

# Uncertainty, Credit Spreads, and Investment Dynamics

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## Abstract

In the standard bond-pricing framework (e.g., Merton [1974]), the return function of holders of risky corporate debt is a concave function of the firm's stochastic return, implying that a mean-preserving spread is associated with an increase in the bond risk premium. This feature of the standard debt contract has two important implications for the relationship between uncertainty and investment. First, to the extent that firms face significant frictions in credit markets, the rise in the bond risk premium implies an increase in the cost of capital and hence a reduction in investment; in such environment, uncertainty can have a significant effect on investment dynamics absent any investment irreversibility or managerial risk aversion. Second, if credit market frictions are an important mechanism through which uncertainty affects investment, then the inclusion of the bond risk premium in an empirical investment specification should attenuate the direct impact of uncertainty on investment. Using both the aggregate time-series and firm-level data, we test these two hypotheses and find strong support for the view that the relationship between uncertainty and investment is influenced importantly by the presence of credit market frictions. We then develop a tractable general equilibrium model in which firms issue risky bonds and equity in imperfect capital markets to finance investment projects. We calibrate the uncertainty process using firm-level estimates of shocks to the firms' profits and show that the model successfully explains the cross-sectional and time-series properties of bond risk premiums and their comovement with uncertainty and aggregate investment.

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# 1 Introduction

A well-documented empirical regularity of U.S. cyclical fluctuations is the fact that the cross-sectional dispersion of economic returns such as labor income, profits, or stock returns increases significantly during economic downturns.<sup>1</sup> These countercyclical changes in the variance of the distribution of returns imply that economic agents make their decisions in an environment of time-varying uncertainty. The notion of investment irreversibility provides the traditional mechanism through which changes in uncertainty, by altering the “option value of waiting,” affect the macroeconomy; see for example, Bernanke [1983], McDonald and Siegel [1986], Dixit and Pindyck [1994], and Caballero and Pindyck [1996]. As emphasized by Abel [1983], Abel and Eberly [1999], and Veracierto [2002], however, the effect of uncertainty on aggregate investment can be theoretically ambiguous, because it depends importantly on the assumptions regarding the initial accumulation of capital, market structure, and the equilibrium setting. As a result, the literature on irreversible investment lacks a consensus regarding the effect on economic activity from fluctuations in uncertainty.

The presence of credit market frictions—reflecting agency problems between borrowers and lenders—offers another potential channel through which changes in uncertainty can influence investment dynamics. According to the standard bond-pricing framework (e.g., Merton [1974]), the return function of holders of risky corporate debt is a concave function of the firm’s stochastic return. Thus a mean-preserving spread in the distribution of the underlying shocks—that is, an increase in uncertainty—will cause lenders to demand a higher risk premium, in order to be compensated for the increase in the downside risk. This feature of the standard debt contract has two important implications for the relationship between uncertainty and investment. First, to the extent that firms face significant frictions in credit markets, a higher corporate bond risk premium implies an increase in the cost of capital and hence a reduction in investment; in such environment, uncertainty can have a significant effect on investment absent any investment irreversibility or managerial risk aversion. Second, if credit market frictions are an important mechanism through which uncertainty affects investment, then the inclusion of the bond risk premium in an empirical investment equation should significantly attenuate the direct effect of uncertainty on investment. Consequently, the well-known “bad news principle” articulated by Bernanke [1983] could reflect, in part, the option-like structure of the standard debt contract as well as some form of investment irreversibility.

In this paper, we analyze the interaction between uncertainty and investment in the presence of financial market frictions. To motivate our theoretical framework, we first doc-

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<sup>1</sup>See, for example, Campbell, Lettau, Malkiel, and Xu [2001], Storesletten, Telmer, and Yaron [2004], Eisfeldt and Rampini [2006], and Bloom, Floetotto, and Jaimovich [2009].

ument a strong statistical relationship between time-varying uncertainty and risk premiums on corporate bonds, using both aggregate time-series and firm-level data. Our empirical results, based on a large firm-level panel data set linking prices of the firm’s outstanding unsecured bonds trading in the secondary market to the firm’s equity valuations and its income and balance sheet data, indicate that an increase in uncertainty leads to the widening of yield spreads on corporate bonds, even when controlling for the firm’s leverage, expected return on its assets, and the firm’s credit rating. Increases in credit spreads—conditional on standard investment fundamentals measuring the marginal product of capital—are strongly associated with declines in firm-level investment. Moreover, when we include credit spreads in an empirical investment equation augmented with measures of uncertainty, the direct effect of uncertainty on investment is significantly attenuated, a confluence of results that supports the notion that the impact of uncertainty on investment reflects in part the presence of significant frictions in credit markets.

In the second part of the paper, we provide a context for our empirical findings by constructing a tractable debt-contracting model of the type analyzed by Bernanke, Gertler, and Gilchrist [1999] and Cooley and Quadrini [2001]. We embed this contracting framework—in which the debt contract is defaultable and renegotiable owing to the limited liability of borrowers—into a capital accumulation problem with time-varying uncertainty. Specifically, we assume that firms employ a technology that is subject to a persistent idiosyncratic productivity shock, the variance of which evolves according to a stochastic process. In this environment of time-varying uncertainty, the firms make investment financing decisions subject to a full range of choices regarding their capital structure: internal funds, debt, and equity financing. We assume that capital markets are imperfect in that financing investment through debt is costly because of moral hazard and limited liability problems, whereas equity financing is costly because the issuance of new shares is subject to asymmetric information problems.

We analyze the implications of our model in a general equilibrium setting, where not only factor prices but also the prices of assets used as collateral are endogenous. As emphasized by Khan and Thomas [2008], the treatment of model dynamics in general equilibrium is important because the predictions made in partial equilibrium are often overturned once general equilibrium effects are taken into account. According to our results, an unanticipated increase in uncertainty generates a cyclical downturn by raising the price of credit risk, which lowers aggregate investment spending. We also show that the responses of the corporate bond risk premium and aggregate investment to changes in uncertainty are comparable in magnitude to those seen in our data. Finally, our model delivers a rich set of empirically consistent dynamics for firms’ financial choices such as countercyclical defaults, procyclical recovery rates, countercyclical equity issuance, and procyclical leverage.

The remainder of the paper is organized as follows: In Section 2, we construct our measures of time-varying uncertainty. We investigate the empirical relationship between investment, credit spreads, and our measures of uncertainty using both aggregate time-series and firm-level data in Section 3. In Section 4, we develop our structural model and analyze its equilibrium dynamics. In Section 5, we calibrate the parameters of the model based on the firm-level evidence and examine the dynamics of key variables. Section 6 concludes.

## 2 Measuring Macroeconomic Uncertainty

To find an empirical counterpart to the concept of time-varying macroeconomic uncertainty used in theoretical models presents a considerable challenge. In this section, we construct four alternative measures of uncertainty for the U.S. (nonfarm) nonfinancial corporate sector over the past four decades, several of which have been used in previous empirical research. Two of our uncertainty measures have no cross-sectional dimensions—that is, they are a pure time series—whereas two of the measures are firm specific, a feature that allows us to match them with firm-level data on investment and corporate bond spreads. We discuss both in turn.

**Cross-sectional dispersion of sales growth:** From quarterly firm-level Compustat data, we selected all nonfinancial firms over the period 1969:Q4 to 2008:Q3. For these firms, we computed their annualized quarterly growth rate of real sales.<sup>2</sup> For each quarter  $t$ , we then computed the weighted standard deviation of sales growth—denoted by  $\sigma_{t,S}$ —using the level of real sales in quarter  $t - 1$  as weights. (To mitigate the effect of extreme observations, we eliminated all growth rates below the 1st percentile and above 99th percentile of the entire distribution.) We then seasonally adjusted the resulting time series using the standard X12-ARIMA quarterly filter.

**Cross-sectional dispersion of equity returns:** From daily firm-level CRSP data, we selected all nonfarm, nonfinancial firms over the period 1969:Q4 to 2008:Q3. For these firms, we computed their annualized quarterly total stock return.<sup>3</sup> For each quarter  $t$ , we then computed the weighted standard deviation of returns—denoted by  $\sigma_{t,R}$ —using the market value of equity in quarter  $t - 1$  as weights. (To mitigate the effect of extreme observations, we eliminated all returns below the 1st percentile and above 99th percentile of the entire distribution.)

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<sup>2</sup>Nominal sales were deflated by the output price deflator (c-w, 2000=100) for the U.S. nonfarm business sector.

<sup>3</sup>Returns include both the capital gains and dividends.

Although based on the firm-level data, the two measures of macroeconomic uncertainty discussed above have no cross-sectional dimension. We now consider two alternative estimates of time-varying *firm-specific* uncertainty, both of which preserve the cross-sectional dimensions but nonetheless can easily be aggregated to yield a measure of time-varying aggregate uncertainty.

**Merton DD-model:** Our third measure of uncertainty is based on the seminal work of Merton [1973, 1974]. The key insight of the Merton “distance-to-default” (DD) framework is that the equity of the firm can be viewed as a call option on the underlying value of the firm with a strike price equal to the face value of the firm’s debt. Although neither the underlying value of the firm nor its volatility can be directly observed, they can, under the assumptions of the model, be inferred from the value of the firm’s equity, the volatility of its equity, and the firm’s observed capital structure.

The first critical assumption underlying the DD-framework is that the total value of the a firm—denoted by  $V$ —follows a geometric Brownian motion:

$$dV = \mu_V V dt + \sigma_V V dW, \tag{1}$$

where  $\mu_V$  is the expected continuously compounded return on  $V$ ;  $\sigma_V$  is the volatility of firm value; and  $dW$  is an increment of the standard Weiner process. The second critical assumption pertains to the firm’s capital structure. In particular, it is assumed that the firm has just issued a single discount bond in the amount  $D$  that will mature in  $T$  periods. Together, these two assumption imply that the value of the firm’s equity  $E$  can be viewed as a call option on the underlying value of the firm  $V$  with a strike price equal to the face value of the firm’s debt  $D$  and a time-to-maturity of  $T$ . According to the Black-Scholes-Merton option-pricing framework, the value of the firm’s equity then satisfies:

$$E = V\Phi(\delta_1) - e^{-rT}D\Phi(\delta_2), \tag{2}$$

where  $r$  is the instantaneous risk-free interest rate,  $\Phi(\cdot)$  is the cumulative standard normal distribution function, and

$$\delta_1 = \frac{\ln(V/D) + (r + 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}} \quad \text{and} \quad \delta_2 = \delta_1 - \sigma_V\sqrt{T}.$$

According to equation 2, the value of the firm’s equity depends on the total value of the firm and time, a relationship that also underpins the link between volatility of the firm’s

value  $\sigma_V$  and the volatility of its equity  $\sigma_E$ . In particular, it follows from Ito’s lemma that

$$\sigma_E = \left[ \frac{V}{E} \right] \frac{\partial E}{\partial V} \sigma_V. \quad (3)$$

Because under the Black-Scholes-Merton option-pricing framework  $\frac{\partial E}{\partial V} = \Phi(\delta_1)$ , the relationship between the volatility of the firm’s value and the volatility of its equity is given by

$$\sigma_E = \left[ \frac{V}{E} \right] \Phi(\delta_1) \sigma_V. \quad (4)$$

The most critical inputs to the Merton DD-model are clearly the market value of the equity  $E$ , the face value of the debt  $D$ , and the volatility of equity  $\sigma_E$ . Assuming a forecasting horizon of one year ( $T = 1$ ), we implement the model in two steps: First, we estimate  $\sigma_E$  from historical daily stock returns. Second, we assume that the face value of the firm’s debt  $D$  is equal to the sum of the firm’s current liabilities and one-half of its long-term liabilities.<sup>4</sup> Using the observed values of  $E$ ,  $D$ ,  $\sigma_E$ , and  $r$  (1-year Treasury yield), equations 2 and 4 can be solved for  $V$  and  $\sigma_V$  using standard numerical techniques. However, as pointed out by Crosbie and Bohn [2003] and Vassalou and Xing [2004], the excessive volatility of market leverage ( $V/E$ ) in equation 4 causes large swings in the estimated volatility of the firm’s value  $\sigma_V$ , which are difficult to reconcile with the observed frequency of defaults and movements in financial asset prices. To resolve this problem, we implement an iterative procedure recently proposed by Bharath and Shumway [2008].<sup>5</sup> This procedure yields a monthly estimate of the firm-specific uncertainty—denoted by  $\sigma_{it,V}$ —for all publicly traded companies in the U.S. nonfarm, nonfinancial sector (11,221 firms) covered both by CRSP and Compustat from January 1970 to September 2008 (1,238,466 firm/month observations). Our third measure of macroeconomic uncertainty—denoted by  $\bar{\sigma}_{t,V}$ —is then simply the weighted cross-sectional average of the firm-specific volatilities in month  $t$ , with weights equal to the estimated value of the firm in month  $t - 1$ .

We also use our solutions of the Merton DD-model to calculate the firm-specific distance-

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<sup>4</sup>This assumption for the “default point” is also used by Moody’s/KMV in the construction of their Expected Default Frequencies (EDFs) based on the Merton DD-model, and it reflects the finding that most defaults occur when the market value of the firm’s assets drops below the sum of its current liabilities and one-half of its long-term liabilities. Both current and long-term liabilities are taken from quarterly Compustat files.

