

# Critical or Tricritical Point in Mixed-Action SU(2) Lattice Gauge Theory?

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## Abstract

An analysis of scaling along the first-order bulk transition line in fundamental-adjoint SU(2) lattice gauge theory strongly supports the first-order endpoint being a tricritical point, and is inconsistent with it being an ordinary critical point as is usually assumed. If tricritical, the transition must continue from the endpoint further into the phase diagram as a second-order bulk transition and extend to and beyond the Wilson axis. Observations indicate that this is most likely the same transition that has been traditionally considered a finite-temperature transition.

## 1 Introduction

The characterization of phase transitions has often been made clearer by considering higher-dimensional coupling spaces, especially ones that become more-familiar or exactly-known theories at one or more edges of the phase diagram. Then one can see how the various phase boundaries and critical points attach to better-known transitions. In SU(2) and SU(3) lattice gauge theory, the fundamental-adjoint plane has provided interesting insights. The SU(2) case was first studied by Bhanot and Creutz [1], who found two lines of first-order transitions which joined at a triple point and then continued as a single first-order line until ending at a presumed critical point (Fig. 1). They argued that since the transition apparently ended, the strong coupling confining phase could be continued around the endpoint resulting in a confining continuum limit, since a connecting path which encounters no phase transition could be found. The fact that the Polyakov loop, an order parameter for deconfinement (or disorder parameter for confinement), appeared to undergo a sudden jump to non-zero values across this line would seem to be inconsistent with the idea that both sides of the transition were confining, however it was assumed that in the limit of an infinite lattice the deconfinement signal would disappear.

One place where it cannot disappear, however is along the top line of the phase diagram where  $\beta_A = \infty$ . This is the well-known Z2 lattice gauge theory, which has a bulk first-order transition at  $\beta_F = \frac{1}{2} \ln(1 + \sqrt{2}) \approx 0.44$  (determined exactly from self-duality)[2]. This transition is deconfining with the Polyakov loop as order parameter. Therefore the line  $\overline{AB}$  on Fig. 1 is definitely deconfined even on the 4-d infinite lattice. In the conventional interpretation of lattice gauge theory the entire rest of the phase diagram is confining on such a lattice.

The situation became clearer when it was realized that finite lattices were at a finite 3-d (ordinary physical) temperature which increased as  $\beta$  increased. A deconfinement transition due to physical temperature - a so called finite-temperature transition - should occur on a finite lattice. For SU(2) this is a second-order transition and was studied extensively on the Wilson axis ( $\beta_A = 0$ ). In Ref. [3] the finite-temperature transition was studied on the fundamental-adjoint plane on lattices with temporal extent  $N_\tau = 4$ . Some couplings were further studied at  $N_\tau = 6$  and 8 [4]. The rather surprising result of these studies was that the line of second-order finite-temperature transitions seemed to join up with the first-order bulk transition at its endpoint. The finite-temperature deconfinement transition also became first-order at this point (point D in Fig. 1). If these transitions truly were joined that would call into question the finite-temperature interpretation of the second-order transition. This is because a finite-temperature transition should move all the way to the right of the phase diagram as  $N_\tau$  is increased (similar to the hypothetical dashed lines 1-4 on Fig. 1). However this would not be possible if one end was tied down at point D. In this case it would be unlikely for the second-order transition to move beyond line 1 (this is a line of constant physics as determined by continuum two-loop perturbative renormalization-group theory, as are lines 2-4). Line 1 is constructed to join at the endpoint of the first-order line, as determined by extrapolating the latent heat (which appears to vary linearly with  $\beta_A$ ) to zero using the  $12^4$  lattice data from the current study. This is at  $(\beta_F, \beta_A) = (1.38 \pm 0.03, 1.04 \pm 0.01)$ , a somewhat higher  $\beta_A$  (and lower  $\beta_F$ ) than the original low-statistics Bhanot-Creutz result. This is in agreement with the results of Refs. [3, 4], although not with [5] (more on this later). In the current paper, this point will be referred to as the “first-order-endpoint” (FOE), as its identification as either a critical point or a tricritical point is the question under consideration. To the right of the phase line the coupling quickly becomes weak enough for these perturbative lines of constant physics to be accurate. Near the phase transition they are more hypothetical. Putting aside the unreasonable possibility of a phase line that curves up and then down, one can therefore conclude that if the second-order deconfinement transition continues to join the bulk transition at point D as  $N_\tau \rightarrow \infty$  then, at least for  $\beta_A > 1$ , the zero-temperature continuum limit (right hand side of the phase diagram) would be deconfined. In other words, the entire transition would be bulk. The movement of the second-order line with changing lattice size, seen on the Wilson axis, would be explained as an ordinary finite-size shift in the critical point, perhaps with an

unusually large shift-exponent or following something other than a power law. In this case the transition point would converge to some finite value of  $\beta_F$ , probably in the range 3.0 to 4.0, as  $N_\tau \rightarrow \infty$ .

Another hypothesis consistent with the original Bhanot-Creutz scenario is that the two transitions join, but *not* at the FOE. There are a few systems known in which a second-order line meets a first-order line at a point other than its endpoint, such as the Blume-Emery-Griffiths model[6] and certain metamagnets[7, pp175-181]. These systems each have two different order parameters and two corresponding correlation lengths. However, in these systems both transitions are bulk.

Applying this scenario to the lattice gauge theory, where the second-order transition is finite-temperature, the point where the second-order transition joins the first-order line is hypothesized to slowly move up the diagram as  $N_\tau$  increases (lines 2-4 in Fig. 1). The lower part of the bulk transition would no longer be deconfining. Eventually as  $N_\tau$  became infinite the entire phase diagram except for the line AB would be confined. The bulk transition would have nothing to do with confinement. The observed deconfinement across the bulk line on small lattices would be due to the coincident finite-temperature transition, which somehow becomes first-order due to the influence of the bulk transition. The bulk transition would have its own order parameter which was not symmetry-breaking, similar to the liquid-gas transition. This order parameter would have a correlation length associated with it which would become infinite at the critical point at the end of the first-order line. This would happen at a place within the confining region, where the string tension (and correlation length associated with it) is finite. In other words the theory would have to have two independent correlation lengths. This seems a bit odd in that there is no evidence for the more-complicated scaling laws that would normally result from a theory with more than one correlation length. However, otherwise this interpretation is consistent with the conventional interpretation of Lattice Gauge Theory - in particular with a confining zero-temperature continuum limit.

