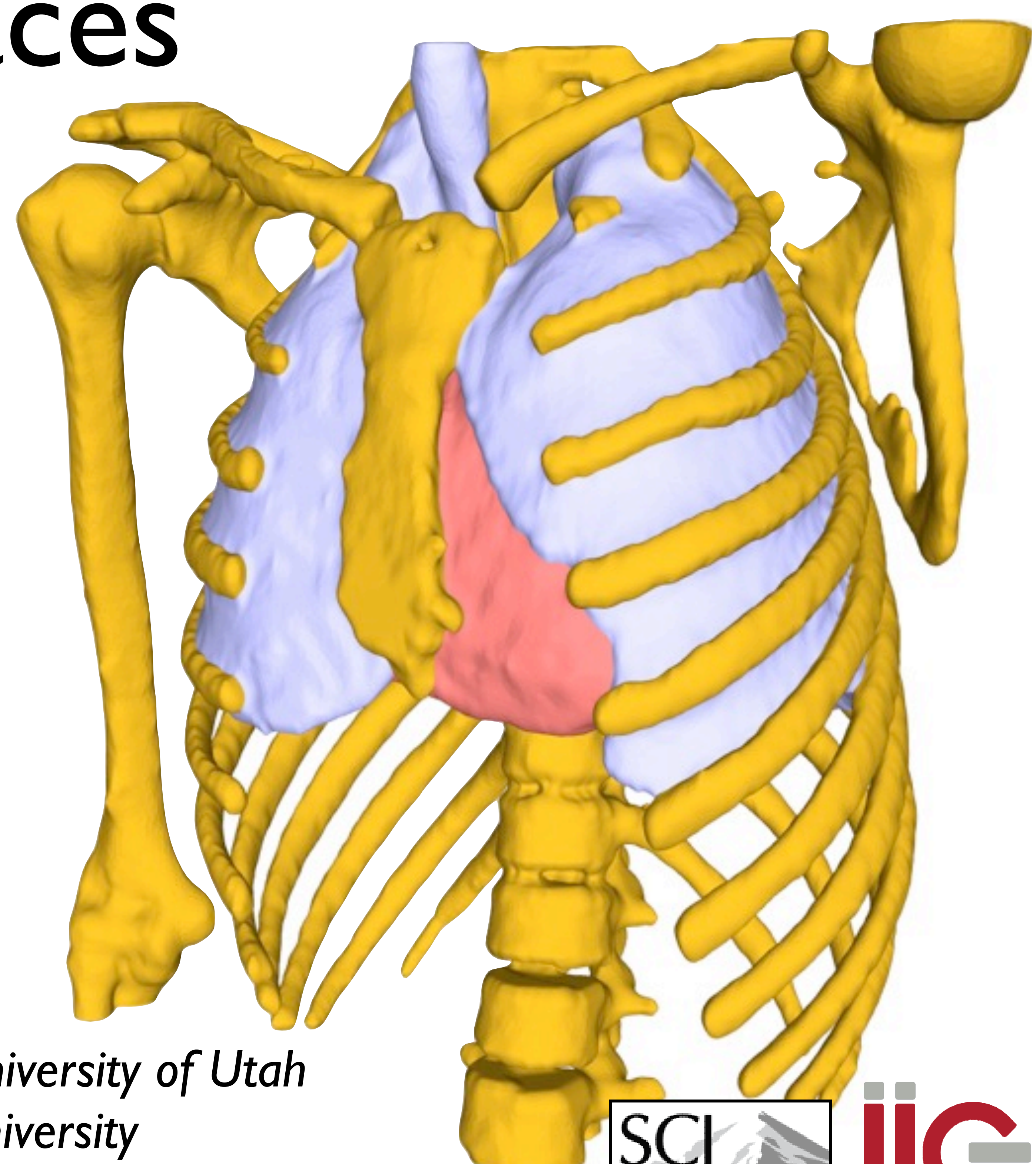
Particle-based Sampling and Meshing of Surfaces in Multimaterial Volumes



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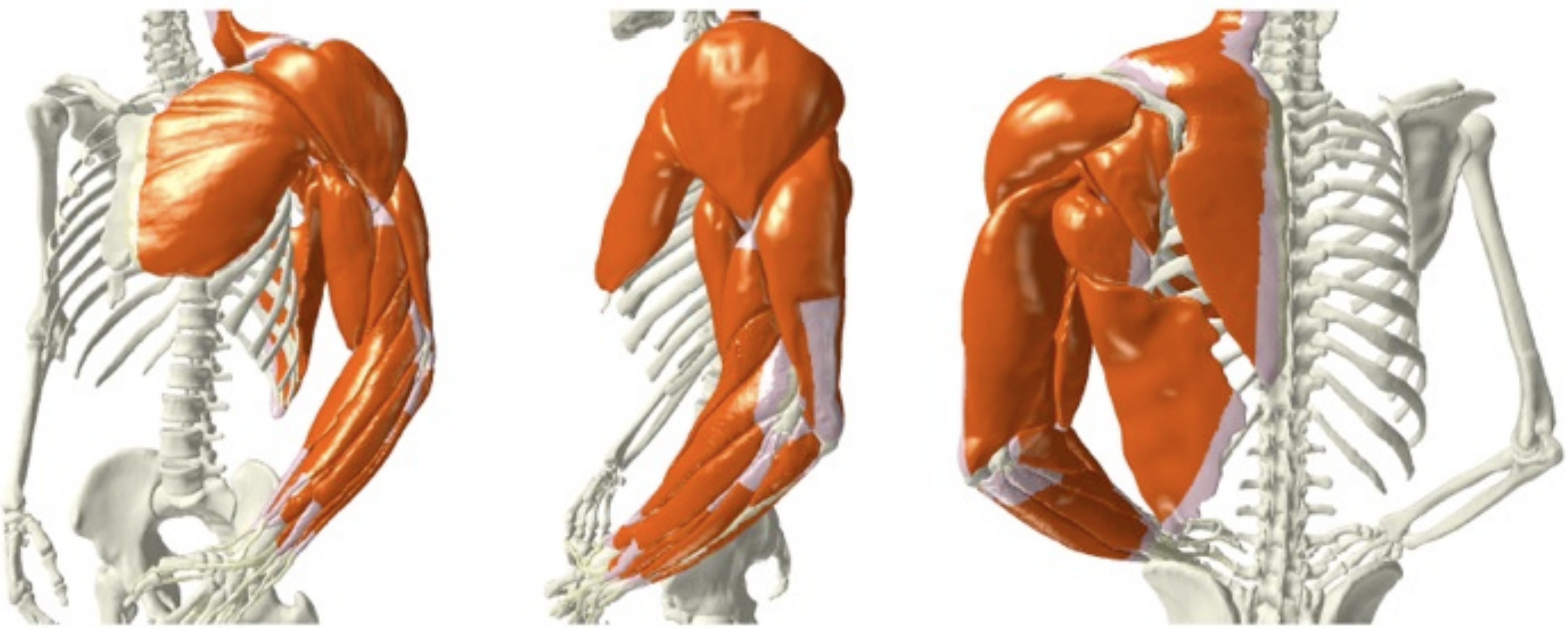
Multimaterial Data

Realistic physically-based simulations



Teran *et al.* 2005

Data from volumetric scanning devices

Challenges

Simulations require efficient sets of nearly-regular elements that accurately capture geometry and topology

Boundaries between materials are typically not smooth manifolds

Materials represented as discrete labels

Previous Work

Grid-based multimaterial

[Neilson97] [Bonnell03] [Bertram05] [Dillard07] [Zhang07] ...