<sup>5</sup>Briefly, the procedure involves the following steps. Initialize the procedure by letting  $\sigma_V = \sigma_E[D/(E + D)]$ . Use this value of  $\sigma_V$  in equation 2 to infer the market value of the firm’s assets  $V$  for every day of the previous year. Then calculate the implied daily log-return on assets (i.e.,  $\Delta \ln V$ ) and use the resulting series to generate new estimates of  $\sigma_V$  and  $\mu_V$ . Iterate on  $\sigma_V$  until convergence; see Bharath and Shumway [2008] for details.

to-default over the one-year horizon as

$$DD = \frac{\ln(V/D) + (\mu - 0.5\sigma_V^2)}{\sigma_V}, \quad (5)$$

where  $\mu$  is an estimate of the expected annual return of the firm’s assets. The corresponding implied probability of default—the so-called “expected default frequency” (EDF)—is given by

$$EDF = \Phi\left(-\left(\frac{\ln(V/D) + (\mu - 0.5\sigma_V^2)}{\sigma_V}\right)\right) = \Phi(-DD), \quad (6)$$

which, under the assumptions of the Merton model, should be a sufficient statistic for predicting defaults.

**Simple model:** Our fourth and final measure of uncertainty is also firm specific and equals the rolling 250-day standard deviation of (annualized) daily equity returns, a measure that we denote by  $\sigma_{it,E}$ . (The choice of the 250-day window ensures that  $\sigma_{it,E}$  spans the same one-year horizon as does the uncertainty measure from the Merton DD-model.) Note that this simple firm-specific uncertainty measure is closely related to the volatility estimate used to initialize the Bharath and Shumway [2008] iterative procedure in the Merton DD-model. Although much simpler to compute than the volatility of the firm’s total assets ( $\sigma_{it,V}$ ) that underlies the Merton DD-model, this simple estimate of uncertainty, by abstracting from the firm’s capital structure decisions, may be more affected by excess volatility or “irrational exuberance” in the stock market, a concern emphasized by previous research on the relationship between equity valuations, expected profits, and investment; see, for example, Shiller [1981, 2000] and Bond and Cummins [2001]. However, we include  $\sigma_{it,E}$  in our analysis because it was introduced into empirical investment literature by Leahy and Whited [1996] and Bloom, Bond, and Van Reenen [2001]. Using this simple estimate, our last measure of macroeconomic uncertainty—denoted by  $\bar{\sigma}_{t,E}$ —is the weighted cross-sectional average of the firm-specific volatilities in month  $t$ , with weights equal to the market value of the firm’s equity in month  $t - 1$ .

Figure 1 depicts the time-series evolution of our four measures of aggregate uncertainty. Regarding the average level of uncertainty over the past four decades, all of our uncertainty measures indicate an average level of uncertainty of around 30 percent. Moreover, the general contours—both at low and high frequencies—are quite similar across the four measures. Indeed, according to the entries in Table 1, all four series are positively correlated, with correlations being especially strong for measures of uncertainty based on equity valuations. The common pattern that emerges from Figure 1 is that aggregate uncertainty in the U.S. nonfinancial corporate sector was generally low during the 1970s and then rose gradually over the subsequent two decades. According to all four measures, the level of uncertainty

almost doubled during the late 1990s, reaching a record level in 2000, prior to the onset of the 2001 recession. The uncertainty then retreated markedly as the economy recovered but uncertainty has moved up significantly since the middle of 2007, again several quarters before the cyclical peak in economic activity. More generally, macroeconomic uncertainty tends to increase noticeably before economic downturns, suggesting that large swings in uncertainty may play an important role in cyclical fluctuations.

We now turn to the relationship between corporate bond spreads and macroeconomic uncertainty. For the ease of comparison, we summarize the information content of our different uncertainty measures in a single indicator, defined as the first principal component of the four measures shown in Figure 1.<sup>6</sup> Figure 2 depicts the relationship between this measure of aggregate uncertainty and the credit spread on Baa-rated long-term corporate bonds. The two series appear to be closely correlated, with both of them rising prior to and during cyclical downturns. Nevertheless, the correlation is far from perfect and the two series can diverge markedly. For example, between the end of 1993 and the middle of 1995, the Baa credit spread declined almost 100 basis point despite no discernible change in the level of uncertainty; in 2001, by contrast, credit spreads continued to surge despite a significant drop in macroeconomic uncertainty. Since the onset of the most recent financial crisis in the summer of 2007, however, both the the Baa credit spread and uncertainty have surged in a pattern consistent with previous economic downturns.

To investigate more formally the lead-lag relationship between credit spreads and macroeconomic uncertainty, we perform a series of Granger causality tests, the results of which are summarized in Table 2. As evidenced by the entries in the table, we do not reject the null hypothesis that movements in credit spreads do not Granger-cause changes in uncertainty in the baseline bivariate specification and in a trivariate specification that controls for movements in lagged output growth. In contrast, we overwhelmingly reject the hypothesis that fluctuations in macroeconomic uncertainty do not Granger-cause changes in corporate credit spreads. These results thus suggest that changes in macroeconomic uncertainty tend to precede changes in credit spreads, a finding consistent with the transmission mechanism in which shocks to uncertainty affect investment and output by influencing the level of credit spreads in the economy.

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<sup>6</sup>Given the strong positive correlations between our four measures of uncertainty, the first principal component explains almost 80 percent of the total variance among the four series. In order to cast this uncertainty index in units that are easily interpretable, we scaled the principal component such that its mean equals the time-series mean of the average level of uncertainty across the four measures.



### 3 Investment, Uncertainty, and Credit Spreads

In this section we examine the dynamic relationship between investment, uncertainty, and credit spreads. First, we take a look at the aggregate time-series evidence. Second, we turn to firm-level panel data. In particular, we construct a new data set that links income and balance sheet information for about 900 large U.S. nonfinancial corporations to interest rates on their publicly-traded debt. Covering the last three decades, this new data set enables us to evaluate and quantify empirically the relationship between firms' investment decisions, fluctuations in uncertainty, and movements in credit spreads as measured by the changes in secondary market prices of firms' outstanding bonds.

#### 3.1 Aggregate Time-Series Evidence

We now examine the extent to which changes in macroeconomic uncertainty help predict the growth in business investment. Specifically, we consider the following reduced-form investment specification:

$$\Delta \ln I_{t+1} = \alpha + \sum_{s=0}^3 \beta_s \Delta \ln I_{t-s} + \sum_{s=0}^3 \gamma_s \Delta \ln Y_{t-s} + \theta_1 \sigma_t + \theta_2 \text{Baa}_t + \epsilon_{t+1}, \quad (7)$$

where  $\Delta \ln I_t$  is the log-difference in real business fixed investment;  $\Delta \ln Y_t$  is the log-difference in real business sector output;  $\sigma_t$  is our measure of time-varying macroeconomic uncertainty based on the first principal component; and  $\text{Baa}_t$  is the Baa corporate bond spread.

Table 3 contains results of this exercise: column 1 considers a specification that omits credit spreads (i.e.,  $\theta_2 = 0$ ); column 2 omits the uncertainty measure (i.e.,  $\theta_1 = 0$ ); and column 3 considers both effects. As evidenced by the entries in the first column, uncertainty has an economically significant effect on the growth of aggregate business fixed investment. A 10-percentage-point increase in our measure of uncertainty in quarter  $t$  is associated with about a 2-percentage-point reduction (at an annual rate) in investment growth over the subsequent quarter, although this effect is not estimated very precisely ( $p$ -value = 0.059). The economic effect of the change in credit spreads, by contrast, is considerably larger: a 10-percentage-point jump in the Baa credit spread leads to an almost 20-percentage-point drop (at an annual rate) in the rate of growth of investment, though again, this effect is estimated relatively imprecisely ( $p$ -value = 0.066). Finally, as indicated in column 3, when both variables are included in the specification, the Baa credit spread remains a marginally significant predictor of future investment growth ( $p$ -value = 0.086), whereas the statistical significance of our uncertainty measure vanishes in its entirety. All told, these time-series results, together with the Granger causality tests reported in Table 2, provide a modest

support for the contention that changes in uncertainty lead to changes in corporate credit spreads, which, in turn, influence investment spending.

### 3.2 Firm-Level Panel Data Evidence

Our empirical strategy involves regressing investment on measures of economic fundamentals and a firm-specific estimate of uncertainty and a firm-specific credit spread. Our firm-specific credit spreads are based on the month-end market prices of outstanding long-term corporate bonds from the Lehman/Warga (LW) and Merrill Lynch (ML) databases. These two data sources include prices for a significant fraction of dollar-denominated bonds publicly issued in the U.S. corporate cash market. The ML database is a proprietary data source of *daily* bond prices that starts in 1997. Focused on the most liquid securities, bonds in the ML database must have a remaining term-to-maturity of at least two years, a fixed coupon schedule, and a minimum amount outstanding of \$100 million for below investment-grade and \$150 million for investment-grade issuers. By contrast, the LW database of *month-end* bond prices has a somewhat broader coverage and is available from 1973 through mid-1998 (see Warga [1991] for details).

To ensure that we are measuring long-term financing costs of different firms at the same point in their capital structure, we limited our sample to only senior unsecured issues. For the securities carrying the senior unsecured rating and with market prices in both the LW and LM databases, we spliced the option-adjusted effective yields at month-end—a component of the bond’s yield that is not attributable to embedded options—across the two data sources. To calculate the credit spreads at each point in time, we matched the yield on *each* individual security issued by the firm to the estimated yield on the Treasury coupon security of the same maturity. The month-end Treasury yields were taken from the daily estimates of the U.S. Treasury yield curve reported in Gürkaynak, Sack, and Wright [2006]. To mitigate the effect of outliers on our analysis, we eliminated all observations with credit spreads below 10 basis points and with spreads greater than 5,000 basis points. This selection criterion yielded a sample of 5,269 individual securities between January 1973 and September 2008. We matched these corporate securities with their issuer’s annual income and balance sheet data from Compustat, yielding a matched sample of 926 firms.

Table 4 contains summary statistics for the key characteristics of bonds in our sample. Note that a typical firm has only a few senior unsecured issues outstanding at any point in time—the median firm, for example, has two such issues trading at any given month. This distribution, however, exhibits a significant positive skew, as some firms can have as many as 75 different senior unsecured bond issues trading in the market at a point in time. The distribution of the real market values of these issues is similarly skewed, with the range running from \$1.1 million to more than \$6.6 billion. Not surprisingly, the maturity of

these debt instruments is fairly long, with the average maturity at issue of about 14 years. Because corporate bonds typically generate significant cash flow in the form of regular coupon payments, the effective duration is considerably shorter, with both the average and the median duration of about 6 years. Although our sample spans the entire spectrum of credit quality—from “single D” to “triple A”—the median bond/month observation, at “A3,” is solidly in the investment-grade category. Turning to returns, the (nominal) coupon rate on these bonds averaged 7.77 percent during our sample period, while the average total nominal return, as measured by the nominal effective yield, was 8.25 percent per annum. Reflecting the wide range of credit quality, the distribution of nominal yields is quite wide, with the minimum of about 1.2 percent and the maximum of about 57 percent. Relative to Treasuries, an average bond in our sample generated a return of about 175 basis points above the comparable-maturity risk-free rate, with the standard deviation of 165 basis points.

Before analyzing the relationship between investment, uncertainty, and credit spreads, we examine the link between credit spreads and uncertainty. In particular, we estimate the following reduced-form bond-pricing equation using monthly data on credit spreads and our monthly firm-specific estimates of uncertainty:

$$\ln \text{SPR}_{it}^j = \theta_1 \ln \text{LEV}_{i,t-1} + \theta_2 \mu_{i,t-1} + \theta_3 \ln \sigma_{i,t-1} + \beta' \mathbf{x}_{it}^j + \epsilon_{it}^j, \quad (8)$$

where  $\text{SPR}_{it}^j$  is the credit spread on bond  $j$  in month  $t$  issued by firm  $i$ ;  $\text{LEV}_{it}$  is a measure of market leverage of firm  $i$  at the end of month  $t$ ;  $\mu_{it}$  is a measure of the expected return on firm  $i$ 's assets at the end of month  $t$ ;  $\sigma_{it}$  is a measure of uncertainty for firm  $i$  at the end of month  $t$ ; and  $\mathbf{x}_{it}^j$  is vector of firm and/or bond-specific control variables. We consider two variants of equation 8: in the first specification, the key explanatory variables (i.e.  $\text{LEV}_{i,t-1}$ ,  $\mu_{i,t-1}$ ,  $\sigma_{i,t-1}$ ) are all derived from the Merton DD-model, whereas in the second specification, they are based on the simple alternative.

Tables 5–6 report the results of this exercise. In both tables, all the coefficients have a correct theoretical sign and are economically and statistically highly significant: an increase in market leverage is associated with an increase in credit spreads; an increase in the firm's expected return is associated with a decrease in spreads; and an increase in uncertainty is associated with an increase in spreads. According to columns 2–4 in both tables, these results are robust to the inclusion of fixed credit rating effects (column 2) that control of additional aspects of credit quality that are not captured by the Merton DD-model or its simple counterparts. In addition, as indicated in columns 3–4, the results are robust to the inclusion of fixed industry effects and fixed time effects. All told, these results provide strong evidence that the link between uncertainty and credit spreads—conditional on firm-specific measures of credit quality—is not a reflection of omitted industry characteristics or common shocks. Moreover, the reduced-form pricing models that include all the control

variables (column 4) explain about two-thirds of the variation in credit spreads, an impressive goodness-of-fit statistic given the large dispersion of credit spreads both across firms and across time.