It is difficult to distinguish these two hypotheses simply by looking for a joining away from the FOE, because one would have to go to rather high  $N_\tau$  to get a convincing separation. A small separation was reported for the SU(3) case [8] which has a similar phase diagram, except that both transitions (bulk and deconfining) are first-order. However, critical points determined on finite lattices from different quantities or by different techniques can differ substantially from one another. This could lead to a small apparent separation of the critical point and the end of the finite-temperature line, since one is determined from the plaquette and the other from the Polyakov loop. A more convincing demonstration of separation would be the observation of uncorrelated tunneling events in two different order parameters.

There is actually a much easier way to determine which of these cases is correct, based on a study of the bulk transition itself. In the conventional scenario just described, the line of first-order transitions ends in an ordinary critical point. Its order parameter therefore cannot be associated with spontaneous symmetry break-

ing, because otherwise the transition would have to continue, in order to divide the plane into symmetry-broken and unbroken parts. First-order transitions ending in a critical point are characterized in the Landau theory as having a cubic term in the free energy which explicitly breaks the symmetry. Only at the critical point, where the cubic term is zero, is the symmetry accidentally realized, allowing for a single point of criticality. The other scenario, where the second-order line joins the first-order at the endpoint, is exactly what happens at a *tricritical* point. A tricritical point is associated with a symmetry-breaking phase transition. Its Landau free-energy has only even-order terms but must be considered out to sixth order, because in part of the phase diagram the quartic term is negative. Here there is a first-order transition, which becomes second-order when the quartic term becomes positive. The tricritical point is simply where the change in order takes place, when the quartic term vanishes. If the endpoint of the fundamental-adjoint bulk transition is tricritical, then the transition *must* continue as second-order, and it must be symmetry breaking.

A favorable aspect of this study is that the bulk transition should not, by its very nature, depend much on lattice size. Bhanot and Creutz's study was done on a  $5^4$  lattice, and more recent results, such as those presented here for  $12^4$  and  $20^4$  lattices, do not differ much in the location of the transition or other parameters such as latent heat. Except for an expected reduction in variance from simple statistics, no significant differences are seen between our runs on  $12^4$  and  $20^4$  lattices (see Fig. 3 below). Similarly, because it is a bulk quantity, one would expect the Landau free energy function to be accurately determined by modest lattices with very minor corrections from surface effects. Any results linked to the behavior of this free energy are therefore unlikely to change much on larger lattices.

The lack of finite size dependence seen in the data shown below contrast with what was reported by Gavaï who also studied the bulk transition on symmetric lattices[5]. This can be traced to a difference in measurement technique. Gavaï found a significant decrease in latent heat as lattice size was increased at  $\beta_A = 1.25$ . The latent heat decreased by a factor of three as the lattice size was varied from  $6^4$  to  $16^4$ . This information was extracted from runs very close to the phase transition for each lattice, from which the latent heat was taken from the peak separation of the apparently bimodal distribution. However, this method has a particular problem when used on the symmetric lattice due to the symmetry actually being  $(Z_2)^4$  rather than just  $Z_2$  (assuming periodic boundary conditions in all four directions). As the phase transition is approached on the finite lattice, the lattice goes from having four broken directions (and no unbroken), first to three broken (and one unbroken), then to two broken, then one, and finally to the fully unbroken symmetry case. The reason for this is simply the entropy factor associated with each. There are 16 states with four broken directions,  $4 \cdot 8 = 32$  with three broken directions, 24 with two, and 8 with one and only one corresponding state in the fully symmetric state. At the critical point where these all have equal energy, the fully unbroken state would occur

only 1/81 of the time. It is easy to see how this state could be missed. If one set  $\beta$  so that this state occurred 50% of the time, one would be below the critical point (in  $\beta$ ). The Boltzmann factor would then suppress the multiple-broken direction cases, which might not appear at all on larger lattices where the energy fluctuations are smaller. Thus, from a practical point of view, it is very difficult to display the full range of symmetry-broken cases in a single simulation. The multiple symmetry breakings appear to be associated with nearly equal jumps in the plaquette (see Fig. 2). On smaller lattices close to the transition, tunneling will occur between all of these states, showing nearly the full latent heat. However, larger lattices, with their smaller energy fluctuations (requiring hitting the critical point more accurately) and longer tunneling times, may only tunnel between two or three of the five levels in a reasonable-length run, showing an apparently smaller latent heat. Fig. 2a shows a time history on an  $8^4$  lattice at  $\beta_A=1.25$  and  $\beta_F=1.2185$  along with the Polyakov loop histories. There appear to be several energy plateaus in between the upper and lower, associated with only some of the Polyakov loop directions breaking. A similar simulation on a  $12^4$  lattice with  $\beta_F = 1.2183$  (closer to the critical point) populates only four of the five levels. Its plaquette histogram, shown in Fig. 2b, shows four peaks. The missing peak in this case is from all-four Polyakov loops unbroken. A similar  $16^4$  simulation at the same couplings populated only three peaks. The multimodal nature of the distribution is not always as apparent as in Fig. 2b. If the peaks are unequally populated then only shoulders will be seen. Thus it is risky to try to measure the latent heat from the widths of these distributions unless all five peaks of the multimodal distribution can be resolved. This multiple symmetry-breaking effect could easily explain the decrease in latent heat with lattice size seen in Ref. [5].

The data presented in the current study are from hysteresis loops, which do not suffer from this problem, as not much time is spent at  $\beta_c$ . As shown below, the hysteresis loops on  $12^4$  and  $20^4$  lattices are nearly identical, suggesting almost no finite size effects. Our values of latent heat agree closely with the values Gavai and coworkers found on *asymmetric* lattices, for which no finite size effect was found. Asymmetric lattices with one short direction do not suffer from the above problem, as the symmetry broken at the transition in question is then just the single  $Z_2$  of the short direction.