High-quality, single material

[Crossno97] [Amenta98] [Dey03] [Schreiner06] [Meyer07] ...

Delaunay refinement

[Boissonnat05] [Oudot05] [Pons07] ...

Approach

Generate a continuous representation of the multimaterial volume

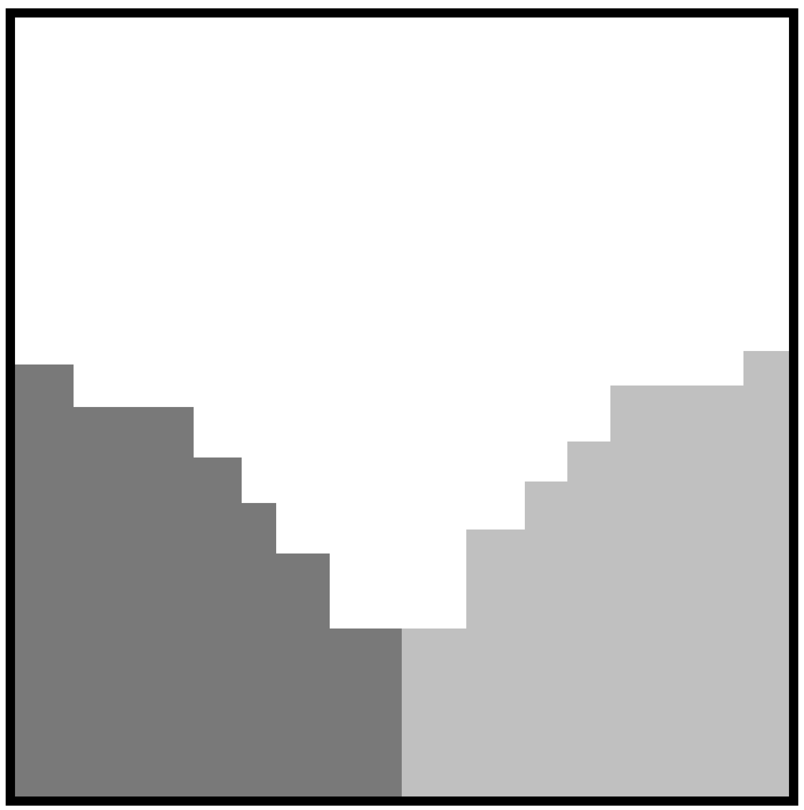
Describe material junctions analytically