We now turn to the link between investment, uncertainty, and credit spreads. Our baseline empirical investment equation is given by the following specification:

$$\left[ \frac{I}{K} \right]_{jt} = \beta_1 Z_{jt} + \beta_2 \sigma_{j,t-1} + \beta_3 \text{SPR}_{j,t-1} + \eta_j + \lambda_t + \epsilon_{jt}, \quad (9)$$

where  $[I/K]_{jt}$  denotes the investment rate of firm  $j$  in period  $t$  (i.e., the ratio of capital expenditures in period  $t$  to the capital stock at beginning of the period),  $Z_{jt}$  is a variable that measures firm  $j$ 's future investment opportunities (i.e., economic fundamentals),  $\sigma_{jt}$  is the firm-specific measure of uncertainty,  $\text{SPR}_{jt}$  is the credit spread on the portfolio of bonds issued by firm  $j$ ,  $\eta_j$  is the firm-specific fixed effect, and  $\lambda_t$  is a time dummy. In our baseline case, we assume that the error term  $\epsilon_{jt}$  is orthogonal to current and past values of  $Z_{jt}$  as well as past values of  $\sigma_{jt}$  and  $\text{SPR}_{jt}$ . We measure investment fundamentals with either the current log of the sales-to-capital ratio—denoted by  $\ln[S/K]_{jt}$ —or with the Tobin's  $Q$  at the end of the previous period, denoted by  $Q_{j,t-1}$ .<sup>7</sup>

Table 7 contains the results from the estimation of the investment specification using the measure of uncertainty based on the Merton DD-model, whereas Table 8 contains estimates based on the simple measure. According to the entries in Table 7, uncertainty—as measured by the Merton DD-model—has no discernible effect on investment, a result that holds regardless of whether or not the specification includes credit spreads. Note, however, that credit spreads are economically and statistically highly significant predictors of future investment using both sales-to-capital ratio and the Tobin's  $Q$  as a measure of investment fundamentals. Results in Table 8, by contrast, do indicate some role for uncertainty in the investment process. Using either the sales-to-capital ratio or the Tobin's  $Q$  as a measure of investment fundamentals, the estimated coefficient on the simple measure of uncertainty is statistically significant and about -0.10, indicating that a 10-percentage-point increase in uncertainty leads to a 1-percentage-point decline in the investment rate over the subsequent year. By contrast, the effect of changes in credit spreads is considerably larger—a 10-percentage-point rise in credit spreads is associated with an almost 10-percentage-point plunge in investment. Note also, that the effect of uncertainty on investment is attenuated by almost a factor of one-half, when we include both uncertainty and credit spreads in the regression specification, a finding consistent with the time-series evidence presented earlier

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<sup>7</sup>We consider two alternative measures of Tobin's  $Q$ . The first measure, denoted by  $Q_{V,jt}$ , is given by the ratio of the market value of the firm's assets  $V_{jt}$  from the Merton DD-model to the book value of assets as reported on the firm's balance sheet. In the second measure, denoted by  $Q_{E,jt}$ , we assume that the market value of the firm's assets is given by sum of the the market value of its equity and the face value of its debt.

and which adds further support to the notion that a significant portion of the effect of uncertainty on investment is due to the presence of financial market imperfections.

## 4 Structural Model

### 4.1 Environment

The model economy is composed of four types of agents: (i) a representative consumer (ii) a continuum of final-good firms (iii) capital-good firms and (iv) bond investors. The representative consumer lives forever and maximizes an expected sum of discounted,  $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$ . The representative earns competitive wages ( $w$ ) by working  $h$  hours and saves by investing in final-good firms' share.

The final-good firms produce outputs using capital ( $k$ ) and labor ( $h$ ) as inputs. The final goods can be used for direct consumption or production of new capital goods ( $i$ ) as inputs. The production technology in the final-good sector is characterized by decreasing-returns-to-scale (DRS) and Cobb-Dougllass production function. Production is subject to a persistent idiosyncratic shock ( $z$ ),

$$\ln z' = \rho \ln z + \ln \varepsilon', \quad \ln \varepsilon' \sim N(-0.5\sigma^2, \sigma^2) \quad (10)$$

The assumptions underlying the production technology can be summarized by a function  $y = z^\nu (k^\alpha h^{1-\alpha})^\gamma$  where  $\alpha$  is the value-added share of capital and  $\gamma$  is s DRS parameter ( $0 < \gamma < 1$ ).  $\nu$  is a normalization parameter that makes the profit function  $\pi(z, k)$  of the firm linear in  $z$ , i.e.,  $z\pi(k)$ .

The capital-good firms combine old capital and final goods to produce new capital. The production technology is assumed to be a constant-returns-to-scale (CRS) kind. The specific functional form will be described below. The newly produced capital goods are homogeneous and are sold to the final-good firms at a competitive market. As a consequence, the capital-good firms earn zero profit in equilibrium and the firm size distribution is not well defined. Without loss of generality, therefore, the sector can be assumed as centralized by one large firm.<sup>8</sup> The price of final goods is taken as a numeraire and the relative price of capital is denoted by  $Q$ .

The investors provide debt financing to the final-good firms. A CRS technology is available to any bond investor and the financing industry is assumed to be competitive. As a result, the bond investors earn zero profit in equilibrium. Risk pooling across different bond contracts is not allowed by assumption and the zero profit condition must be satisfied for each bond contract.

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<sup>8</sup>For this reason, in what follows, the term, "firm" is reserved for a final-good firm.

To model a time-varying economic uncertainty, we assume that the idiosyncratic uncertainty level ( $\sigma$ ) associated with the production technology of the final-good industry is changing over time according to a persistent Markov process. More specifically, we assume a Markov Chain process with  $N$  states and a transition matrix  $p(\sigma, \sigma')$ . A shock to the uncertainty level is an aggregate shock in that all firms have the same uncertainty level. However, the aggregate shock to the uncertainty level does not alter the economy-wide mean productivity level, hence no aggregate technology shock. In what follows, we let  $s$  denote the set of aggregate state variables, which will be defined shortly.

## 4.2 Bond Contract

To finance investment projects, the final-good firms use the combination of internal and external funds. The source of internal funds is the net-worth of the firm defined as

$$n \equiv z\pi(k, s) + Q(s)(1 - \delta)k - b \quad (11)$$

where  $Q(s)(1 - \delta)k$  is the resale value of installed capital and  $b$  is the face value of the bond issued by the firm. External funds are composed of debt and equity. We describe the structure of debt-financing first. It is assumed through the paper that only one period discount bond is available.

In the beginning of each period, all economic agents in the model observe the realization of idiosyncratic shock ( $z_t$ ) and uncertainty shock ( $\sigma_t$ ). An important assumption regarding the timing convention is that  $\sigma_t$  is the level of the standard deviation of the distribution of technology shock at time  $t + 1$ . Based on this information, the bond contract specifies the amount of bond issue and the price of bond. The price of bond incorporates all information including conditions under which the firm defaults and how much of the original debt obligation will be recovered in a default state.

Suppose that the firm issues  $b_{t+1}$  today. Let  $q_t$  denote the price of the bond issued at time  $t$ . We can then express the actual amount of debt financing today as  $q_t b_{t+1}$ . The firm purchases capital using the debt-financing together with other source of funds. In the next period, after observing the realization of shocks, the firm decides whether or not to fulfill the debt obligation. If the firm decides not to default, it pays the face value of the debt  $b_{t+1}$  to the lender and makes real and financial decisions for the next period and the process continues.

If the firm does default, it enters a debt renegotiation process with the investor. For the structure of the renegotiation process, we adopt the assumption of limited liability. More specifically, we assume that there is a lower bound to the net-worth of the firm,  $\underline{n}$  below which the firm cannot pledge itself to pay back any outstanding liability. We also assume

that the firm does not have any better outside option than the current value of the firm unless the net-worth of the firm goes down below  $\underline{n}$ .

Two assumptions together imply that the firm defaults if and only if the realized net-worth is lower than the lower bound. This simplifies the analysis of bond contract considerably. In particular, it allows us to derive an analytical expression for the bond price, expediting the speed of computation substantially. Given the price of capital, the amounts of capital and debt, the firm defaults if and only if the realized technology is lower than the threshold level, which is implicitly defined as

$$\underline{n} = \underline{z}_{t+1}\pi(k_{t+1}, s_{t+1}) + Q(s_{t+1})(1 - \delta)k_{t+1} - b_{t+1} \quad (12)$$

In words, the default threshold technology is the one that makes the firm's net-worth equal to the lower bound.

Under the limited liability assumption that we made, the new debt level renegotiated by the firm and the investor, denoted by  $b_{t+1}^R$ , cannot exceed the upper bound of debt  $\bar{b}_{t+1}$  that is consistent with the lower bound of the net-worth, i.e.,

$$b_{t+1}^R \leq \bar{b}(z_{t+1}, k_{t+1}, s_{t+1}) \equiv z_{t+1}\pi(k_{t+1}, s_{t+1}) + Q(s_{t+1})(1 - \delta)k_{t+1} - \bar{n}$$

In other words, the maximum recovery of the bond holder is the level of debt that brings the firm's net-worth back to the lower bound. Note that the maximum recovery depends on firm-specific conditions as well as macroeconomic conditions.

For simplicity, we also assume that the firm does not have any bargaining power during the renegotiation process. With this assumption, it is straightforward to show that the renegotiated debt is set equal to the upper bound of the debt recovery, i.e.,  $b_{t+1}^R = \bar{b}(z_{t+1}, k_{t+1}, s_{t+1})$ . Given the default condition and the default recovery, the price of debt can be computed in a straightforward way. Since the bond investors face perfect competition, their expected profits are driven down to zero in each period. The bond price is an weighted average of discounted returns in default and non-default states, i.e.,

$$q(z, k', b', s)b' = \sum_{\sigma'} p(\sigma, \sigma')m(s, s') \left[ \int_{z' \geq \underline{z}} b' dF(z'|z, \sigma) + \int_{z' \leq \underline{z}} (1 - \xi)\bar{b}(z', k', s') dF(z'|z, \sigma) \right] \quad (13)$$

where  $m(s, s')$  is a pricing kernel and  $F$  is the conditional c.d.f. of  $z'$  given  $z$  and  $\sigma$ . When the firm does not default, the investor is paid the face value of the bond  $b'$ . When the firm defaults, the investor recovers  $\bar{b}(z', k', s')$  the value of which is equal to the value of the firm's total asset minus the minimum level of net-worth. Note that we add one more assumption that the lender only recovers a fraction  $1 - \xi$  of  $\bar{b}(z', k', s')$ . We interpret  $\xi$  as bankruptcy cost. As in the literature motivated by costly state verification framework of

Townsend [1979], the bankruptcy cost captures the welfare loss stemming from the moral hazard of the firm. The bankruptcy cost ensures that the firm does not have incentive to falsify the state of the firm under the default.

An important aspect of the bond price is that the default recovery depends on the value of the collateral asset. When the price of capital is high, the borrowing capacity is boosted and the market value of bond increases. This allows a greater amount of financing for investment. We allow this collateral channel to enrich the dynamic interaction between the financial condition of firms and their investment along the line of Shleifer and Vishny [1992], Kiyotaki and Moore [1997] and Bernanke, Gertler and Gilchrist [1999].

Given the shock process for (10) and the current technology level  $z$ , the default threshold  $\underline{z}' = \underline{z}(k', b', s')$  implies a threshold value for  $\varepsilon$ , denoted by  $\underline{\varepsilon}' = \underline{\varepsilon}(z, k', b', s')$ , which depends on the current technology  $z$  as well as  $(k', b', s')$ . Let  $\underline{\theta}$  denote the standardized value of  $\underline{\varepsilon}(z, k', b', s')$ , i.e.,

$$\underline{\theta}(z, k', b', s') \equiv \sigma^{-1} [\ln \underline{\varepsilon}(z, k', b', s') + 0.5\sigma^2]$$

Using this standardization and the properties of lognormal distribution, the bond price equation (13) can be shown to be equal to

$$q(z, k', b', s) = \sum_{\sigma'} p(\sigma, \sigma') m(s, s') \Phi(\underline{\theta}(z, k', b', s')) \left[ \frac{1 - \Phi(\underline{\theta}(z, k', b', s'))}{\Phi(\underline{\theta}(z, k', b', s'))} + (1 - \xi) \left( \frac{\Phi(\underline{\theta}(z, k', b', s') - \sigma)}{\Phi(\underline{\theta}(z, k', b', s'))} \frac{z^\rho \pi(k', s')}{b'} + \frac{Q(s')(1 - \delta)k' - \bar{n}}{b'} \right) \right]$$

where  $\Phi(\cdot)$  is the standard normal c.d.f.. Given the firm's choice  $k'$  and  $b'$ , and the exogenous state variables, the above expression for the bond price can be computed analytically.