## 2 Critical vs. Tricritical

One needs to find an easily-measured quantity which can distinguish the critical from the tricritical cases. As shown below, it turns out that the size of the hysteresis region,  $(T^{**} - T^*)$  is a linear function of the latent heat for the critical case and a quadratic function for the tricritical case. Here  $T^*$  is the lower (supercooling) metastability limit and  $T^{**}$  is the upper (superheating). Between these temperatures, there are two minima of the free energy and tunneling exists. Outside of this

region there is only one local minimum and no tunneling exists. This gross difference in scaling behavior follows from basic dimensional analysis of the powers in the Landau free energy, but a detailed derivation is also given below. Both the latent heat and metastability regions are easily determined from hysteresis sweeps. By plotting vs. latent heat no assumptions need be made concerning the possibly complex relationship between  $(\beta_F, \beta_A)$  and the temperature and next higher coefficient in the free energy. It is interesting that this method is able to distinguish between symmetry-breaking and non-symmetry-breaking first-order transitions from energetics alone, without the need to identify the symmetry or order parameter.

In the Landau theory, the free energy is given by a power series in the order parameter.

$$f = \frac{1}{2}r\phi^2 - w\phi^3 + u_4\phi^4 + u_6\phi^6. \quad (1)$$

The quantity  $r$  is an increasing function of temperature, which can be defined as  $r = a(T - T^*)$ . For an ordinary 1st order transition that ends in a critical point,  $w > 0$  and  $u_4 > 0$  ( $w$  becomes zero at the critical point, whereas  $u_4$  remains positive everywhere). The sixth order term can be ignored. The critical point occurs when  $f = 0$ ,  $\partial f/\partial\phi = 0$  for the minimum away from  $\phi = 0$ . These are easily solved for  $\phi_c = w/2u_4$  and  $r_c = w^2/2u_4$ . The latent heat can be obtained from the change in entropy between phases. Taking  $s = -df/dT$  and expanding  $f$  to lowest order in  $r$  about the two minima gives[7]  $\Delta s = (a/2)\phi_c^2$ . Therefore the latent heat is given by

$$q = T\Delta s = (aT_c/2)(w/2u_4)^2. \quad (2)$$

Further details can be found in Ref. [7, pp. 168-175] from which this and the following derivations are abstracted. Note that  $q$  is quadratic in  $w$ , the parameter which is rapidly varying as one moves along the critical line, away from the critical point (rapidly varying because it must vanish at the critical point). The metastability limit on superheating,  $T^{**}$  occurs when the local minimum away from  $\phi = 0$  becomes an inflection point instead, i.e.  $\partial f/\partial\phi = 0$  and  $\partial^2 f/\partial\phi^2 = 0$ . Solving these gives

$$r^{**} = 9w^2/16u_4, \quad (3)$$

also quadratic in  $w$ . Taking the usual assumption that  $u_4$  is slowly varying, results in the prediction that

$$T^{**} - T^* = a^{-1}r^{**} \propto q. \quad (4)$$

In contrast, for the first-order transition that ends in a tricritical point,  $w = 0$  is enforced by symmetry. The quantity  $u_4 < 0$  and one needs the positive  $u_6$  term for stability. In this case *two* additional minima away from  $\phi = 0$  occur, one for positive and one for negative  $\phi$ . If these dip below the minimum at  $\phi = 0$  a first-order phase transition occurs. The tricritical point occurs when  $u_4 = 0$ . Beyond this is a line of second-order transitions ( $u_4 > 0$ ). In this case the transition changes from first-order to second-order, rather than disappearing. In fact it must continue

on, in order to divide the entire coupling plane into symmetry-broken and symmetry unbroken phases. Here  $u_4$  is the rapidly changing parameter as one moves along the transition line near the tricritical point, and  $u_6$  is assumed to be slowly varying. Following the same procedure given above results in [7]

$$\phi_c = \pm[|u_4|/2u_6]^{\frac{1}{2}} \quad (5)$$

$$q = aT_c|u_4|/(4u_6) \quad (6)$$

(linear in  $u_4$ ), and

$$r^{**} = 2u_4^2/(3u_6) \quad (7)$$

(quadratic in  $u_4$ ). Therefore the prediction for the tricritical case is

$$T^{**} - T^* \propto q^2. \quad (8)$$

### 3 Hysteresis Loops

Rather slow hysteresis sweeps were performed to determine  $q$  and  $\Delta\beta \equiv \beta_F^{**} - \beta_F^*$ , for various fixed  $\beta_A$ . One has a fair degree of flexibility in deciding which parameter to choose as the temperature. Here  $\beta_F$  is being treated as the inverse (4-d) temperature in the partition function. The  $\beta_A$  term with  $\beta_A$  held fixed can be thought of as either a temperature dependent external field term (with coefficient  $\beta_A/\beta_F$ ) or a temperature independent modification to the measure (contributing to the entropy). Because  $\Delta\beta \ll \beta_F$ ,  $\Delta\beta \propto T^{**} - T^*$  to lowest order. Each run was begun with 500 equilibration sweeps, followed by 2000-4000 sweeps where  $\beta_F$  is changed by 0.0001 on each sweep. This is slow enough that no hysteresis can be detected away from critical regions. Measurements were performed after each sweep. Runs were performed on a  $12^4$  lattice, except for five additional runs performed on a  $20^4$  lattice to test for finite size effects. Typical results for three different  $\beta_A$  are shown in Fig. 3. Each  $12^4$  sweep was performed three times to test for repeatability and to estimate errors (only one is shown). The latent heat was measured as the jump in fundamental plaquette,  $\langle p \rangle$ , at the position of maximum vertical distance between the hysteresis curves (with the definition of temperature above, the internal energy is given by  $1 - \langle p \rangle$ ). The quantity  $\Delta\beta$  was measured as the maximum width of the hysteresis curve. One can also take  $\beta_F^*$  and  $\beta_F^{**}$  to be the points of maximum slope of the hysteresis curves, and compute  $\Delta\beta$  from the difference - the results are nearly identical. Multiple runs are remarkably similar, indicating modest statistical errors (detailed later). A worrisome systematic error from hysteresis sweeps is the possibility of premature tunneling. If one sweeps too slowly, the system could tunnel to the other phase before the metastability limit is reached. In rare instances it could happen a considerable distance away. However, the fact that the  $20^4$  sweeps were nearly identical to the  $12^4$  at the same  $\beta$ 's (see Figs. 3 and 4), would seem to indicate this is not a problem. Tunneling times on the larger lattice are much