Define projection operators

Sample junctions with sets of particles in an ordered-scheme

Mesh particles with a Delaunay-based labeling algorithm

Multimaterial Data



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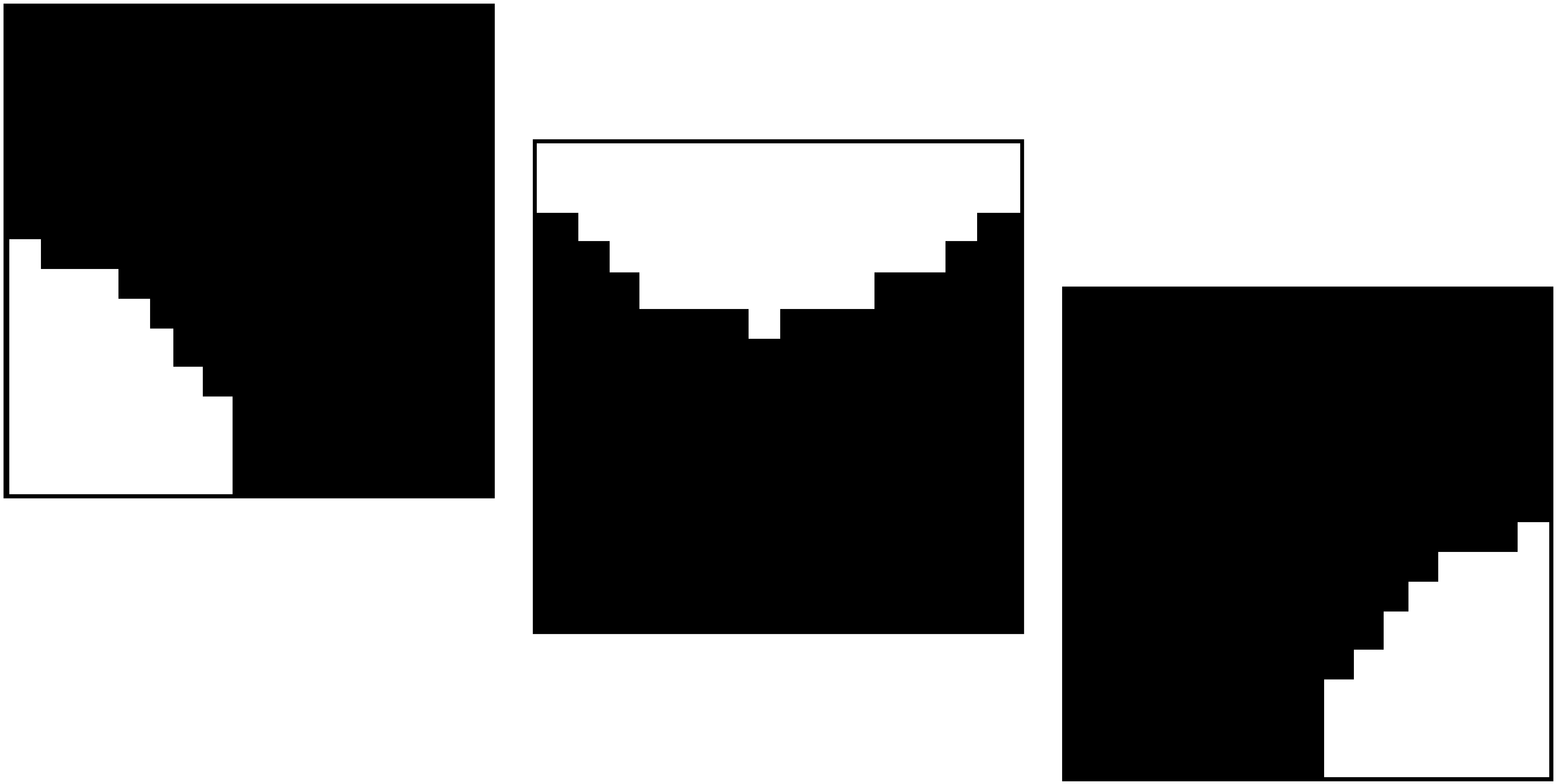
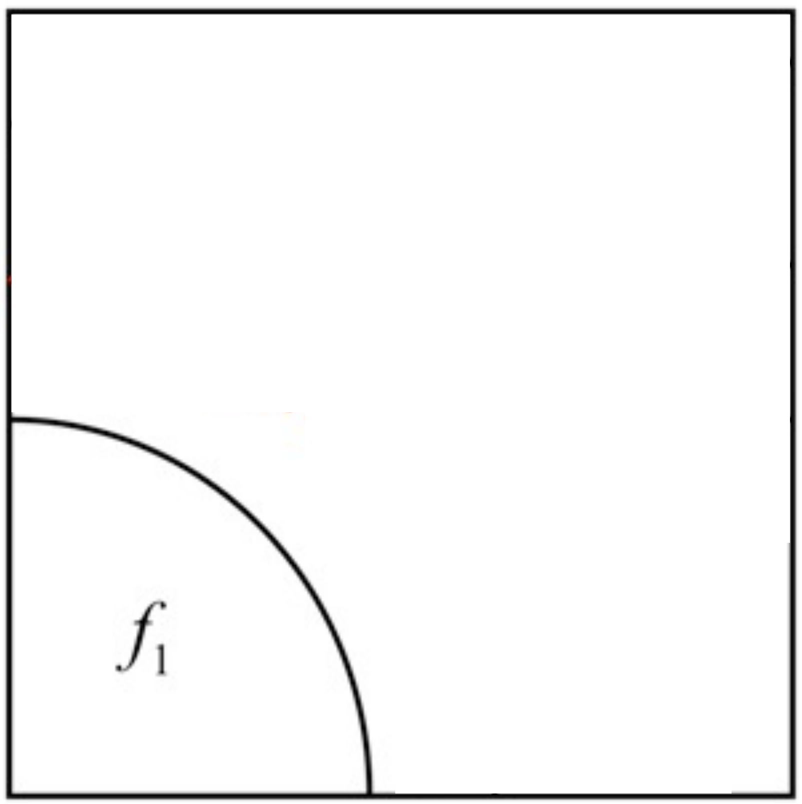
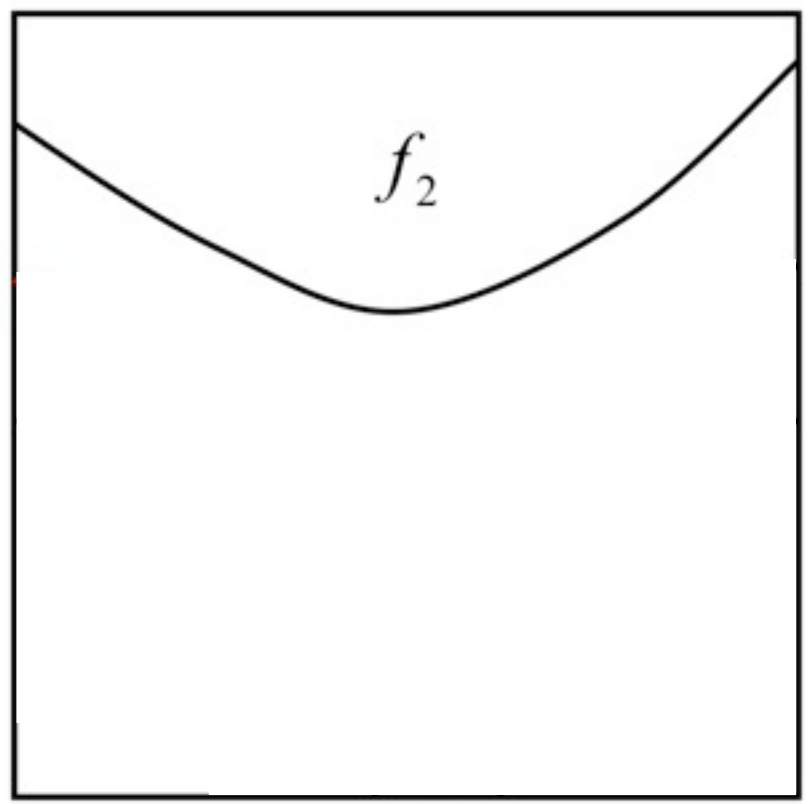
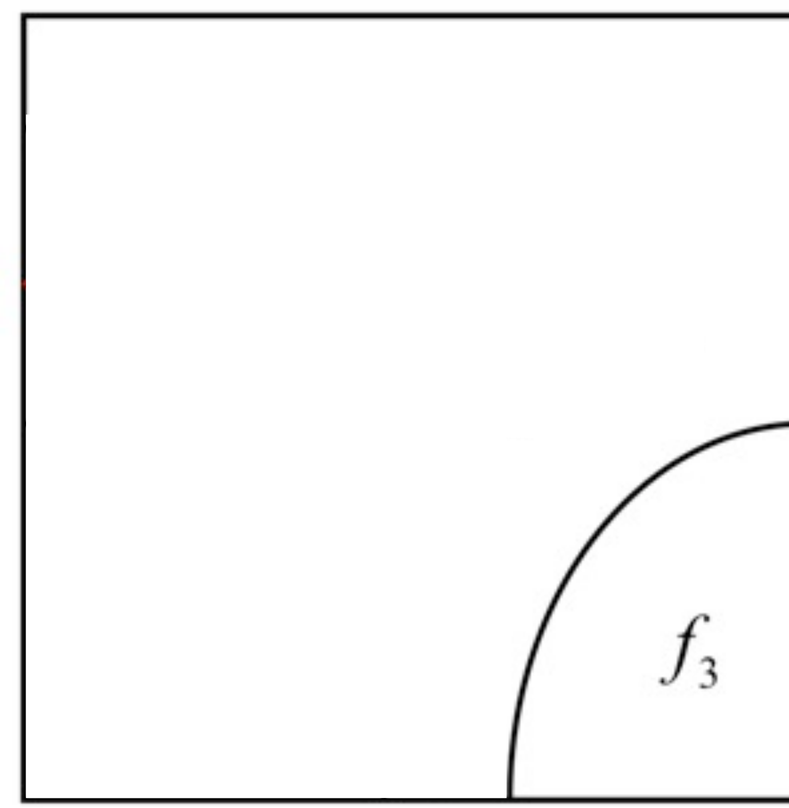