The structure of the bond contract is similar to Cooley and Quadrini [2001] and Hennessey and Whited [2007] in corporate finance literature. An important difference is the condition under which defaults arise: a default occurs when the net-worth  $n$  of the firm hits the lower bound ( $\underline{n}$ ) in the current setup while a default arises when the value of equity,  $V(n)$  hits its lower bound,  $\underline{V}$  in the aforementioned literature. If the technology shock follows an iid process, the two assumptions do not make any difference because the value function is monotonically increasing with respect to net-worth and there is a unique level of net-worth corresponding to the assumed lower bound for the equity  $\underline{V}$ . However, if the technology shock is persistent or the value function has other arguments such as aggregate state variables like in this paper, the two assumptions are not equivalent because there can be a range of net-worth levels  $\underline{n}(z, s)$  which delivers the same lower bound equity level  $\underline{V}$ . Therefore, we admit that the uniqueness of the default threshold in net-worth is a simplifying assumption.<sup>9</sup> The simplification, however, allows us to avoid the computationally intensive

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<sup>9</sup>However, the uniqueness of the default threshold in equity is also strong an assumption as well in reality.



task of inverting value function to compute the default boundary  $\underline{n}(z, s)$  in each round of dynamic programming routine.

### 4.3 Firm's Problem

In this section, we describe the dynamic optimization problem of final-good firms given the structure of the bond contract and the pricing function. We assume that all firms are owned by the household. This implies that a firm discounts future cash-flows with the intertemporal substitution rate of the owner's consumption,  $m(s, s') = \beta u_c(s')/u_c(s)$ . However, we assume that there is informational asymmetry between the insider (managers) and the outsider of the firm. Under the information asymmetry, issuing new equity can involve costs that are used to overcome this informational frictions. For instance, Myers [2000] shows that the information asymmetry forces an entrepreneurial firm to sell new equity at a substantial discount. Although we do not model the firm as an entrepreneurial entity, we assume that a similar type of information asymmetry and the conflicts of interests exist between the manager and the investors. The equity issuance cost in our model economy is a welfare loss that does not accrue to any agent's economic accounts.

In the literature, a convex cost of equity issuance has been frequently used (see Cooley and Quadrini [2001], Hennessey and Whited [2007] and Jermann and Quadrini [2007]). We also follow this approach. In particular, we parameterize the cost as a quadratic form,

$$\lambda(d) = \begin{cases} 0 & \text{if } d \geq 0 \\ \lambda_1|d| + \lambda_2|d|^2 & \text{if } d < 0 \end{cases} \quad (14)$$

where  $\lambda_1, \lambda_2 > 0$  and  $d$  is the dividend of the firm defined as

$$d = n - Q(s)k' + qb' \quad (15)$$

The fact that the corporate debt is defaultable and renegotiable makes the firm's problem more dynamic and complicated because the firm's continuation value is truncated by the default pay-off. Despite this additional complication, however, the firm's problem can still be analyzed along the line of standard optimization theory. We can express the firm's problem in a recursive way:

$$\begin{aligned} V(z, n, s) = & \max_{i, b', d} \left\{ d - \lambda(d) + (1 - \eta) \sum_{\sigma'} p(\sigma, \sigma') m(s, s') \right. \\ & \left. \times \int \max \{ V(z', n(z', k', b', s'), s'), V(z', \bar{n}, s') \} dF(z'|z, \sigma) \right\} \quad (16) \end{aligned}$$

subject to (11), (15), (14), (16), the law of motion for capital,  $k' = i + (1 - \delta)k$ , and the law

of motion for aggregate state variables. In the next section, we describe the law of motion for the aggregate state variables in detail. Note that the firm has an additional discount factor  $1 - \eta$ . We assume that the firm faces a time-invariant death probability  $\eta$ . The additional discount factor makes the firm accumulate a strictly positive amount of debt in the steady state. We introduce this additional discount factor as a short-cut to tax-benefit. Regarding the timing of the death shock, we assume that the death shock occurs only after the firm makes debt payment/default decision so that the zero profit condition for the bond investor is not affected by the death probability.

The continuation value of the firm is truncated by the default value  $V(z', \bar{n}, s')$ . Using the definition of the default threshold shock, the continuation value can be decomposed into

$$\sum_{\sigma'} p(\sigma, \sigma') m(s, s') \left[ \int_{z' \leq \underline{z}(k', b', s')} V(z', \bar{n}, s') dF(z'|z, \sigma) + \int_{z' \geq \underline{z}(k', b', s')} V(z', n(z', k', b', s'), s') dF(z'|z, \sigma) \right]$$

The truncated continuation value has important consequences for the properties of investment and debt policies of the firm. To show the consequences of the truncation, we derive the first order condition (FOC) for capital as follows.

$$\begin{aligned} Q(s) &= (1 - \eta) \sum_{\sigma'} p(\sigma, \sigma') m(s, s') \int \frac{1 - \lambda'(d')}{1 - \lambda'(d)} \left[ z' \pi_k(k', s') + Q(s')(1 - \delta) \right] dF(z'|z, \sigma) \\ &+ q_k(z, k', b', s) b' - (1 - \eta) \sum_{\sigma'} p(\sigma, \sigma') m(s, s') \int_0^{\underline{z}(k', b', s')} \frac{1 - \lambda'(d')}{1 - \lambda'(d)} z' \pi_k(k', s') dF(z'|z, \sigma). \end{aligned}$$

The left hand side of the above measures the marginal cost of investment, which is equal to  $Q(s)$ . The right hand side measures the marginal benefit of investment. The investment Euler equation has a few non-neoclassical features: first, investment not only affects tomorrow's profitability of the firm but also affects the terms of borrowing today. This is for two reasons. For any given level of borrowing  $b'$ , increase in capital lowers the default boundary in the productivity shock in (12), raising the price of bond. The increase in capital also boosts the value of default recovery and therefore, lowers the default premium, raising the price of bond. This unusual element of marginal benefit from investment is captured by the term,  $q_k(z, k', b', s) b'$ .

Second, the future cash-flow is discounted by a firm-specific stochastic discount factor given by  $[1 - \lambda'(d')]/[1 - \lambda'(d)]$ . Since  $1 - \lambda'(d) = 1 + \lambda_1 - \lambda_2 d > 0$  when  $d < 0$  and  $1 - \lambda'(d) = 1$  when  $d \geq 0$ , the firm discounts future cash-flow more heavily when issuing equity. Therefore, the ratio  $[1 - \lambda'(d')]/[1 - \lambda'(d)]$  measures the relative value of non-equity

financing today vs. tomorrow. In equilibrium, the firm is induced to equalize the costs of debt financing and equity financing. We will show this when we derive the FOC for  $b'$

Third, the marginal profitability of the firm is truncated by the default boundary  $\underline{z}(k', b', s')$ . This is a natural consequence of introducing default into the firm's optimization problem—the firm does not care about the downside risk that is beyond the default boundary. Finally, the market value of capital affects the firm's investment policy in two opposing ways. On one hand, a higher price of capital hinders the firm from increasing capital stock through the traditional cost channel. On the other hand, a higher market value of capital accelerates investment through the collateral channel.

When  $q_k(z, k', b', s)b' = 0$ ,  $[1 - \lambda'(d')]/[1 - \lambda'(d)] = 1$  and  $\underline{z}(k', b', s') = 0$ , the Euler equation for capital collapses into the neoclassical one. This means that when the firm has a strong balance sheet position, the firm's investment policy is very much similar to that of a neoclassical firm, responding only to the fundamentals of the firm. In contrast, when the firm has a weak balance sheet position, the firm's investment policy is very sensitive to the financial conditions of the firm including cash-flow and collateral asset.

The efficiency condition for debt also has several non-standard features. The FOC is given by

$$q(z, k', b', s) + q_b(z, k', b', s)b' = (1 - \eta) \sum_{\sigma'} p(\sigma, \sigma') m(s, s') \int_{\underline{z}(k', b', s')}^{\infty} \frac{1 - \lambda'(d')}{1 - \lambda'(d)} dF(z'|z, \sigma).$$

The marginal cost of borrowing, the right hand side, is different from that of standard Euler equation. First, the right hand side has an additional discounting factor,  $1 - \eta$ . Second, as in the investment Euler equation, the firm discount the future more heavily than usual when issuing equity. Third, the probability weight given to the marginal cost is less than one owing to the default probability.

The left hand side measures the marginal benefit of debt financing while the right hand side measures the marginal cost of debt financing. For a given level of arbitrary capital policy  $k'$ , increase in debt financing raises the cost of borrowing, i.e.,  $q_b(z, k', b', s) < 0$ , for two reasons. It increases the default boundary in (12) and raises the default probability. It also lowers the recovery ratio,  $\bar{b}(z', k', s')/b'$  thereby bringing down the market value of debt. Note that the FOC can be interpreted from another perspective. The left hand side of the FOC measures the marginal cost of debt financing. The right hand side measures the marginal cost of equity financing. The FOC then requires that the marginal costs of two funding methods be equalized.

Given the assumption made for the timing of the death shock, the financial frictions are essential for a well-defined problem. To see this point, assume that the default probability is close to zero,  $\underline{z}(k', b', s') \simeq 0$ . In a non-stochastic steady state without financial frictions,

the price of bond is equalized to the time discount factor,  $q = E[m(s', s)] = \beta$ . Absent any financial frictions (because  $\underline{z}(k', b', s') \simeq 0$ ),  $q_b = 0$ . With no default probability and  $E[1 - \lambda'(d')]/[1 - \lambda'(d)] = 1$ , the marginal cost becomes  $\beta(1 - \eta)$ . In this environment, therefore, the firm wants to increase the debt indefinitely and the problem is not well defined. With financial frictions, however, the firm increases the debt financing until  $q_b(z, k', b', s)b'$  takes a sufficiently large negative number until the left hand side is equalized to the right hand side and the problem is well defined.

The fact that the recovery rate is reciprocal to the debt level does not imply that the debt financing  $qb'$  cannot be raised by increasing  $b'$ . In other words,

$$q(z, k', b', s) + q_b(z, k', b', s)b' > 0$$

in general. This is because the default recovery rate is given a weight,  $\Phi(\underline{\theta}(z, k', b', s')) < 1$ . However, as the level of borrowing is increased indefinitely, the default boundary  $\underline{\theta}$  eventually reaches a sufficiently high level so that the effective default probability is close to zero. In this limit, the amount of debt financing cannot be raised further even when the firm increases the face value of the debt indefinitely, hence  $q(z, k', b', s) + q_b(z, k', b', s)b' = 0$ . In other words, the debt financing is bounded. This implies that if the default boundary responds too sensitively to the increase in debt issue, the firm can run into the debt ceiling rather quickly. For the problem to be well-conditioned, the firm must be given a sufficient amount of upside potential.

Figure 3 shows a general shape of the bond pricing function. As predicted, the bond price is shown to increase in capital and to decrease in debt.<sup>10</sup> An important fact about the shape of the bond pricing function is that the price declines gradually as the debt issuance is increased initially. As a result, the amount of debt financing  $qb'$  increases as  $b'$  increases. This means that the debt instrument can be used to fill the financial needs of the firm as long as the balance sheet condition is relatively healthy. However, as the firm continues to increase debt issuance, the the market value of debt eventually becomes so low that the debt financing cannot be increased.

#### 4.4 Price of Capital

The price of capital is determined by supply and demand for capital. The firm problem determines the structure of individual demand for capital described in the above section. Aggregate demand for capital can then be constructed by simply aggregating the individual demands, i.e.,

$$I(s) = \int_{N,A} k'(z, \max(n, \bar{n}), s) d\mu - (1 - \delta)K$$

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<sup>10</sup>For the calibration underlying this picture, see section 5.1.

where  $\mu$  is the joint distribution of productivities and net-worths of heterogeneous firms. The joint distribution is an essential element of aggregation. Therefore, it is one of the aggregate state variables that economic agents need to keep track of. Note that the firm's capital demand is truncated owing to the limited liability assumption.

As mentioned earlier, we assume that there is a competitive investment-good sector with a CRS technology. The sector takes the aggregate undepreciated capital  $K$  and consumption goods  $I$  as inputs to produce new capital stock  $K'$ . The investment goods sector sell the capital stock back to the final-good firms at a unit price  $Q(s)$ .

The traditional accumulation equation  $K' = I + (1 - \delta)K$  can be thought of as a production function. In addition to input costs, we assume a convex adjustment cost  $\Xi(I(s)/K)$  as well in order to endogeneize the price of capital asset. The investment good sector problem can then be formulated as

$$\max_{I(s)/K} \left\{ Q(s) \left[ \frac{I(s)}{K} + (1 - \delta) \right] - \frac{I(s)}{K} - Q(s)(1 - \delta) - \Xi \left( \frac{I(s)}{K} \right) \frac{1}{K} \right\}$$

where we normalize the size of the problem by  $K$  given the CRS structure. The zero profit condition and market clearing imply that the unit price of capital in this economy must be equal to

$$Q(s) = 1 + \Xi' \left( \frac{I(s)}{K} \right) \frac{1}{K}$$

Note that the asset value of existing capital not only depends on the distribution of net-worth but also depends on the aggregate capital stock  $K$ . For this reason, the vector of aggregate state variable must be expanded to  $s = [\sigma, \mu, K]$ . The last element is due to the capital adjustment friction at the macroeconomic level. The individual investment and debt policies are also functions of aggregate capital, but not of individual undepreciated capital. This is because we assume that the capital adjustment friction exists at the macro-level, not at the firm-level.