longer, so if the smaller lattice were tunneling prematurely by a significant amount then some difference between these different-size lattice runs would be expected. The main result is shown in Fig. 4, where  $\Delta\beta$  is plotted against the square of the jump in average plaquette,  $(\Delta \langle p \rangle)^2$  (proportional to  $q^2$ ). Although three independent measurements for each point are not sufficient to accurately determine individual error bars, an overall error estimate for the entire dataset can be made, which indicates error bars of about one-third the size of plotted points vertically and twice this horizontally. If one does compute individual error bars, no particular trend is observed - they are consistent with approximately equal error bars for all couplings. A linear trend (which on these axes is a pure quadratic) is observed to fit the data well. Thus the data agree with the prediction of the tricritical case. If one tries to fit to a linear function of  $\Delta \langle p \rangle$ , the result is not satisfactory, with  $\chi^2/\text{d.f.} = 53$ , whereas the pure quadratic shown in Fig. 4, plotted as a straight line with axes given, has  $\chi^2/\text{d.f.} = 0.6$ . A linear+quadratic fit was also performed. In the possible case that the trend is linear, but the region of validity of the Landau theory is small, this should be able to pick up the linear term. This fit, however, gives a linear coefficient of  $0.008 \pm 0.016$ , consistent with zero. The quadratic term (with coefficient  $0.839 \pm 0.039$ ) dominates already at  $(\Delta \langle p \rangle)^2 = 0.0001$ . Thus the data are not consistent with “beginning linear” over any reasonable region of validity. Therefore, the data appear to be strongly inconsistent with the possibility of there being an ordinary critical point at the FOE but entirely consistent with it being a tricritical point.

These results rely on certain assumptions inherent to the Landau theory, namely that higher order terms in the free energy are slowly varying. However if this assumption were not valid it would be unlikely to obtain such a clean result as pure quadratic scaling. Much more likely in this case would be a less conclusive result requiring a multiple-term fit. Also, mean field theory is much more likely to give valid results in four dimensions than in three, where it still provides useful predictions in many cases.

## 4 Identification of the order parameter

The tricritical behavior described above requires a symmetry breaking order parameter. So far, in this analysis, it has not been necessary to identify this symmetry or the associated order parameter. This was deliberately done in order to make as few assumptions as possible. However, now that the tricritical case seems to be established from energetics alone, it makes sense to try to identify the associated broken symmetry. There are many reasons to believe this is no other than the familiar Z2 Polyakov-loop symmetry. For one thing, this is the symmetry that is broken in the attached Z2 lattice gauge theory at the top of the phase diagram. Secondly, the Polyakov loop is seen to break along the bulk line, not just at the same couplings, but also tunneling at the same times. Fig. 5 shows Polyakov-loop histories for heat-



ing and cooling sweeps together with plaquette histories for  $\beta_A = 1.7, 1.25$  and  $1.0$  on the  $20^4$  lattice. These are well above, moderately above, and slightly below the tricritical point. In the first-order region, the metastability in Polyakov loop matches exactly with that of the plaquette. However even in the second-order region the small hysteresis signal in the plaquette from critical slowing-down appears to be associated with the symmetry breaking of the Polyakov loop. This, together with the correlations seen in Fig. 2 would seem to indicate not merely coincident phase transitions, but an intimate locking of order parameters as well, since the tunnelings in plaquette and Polyakov loops are observed to take place at the same Monte Carlo times.

The  $20^4$  data employ moving averages in the Polyakov loops to reduce the variance enough to see the symmetry breaking. The Polyakov loop values in the broken region on such a large lattice are tiny. The individual datapoints are swamped by random fluctuations. Moving averages of, say 100 points, reduce these random fluctuations by a factor of 10, allowing the small nonzero average value in the broken phase to show through remarkably well. Tunneling times in the broken region are generally much longer than this, so the average values deduced are fairly accurate.

One important objection to the Polyakov loop being an order parameter for a bulk transition is that its very definition depends on there being periodic boundary conditions. A bulk transition, on the other hand, should exist for any boundary conditions, such as open boundary conditions for which the Polyakov loop is not defined. However, it is important to realize that this same objection can be made for the Z2 lattice gauge theory, for which there is no controversy about the existence of a bulk deconfining transition. Therefore, at least in this theory, there must be a second hidden order parameter and associated broken symmetry that exists for the case of open boundary conditions. It is reasonable to expect that this same situation may also exist in the SU(2) case. A possible symmetry and order parameter which exist for both theories have been identified. If one employs a partial axial gauge fixing that leaves unbroken a global gauge symmetry on each 3-d layer of the lattice perpendicular to a certain direction, then these remaining global gauge symmetries appear to break spontaneously at weak coupling, apparently in concert with the Polyakov loop[11]. The order parameters are just the average perpendicular links in each layer.

The existence of a tricritical point implies that a bulk (4d zero-temperature) second-order transition exists below it, down to and even below the Wilson axis (because it must divide the plane into two non-connected regions with different symmetry). If the Polyakov loop is the order parameter then there is no finite-temperature deconfinement transition. Deconfinement is a bulk transition and the zero-temperature continuum theory is not confining. In order for the deconfinement transition to be a finite-temperature one as is usually assumed, it must decouple from the bulk transition as described in the introduction. However, this has apparently not happened yet, even on the  $20^4$  lattice, in the coupling regions studied. If

a separation does occur, then a *new* symmetry and order parameter must be found for the second-order bulk transition emanating from the tricritical point. It will be important to locate this new phase transition on the Wilson axis. In order for lattice gauge results to be analytically connected to the continuum limit, one may only run simulations on the weak-coupling side of any bulk transition. If it occurs near the deconfinement transition on accessible lattices, as is likely the case, then there would only be a narrow region of valid couplings in which reliable confining simulations using the Wilson action could be run, lying between the new second-order bulk transition and the previously known finite-temperature transition. However, the fact that no second-order bulk transition separate from the deconfinement transition has ever been seen would seem to make the entire scenario of two separate transitions unlikely. The order parameter studied in ref. [11] breaks both the global gauge symmetry and the Z2 Polyakov loop symmetry. If this is driving the bulk transition, then it will only be possible for the Polyakov loop deconfinement transition to split off to the strong coupling side of this transition, because on the weak coupling side the Z2 symmetry will already be broken by the bulk transition. In this case there would be no region of validity for the confining theory (always separated from the continuum limit by the bulk transition).

