Abstract— Direct-sequence/code-division multiple-access (DS/CDMA) communication systems equipped with adaptive antenna arrays offer the opportunity for jointly effective spatial and temporal (code) multiple-access interference (MAI) and channel noise suppression. This work focuses on the development of fast joint space–time (S–T) adaptive optimization procedures that may keep up with the fluctuation rates of multipath fading channels. Along these lines, the familiar S–T RAKE processor is equipped with a single orthogonal S–T auxiliary vector (AV) selected under a maximum magnitude cross-correlation criterion. Then, blind joint spatial/temporal MAI and noise suppression with one complex S–T degree of freedom can be performed. This approach is readily extended to cover blind processing with multiple AV’s and any desired number of complex degrees of freedom below the S–T product. A sequential procedure for conditional AV weight optimization is shown to lead to superior bit-error-rate (BER) performance when rapid system adaptation with limited input data is sought. Numerical studies for adaptive antenna array reception of multiuser multipath Rayleigh-faded DS/CDMA signals illustrate these theoretical developments. The studies show that the induced BER can be improved by orders of magnitude, while at the same time significantly lower computational optimization complexity is required in comparison with joint S–T minimum-variance distortionless response or equivalent minimum mean-square-error conventional filtering means.

Index Terms— Adaptive equalizers, antenna arrays, code-division multiple access, fading channels, interference suppression, multipath channels, spread-spectrum communication.

I. INTRODUCTION

While signal spreading (DS/CDMA) may lead to significant capacity increases for wireless cellular and PCS (personal communications services) networks [1], additional large capacity gains are available from exploiting the spatial location of cellular system users [2]. This is possible when the base stations are equipped with multiple antenna elements. Usually, an antenna array is a set of \( M \) identical omnidirectional transceivers arranged in a straight line (linear array) or in a circle (circular array) [3]. Certainly, other geometric configurations are possible, and nonidentical orthogonally polarized elements can also be deployed [4]. Theoretically, an antenna array of \( M \) elements can provide a mean signal power gain of \( M \) over spatially and temporally white additive Gaussian noise (linear signal-to-noise ratio (SNR) increase by a factor of \( M \)) [5].

In contrast to fixed sectorized antenna arrays (also known as switched-beam systems) and phased-arrays where only the signal phase is adjustable, \( M \)-element adaptive antenna arrays are supposed to be gain- and phase-adjustable and able to perform online beamforming with full \( M - 1 \) degrees of freedom [2], [4], [6]. Incorporation of adaptive antenna arrays in direct-sequence/code-division multiple-access (DS/CDMA) cellular systems, which is the topic of this paper, is presently gaining significant interest. Examples of recent work include signature matched-filtered code processing combined with a lightweight principal component analysis approach for space processing (in [7] and [8] and with capacity studies in [9]) and static (that is nonadaptive) space–time (S–T) processing in the form of a bank of decorrelators, one per antenna element [10]. Adaptive processing proposals involve disjoint space minimum mean-square error (MMSE) [minimum-variance distortionless response (MVDR)] optimum and RAKE temporal processing, or disjoint MMSE (MVDR) optimum spatial and MMSE (MVDR) optimum temporal, or jointly optimum MMSE (MVDR) S–T processing [11] with post-filtering Viterbi decoding in [12], [13]. Other recent contributions include joint S–T RAKE processing [14], low-complexity S–T RAKE receivers [15], and joint S–T adaptive MVDR processing [16]–[18] for systems with aperiodic spreading codes.

In this work we wish to maintain the principles of joint S–T adaptive processing for jointly effective S–T detection of the information bits of the user(s) of interest in the presence of DS/CDMA multiple-access interference (MAI), multipath fading phenomena, and additive ambient multichannel noise. We recognize that attempts for disjoint space and time optimization may fall far behind the potential of joint processing as shown in [19] in the context of array radar detection.

On the other hand, for a DS/CDMA system with \( M \) antenna elements, system processing gain \( L \) and \( N \) resolvable multipaths (usually \( N \) is between 2 and 4 including the direct path, if any [4]), jointly optimum processing under minimum mean-square criteria requires processing in the \( M(L + N - 1) \) S–T product space. As an example, if \( M = 5 \), \( L = 64 \), and \( N = 3 \), filter optimization needs to be carried out.
in the complex $C^{330}$ vector space. Adaptive sample-matrix-inversion (SMI) implementations of the MMSE/MVDR filter solution are known to require data samples several times the \(S-T\) product \(M(L + N - 1)\) to approach the performance characteristics of their ideal counterparts [20], [21]. In fact, theoretically, system optimization with less than \(M(L+N-1)\) data samples is not even possible due to the singularity of the underlying sampled autocovariance matrix. With CDMA chip rates at 1.25 MHz [4], \(L = 64\), and typical fading rate measurements between 4.5 Hz for mobiles on foot and 70 Hz for vehicle mobiles (carriers are assumed at 900 MHz) [2], the fading channel may fluctuate significantly—as quickly as every 280 data symbols. In this context, adaptive SMI filter optimization in the $C^{330}$ vector space becomes an unrealistic goal.