## 4.5 Market Clearing

The household's problem is standard. Since we assume that all informational frictions between the owner and the manager of a firm are controlled by the equity issuance cost, the household problem can be analyzed in a standard way. The efficiency conditions for the household can then be summarized by a complete set of asset pricing equations for the continuum of final good firms and a FOC linking the marginal disutility of work hours to the valuation of marginal consumption.

Using the zero profit condition of the capital-good sector, we can impose  $Q(s)I(s) = I(s) + \Xi[I(s)/K]$  to the market clearing condition for consumption-good market. Also im-

posing the stock market clearing conditions, the market clearing condition for consumption goods becomes

$$c(s) = Y(s) - I(s) - \Xi(I(s)/K) - \underbrace{\xi \int 1(n(i, s) \leq \bar{n}) \bar{b}(i, s) di}_{\text{bankruptcy costs}} - \underbrace{\int 1(d(i, s) \leq 0) \lambda(d(i, s)) di}_{\text{equity issuance costs}}$$

where  $\int y(i) di$ .<sup>11</sup> Compared with a frictionless real business cycle model, the consumption-good market clearing condition has two non-standard components: the bankruptcy costs and equity issuing costs. These costs measures the resource loss stemming from the capital market frictions. Although the costs are very small relative to aggregate output, the capital market frictions modify macroeconomic equilibrium mainly by altering the first order principles of the agents.

In order to solve the problem, economic agents need to understand how the aggregate state variables are evolving over time. One of the aggregate state variables that they need to keep track of is the joint distribution of net-worth and technology among heterogenous firms. The exact law of motion of the joint distribution is given by

$$\mu(N_0, Z_0) = \int_{N_0, Z_0} \left\{ \int_{N, A} 1[n' = \max\{\bar{n}, n(z', k', b', s')\}] f(z'|z, \sigma) d\mu \right\} dn' dz' \quad (17)$$

where  $k' = k'(z, \max(n, \bar{n}), \sigma, \mu, K)$ ,  $b' = b'(z, \max(n, \bar{n}), \sigma, \mu, K)$  and  $\mu(N_0, Z_0)$  measures the number of firms which have net-worth level in the set  $N_0$  and technology level in the set  $Z_0$  next period.

Note that tomorrow's distribution of net-worth depends on tomorrow's price of capital  $Q(s')$  since the collateral value of tomorrow's capital depends on  $Q(s')$ .  $Q(s')$  in turn depends on today's joint distribution of net-worth and technology because the joint distribution determines the aggregate capital stock tomorrow ( $K'$ ) and the price of capital tomorrow depends on  $K'$ .

Since the distribution is an infinite dimensional object, it is not feasible to solve the problem exactly. Rather, following the literature on the general equilibrium with heterogenous agents (see Krusell and Smith [1998], Rios-Rull [1995]), we adopt the assumption of bounded rationality—the agents carry only a finite number of moments of the distribution and use them in log-linear functional forms to forecast equilibrium prices. There are three prices that the agents need to forecast in order to solve their optimization problem: the

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<sup>11</sup>In the resource constraint, we assume that there is no output loss due to exogenous deaths of firms. This implies that the death shocks realize after firms produce output and the old firms that are hit by the shocks are replaced by immediate entries of new firms. We assume that the new firms inherit the financial positions of old firms. We adopt this simplifying assumption because the death probability is introduced to provide a long term incentive to accumulate debt.

marginal utility of the representative consumer ( $u_c(s)$ ), wage ( $w(s)$ ) and the price of capital ( $Q(s)$ ). The approximate laws of motion are specified as

$$\ln \mathbf{y} = \alpha(\sigma_{-1}, \sigma) + \beta \ln \mathbf{y}_{-1}$$

where  $\mathbf{y} = [u_c(s) \ w(s) \ N(s) \ K(s)]$ . Note that we allow the constant terms to depend not only on the current uncertainty regime, but also on yesterday's regime. This is to allow asymmetric responses of the economy to low-to-high vs. high-to-low regime changes in uncertainty.

## 5 Results

### 5.1 Calibration

We calibrate the parameters of the model using firm-level data. This is mainly because we model the uncertainty as a time-varying volatility of the distribution of idiosyncratic productivity. We also choose annual frequency for the ease of comparison between the data and the model counterpart.

For the calibration of DRS technology, we estimate the profit function using the panel data set. The elasticity of the profit function with respect to capital is estimated as 0.61. Given our calibration of relative capital share 0.30 in the Cobb-Douglas production function, the elasticity implies a DRS 0.84. Obviously, the curvature of the profit function may be interpreted as the result of monopolistic competition, rather than the result of a DRS technology. For simplicity, however, we choose the environment of homogeneous goods and DRS technology.

We use the residuals from the profit function estimation to calibrate the process of idiosyncratic technology shock. In this estimation, we allow the residual to follow an AR(1) process, using Baltagi and Wu [1999]'s unequally spaced panel data regressions with AR(1) disturbances. The persistence of the AR(1) disturbances is estimated as  $\rho_z = 0.89^4 = 0.62$ .

We also use the dispersion of the residuals to calibrate the uncertainty process. We assume that the uncertainty process follows a two-state Markov Chain process. The two uncertainty states can be thought of as low and high uncertainty regimes. To calibrate the Markov Chain process, we first estimate an AR(1) process for the standard deviation of the disturbances from the profit function estimation. We then use the approach of Tauchen [1986] to discretize the process. We set  $\sigma_L = 0.25$  and  $\sigma_H = 0.52$  with the steady state level of dispersion  $\bar{\sigma} = 0.36$ . The probability of tomorrow's uncertainty regime remaining the same as today is set at 0.69, which has an AR(1) representation,  $\rho_\sigma = 0.80^4 = 0.41$ .

We use the parameters of the equity cost function, the bankruptcy cost parameter and

the survival probability to match the historical average of credit risk spread. We use Baa-rated corporate bond spread over 10 year Treasury yield. The historical mean of the spread is about 170 bps since mid 1950s. We set  $1 - \eta = 0.95^4 = 0.80$  for the survival probability. Given our assumption on the timing of the death shock, the survival probability does not directly determine the credit risk spread. However, the survival probability determines the long term leverage ratio of the firm and indirectly determines the credit spread by this channel. We specify the bankruptcy cost as 10% of default recovery. Increasing the bankruptcy cost parameter too much from this level raises the mean credit spread of the model while dampening the volatility of the model dynamics.

The equity issuance cost also is important for the determination of the credit spread. Without the equity issuance cost, the firm would try to avoid high premiums on debt issue using equity channel more frequently. We set the linear and the quadratic cost parameters equal to 0.15 and 0.50. The parameterization implies a substantial discount on equity issuance and that the equity is not a preferable source of funds unless the firm's balance sheet condition is substantially deteriorated and is facing a considerable amount of credit risk premium already. When calibrating the parameters of equity issuance cost, we also takes into account the fact that equity financing covers approximately only 8% of total external financing in U.S. (Bolton and Scharfstein [1996]). We show that the model with so calibrated parameters generates about 11% share of equity financing in total external financing, roughly in line with the empirical counterpart in the data.

Based on COMPUSTAT data, we calibrate the annual depreciation rate of capital stock as 0.18. For the capital adjustment cost, we adopt a conventional quadratic specification. We calibrate the capital adjustment cost parameter so that the elasticity of the price of capital with respect to investment/capital ratio is set approximately 0.20.<sup>12</sup>

For the representative household, we choose the simplest possible specification for the ease of computation. In particular, we assume a log utility for consumption and linear disutility for hours. With this specification, the wage is proportional to consumption and economic agents do not need to predict wage separately from consumption. This substantially simplifies the computation. (see Krusell and Smith [1998]). Finally, for the subjective discounting factor, we choose  $0.99^4 = 0.96$  so that the real risk-free rate in the steady state is equal to 4%.

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<sup>12</sup>Traditionally, macroeconomic literature has used much higher values for the adjustment cost parameter (see, for instance, Chirinko [1993]). We choose a relatively modest degree of capital adjustment friction, considering recent findings based upon micro-level studies (see, for instance, Cooper and Haltiwanger [2006]).



## 5.2 Model Dynamics

In this section, we analyze how the aggregates of the model economy respond to the fluctuation in the uncertainty level. In particular, we show how default risk, borrowing cost, the choice of capital structure, the price of capital and aggregate investment interact with each others to create a full-blown business cycle when the uncertainty regime changes.<sup>13</sup>

Since the relationship between the uncertainty and the price of bond is the essential mechanism that drives aggregate investment cycle in this paper, we start by showing the implication of the regime change in uncertainty on the price of bond. In Figure 4, we plot the bond prices for firms with different qualities under two different uncertainty levels. In all cases, the price of bond effectively stays at the price of risk-free bond when the leverage ratio is relative low. As firms continue to increase their leverage ratios, the probability of default increases, the default recovery ratio declines and the price of bond starts to fall from the risk-free rate to compensate for the expected default loss.

In this picture, thick lines are for a “good” firm which has a productivity 50% higher than the average firm. When the uncertainty level increases from the low state (25%) to the high state (52%), the price of bond shifts down. However, the pattern is highly non-linear: at low enough level of leverage, the effect is almost nil. The vertical difference starts to increase as the firm’s leverage ratio goes up higher than 90% level. At around 120% leverage, the effect of increased on the bond price begins to decline. This is because at a high enough level of borrowing, the default probability is already very close to unity and the price of bond becomes the reciprocal of the quantity of bond issued, i.e.,  $q \simeq 1/b'$ . This also explains why the effect of increase in uncertainty is smaller for a “bad” firm that has a productivity level 50% lower than the average firm. The thin lines in the same figure show this aspect. The price of bond for this firm starts to fall off from the risk-free rate at a relatively smaller level of leverage. As the uncertainty level goes up, the bond price shifts down, but to a lesser degree relative to the case of a good firm.

Figure 5 plots the response of the credit risk spread with respect to the change in uncertainty regime. The difference between Figure 4 and 5 is that Figure 5 shows the average level of credit risk premia that are results of the optimal choice of firms regarding capital structure while Figure 4 exhibits the response of bond prices under an arbitrary set of leverage ratios. The red line in the picture shows which uncertainty regime the economy is in. The blue line depicts the movement in credit risk premium. The model generates approximately 150bps of credit spread on average, comparable to 170 bps on average in the post-war U.S. data. As the uncertainty regime changes from low to high, the

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<sup>13</sup>All aggregate variables in this section are constructed as the averages of simulation of 10,000 firms. We run the simulation for 1100 time periods and delete the first 100 observation. In the figures that we show below, the first 100 time periods are shown after the deletion of initial 100 observations.

credit risk spread immediately jumps up by 70 bps, thereby validating the prediction that a mean-preserving spread is associated with an increase in the bond risk premium even in equilibrium business cycles.

Note that the change in credit spread ensuing the regime change in uncertainty is very persistent. In fact, the credit risk premium continues to climb for a few more periods even after the immediate change in the uncertainty regime. This is because the initial decline in investment further deteriorates the collateral position of firms. Figure 6 shows the response of aggregate investment/capital ratio as the uncertainty regime changes. Aggregate investment/capital ratio immediately plunges about 22% from its steady state ratio. The aggregate investment only slowly recovers to the normal level and the collateral value of firms continue to decline while the high uncertainty regime lasts.

If the equity financing were not costly, firms could avoid the high credit risk premium caused by the regime change to the higher uncertainty by deleveraging balance sheet position fast enough to lower the default risk sufficiently. This does not mean that the equity financing is not actively used by firms. In fact, the firms in the model economy sharply increase equity financing during the down-turn. Figure 7 shows how the amount of equity financing changes over time as the uncertainty level jumps from one to the other. On average, total value of equity issuance (before netting out issuance cost) increases about 50 ~ 100% during the high uncertainty regime relative to the mean level.

The sharp increase in equity issuance is not simply driven by large issuances of a small number of big firms. As shown in Figure 8, the fraction of firms that are issuing equity jumps from 4% to 10 ~ 20% as the uncertainty regime changes from low to high. Both the aggregate quantity and the frequency of equity financing are highly countercyclical.<sup>14</sup>

However, there is a clear limit to the extent that firms can avoid higher default risk premium by simply replacing debt by equity. In Figure 9, we show the fluctuation in the share of equity relative to debt financing. As mentioned earlier, the model generates roughly 11% of equity share in total external financing on average. Despite the sharp increase in both the volume and the frequency of equity financing, however, the share of equity financing only goes up by about 7% points during the high uncertainty regime. With the stiffly increasing marginal cost of equity issuance cost, firms only end up equalizing two marginal costs of external financing, rather than avoiding one source for the other. With our calibration, the firms are led to maintain the balance between equity and debt either at 6% or at 17% depending on the uncertainty regime that they are in.

The source of aggregate fluctuation in this paper is the time-varying economic uncertainty that change the “riskiness” of potential borrowers. It is interesting to see how much

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<sup>14</sup>See Covas and Den Haan [2007] to see the pro/countercyclicity issue of debt and equity. They show that debt and equity issuance are procyclical in the micro data, but countercyclical in the macro data. They explain the discrepancy from the influence of disproportionately large firms.

fluctuation in default rate and recovery rate the uncertainty shock in the model can generate and to check if their properties are consistent with those of the data.