The evidence for the Polyakov-loop transition being a finite-temperature one stems mostly from an observed shift in transition point with temporal lattice size,  $N_\tau$ , on asymmetric lattices with  $N_\sigma > N_\tau$ . The size of the shift is larger than one usually expects for a bulk transition. However the possibility exists that the four-dimensional non-abelian gauge theory could just have an unusually large finite-size shift. Fig. 6 shows data for the finite-temperature deconfinement transition point,  $\beta_c$  for the Wilson action on different size asymmetric lattices (data from Ref. [9]). Also shown is data from a different action used by Gavai[10], in which Z2 monopoles and vortices are suppressed. This action has the same  $\Lambda$ -parameter and therefore the same perturbative scaling as the Wilson action, but the scaling is much different in the region of the deconfinement transition. Indeed, although the Wilson-action data appear to fit roughly to the weak-coupling renormalization group scaling law (though not acceptably, with a  $\chi^2/d.f. = 28$ ), it does not fit at all to the Z2 monopole and vortex suppressed action data ( $\chi^2/d.f. = 100$ ). However, eliminating strong coupling lattice artifacts would be expected to improve scaling. This suggests that the rough fit of the Wilson-action data may be accidental. A linear fit to  $1/\ln(N_\tau)$  also produces a rough fit in the Wilson-action case (but also unacceptable considering the very small errors quoted for these points, with  $\chi^2/d.f. = 64$ ). A linear fit fares better with the Gavai data with  $\chi^2/d.f. = 4.6$  (none of the fits discussed thus far include the lowest  $N_\tau = 4$  points). The apparent intersection of these lines at  $N_\tau = \infty$  is probably fortuitous, but it is interesting that they would agree on the infinite lattice critical point. The scaling behavior of lattices with the same  $\Lambda$ -parameter need only match in the weak coupling region. They do not need to match exactly at  $\beta_{c\infty}$  (the critical coupling for an infinite lattice), but since this

$\beta$  is close to the perturbative region they should be close. Above a phase transition there is no reason for them to match at all, which could explain the rather different slopes. The dashed line fits include a quadratic as well as a linear term (here the  $N_\tau = 4$  point was included in the Gavai-data fit). This is able to accommodate the small curvature in the data rather well, and still has near-agreement for the infinite lattice critical point of around  $\beta = 3.9$ . The  $\chi^2/d.f.$  for these fits are 3.5 and 2.3 which are coming rather close to acceptability, considering the quadratic term is probably just approximating a more complex non-linearity. A possible reason for approximate but not exact  $1/\ln(N_\tau)$  scaling is as follows.

The behavior pictured in Fig. 6 can be understood if the transition is associated with percolation of abelian-monopole loops in the maximal abelian gauge. It has been shown that confinement seems to be associated with the existence of a monopole loop that wraps through the periodic boundary, or with the closely-related existence of a percolating cluster of such loops. The deconfinement transition seems to occur when this is no longer the case. The probability of a monopole loop of size  $l$  (for  $l < N$ ) existing on a lattice, normalized per lattice site, has been shown to be proportional to  $l^{-\gamma}$  where  $\gamma$  is a  $\beta$ -dependent quantity[12, 13]. Here  $N$  is the linear size of a symmetric lattice.  $\gamma$  is about 3 in the crossover region and becomes equal to 5 around  $\beta = 2.9$ [13]. The scaling law may be somewhat different for  $l > N$ , but the results are insensitive to this so long as the exponent  $\gamma \geq 1$  for all  $l > N$ [13]. To have a wrapping loop, one requires at least one loop of size of order  $N^{1+\epsilon}$  or larger, where  $\epsilon$  is a fractal dimension between 0 and 1, which has not yet been accurately measured. Probably  $\epsilon > 0$  because the loops are generally somewhat crumpled but  $\epsilon = 0$  is also a possibility. The probability of finding such a loop on an  $N^4$  lattice is  $CN^{(5+\epsilon)-\gamma(1+\epsilon)}$  where  $C$  is some constant. This expression results from integrating the probability over loop sizes beginning at the critical loop size for a wrapping loop. Setting this probability to  $\frac{1}{2}$  results in an estimate for the critical value of  $\gamma$  for that lattice, from which  $\beta_c$  can be determined. This therefore gives a model for the dependence of  $\beta_c$  on  $N$ . If one assumes that the asymmetric lattices, for which most of the data has been taken, have a similar scaling law, and also assuming that  $\gamma$  can be taken to be a linear function of  $\beta$  in the region of interest (a simplification), then one obtains the finite-lattice scaling law

$$\beta_c = \beta_{c\infty} - \frac{c}{\ln(N)} \quad (9)$$

where  $c$  is a constant. This contrasts with the usual finite-lattice shift for a thermal transition

$$\beta_c = \beta_{c\infty} - \frac{c}{N^{1/\nu}} \quad (10)$$

which converges much more rapidly to  $\beta_{c\infty}$  as  $N \rightarrow \infty$ . The idea that this transition may be a 4-d percolation transition could explain why it has been so hard to identify, as there are not many examples of such transitions. The fact that  $\gamma$  is a somewhat non-linear function of  $\beta$  [13] could explain the need for a quadratic term in the fits.

Better determination of  $\gamma(\beta)$  and measurements on asymmetric lattices would be needed to come to a definitive conclusion on the higher-order terms.

It has been shown by Gavai and Mathur that the bulk first-order transitions are lattice artifacts that can be removed through a judicious choice of action, leaving only a second-order deconfining transition[14]. Indeed all bulk transitions are lattice artifacts in that they do not affect the continuum limit. However, the question still exists whether this remaining second-order transition is bulk or finite-temperature. By using the original Bhanot-Creutz action, one can learn more about this transition by its apparent connection to the first-order bulk transition at a tricritical point. The existence of a tricritical point, surmised above from the behavior of the free energy around the first-order transition, strongly implies that the second-order line is also bulk (i.e. a 4-d zero-temperature transition), due to its attachment to the bulk first-order endpoint. If this is true in the original action, then the continuum limit is deconfined for this action, and weak-coupling universality would imply this is also true for all other actions. Thus this work supports the hypothesis made some time ago that the continuum limit of SU(2) pure-gluon lattice gauge theory may not be confining[15, 16].