This paper focuses on fast adaptive joint S–T optimization through small data sets that can “catch up” with multipath fading communications channels. In this direction, first we extend the auxiliary-vector (AV) framework—originally introduced in [22] for time-domain processing of DS/CDMA signals—to cover joint S–T processing in complex data vector spaces. In this context, the familiar joint S–T RAKE filter is accompanied by a single orthogonal auxiliary vector selected under a maximum magnitude cross-correlation criterion for blind S–T MAI and noise suppression with one degree of freedom. Further generalization leads to adaptive generation of multiple auxiliary vectors, which are orthogonal to each other and to the S–T RAKE filter, for blind S–T MAI and noise suppression with any desirable number of degrees of freedom below the S–T product. Interestingly enough, this generalization parallels well-known “blocking-matrix” processing techniques that have been used successfully for radar and antenna array applications in the past [23]–[26] and recently for time-domain-only CDMA signal processing [27]. However, to avoid SMI altogether even for reduced $P$-dimension ($P < M(L + N - 1)$) blocking-matrix processing, here we generate a data-dependent sequence of $P$ auxiliary vectors through successive conditional optimization. Performancewise, we maintain that in small-sample support situations, conditional optimization outperforms $P$-dimension blocking-matrix processing without ever inverting the underlying $P \times P$ covariance matrix.

In summary, the developments in this work cover adaptive joint S–T processing procedures with any desirable number of degrees of freedom, from one up to the full S–T product. The choice, of course, depends on the size of the available data set, which in turn depends on the channel fading rate and other pertinent considerations. Given the prevailing DS/CDMA system parameters and mobile channel fading rates, we have to argue in favor of very few degrees of freedom with conditional optimization, as we discuss in the sequel.

The rest of this paper is organized as follows. The all-important received signal model is presented in Section II. The core concept of AV filtering is developed in Section III. Extensions in the form of blocking-matrix processing and sequences of conditionally optimized weighted auxiliary vectors are pursued in Section IV. Numerical results are presented in Section V, and a few concluding remarks are drawn in Section VI.

II. SIGNAL MODEL

Receiver design strategies and signal processing algorithms are dictated by signal and channel modeling assumptions. As a brief overview, in this work we consider mobile-to-base transmissions of $K$ DS/CDMA users simultaneous in time and frequency with processing gain $L$. The transmissions take place over multipath fading additive white Gaussian noise (AWGN) channels. The fading is modeled by multiplicative complex Gaussian random variables (Rayleigh-distributed amplitude and uniformly distributed phase). The multipath fading channels are modeled by tapped-delay lines with independent fading per path. The received signal is collected by a narrow-band antenna array of $M$ elements. For illustration purposes, we consider uniform linear arrays. Identical fading is assumed to be experienced by all antenna elements for each path of each user signal (no antenna diversity). Details and notation are given below.

The contribution of the $k$th user, $k = 0, \ldots, K - 1$, to the transmitted signal is denoted by

$$u_k(t) = \sum_i b_k(i) \sqrt{E_k} s_k(t - i T) e^{j(2\pi f_c t + \phi_k)}$$

where $b_k(i) \in \{-1, 1\}$ is the $i$th transmitted data (information) bit, $E_k$ denotes energy, $\phi_k$ is the carrier phase with carrier frequency $f_c$ and $s_k(t)$ is the assumed normalized user signature waveform given by

$$s_k(t) = \sum_{l=0}^{L-1} d_k[l] \psi(t - lT_c)$$

where $d_k[l] \in \{-1, 1\}$ is the $l$th bit of the spreading sequence of the $k$th user, $\psi(t)$ is the chip waveform, $T_c$ is the chip period, and if $T$ in (1) denotes the information bit duration then $T/T_c = L$ is the spreading processing gain.

After multipath fading channel “processing,” the total signal due to all users received at the input of a narrow-band uniform linear array of $M$ elements is given by

$$x_c(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \alpha_{k,n} u_k \left( t - \frac{n}{B} - \tau_k \right) a_{k,n} + n(t)$$

where $N$ is the total number of resolvable multipaths (without loss of generality, the number of resolvable multipaths is assumed to be the same for all users) and $\alpha_{k,n}, k = 0, \ldots, K - 1, n = 0, \ldots, N - 1$, are independent zero-mean complex Gaussian random variables that model the fading phenomena and are assumed to remain constant over several bit intervals. We recall that, in fact, measurements have shown that mobiles on foot operating at $f_c = 900$ MHz induce a typical fading rate of about 4.5 Hz, while fast-moving vehicles may cause fading rates as high as 70 Hz [2]. Moreover, $\tau_k$ in (3) denotes the relative transmission delay of user $k$ with respect to user 0 with $\tau_0 = 0$ and with $x_c(t)$ bandlimited to $B = 1/T_c$, the tap-delay line channel model has taps spaced at chip intervals $T_c$. The $M \times 1$ array response vector $a_{k,n}$ for the $n$th path of the $k$th user signal is defined by

$$a_{k,n}[m] = e^{j2\pi(m-1)(d/\lambda) \sin \theta_k, n}, \quad m = 1, \ldots, M$$
where \( \theta_k \) identifies the angle of arrival of the \( k \)th multipath signal from the \( k \)th user, \( \lambda \) is the carrier wavelength, and \( d \) is the element spacing of the antenna (usually \( d = \lambda / 2 \)). Finally, \( \mathbf{n}(t) \) in (3) denotes an \( M \)-dimension complex Gaussian noise process that is assumed white both in time and space.