In U.S. data, the realized default rate in annual frequency is about 2.5% on average while the recovery rate is on the order of 40%.<sup>15</sup> The default rate in the data is highly countercyclical whereas the recovery rate is strongly procyclical, implying a strong negative correlation between them (see, Levin et al [2008], Altman et al [2005] and Acharya et al [2007]). Figure 10 and 11 show the model generated default rate and recovery rate. The model is very successful in generating countercyclical default rate and procyclical recovery rate. The correlation between the default and recovery rates is close to -1 as in the data.

However, the model is less successful in matching the default rate and the recovery rate of the data: the model generated default rate is close to 8% on average, obviously too high a level relative to that of the data while the recovery rate in the model is about 87%, again too high a level relative to the counterpart of the data. It is also noteworthy that the model cannot generate enough time-variation in the recovery rate although the model can generate large enough volatility in the default rate comparable to that of the data.

In order to generate a sufficiently low recovery rate especially during the downturn, one could use a much higher bankruptcy cost parameter. Currently the bankruptcy cost parameter is calibrated as 10% of the total asset of a firm. While much higher bankruptcy cost parameters can result in lower recovery rates for a given leverage ratio, the firms in the model economy tend to endogenously lower their leverage positions sufficiently enough to avoid the resulting increase in the marginal cost of borrowing. To avoid a stiff increase in the borrowing cost, we must allow the default probability to go down simultaneously by calibrating a sufficiently lower default boundary. This, however, tends to increase recovery rate because this implies that a greater portion of the default loss is taken by a firm.<sup>16</sup>

Figure 12 shows the fluctuation in the leverage ratio. The black-starred line shows the response of market value leverage ratio ( $b/Qk$ ) and the blue-circled line shows the response of book leverage ratio ( $b/k$ ). Both leverage ratios show a strong procyclicality. The figure shows that while the difference is not large, the volatility of book leverage cycle is greater than that of market leverage cycle. The difference is explained by the procyclicality of

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<sup>15</sup>From the point of view of a standard bond pricing framework such as the one in this paper, the average default rate and the average recovery rate in the data would imply a credit risk spread of 160 bps. Assuming a discount factor (risk-free rate) 0.96, the zero profit condition would imply an average credit risk premium,  $160 \text{ bps} \approx 10000 \times 1/0.96 \times \{[1/(1 \times 0.975 + 0.4 \times 0.025)] - 1\}$ . While this level of credit risk premium is consistent with the historical mean of Baa corporate spread over 10 year Treasury yield, a direct comparison is not possible because the empirical default and recovery rates are computed over the samples that include high-yield bonds. Using Merton's distance to default model, we compute the implied default frequency of our data sample as 5%. In this case, the mean credit spread is computed as 320 bps.

<sup>16</sup>Another way of generating much more procyclical recovery rate is to increase the elasticity of the price of capital with respect to aggregate investment ratio by raising capital adjustment cost parameter. In principle, this increases the sensitivity of collateral price. We are currently exploring this possibility.

collateral value. When the uncertainty regime is low, the price of collateral is high. This lowers the borrowing costs of firms and allow them to expand capital expenditure more than they would under fixed price of collateral. Increased capital expenditure then further strengthens the collateral position, allowing even greater expansion until the incremental credit multiplier effect converges to zero. The opposite multiplier works as well when the uncertainty regime changes from low to high. This shows a potential destabilizing role of endogenous market value of firms' collateral assets. The book-to-market leverage makes a firm's balance sheet position look stronger in the upturn and look weaker in the downturn. As a result, the credit risk spread falls too much in the upturn while it goes up too much in the downturn, amplifying the magnitude of the uncertainty-driven business cycle.

We conclude this section by evaluating whether or not the strength of the relationship among uncertainty shock, credit risk spread and investment generated by the model is comparable to empirical evidence. The empirical findings in the earlier analysis of firm-level data suggest that the elasticity of credit risk premium with respect to uncertainty shock in the data is close to unity. In our quantitative exercise, the uncertainty goes up by about 100% when the regime changes from low to high. In response to this increase in uncertainty, the credit risk premium in our model goes up from 100 bps to 200 bps, exactly matching the empirical elasticity. The empirical analysis of the firm-level data also suggests that approximately 100% increase in uncertainty (from  $\sigma_L = 0.25$  to  $\sigma_H = 0.52$ ) lowers the investment/capital ratio by 2 to 3 percentage points (see Table 8). The quantitative model in this paper generates a similar magnitude for the response of aggregate investment: the investment/capital ratio is lower by 2.5 percentage lower on average while the uncertainty regime is high. This shows that the model successfully replicates the empirical regularity found among uncertainty measure, credit spread and investment.

## 6 Conclusion

In the aggregate investment literature, uncertainty is related with aggregate fluctuation through the channel of investment irreversibility. While the relationship between the uncertainty and irreversible investment is theoretically appealing, it is not easy to make a robust prediction regarding the sign of the uncertainty effect on capital accumulation especially in general equilibrium settings. This paper shows that it is possible to link the fluctuation in economic uncertainty to the aggregate investment cycle by exploiting the implication of uncertainty for the price of credit risk in general equilibrium. By using both empirical and quantitative approaches, we could show that increase in economic uncertainty can bring potentially large detrimental effects on aggregate investment by increasing credit risk premium required for bond investor's participation constraint. The magnitude of the responses in

credit risk premium and investment ensuing the increase in uncertainty seems comparable to that of the data. The model also generates ample dynamics for capital structure choice at the time of heightened uncertainty. Our research suggests that more attention should be paid to the implication of changing uncertainty for aggregate activity and the impact on credit market is one of the place where to start from.

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Table 1: Cross-Correlations of Uncertainty Measures

Uncertainty Measure	$\sigma_S$	$\sigma_R$	$\bar{\sigma}_V$	$\bar{\sigma}_E$
$\sigma_S$	1.000	0.569	0.567	0.641
$\sigma_R$	-	1.000	0.772	0.790
$\bar{\sigma}_V$	-	-	1.000	0.960
$\bar{\sigma}_E$	-	-	-	1.000

NOTE: Sample period: quarterly data from 1970:Q1 to 2008:Q3 ( $T = 155$ ). The entries in the table are the contemporaneous correlation coefficients between the four measures of time-varying macroeconomic uncertainty shown in Figure 1:  $\sigma_S =$  (weighted) cross-sectional standard deviation of real sales growth;  $\sigma_R =$  (weighted) cross-sectional standard deviation of equity returns;  $\bar{\sigma}_V =$  (weighted) cross-sectional mean of the volatility of the firm's assets computed using the Merton-DD model; and  $\bar{\sigma}_E =$  (weighted) cross-sectional mean of the volatility of the firm's equity computed using the simple model.

Table 2: Macroeconomic Uncertainty and Credit Spreads  
(Granger Causality Tests)

Null Hypothesis	VAR Specification	
	Bivariate	Trivariate <sup>a</sup>
Baa spread does not GC uncertainty	0.271	0.319
Uncertainty does not GC Baa spread	0.004	0.002

NOTE: Sample period: quarterly data from 1970:Q1 to 2008:Q3 ( $T = 151$ ). The entries in the table are the (asymptotic)  $p$ -values associated with the  $\chi^2$  test of the specified null hypothesis. The baseline (bivariate) VAR specification includes four lags of the Baa corporate bond spread and four lags of the uncertainty measure shown in Figure 2.

<sup>a</sup>The baseline VAR specification augmented with four lags of the growth in real nonfarm business sector output.



Table 3: Investment, Credit Spreads and Uncertainty

Parameter: Explanatory Variable	Specification		
	(1)	(2)	(3)
$\sum \beta: \Delta \log I_{t-s}$	0.144 (0.148)	0.112 (0.134)	0.000 (0.166)
$\sum \gamma: \Delta \log Y_{t-s}$	1.612 (0.408)	1.278 (0.347)	1.640 (0.403)
$\theta_1: \sigma_t$	-0.191 (0.100)	-	-0.150 (0.104)
$\theta_2: \text{Baa}_t$	-	-1.916 (1.035)	-2.543 (1.472)
Adj. $R^2$	0.396	0.365	0.403
$\text{Pr} > E_p^a$	0.076	0.017	0.068

NOTE: Sample period: quarterly data from 1970:Q1 to 2008:Q3 ( $T = 154$ ). Dependent variable is  $\Delta \log I_{t+1}$ , the log-difference in real business fixed investment in quarter  $t + 1$ . All specifications include a constant (not reported) and are estimated by OLS. Heteroscedasticity- and autocorrelation-consistent (of order 4) asymptotic standard errors are computed according to Newey and West [1987] and are reported in parentheses.

<sup>a</sup> $p$ -value for the Doornik and Hansen [1994] omnibus test statistic for the normality of OLS residuals.

Table 4: Summary Statistics of Corporate Bond Characteristics

Bond Characteristic	Mean	SD	Min	P50	Max
# of bonds per firm/month	5.77	8.48	1.00	2.00	75.0
Mkt. Value of Issue <sup>a</sup> (\$mil.)	290.0	305.7	1.02	217.9	6,657
Maturity at Issue (years)	14.0	9.4	1.0	10.0	50.0
Term to Maturity (years)	11.8	8.6	0.01	8.63	30.0
Duration (years)	6.25	3.14	0.00	6.01	24.4
S&P Credit Rating	-	-	D	A3	AAA
Coupon Rate (pct.)	7.77	2.19	0.00	7.50	17.5
Nominal Effective Yield (pct.)	8.25	3.32	1.20	7.75	57.4
Credit Spread <sup>b</sup> (bps.)	174	264	10	109	4,995

*Panel Dimensions*

Obs. = 358,638  $N$  = 5,269 bonds

Min. Tenure = 1 Median Tenure = 59 Max. Tenure = 294

NOTE: Sample period: Monthly data from January 1973 to September 2008 for a sample of 926 nonfinancial firms. Sample statistics are based on trimmed data (see text for details).

<sup>a</sup>Market value of the outstanding issue deflated by the CPI.

<sup>b</sup>Measured relative to comparable-maturity Treasury yield (see text for details).

Table 5: Credit Spreads and Uncertainty  
(Merton DD-Model Estimate of Uncertainty)

Explanatory Variable	Specification			
	(1)	(2)	(3)	(4)
$\log[V/D]_{t-1}$	-0.474 (0.025)	-0.182 (0.017)	-0.173 (0.016)	-0.115 (0.016)
$\mu_{V,t-1}$	-0.282 (0.041)	-0.384 (0.025)	-0.387 (0.025)	-0.272 (0.020)
$\log \sigma_{V,t-1}$	0.747 (0.041)	0.396 (0.031)	0.389 (0.028)	0.107 (0.029)
$\log \text{PAR}_t$	0.041 (0.015)	-0.020 (0.012)	-0.017 (0.009)	-0.006 (0.011)
$\log \text{DUR}_t$	-0.049 (0.013)	0.041 (0.009)	0.042 (0.009)	0.029 (0.009)
Adj. $R^2$	0.297	0.527	0.534	0.653
Rating Effects <sup>a</sup>	no	yes (0.000)	yes (0.000)	yes (0.000)
Industry Effects <sup>b</sup>	no	no	yes (0.000)	yes (0.000)
Time Effects <sup>c</sup>	no	no	no	yes (0.000)

NOTE: Sample period: monthly security-level data from January 1973 to September 2008 (Obs. = 352,156). Dependent variable is  $\log \text{SPR}_{jt}^i$ , the logarithm of the credit spread of bond  $i$  (issued by firm  $j$ ) in month  $t$ . All specifications include a constant (not reported) and are estimated by OLS. Heteroscedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses.

<sup>a</sup> $p$ -values for the test of the null hypothesis of the absence of fixed credit rating effects are reported in parentheses.

<sup>b</sup> $p$ -values for the test of the null hypothesis of the absence of fixed industry effects are reported in parentheses.

<sup>c</sup> $p$ -values for the test of the null hypothesis of the absence of fixed time effects are reported in parentheses.

Table 6: Credit Spreads and Uncertainty  
(Simple Estimate of Uncertainty)

Explanatory Variable	Specification			
	(1)	(2)	(3)	(4)
$\log[E/D]_{t-1}$	-0.364 (0.024)	-0.129 (0.018)	-0.111 (0.017)	-0.095 (0.017)
$\mu_{E,t-1}$	-0.164 (0.024)	-0.309 (0.018)	-0.307 (0.017)	-0.232 (0.013)
$\log \sigma_{E,t-1}$	0.985 (0.035)	0.513 (0.027)	0.523 (0.025)	0.302 (0.033)
$\log \text{PAR}_t$	0.017 (0.014)	-0.029 (0.011)	-0.022 (0.009)	-0.009 (0.011)
$\log \text{DUR}_t$	-0.029 (0.011)	0.042 (0.009)	0.043 (0.009)	0.030 (0.009)
Adj. $R^2$	0.383	0.546	0.553	0.660
Rating Effects <sup>a</sup>	no	yes (0.000)	yes (0.000)	yes (0.000)
Industry Effects <sup>b</sup>	no	no	yes (0.000)	yes (0.000)
Time Effects <sup>c</sup>	no	no	no	yes (0.000)

NOTE: Sample period: monthly security-level data from January 1973 to September 2008 (Obs. = 352,156). Dependent variable is  $\log \text{SPR}_{jt}^i$ , the logarithm of the credit spread of bond  $i$  (issued by firm  $j$ ) in month  $t$ . All specifications include a constant (not reported) and are estimated by OLS. Heteroscedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses.