One should, of course, remember that the SU(2) non-abelian gauge theory is not Quantum Chromodynamics (QCD) but an approximation to it. Lack of confinement in the SU(2) theory does not imply that quarks are unconfined in the real world. However, it does shed important light on the possible confinement mechanism. Of course this result must first be checked in the SU(3) case. If it holds up there, then suspicion must be cast on light quarks as the source of confinement in QCD, a position held by Gribov[17, 18] and others[15, 19, 20]. It could be that confinement is a byproduct of chiral symmetry breaking rather than the other way around as sometimes stated. One possibility is that strong color fields disrupt the chiral condensate, creating a bag of diminished and polarized chiral condensate around a hadron, carrying an energy proportional to the volume of excluded condensate. Supporting this conjecture is the observation in lattice simulations that the strength of the chiral condensate is reduced from its vacuum value in the presence of a quark source[21].

A way to conclusively demonstrate confinement due to light quarks would be to find an action that completely erases the bulk confinement transition, including the second-order one, but agrees with the Wilson action at weak coupling. Since all bulk transitions are lattice artifacts, this should be possible. An action that suppresses both Z2 monopoles and vortices as well as another gauge-invariant topological lattice artifact, the SO(3)-Z2 monopole [22] appears promising. Simulations on lattices up to  $30^4$  remain deconfined with this action for all couplings [23]. If confinement were to return when light quarks are added to this theory, one would then have clear evidence of their essential role in confinement.

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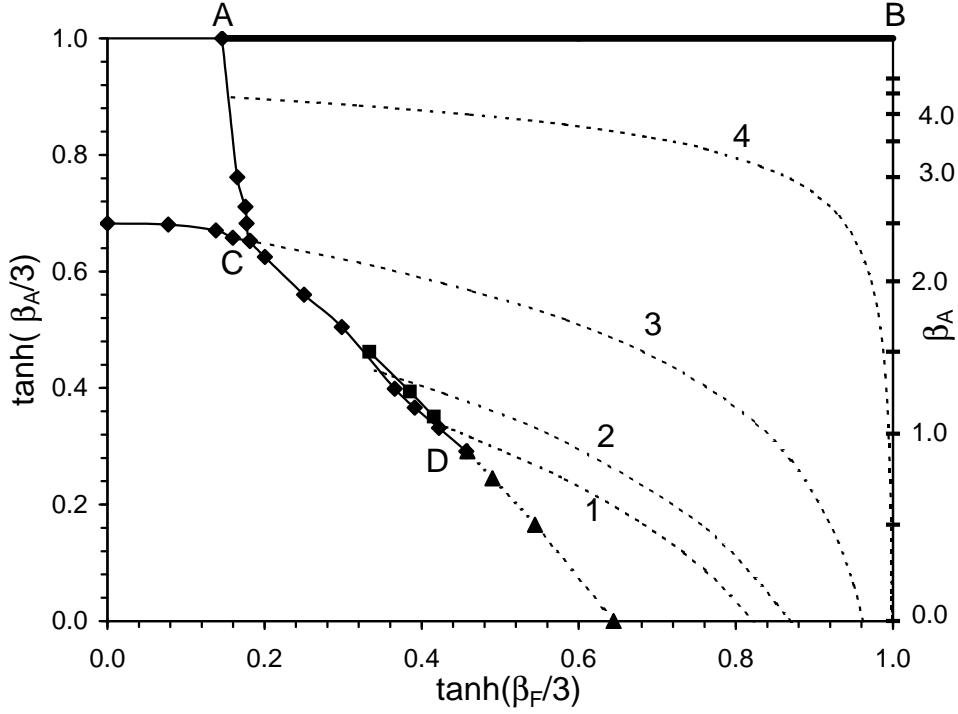


Figure 1: Phase transitions on the fundamental-adjoint plane. Nonlinear axes have been chosen so the entire coupling plane, including continuum limit at right and  $Z_2$  lattice gauge theory at top can be seen. Scaling of couplings has been chosen to reproduce “look” of usual plot on linear axes. Nonlinear scale for  $\beta_A$  is shown at right. Diamonds are Bhanot-Creutz data [1], triangles are second-order  $8^3 \times 4$ , and squares are  $8^3 \times 4$  first-order data from [3]. Solid lines are first-order, dashed second-order. Lines 1-4 are hypothetical second-order lines for very large lattices, following perturbative lines of constant physics. The line  $\overline{AB}$  is the deconfined phase of the 4-d  $Z_2$  lattice gauge theory.

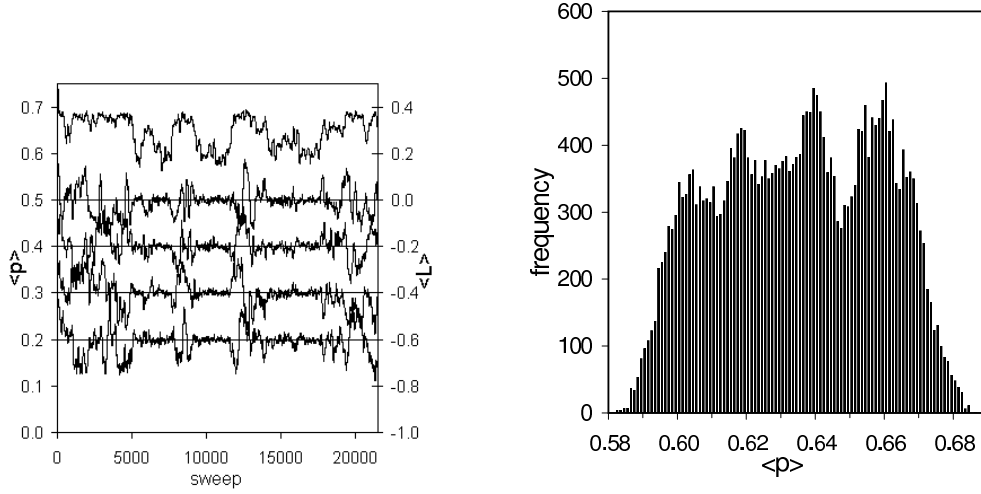


Figure 2: (a) Time history of an  $8^4$  lattice run at  $\beta_F = 1.2185$ ,  $\beta_A = 2.25$ . Upper trace is average plaquette, lower four traces are Polyakov loops. Each successive Polyakov loop trace is offset by 0.2 for clarity. Steplike structure in plaquette appears associated with the number of loops which show spontaneous symmetry breaking at any time. (b) Plaquette histogram on a  $12^4$  lattice at  $\beta_F = 1.2183$ ,  $\beta_A = 2.25$  showing clear multimodal distribution.