After carrier demodulation

\[
\mathbf{x}(t) = \sum_{k=0}^{K-1} \sum_{i=0}^{N-1} c_k \sin(\omega_c t + \varphi_k) \exp(-j2\pi f_c t) + \mathbf{n}(t)
\]

(5)

where \( c_k \sin(\omega_c t + \varphi_k) \) represents the received symbol at the carrier frequency \( \omega_c \) and \( \varphi_k \) is the total carrier phase absorbed into the channel coefficient. Assuming synchronization at the reference antenna element \( (m = 1) \) with the signal of the user of interest, for example, user 0, direct sampling of \( \mathbf{x}(t) \) at the chip rate \( 1/T_c \) (or chip-matched filtering and sampling at the chip rate) over the multipath extended \( L + N - 1 \) chip period prepares the data for one-shot detection of the \( k \)th information bit of interest \( k_0 \). We visualize the S-T data in the form of a \( M \times (L + N - 1) \) matrix

\[
\mathbf{X}_{M \times (L + N - 1)} = [\mathbf{x}(0) \mathbf{x}(T_c) \cdots \mathbf{x}((L + N - 2)T_c)].
\]

(6)

To avoid cumbersome two-dimensional filtering operations and notation, we decide at this time to “vectorize” \( \mathbf{X}_{M \times (L + N - 1)} \) by sequencing all matrix columns in the form of a vector

\[
\mathbf{X}_{M \times (L + N - 1) \times 1} = \text{Vec} \{ \mathbf{X}_{M \times (L + N - 1)} \}.
\]

(7)

From now on, \( \mathbf{X} \) denotes the joint S-T data in the \( C^{M(L + N - 1)} \) complex vector domain.

The cornerstone for any form of joint S-T filtering is the S-T RAKE filter that we define compactly for user 0 as the cross correlation between the received S-T data \( \mathbf{X} \) and the desired information bit \( k_0 \)

\[
\mathbf{V}_0 \triangleq E_{k_0} \{ \mathbf{X} k_0 \}
\]

(8)

where the statistical expectation operation \( E \{ \cdot \} \) is taken with respect to \( k_0 \) only. Equation (8), in the form of sample averaging, can also serve as a supervised estimator of the S-T RAKE filter for slowly fading channels

\[
\hat{\mathbf{V}}_0 \triangleq \frac{1}{J} \sum_{j=1}^{J} \mathbf{X}_j k_0(j)
\]

(9)

where \( \{ \mathbf{X}_j \}_{j=1}^{J} \) is a sequence of \( J \) S-T received data vectors. We recognize that if sample averaging extends over multiple multipath channel realizations \( (J \rightarrow \infty) \), then \( \hat{\mathbf{V}}_0 \) converges either to the direct path S-T signature of user 0 when Ricean fading is assumed or to the \( \mathbf{0} \) vector when no direct path exists, i.e., Rayleigh fading is assumed. In either case, in order to capture the prevailing full S-T effective signature of user 0, (9) needs to be continuously reinitialized at a rate that is consistent with the fading rate or a corresponding forgetting factor needs to be embedded in the sample average procedure [28]. We are careful to point out that estimation of the S-T RAKE filter through supervised or blind estimation procedures as in [8], [9], [14], [15], or [29] is not the primary subject of this work.

What we are concerned with in this paper is to minimize the induced bit-error rate (BER) of the overall system with a given S-T RAKE vector and a few data samples \( \mathbf{x}_{j, j = 1, \ldots, J} \). Cellular system designers may then take the available lower BER to increase the number of active users for a given BER quality threshold, or to reduce the transmit power of the mobile handset, or to increase the range of the base station transceiver (cell size).

### III. Blind Auxiliary-Vector Joint Space-Time (S-T) Processing

With signal model and notation already in place, we begin this section with a direct statement of the problem that we investigate. With respect to a specific DS/CDMA user of interest (user 0), we assume that we have available the joint S-T RAKE filter \( \mathbf{V}_0 \) as defined in the previous section and a finite set of joint S-T data \( \mathbf{x}_{j, j = 1, \ldots, J} \). No knowledge of the information bits of the user of interest in \( \mathbf{x}_{j, j = 1, \ldots, J} \) is assumed and the MAI S-T population is considered unknown (unknown interfering user signatures, locations, and multipaths). We wish to determine the best detection strategy for a DS/CDMA system with \( M \) antenna elements, spreading gain \( G \) and \( L \) resolvable multipaths per user, given the S-T RAKE vector \( \mathbf{V}_0 \) and a total number of \( J \) S-T data vectors.

The first well-known approach that we may consider is the straight S-T RAKE-based [11], [30] detector that does not utilize the available data set

\[
\hat{k}_0 = \text{sgn} (\text{Re} \{ \mathbf{V}_0^H \mathbf{X} \})
\]

(10)

where \( \hat{k}_0 \) denotes the information bit decision for the user of interest, \( \text{sgn} (\cdot) \) identifies the sign operation (zero-threshold hard limiter), \( \text{Re} \{ \cdot \} \) extracts the real part of a complex number, and \( \mathbf{V}_0^H \) is the Hermitian of \( \mathbf{V}_0 \). We note that joint S-T RAKE processing is equivalent to cascade space and time RAKE processing, for cases in which cascade realizations are preferred for implementation purposes. In any case, the output variance of the S-T RAKE filtered data is

\[
\text{VAR}_{\text{RAKE}} \triangleq E \{ (\mathbf{V}_0^H \mathbf{X})^2 \} = \mathbf{V}_0^H R \mathbf{V}_0
\]

(11)

where \( R_{M \times (L + N - 1) \times M \times (L + N - 1)} = E \{ \mathbf{X} \mathbf{X}^H \} \).