Multimaterial Data



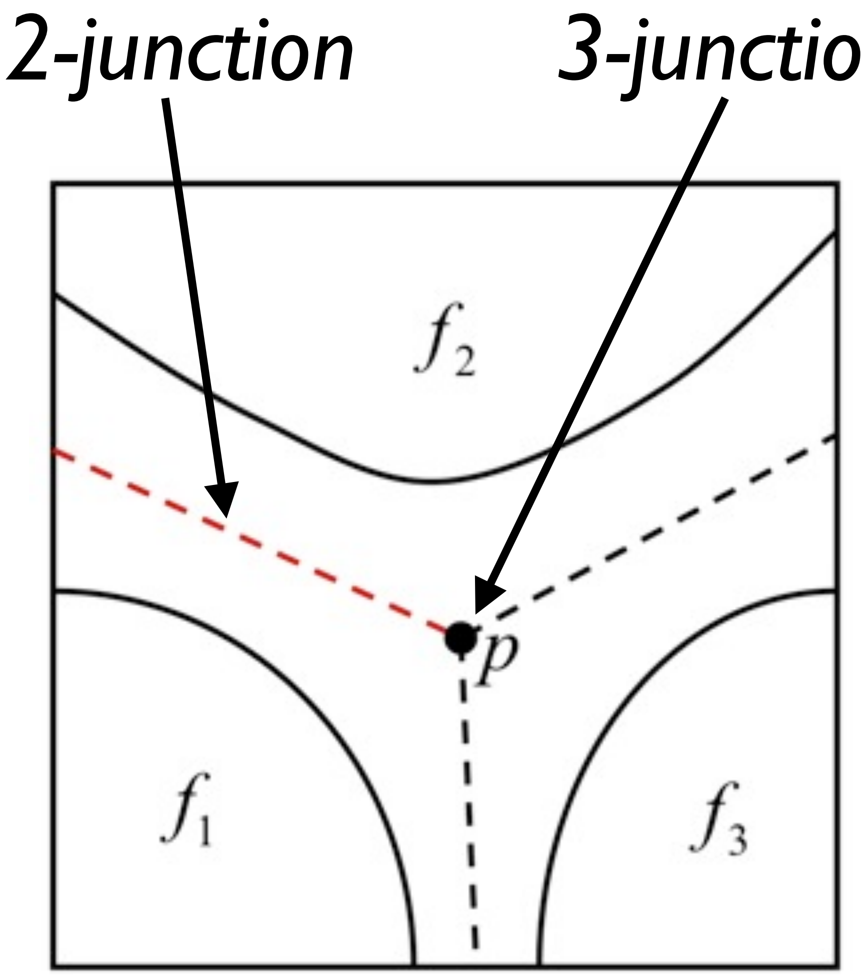




Tightening

Williams *et al.* 2005

Multimaterial Model



Multimaterial Representation

$$F = \{f_i | f_i : V \mapsto \Re\}$$

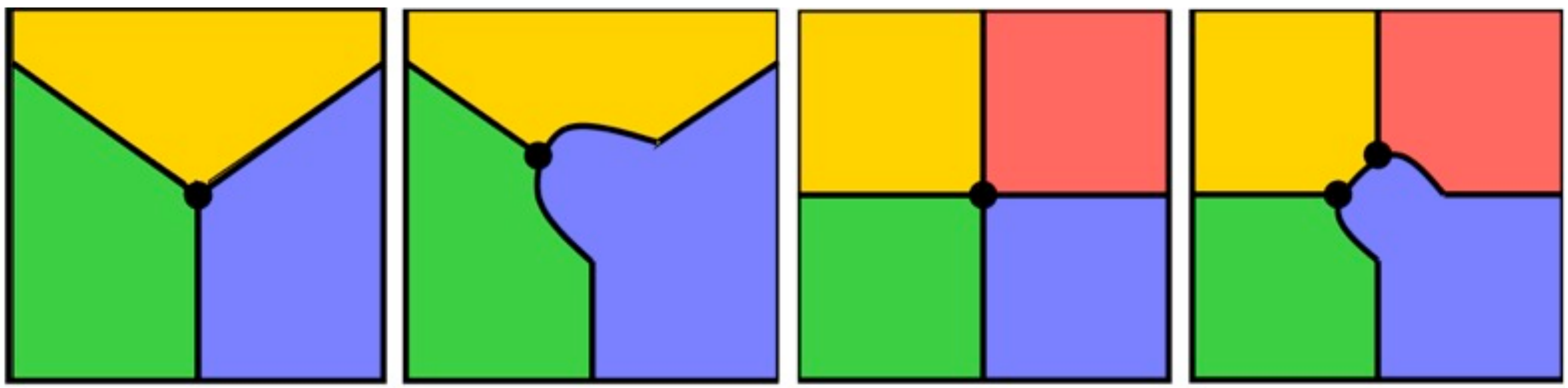
$$x \in V \rightarrow i$$

$$f_i(x) > f_j(x) \;\; \forall \;\; j \neq i$$

Intersections occur at maximal material transitions

Multimaterial Topology

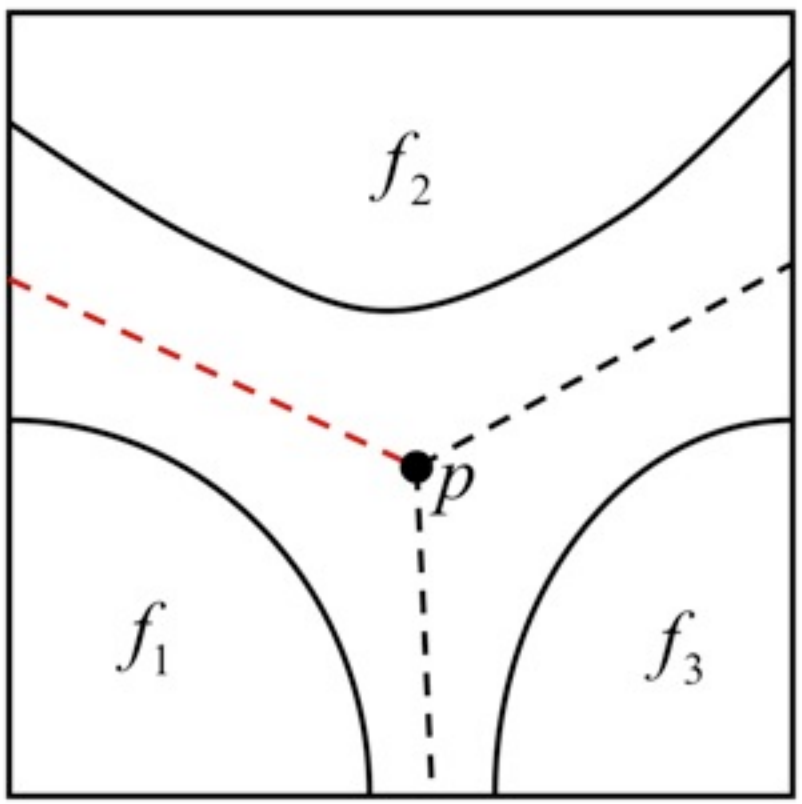
General Position and Generic Junctions



(a) (b) (c) (d)

in $\Re^3$ $\begin{cases} \textit{2-junctions}\text{: sheets} \\ \textit{3-junctions}\text{: edges} \\ \textit{4-junctions}\text{: points} \end{cases}$

