, India for his advice to improve the paper.

### References


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#### Fig. 13. \( \theta \) vs \( M \) for Model-4.

#### Fig. 14. TSC changes with rate of deterioration in Model-3 and Model-4.

(i) The total cost (TSC) for the integrated system increases almost linearly with the increase of deterioration rate (\( \theta \)) for each item (cf. Fig. 4).

(ii) Credit period (\( M_i \)) and order quantity (\( Q_i \)) decreases when number of cycles (\( n_i \)) increases with deterioration rate (\( \theta \)), but for fixed \( n_i \), it increases very slowly with increase of \( \theta \) for each item (cf. Figs. 6 and 7).

(iii) Again Table 4 shows that \( c_i \) decrease with increase of \( \theta \).

From Table 9 following decision can be made, which are also reflected in Figs. 8–14.

(i) Credit period (\( M_i \)) and order quantity (\( Q_i \)) decreases when number of cycles (\( n_i \)) increases with deterioration rate (\( \theta \)), but for fixed \( n_i \), it increases with increase of \( \theta \) for each item (cf. Figs. 8, 9, 12 and 13).

(ii) The total cost (TSC) for the integrated system increases linearly with the increase of deterioration rate (\( \theta \)) for each item (cf. Fig. 14).

11. Concluding remarks

In this paper, we introduced realistic stock dependent integrated multi-item supply chain models for deteriorating and non-deteriorating items. Until now, no multi-item integrated supply chain models has been formulated with such considerations i.e. multi-item, stock dependent demand, deterioration, credit period, procurement, space constraints, budget constraints, etc. in fuzzy, fuzzy random and bifuzzy environments. The proposed models are optimized via soft computing methods GA. The imprecise models—fuzzy-random and bifuzzy models are normally completely different in their inputs and representations. So these results cannot be compared. But in our models as the inputs are presented about the some mid-values, for these reason we can say that bifuzzy model has given the minimum value than the rest of the models. Thus, this model incorporates some practical features that are likely to relate with some kinds of inventory. The model be used for electronic components, domestic goods, gas, alcohol and other products which are more likely with the characteristics above. The present idea can be extended to multi-objective, multi-supplier, multi-retailer supply chain models. The presents model can also be consider under inflation and variable deterioration for future research work.