Another well-known approach is joint S-T MMSE optimum filtering (proposed in [11] for joint S-T processing and in [31]–[34] for time-only processing of CDMA signals). It is straightforward to obtain the Wiener filter solution [28] for our problem if we view the sequence of information bits of the user of interest as the “desired” filter output when the filter input is the S-T data vectors in (7). The solution is

\[
\mathbf{w}_{\text{MMSE}} = c R^{-1} \mathbf{V}_0
\]

(12)

where \( c \) is an arbitrary positive scalar, \( \mathbf{V}_0 \) is the joint S-T RAKE filter previously defined, and \( R = E \{ \mathbf{X} \mathbf{X}^H \} \). If we choose \( c = [\mathbf{V}_0^H R^{-1} \mathbf{V}_0]^{-1} \), then we obtain the well-known equivalent MVDR filter (which was used in [35] for time
processing of plain asynchronous channels and in [36], [37] for time processing of multipath channels)

\[ \mathbf{w}_{\text{MVDR}}^H = \frac{\mathbf{R}^{-1}\mathbf{V}_0}{\mathbf{V}_0^H \mathbf{R}^{-1}\mathbf{V}_0} \]  

for which \( \mathbf{w}_{\text{MVDR}}^H \mathbf{V}_0 = 1 \) and

\[ \text{VAR}_{\text{MVDR}} \triangleq E\{[\mathbf{w}_{\text{MVDR}}^H \mathbf{X}]^2\} = [\mathbf{V}_0^H \mathbf{R}^{-1}\mathbf{V}_0]^{-1}. \]

Direct comparison of (12) or (13) with the S-T RAKE filter \( \mathbf{V}_0 \) shows that S-T RAKE processing is mean square (MS) optimal only if the joint S-T disturbance (multichannel MAI and noise) is white Gaussian. Certainly, this is not the case for the general multiuser multipath CDMA signal model in (5). To tap, however, the merits of optimum MS processing and to form the \( \mathbf{w}_{\text{MVDR}} \) filter, exact knowledge of the S-T data autocovariance matrix \( \mathbf{R} \) is required. Since \( \mathbf{R} \) is unknown, we may use the available data set to form a sample-average estimate \( \hat{\mathbf{R}} = (1/J) \sum_{j=1}^{J} \mathbf{X}_j \mathbf{X}_j^H \) and then proceed with SMI [20]. In this case, \( \mathbf{w}_{\text{MVDR}}^H = (\hat{\mathbf{R}}^{-1}\mathbf{V}_0)/(\mathbf{V}_0^H \hat{\mathbf{R}}^{-1}\mathbf{V}_0) \) and bit decisions are made as follows:

\[ \hat{b}_0 = \text{sgn} \left( \text{Re} \left\{ \frac{\mathbf{V}_0^H \hat{\mathbf{R}}^{-1} \mathbf{X}}{\mathbf{V}_0^H \hat{\mathbf{R}}^{-1} \mathbf{V}_0} \right\} \right). \]

It is known that batch SMI procedures outperform supervised least mean square (LMS) implementations of the MMSE filter [20] or blind-constraint LMS implementations of the MVDR solution in terms of small-sample convergence characteristics [6]. However, at least \( J = M(L+N-1) \) data samples are required for \( \hat{\mathbf{R}} \) to be invertible with probability one, and in fact, data sizes many times the S-T product \( M(L+N-1) \) are necessary for \( \hat{\mathbf{w}}_{\text{MVDR}} \) to approach the performance characteristics of the ideal \( \mathbf{w}_{\text{MVDR}} \) filter reasonably well. Unfortunately, for typical \( M, L, N \) values and data transmission rates, filter adaptation over the required number of symbol intervals can not keep up with typical channel fluctuation (fading) rates, as explained earlier in Section I.

This discussion brings us to the core concept of this paper. First, without loss of generality, let us assume that the S-T RAKE filter \( \mathbf{V}_0 \) is normalized, that is \( \mathbf{V}_0^H \mathbf{V}_0 = 1 \). Then, we consider an arbitrary fixed “auxiliary” vector \( \mathbf{G} \) that is orthonormal with respect to \( \mathbf{V}_0 \), i.e.,

\[ \mathbf{G}^H \mathbf{V}_0 = 0 \quad \text{and} \quad \mathbf{G}^H \mathbf{G} = 1. \]

We propose S-T adaptive processing with a single joint S-T degree of freedom in the form of \( \mathbf{w}_{\text{AV}}^H \mathbf{X} \), where [22]

\[ \mathbf{w}_{\text{AV}} \triangleq \mathbf{V}_0 - \mu \mathbf{G}. \quad \mu \in \mathbb{C}. \]

Decision making is carried out as seen in Fig. 1

\[ \hat{b}_0 = \text{sgn} \left( \text{Re} \{ \mathbf{w}_{\text{AV}}^H \mathbf{X} \} \right). \]

With an arbitrary but fixed auxiliary vector \( \mathbf{G} \) that satisfies the constraints of (16), the filter in (17) can be optimized with respect to the single complex scalar \( \mu \). The MS-optimum value of \( \mu \) can be identified from two different—yet equivalent—points of view. Since \( \mathbf{w}_{\text{AV}} \triangleq \mathbf{V}_0 - \mu \mathbf{G} \) is by construction distortionless in the \( \mathbf{V}_0 \) direction (\( \mathbf{w}_{\text{AV}}^H \mathbf{V}_0 = 1 \), for any \( \mu \in \mathbb{C} \)), we may seek the value of \( \mu \) that minimizes the output variance (energy) \( E\{[\mathbf{w}_{\text{AV}}^H \mathbf{X}]^2\} \). This way we suppress in the MS-sense the overall disturbance contribution at the filter output (multipath-processed MAI and multichannel ambient noise) without any cancellation (suppression) of the signal of the user of interest. An equivalent interpretation motivated by Fig. 1 is to look for the value of \( \mu \) that minimizes the MS difference between the S-T main-beam (S-T RAKE) processed data \( \mathbf{V}_0^H \mathbf{X} \) and the orthogonal AV processed data \( \mu^* \mathbf{G}^H \mathbf{X} \). The solution to the first minimum-output variance problem can be obtained by direct differentiation of the variance expression with respect to the complex scalar \( \mu \). The solution to the second mean square-error (MSE) problem can be obtained by direct application of the optimum linear MS estimation theorem [28]. The solution is, of course, identical in both cases, and it is identified by the following proposition.