Junction Indicator Functions



Multimaterial Representation

$$\tilde{f}_i = f_i - \max_{j=1, j \neq i}^{n} f_j$$

$$J_{ij} = \tilde{f}_i^2 + \tilde{f}_j^2$$

$$J_{ijk} = \tilde{f}_i^2 + \tilde{f}_j^2 + \tilde{f}_k^2$$

$$J_{ijkl} = \tilde{f}_i^2 + \tilde{f}_j^2 + \tilde{f}_k^2 + \tilde{f}_l^2$$

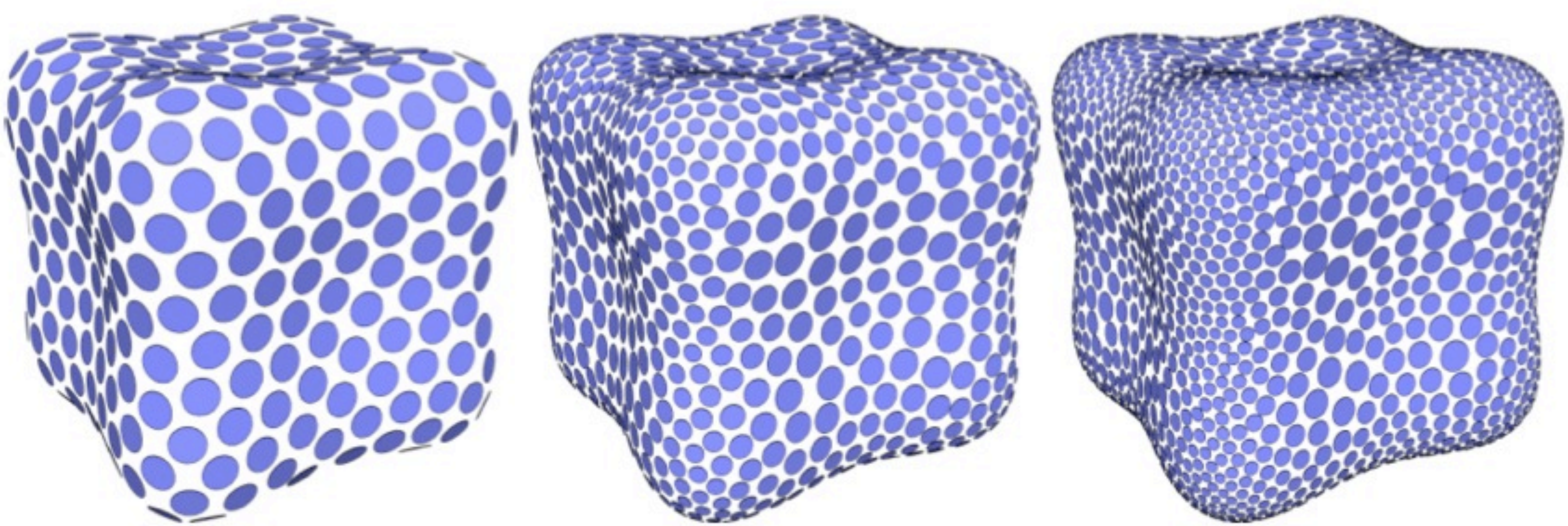
Particle System Basics

Project particles onto surface

Energy kernels induce interparticle forces

Steady state when all forces are equalized