<sup>a</sup> $p$ -values for the test of the null hypothesis of the absence of fixed credit rating effects are reported in parentheses.

<sup>b</sup> $p$ -values for the test of the null hypothesis of the absence of fixed industry effects are reported in parentheses.

<sup>c</sup> $p$ -values for the test of the null hypothesis of the absence of fixed time effects are reported in parentheses.

Table 7: Investment, Uncertainty, and Credit Spreads  
(Merton DD-Model Estimate of Uncertainty)

Explanatory Variable	Static Specification			
	(1)	(2)	(3)	(4)
$\log[S/K]_t$	0.123 (0.008)	0.121 (0.008)	-	-
$Q_{V,t-1}$	-	-	0.049 (0.006)	0.046 (0.006)
$\sigma_{V,t-1}$	-0.007 (0.015)	-0.000 (0.015)	-0.007 (0.017)	-0.001 (0.017)
$\text{SPR}_{t-1}$	-	-1.160 (0.161)	-	-0.973 (0.158)
$R^2$ (within)	0.266	0.284	0.202	0.215
Panel Dimensions	Obs = 6,031 $N = 734$ $\bar{T} = 8.2$			

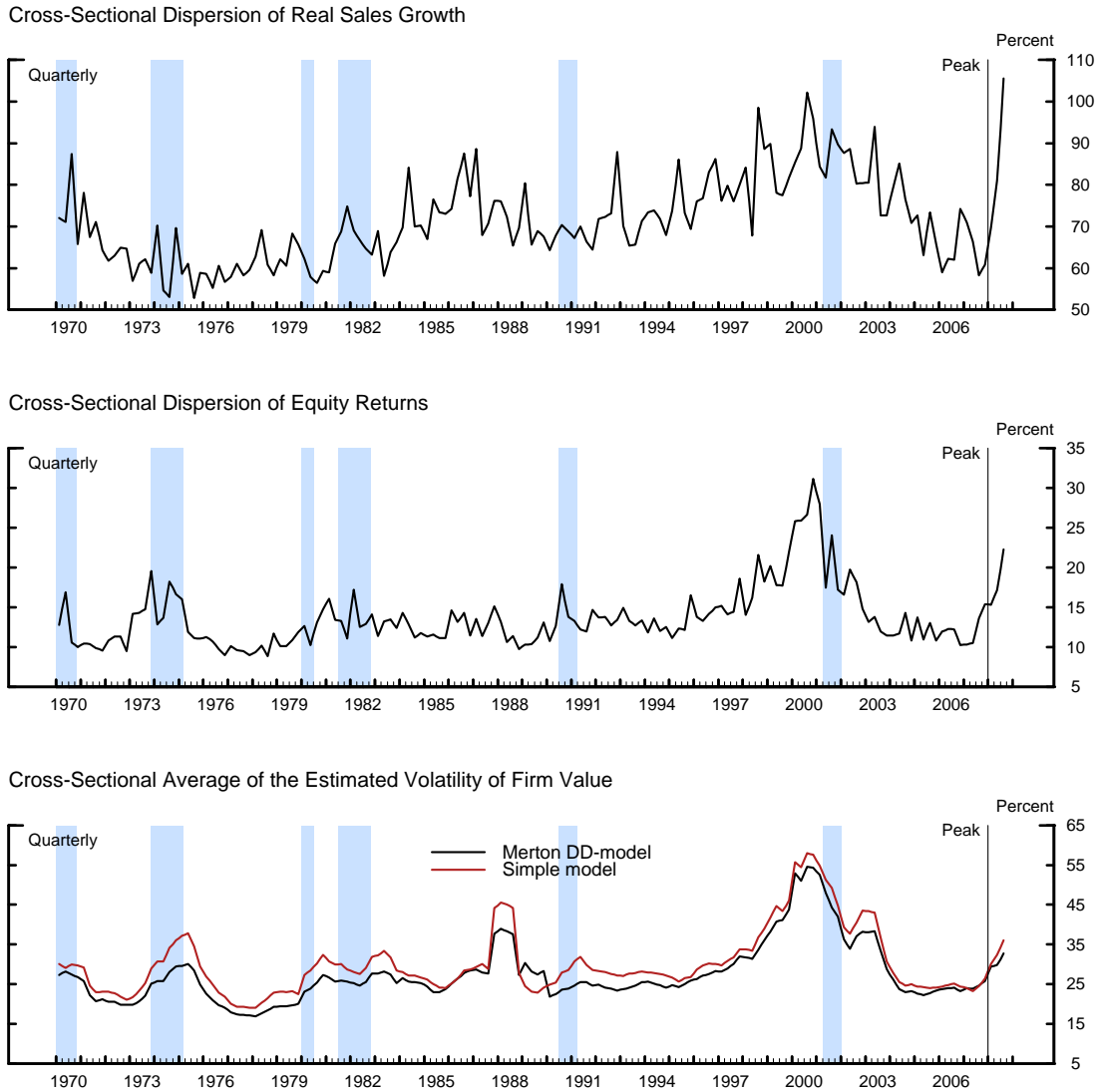
NOTE: Sample period: firm-level annual data from 1973 to 2008. The dependent variable is  $[I/K]_{jt}$ , the investment rate of firm  $j$  in year  $t$ . All specifications include firm fixed effects time fixed effects and are estimated by OLS. Heteroskedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses.

Table 8: Investment, Uncertainty, and Credit Spreads  
(Simple Estimate of Uncertainty)

Explanatory Variable	Static Specification			
	(1)	(2)	(3)	(4)
$\log[S/K]_t$	0.121 (0.009)	0.121 (0.008)	-	-
$Q_{E,t-1}$	-	-	0.046 (0.006)	0.045 (0.006)
$\sigma_{E,t-1}$	-0.090 (0.022)	-0.025 (0.024)	-0.095 (0.021)	-0.042 (0.024)
$SPR_{t-1}$	-	-1.064 (0.172)	-	-0.858 (0.179)
$R^2$ (within)	0.273	0.284	0.203	0.211
Panel Dimensions	Obs = 6,031 $N = 734$ $\bar{T} = 8.2$			

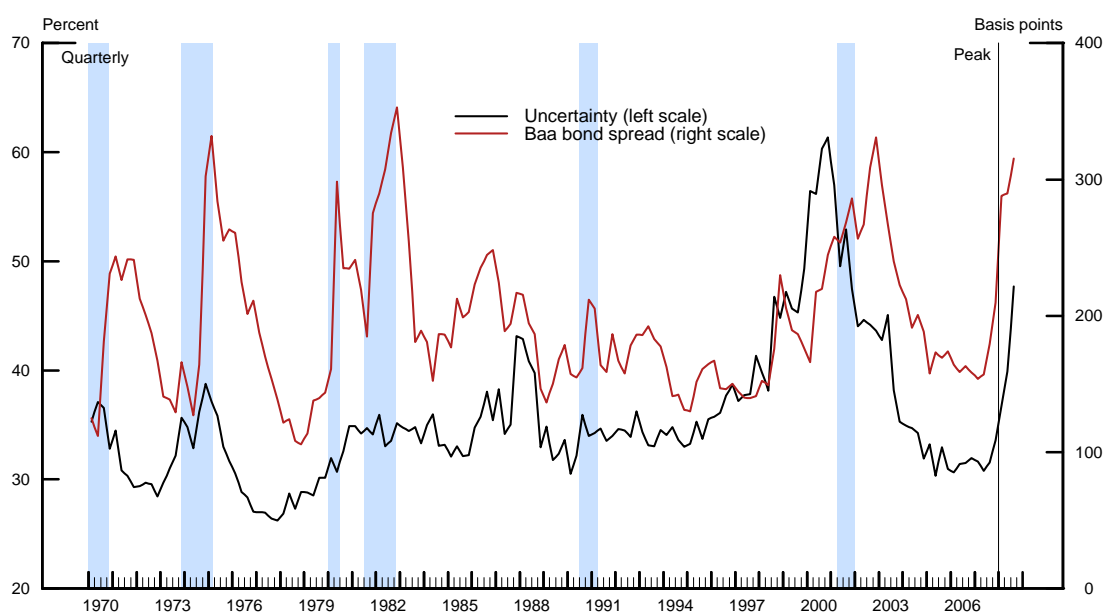
NOTE: Sample period: firm-level annual data from 1973 to 2008. The dependent variable is  $[I/K]_{jt}$ , the investment rate of firm  $j$  in year  $t$ . All specifications include firm fixed effects time fixed effects and are estimated by OLS. Heteroskedasticity- and autocorrelation-consistent asymptotic standard errors are clustered at the firm level and are reported in parentheses.

Figure 1: Measures of Macroeconomic Uncertainty



NOTE: The three panels of the figure depict various measures of time-varying macroeconomic uncertainty for the U.S. (nonfarm) nonfinancial corporate sector (see text for details). The shaded vertical bars denote NBER-dated recession; according to the NBER, the most recent peak in economic activity occurred in 2007:Q4 (the thin vertical line).

Figure 2: Macroeconomic Uncertainty and Credit Spreads



NOTE: The black line depicts a measure of time-varying macroeconomic uncertainty constructed as the first principal component of the four uncertainty measures shown in Figure 1. The red line depicts the spread between the yield an index of Baa-rated long-term industrial bonds and the yield on the 10-year nominal Treasury note. The shaded vertical bars denote NBER-dated recession; according to the NBER, the most recent peak in economic activity occurred in 2007:Q4 (the thin vertical line).