**Proposition 1:** The value of the complex scalar \( \mu \) that minimizes the output variance of the filter \( \mathbf{w}_{\text{AV}} \triangleq \mathbf{V}_0 - \mu \mathbf{G} \) or minimizes the MSE between the main-beam processed data \( \mathbf{V}_0^H \mathbf{X} \) and the AV processed data \( \mu^* \mathbf{G}^H \mathbf{X} \) is

\[ \mu = \frac{\mathbf{G}^H \mathbf{V}_0}{\mathbf{G}^H \mathbf{G}}. \]

Proposition 1 identifies the optimum value of \( \mu \) for any fixed auxiliary vector \( \mathbf{G} \). We now turn our attention to the selection of an auxiliary vector subject to an appropriately chosen criterion. We understand that there exists a vector \( \mathbf{G} \) in the subspace orthogonal to \( \mathbf{V}_0 \) for which \( \mathbf{w}_{\text{AV}} \) and the optimum \( \mathbf{w}_{\text{MVDR}} \) filter coincide. This solution, however, is not desirable since it reverts to processing with full \( M(L+N-1) \) degrees of freedom and requires inversion of the pertinent S-T data autocovariance matrix. The selection criterion for the auxiliary vector \( \mathbf{G} \) that we propose in this work is motivated by the MSE interpretation of the filter in Fig. 1, and it is the maximization of the magnitude of the cross correlation between the \( \mathbf{V}_0 \) processed data \( \mathbf{V}_0^H \mathbf{X} \) and the AV processed data \( \mathbf{G}^H \mathbf{X} \), subject to the constraints in (16)

\[ \mathbf{G} = \arg \max_{\mathbf{G}} \left| E\{[\mathbf{V}_0^H \mathbf{X}(\mathbf{G}^H \mathbf{X})^H]\} \right| \]

subject to \( \mathbf{G}^H \mathbf{V}_0 = 0 \) and \( \mathbf{G}^H \mathbf{G} = 1 \).
In terms of a physical intuitive interpretation of this selection criterion, we might say that the orthonormal to $V_0$ auxiliary vector $G$ that maximizes $|V_0^H RG|$ can capture the most—in the maximum-magnitude cross-correlation sense—of the disturbance contribution present at the S-T RAKE filter output. “Captured” disturbance contributions are then subtracted from the S-T RAKE output as explained in the context of Proposition 1 and shown in Fig. 1. We note that both the criterion function $|V_0^H RG|$ and the orthonormality constraints are phase invariant. In other words, if $G$ maximizes $|V_0^H RG|$ under the given constraints, so does $G e^{j\phi}$ for any phase $\phi$. Therefore, without loss of generality, we may identify the unique auxiliary vector that is a solution to the constraint optimization problem in (20) and places $V_0^H RG$ on the nonnegative real line ($V_0^H RG \geq 0$). The following proposition identifies this optimum auxiliary vector according to our criterion. The proof is given in the Appendix.

**Proposition 2:** The auxiliary vector

$$G = \frac{RV_0 - (V_0^H RV_0)V_0}{||RV_0 - (V_0^H RV_0)V_0||}$$

(21)

maximizes the magnitude of the cross correlation between the main-beam processed data $V_0^H X$ (w.l.o.g. $V_0^H V_0 = I$) and the AV processed data $G^H X$, $|V_0^H RG|$, subject to the constraints $G^H V_0 = 0$ and $G^H G = I$.

Proposition 2 completes the design of the joint S-T auxiliary-vector detector. As a brief summary, the sequence of calculations is as follows: $G$ is given by (21), then $\mu$ is calculated by (19), the filter structure takes the form of (17), and decisions are made according to (18). The S-T data autocovariance matrix $R$ appears in the expression for the ideal $G$ and $\mu$ may be sample average estimated, $\hat{R} = (1/J) \sum_{j=1}^{J} X_j X_j^H$, as usual.

Straightforward algebraic manipulations show that the output variance of the AV filter as a function of the auxiliary vector $G$ is

$$\text{VAR}_{AV} = E\{ |w_{AV}^H X|^2 \} = V_0^H RV_0 - \left| \frac{G^H RV_0}{G^H RG} \right|^2$$

(22)

It is interesting to observe that, as it turns out, the AV selection criterion that we proposed corresponds to the maximization of the numerator of the second term in the $\text{VAR}_{AV}$ expression subject to the constraints in (16). Maximization of the second term as a whole, that is including the denominator, with respect to $G$ with $G^H V_0 = 0$ and unconstrained norm leads to optimization with full $M(L + N - 1)$ degrees of freedom that takes us back to the optimum MVDR solution $\mathbf{w}_{\text{MVDR}}$. Finally, direct comparison of the variance expressions in (11), (14), and (22) shows that

$$\text{VAR}_{\text{MVDR}} \leq \text{VAR}_{AV} \leq \text{VAR}_{\text{RAKE}}$$

(23)

with equalities throughout if $R$ is diagonal (white S-T disturbance). To the extent that linear-filtered (with $M(L + N - 1)$ taps) multichannel multipathed MAI can be closely modeled by Gaussian noise [38], (23) offers a direct ranking of the respective ideal BER’s (that is for perfectly known S-T autocovariance matrices and inverses and perfectly known S-T RAKE vectors).