Adaptive distributions by scaling distances



Sampling Junctions

with particles

Continuous approximation to *max()*

$$\tilde{f}_i = f_i - \max_{j=1, j \neq i}^{n} f_j$$

Sampling Junctions
with particles

Continuous approximation to *max()*

Projection operators

$$J_{ij} = \tilde{f}_i^2 + \tilde{f}_j^2$$

$$J_{ijk} = \tilde{f}_i^2 + \tilde{f}_j^2 + \tilde{f}_k^2$$

$$J_{ijkl} = \tilde{f}_i^2 + \tilde{f}_j^2 + \tilde{f}_k^2 + \tilde{f}_l^2$$

Sampling Junctions
with particles

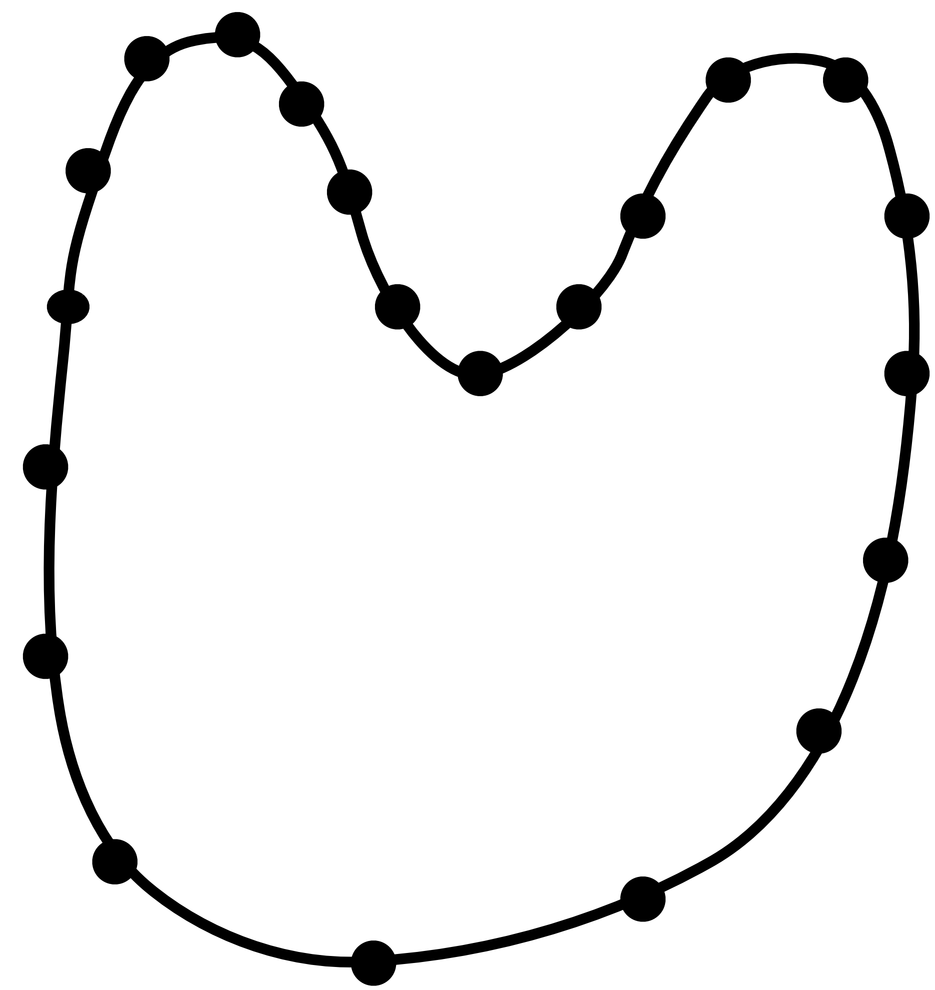
Continuous approximation to *max()*

Projection operators

Ordered-sampling scheme

Labeling Algorithm

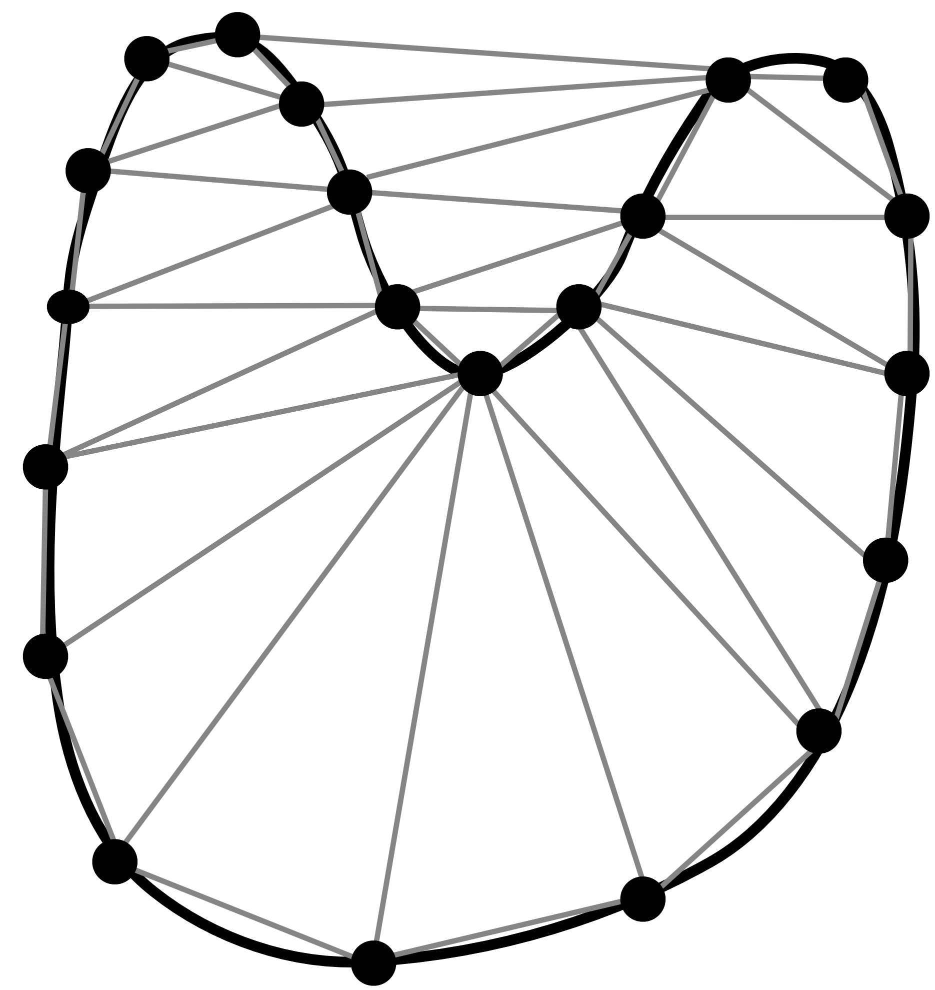
Oudot *et al.* 2005



Labeling Algorithm

Oudot *et al.* 2005

Delaunay triangulation

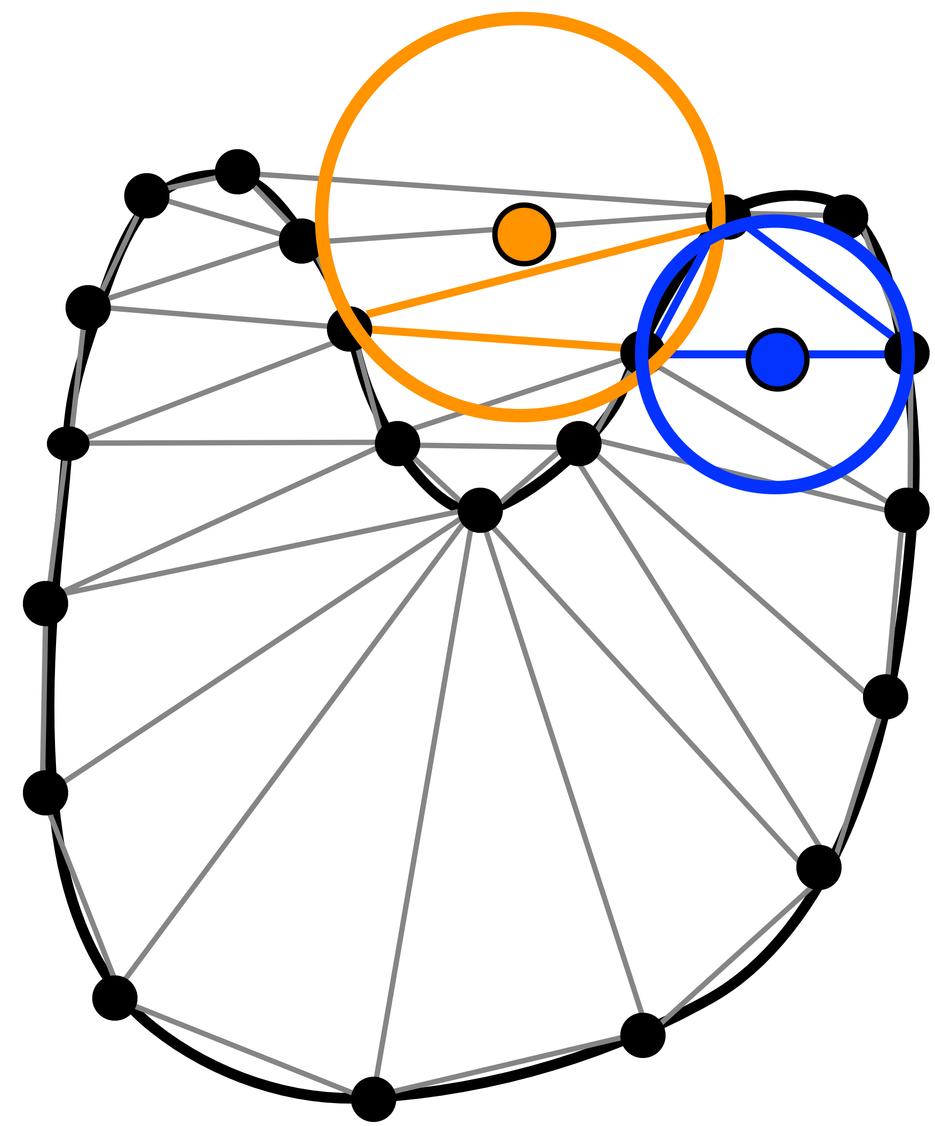


Labeling Algorithm

Oudot *et al.* 2005

Delaunay triangulation

Compute circumcircle centers



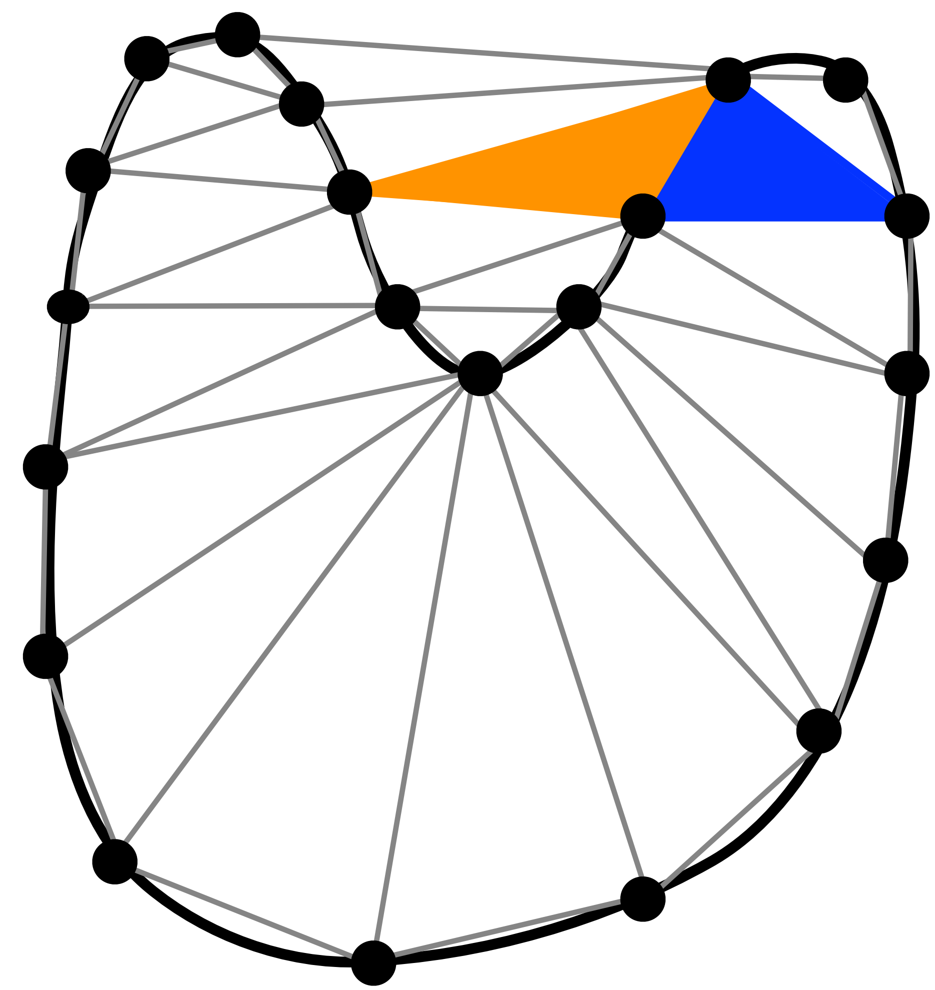