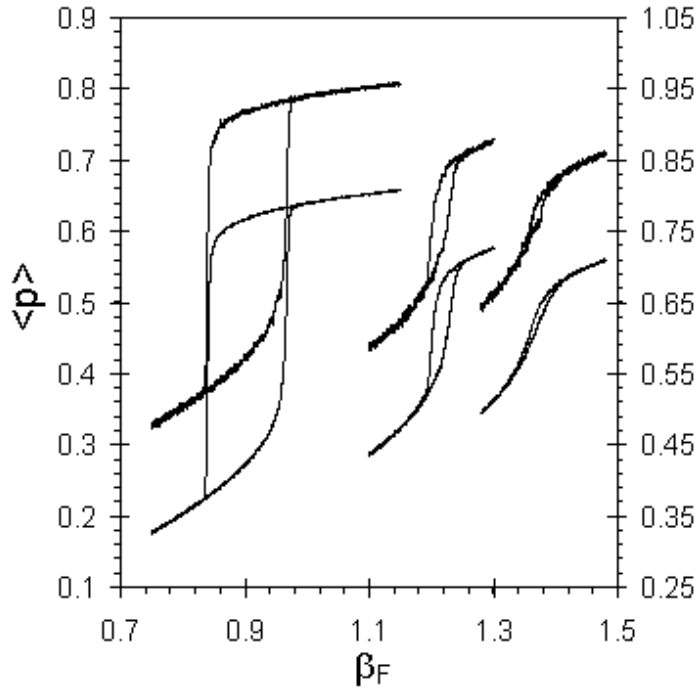


Figure 3: Hysteresis sweeps at  $\beta_A = 1.7, 1.25,$  and  $1.05$ . Upper curves are  $12^4$  lattice, lower offset curves are  $20^4$  lattice (scale at right).  $\beta_F$  is changed by  $0.0001$  after each Monte-Carlo sweep. Near coincidence of curves shows finite lattice size effects are small.

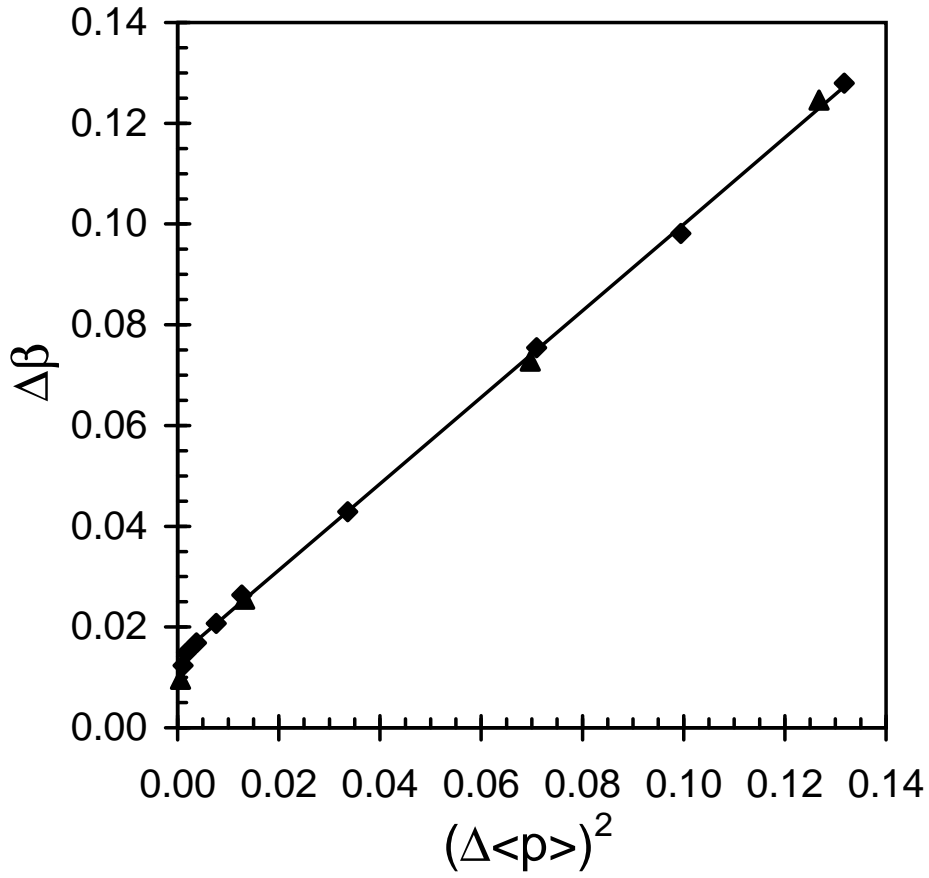


Figure 4: Width of metastable region vs. latent heat squared. Diamonds are for  $12^4$  lattice, triangles for  $20^4$  lattice. Linear fit on these axes is predicted by the tricritical hypothesis. The alternative hypothesis, that of an ordinary critical point, predicts scaling directly with latent heat rather than its square. The points comprise a range in  $\beta_A$  from 1.05 to 1.7.