In the following section, we extend Propositions 1 and 2 to cover joint S-T processing with multiple auxiliary vectors. We focus on sequential conditional auxiliary-vector optimization.

**IV. PROCESSING WITH MULTIPLE AUXILIARY VECTORS**

We consider a set of $P$, $1 \leq P \leq M(L + N - 1) - 1$, auxiliary vectors $G_1, G_2, \ldots, G_P$ that are orthonormal with respect to each other and to the normalized S-T RAKE vector $V_0$. We may organize the auxiliary vectors in the form of an $M(L + N - 1) \times P$ matrix $B$ where $G_p$, $p = 1, \cdots, P$, constitutes the $p$th column of $B$

$$B_{M(L+N-1)\times P} = [G_1 \quad G_2 \quad \cdots \quad G_P].$$

(24)

With this setup, the orthonormality conditions translate to

$$B^H V_0 = 0_{P \times 1} \quad \text{and} \quad B^H B = I_{P \times P}.$$  

(25)

We call $B$ a “blocking” matrix because it blocks signal components in the S-T direction of interest $V_0$. Next, we consider S-T processing of DS/CDMA signals with $P$ joint S-T degrees of freedom in the form of a vector $\mu_{P \times 1}$, as shown in Fig. 2. In this context, the overall linear filter can be written compactly as

$$w_B = V_0 - \sum_{i=1}^{P} \mu_i G_i = V_0 - B \mu$$

(26)

and decisions are made according to

$$\hat{b}_0 = \text{sgn} \left( \text{Re} \{ w_B^H X \} \right).$$

(27)
Blocking matrix processing techniques have been of considerable interest in antenna and radar array processing applications in the past [23]. In fact, some recent research efforts focus on the selection of the blocking processor based on eigenvector metrics [39], [40] or data-independent discrete-Fourier transforms or Walsh-Hadamard transforms [27]. In this present work, instead of eigen analysis or data independent processing, we wish to extend the design criterion introduced in the previous section for processing with a single auxiliary vector to cover blocking-matrix processing with \(1 < P \leq M(L + N - 1) - 1\) auxiliary vectors.

For an arbitrary but fixed blocking matrix \(B\) that satisfies the constraints in (25), the linear filter \(w_B\) can be optimized with respect to the \(P\)-dimension complex vector \(\mu\). There are two equivalent interpretations for the MS-optimum value of the weight vector \(\mu\). Since the overall blocking-matrix filter \(w_B\) in (26) is by construction distortionless in the \(V_0\) direction of interest \((w_B^H V_0 = 1)\), we may search for the weight vector \(\mu\) that minimizes the output variance (energy) \(E\{w_B^H X^2\}\). This would suppress all signal components that are not in the \(V_0\) S-T direction of interest. Equivalently, we may instead identify the vector \(\mu\) that minimizes the MS difference \(E\{[V_0^H X - \mu^H B^H X]^2\}\) between the main-beam processed data \(V_0^H X\) and the blocking-matrix processed data \(\mu^H B^H X\). The solution to the first problem can be obtained by direct differentiation of the variance expression \(E\{w_B^H X^2\}\) with respect to the complex vector \(\mu\). The solution to the second problem falls in the context of optimum linear MS estimation [28]. The solutions, of course, coincide, and the optimum weight vector is identified by Proposition 3 below. This result is a generalization of Proposition 1 of the previous section.

**Proposition 3:** The \(P\)-dimension, \(1 \leq P \leq M(L + N - 1) - 1\), complex weight vector \(\mu\) that minimizes the output variance of the filter \(w_B\) or minimizes the MSE between the main-beam processed data \(V_0^H X\) and the blocking-matrix processed data \(\mu^H B^H X\) is
\[
\mu = [B^H R B]^{-1} B^H R V_0. \tag{28}
\]

With this result, the output variance of the *ideal* filter \(w_B\) as a function of the blocking matrix \(B\) can be seen to be
\[
\text{VAR}_B \triangleq E\{w_B^H X^2\} = V_0^H R V_0 - V_0^H R B [B^H R B]^{-1} B^H R V_0. \tag{29}
\]

Blocking matrix processing with \(P\) joint S-T degrees of freedom requires inversion of the \(P \times P\) blocked data autocovariance matrix \(B^H R B\) as seen in (28). To avoid matrix inversion operations altogether, we may consider recursive conditional optimization of the \(P\) weighted auxiliary vectors.

The anticipated benefits are: 1) significantly lower computational optimization complexity than unconditionally optimum blocking-matrix processing; and 2) superior BER performance for small-sample support optimization. The latter is anticipated due to the curse of dimensionality in data-limited environments, where sequential, one-by-one, conditional optimization of the AV weights is, in general, preferable to joint vector optimization by the numerically unstable operation in (28). Parallel to the blocking-matrix filter definition in (26), let us introduce a new filter \(w_c\) consisting of \(P, 1 \leq P \leq M(L + N - 1) - 1\), weighted auxiliary vectors that are orthonormal with respect to each other and to the normalized S-T RAKE vector \(V_0\)

\[
w_c \triangleq V_0 - \sum_{i=1}^{P} c_i G_i. \tag{30}
\]

\(G_i\) and \(c_i\) are optimized exactly as in (21) and (19) in Section III. For convenience, we repeat the formulas below

\[
G_i = \frac{R V_0 - (V_0^H R V_0) V_0}{\|[V_0^H R V_0] V_0\|}, \tag{31}
\]
\[
c_i = \frac{G_i^H R V_0}{G_i^H R G_i}. \tag{32}
\]