Figure 3: Uncertainty and Credit Spreads

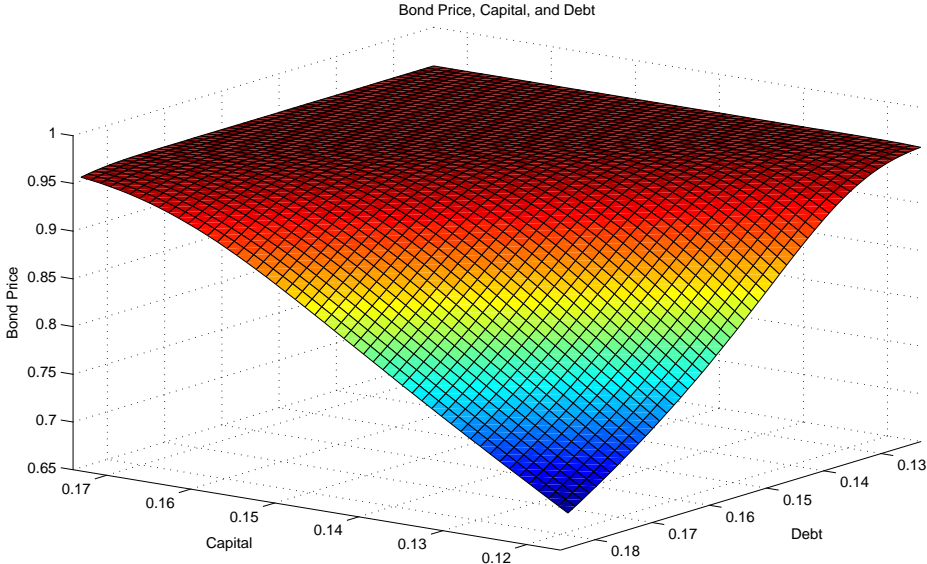


Figure 4: Uncertainty and Bond Prices

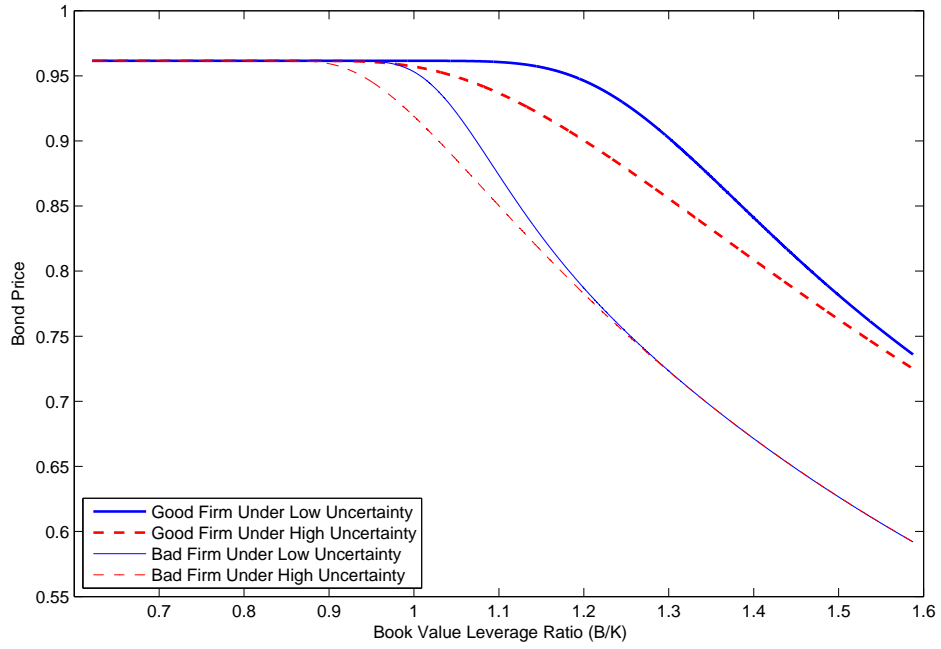


Figure 5: Time-Varying Uncertainty and Credit Spreads

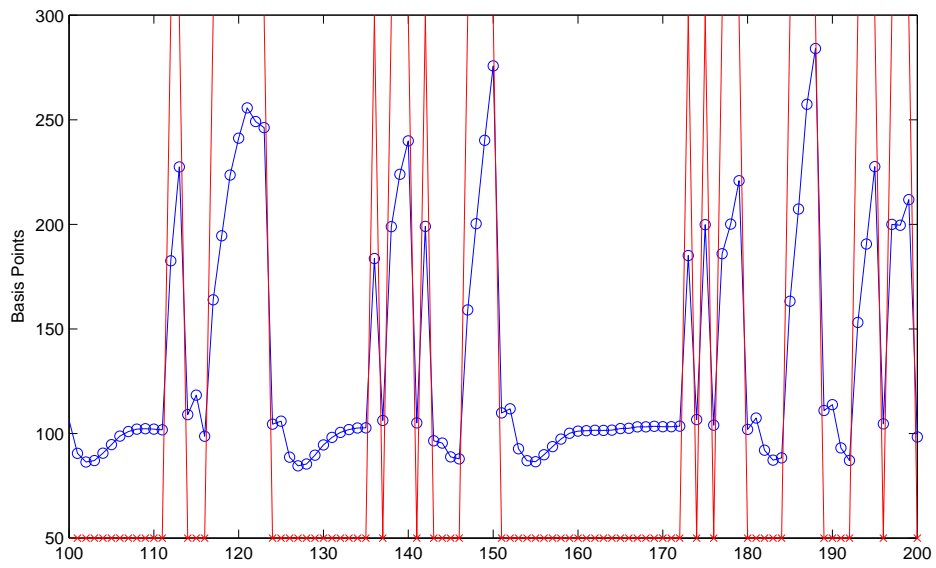


Figure 6: Time-Varying Uncertainty and Investment-to-Capital Ratio

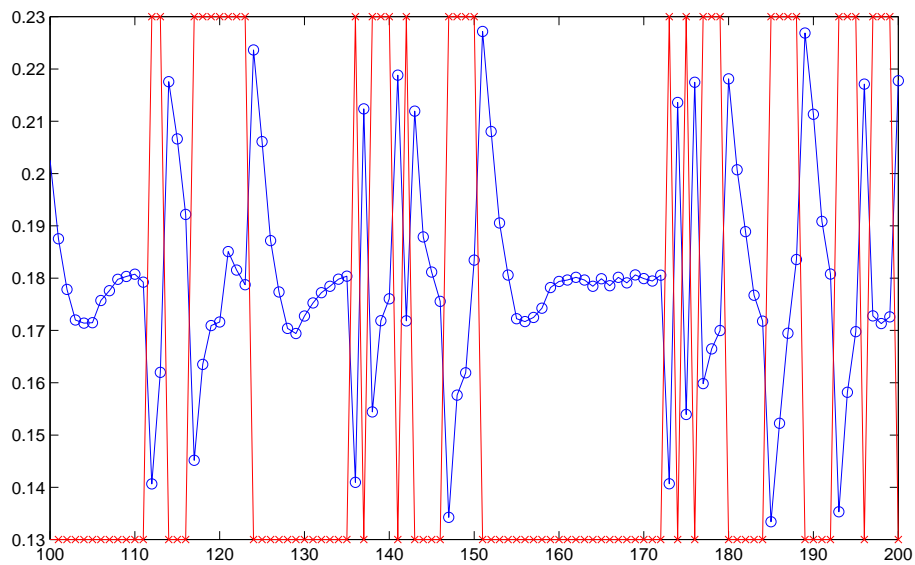


Figure 7: Time-Varying Uncertainty and the Volume of Equity Financing

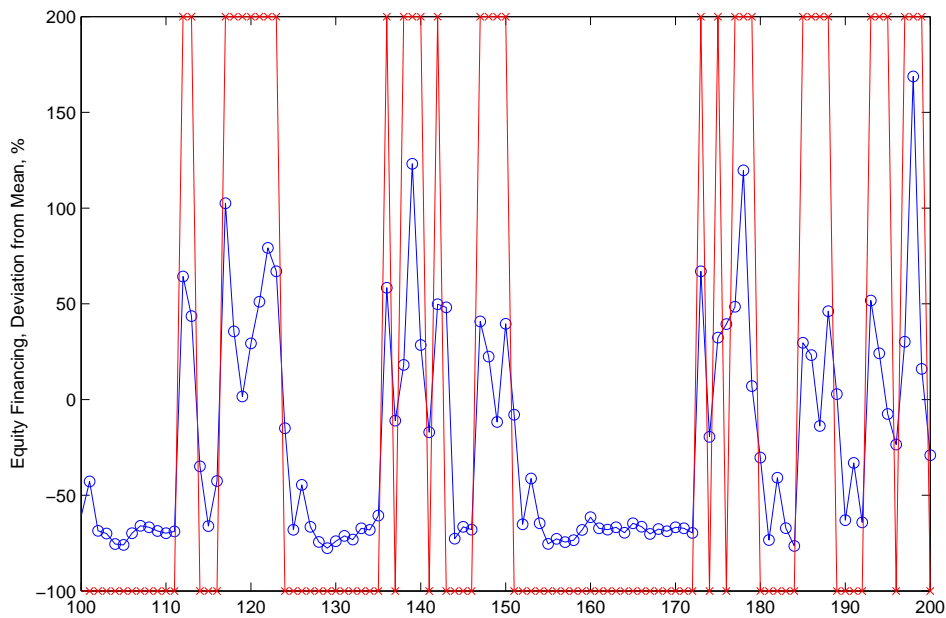


Figure 8: Time-Varying Uncertainty and the Fraction of Firms Issuing Equity

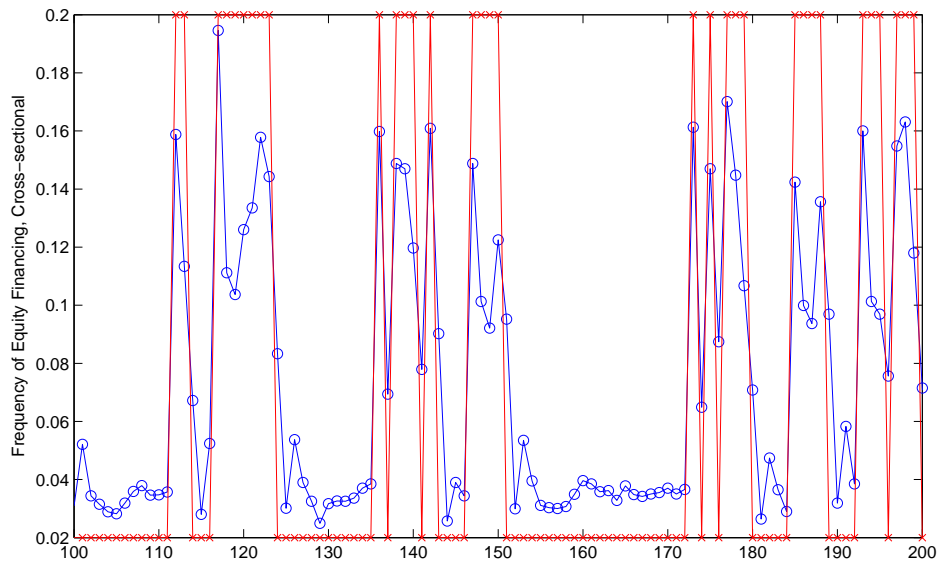


Figure 9: Time-Varying Uncertainty and the Share of Equity Financing

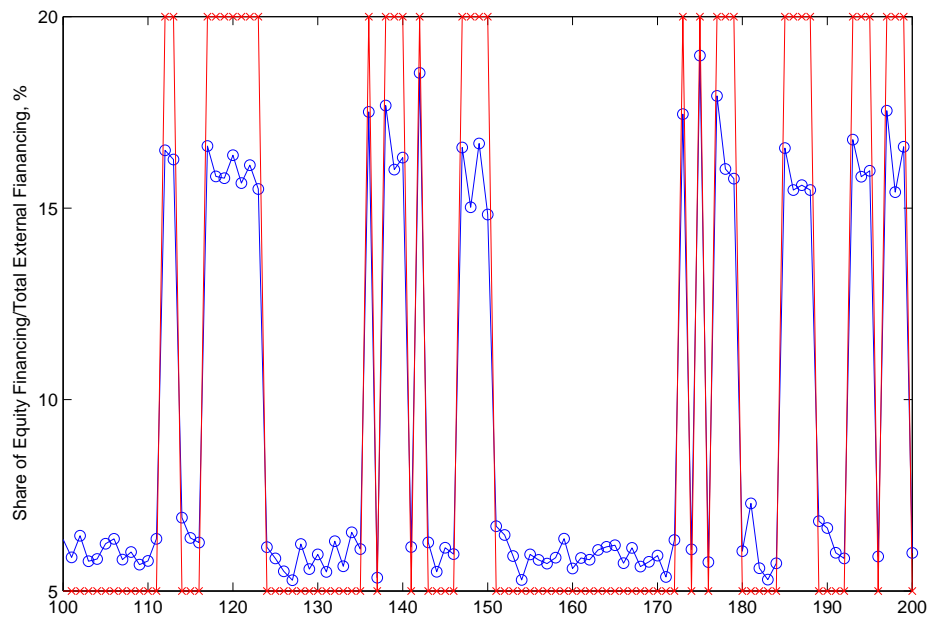


Figure 10: Time-Varying Uncertainty and the Default Frequency

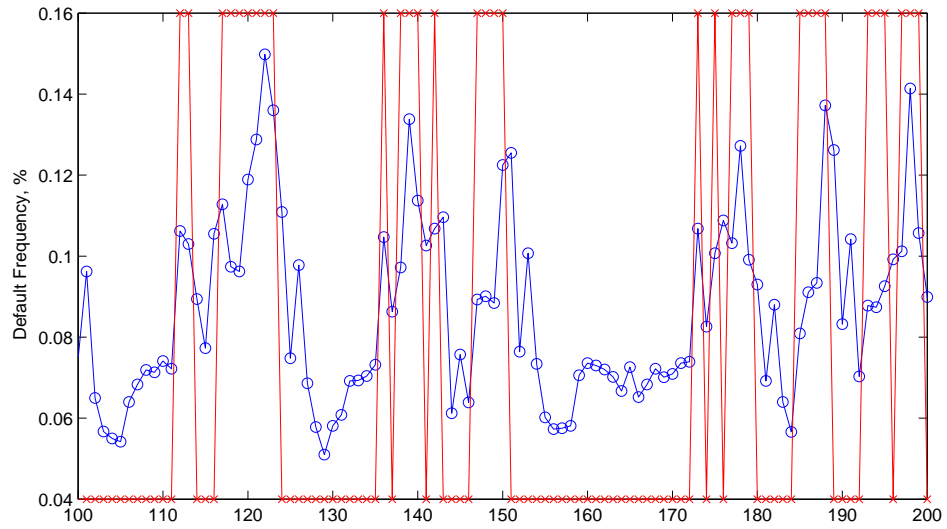


Figure 11: Time-Varying Uncertainty and the Recovery Rate

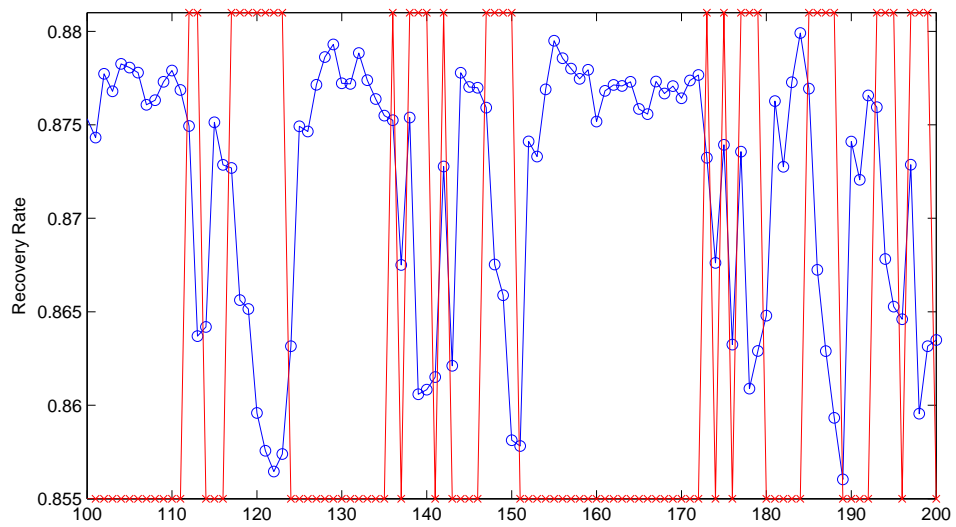


Figure 12: Time-Varying Uncertainty and Leverage