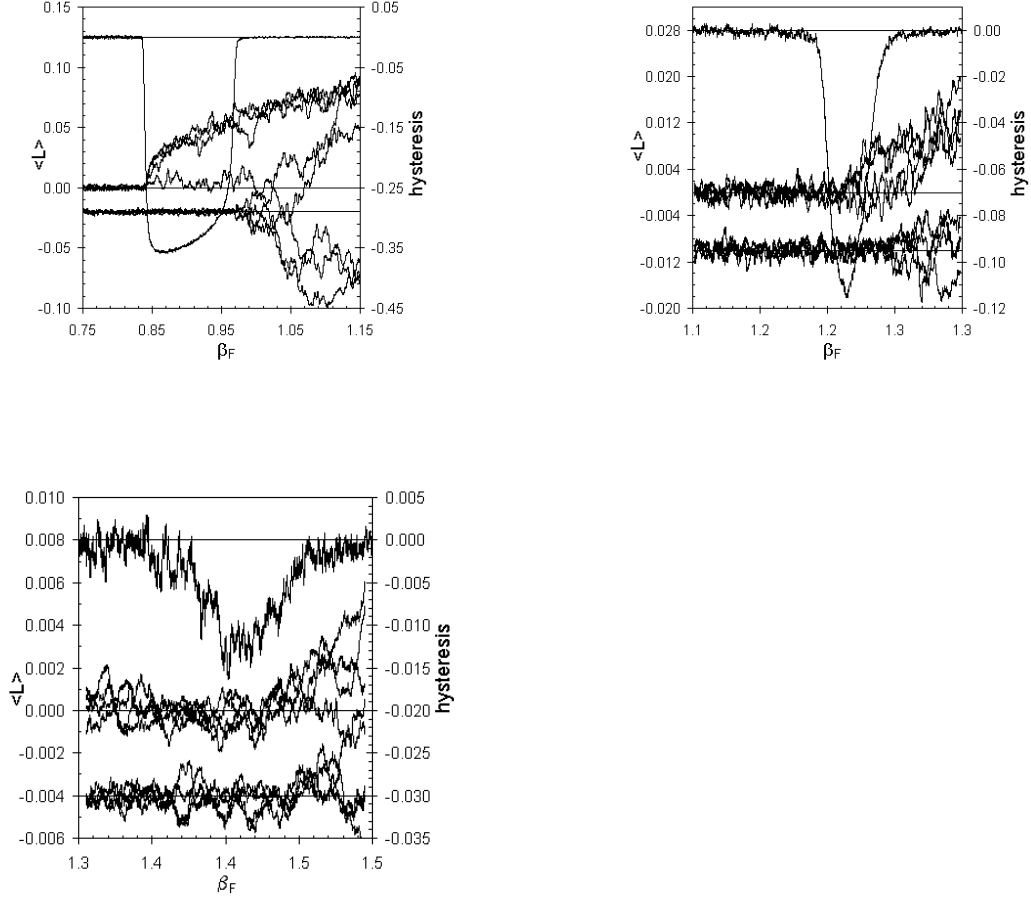


Figure 5: (a) A detailed look at  $20^4$  hysteresis sweep at  $\beta_A = 1.7$ . Upper curve (right scale) is the difference in average plaquettes as measured on cooling vs. heating sweeps. Next four curves are Polyakov loops for heating (beta decreasing) sweep. Final four curves, offset by 0.02 for clarity, are Polyakov loops for cooling sweep. Polyakov loop curves are 25-point moving averages to reduce variance. Spontaneous breaking of Polyakov loop is clearly associated with plaquette tunneling events. (b) Same at  $\beta_A = 1.25$ . (c) Same at  $\beta_A = 1.0$ . This lies below the tricritical point, in the second-order region. Now there is no longer much separation between the symmetry breakings for heating and cooling, but the breaking still seems to be coincident with the rising edge of the plaquette hysteresis. Here 100-point moving averages are used for the Polyakov loops. Note the hysteresis curve has lost the steep sides associated with first-order tunneling, and its magnitude is small compared to the first-order values.

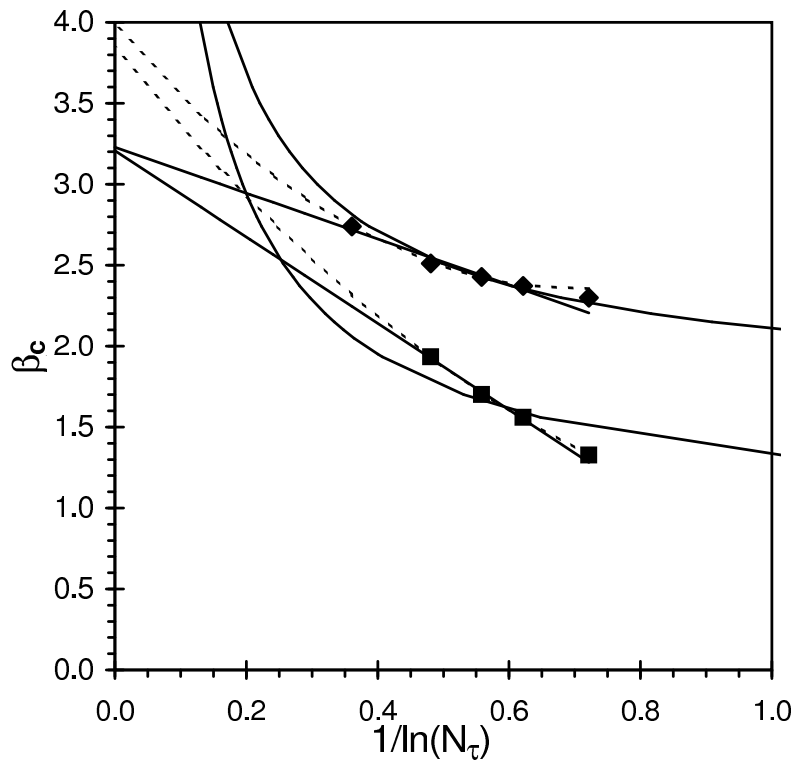


Figure 6: Plot of  $\beta_c$  for deconfinement transition on asymmetric lattices, to test possibility of  $1/\ln(N_\tau)$  scaling law. Diamonds from Ref. [9] are for Wilson action, squares from Ref. [10] are for Z2 monopole and vortex suppressed action. Straight lines are linear fits to the data, excluding the  $N_\tau = 4$  points. Dashed lines add a quadratic term to the fit. Solid curved lines are, for comparison, fits to the normal two-loop scaling formula.