Next, we upgrade the “main-beam” to \(V_0 - c_i G_i\) and seek an auxiliary vector \(G_2\) that maximizes the cross-correlation magnitude \(\|[V_0 - c_i G_i]^H R G_2\|\), subject to the orthonormality constraints \(G_i^H V_0 = 0, G_i^H G_i = 1\), and \(G_2^H G_2 = 1\). Arguing as in Proposition 2, we find that \(G_2\) is as shown in (33) at the bottom of the page. The conditionally MS-optimum value of \(c_2\) for given \(G_1, c_1,\) and \(G_2\) is
\[
c_2 = \frac{G_2^H R (V_0 - c_1 G_1)}{G_2^H R G_2}. \tag{34}
\]

To complete the recursive construction of the filter \(w_c\) in (30), we assume that \(G_i, c_i\) are defined for \(i = 1, \ldots, P\), \(1 \leq P \leq (P-1)\). Then, with “main-beam” \(V_0 - \sum_{i=1}^{P} c_i G_i\), the auxiliary vector \(G_{P+1}\) that maximizes the cross-correlation magnitude \(\|[V_0 - \sum_{i=1}^{P} c_i G_i]^H R G_{P+1}\|\), subject to the orthonormality constraints is \(G_{P+1}\), as shown in (35), at the bottom of the next page. The conditionally MS-optimum coefficient \(c_{P+1}\) becomes
\[
c_{P+1} = \frac{G_{P+1}^H R \left( V_0 - \sum_{i=1}^{P} c_i G_i \right) }{G_{P+1}^H R G_{P+1}}. \tag{36}
\]

When the blocking matrix \(B\) in (24) consists of the \(P\) auxiliary vectors defined recursively by (35) and (36), the variance expressions in (11), (14), (22), and (29) are related.
as follows:¹

$$\text{VAR}_{\text{MVDR}} \leq \text{VAR}_B \leq \text{VAR}_c \leq \text{VAR}_{\text{AV}} \leq \text{VAR}_{\text{RAKE}}.$$  \hspace{1cm} (37)$$

In (37), equality holds when $R$ is diagonal (no MAI and perfectly white S-T disturbance). When $P = 1$, $\text{VAR}_B = \text{VAR}_c = \text{VAR}_{\text{AV}}$. When $P = M(L + N - 1) - 1$, $\text{VAR}_{\text{MVDR}} = \text{VAR}_B$. The latter statement reinforces the known result that MS-optimum filtering in the $C^M(L+N-1)$ vector space can be achieved with filter optimization in a $C^M(L+N-1) - 1$ vector subspace. If we assume that linearly filtered multipath S-T MAI is approximately Gaussian distributed \cite{38}, then (37) identifies the ranking in terms of BER performance of the respective ideal filters (perfect knowledge of the S-T autocovariance matrix and the S-T RAKE vector is assumed). In small-sample support situations, a sequence of conditionally optimized auxiliary vectors is expected to lead to superior BER performance.

In the following section, we compare the plain S-T RAKE filter $V_0$, the full-scale MVDR filter $\mathbf{w}_{\text{MVDR}}$ in (13), the single AV filter $\mathbf{w}_{\text{AV}}$ in (17), the blocking-matrix filter $\mathbf{w}_B$ in (26), and the conditionally optimized sequence of auxiliary vectors in the form of the filter $\mathbf{w}_c$ in (30). The comparisons are focused on the induced BER as a function of the required sample support.

V. NUMERICAL AND SIMULATION STUDIES

We consider the DS/CDMA signal model in (5) for a system with $M = 5$ antenna elements and processing gain $L = 15$. We assume the presence of $K = 6$ active users with fixed signature assignments and the normalized synchronous signature cross correlations between the user of interest and the five interferers are chosen between 0.2 and 0.3. Each user signal experiences independent Rayleigh fading paths with equal average received energy per path and independent angles of arrival uniformly distributed in $(-\pi/2, \pi/2)$. Antenna diversity effects are not pursued. The array interelement spacing is taken to be one-half the wavelength, and identical fading is assumed to be experienced by all antenna elements for each path of each user signal.

In Figs. 3–7, we examine the BER performance of all filters discussed in this work as a function of the total SNR of the user of interest (the sum of the received SNR’s over all paths) and the sample support. Following the notation in (5), the total received SNR for user $k$, $k = 0, \ldots, K - 1$, is defined by $E_k \sum_{k=0}^{K-1} E[|c_k|^2]/\sigma^2$, where $\sigma^2$ is the variance of the sampled AWGN, which was assumed to be identical for every spatial channel (antenna element). All BER’s for all filters are analytically evaluated, that is for every given S-T multipath channel realization and every given filter realization, the induced BER is calculated analytically as the expected probability of error over all possible user–bit combinations. The results that we present are averages over 100 independently drawn multipath Rayleigh fading S-T channels and 10 independent filter realizations per S-T channel.

In Fig. 3, we study the BER characteristics of the MVDR filter as defined by (13). The total SNR of the interferers is fixed at 7, 8, 9.5, 10.5, and 12 dB, respectively. Fig. 3 plots the BER of the user of interest as a function of its total SNR over the 0–12-dB range for various data set sizes. The BER’s of the ideal S-T RAKE and the ideal MVDR filter are included as reference points. While the ideal MVDR filter promises an improvement of many orders of magnitude over the ideal S-T RAKE filter, it is rather disappointing to observe that even for large data sets of size $5M(L+N-1) = 425$, the data-estimated MVDR filter is barely superior to the ideal S-T RAKE filter.

Fig. 4 replicates the studies of Fig. 3 for the blocking-matrix filter $\mathbf{w}_B$ in (26) with $P = 5$ auxiliary vectors selected according to recursion (35), (36). Some improvement with respect to small-sample support BER’s can be seen.

$$G_{p+1} = \left[ R\left(V_0 - \sum_{i=1}^{p} c_i G_i \right) - \left[V_0^H R\left(V_0 - \sum_{i=1}^{p} c_i G_i \right)\right] V_0 - \sum_{j=1}^{P} \left[G_j^H R\left(V_0 - \sum_{i=1}^{p} c_i G_i \right)\right] G_j \right].$$ \hspace{1cm} (35)
Fig. 4. BER versus total SNR under varying sample support for a S–T blocking-matrix filter with five auxiliary vectors and unconditionally optimum AV weight vector $\mathbf{u}$.

Fig. 5. BER versus total SNR for the single auxiliary-vector filter with varying sample support.

Fig. 5 presents the same studies for the single auxiliary-vector filter $\mathbf{u}_{\text{AV}}$ in (17). Significant BER improvement is achieved for data set sizes equal to or a few times the S–T product $M(L + N - 1)$.

In Fig. 6, we maintain the same setup and study the BER behavior of a conditionally optimized sequence of $P = 10$ auxiliary vectors in the form of the filter $\mathbf{u}_c$ in (30). These results indicate superior performance by orders of magnitude compared with the previously examined filters for sample support between $M(L + N - 1) = 85$ and $10M(L + N - 1) = 850$.

Finally, Fig. 7 assumes fixed total SNR for the user of interest at 12 dB and merges the results of Figs. 3–6 in the form of BER plots versus required number of data samples. The superiority of the conditionally optimized sequence of auxiliary vectors (filter $\mathbf{u}_c$ with ten auxiliary vectors) is apparent over the whole $M(L + N - 1) = 85$ to $4M(L + N - 1) = 340$ data-support range of primary interest, even the simple lightweight single auxiliary-vector filter outperforms, by a couple of orders of magnitude or more, its computationally intensive blocking-matrix optimized and full-scale MVDR counterparts.

VI. CONCLUSIONS

Typical fading rate measurements for mobile communication channels with carriers in the vicinity of 900 MHz show that adaptive receiver designs for data communication rates on the order of 19.5 $K$ symbols per second may not rely on more than a couple of hundred data symbols for system adaptation. Linear MS-optimum S–T processing for DS/CDMA antenna array systems requires optimization in the joint S–T product vector space. The dimension of the joint S–T vector space typically exceeds the available data-support size.

In search of adaptive optimization procedures that exhibit low-computational requirements and superior performance in...
small-sample support environments, we introduced the concept of maximum cross-correlation S-T auxiliary-vector (AV) filtering. Then, the extension for adaptive processing with a sequence of conditionally optimized weighted auxiliary vectors is readily available. We saw that simultaneous vector optimization of the AV weights in the context of blocking-matrix processing offers a moderate improvement over standard full-scale MVDR processing. In contrast, the inductive conditional AV optimization procedure leads to greatly improved BER performance characteristics when system adaptation relies on limited data. In this context, conditional AV processing appears a viable approach for adaptive S-T MAI and noise suppression in common multipath fading mobile communication channels.

APPENDIX

Proof of Proposition 2: Without loss of generality, we assume that $V_0^H R G \geq 0$ (real and nonnegative). We form the Lagrangian function

$$L(G) = G^H R V_0 - \lambda_1 G^H V_0 - \lambda_2 (G^H G - 1)$$

and set its conjugate gradient equal to the null vector

$$\nabla_G L(G) = 0 \Rightarrow R V_0 - \lambda_1 V_0 - \lambda_2 G = 0$$

$$\Rightarrow G = \frac{1}{\lambda_2} (R V_0 - \lambda_1 V_0).$$

(39)

The enforcement of the constraints on $G$ in (39) results in a system of equations to be solved for $\lambda_1$ and $\lambda_2$. Indeed, we obtain

$$\lambda_1 = \frac{V_0^H R V_0}{\|V_0\|^2}$$

and

$$\lambda_2 = \left\| R V_0 - \frac{V_0^H R V_0}{\|V_0\|^2} V_0 \right\|_2.$$  

(40)

Substitution of $\lambda_1$ and $\lambda_2$ back in (39) leads to the desired auxiliary vector $G$ that satisfies the constraints in (16) and maximizes the cross-correlation magnitude in (20)

$$G = \frac{-R V_0 + V_0^H R V_0}{\left\| V_0 \right\|^2} V_0.$$  

(41)

This result combined with Proposition 1 shows that the overall filter $w_{AV} = V_0 - \rho G$ is actually given by $w_{AV}$, as shown in (42) at the top of the page, where for simplicity, we assumed $\|V_0\| = 1$.

We also note that the above optimization procedure leads to the same filter $w_{AV}$ regardless of the particular value used for the constraint on the norm of $G$.

ACKNOWLEDGMENT

The authors would like to thank the three anonymous reviewers for their valuable comments and suggestions on the original manuscript. The assistance of G. N. Karystinos in obtaining the numerical and simulation results of Section V is gratefully acknowledged.

REFERENCES


\[
w_{AV} = \frac{(1 - V_0^H R V_0) [V_0 (R V_0)^H R (R V_0)]}{\|-R V_0 + (V_0^H R V_0) V_0\|^2 - \|R V_0\|^2} R V_0
\]  

(42)


Dimitris A. Pados (M’95), for a photograph and biography, see p. 1102 of the July 1999 issue of this TRANSACTIONS.

Stella N. Batalama (S’91–M’94), for a photograph and biography, see p. 917 of the June 1999 issue of this TRANSACTIONS.