Labeling Algorithm

Oudot *et al.* 2005

Delaunay triangulation

Compute circumcircle centers

Label triangles



Labeling Algorithm

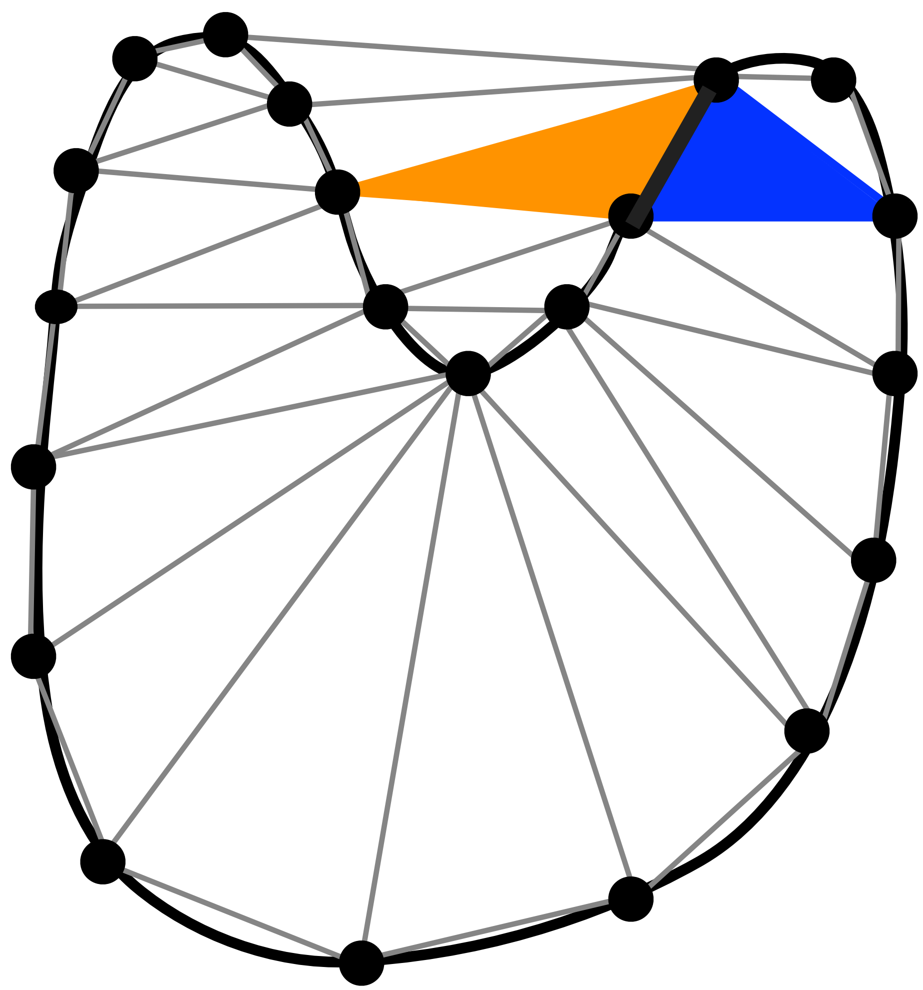
Oudot *et al.* 2005

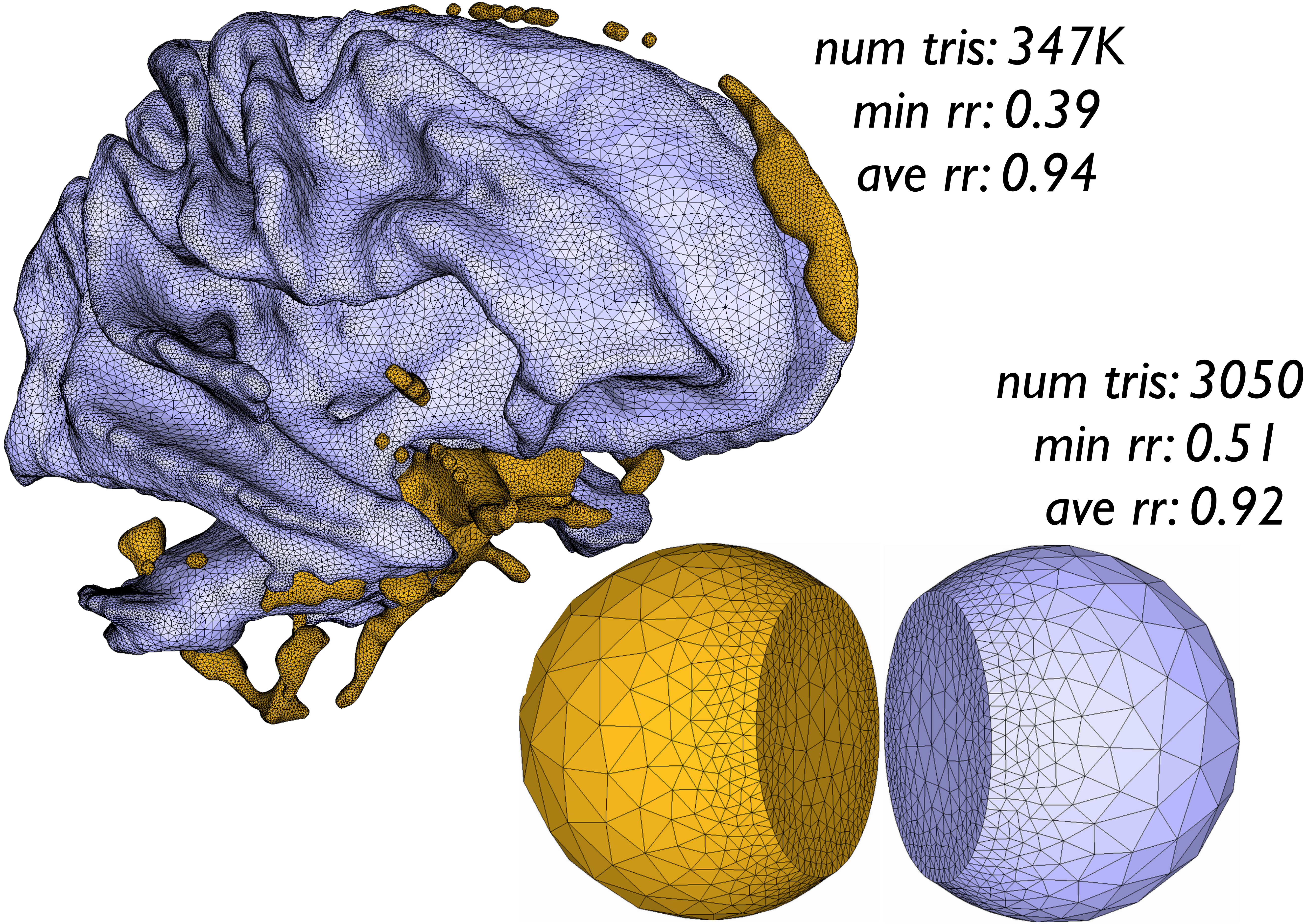
Delaunay triangulation

Compute circumcircle centers

Label triangles

Extract edges

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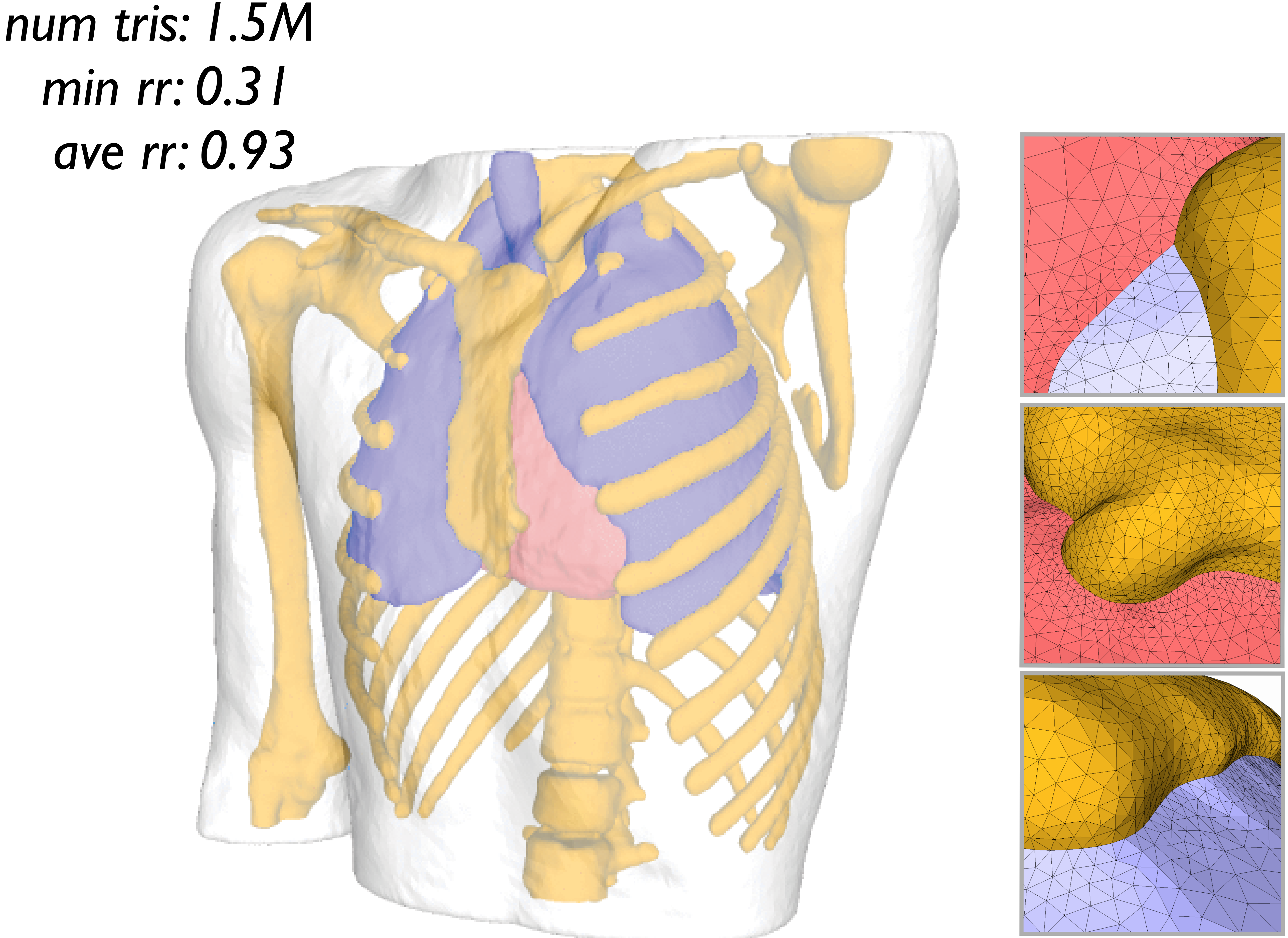
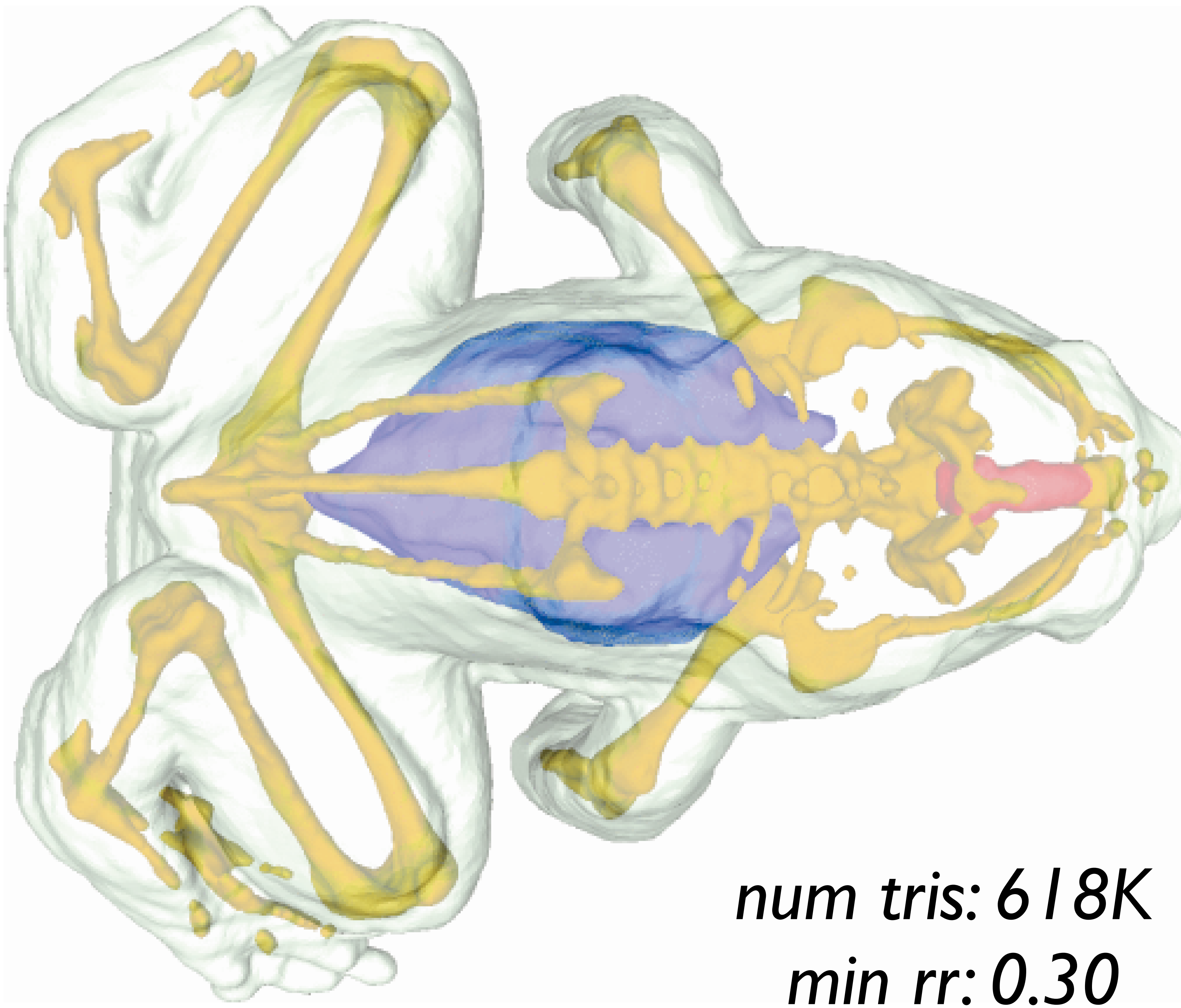
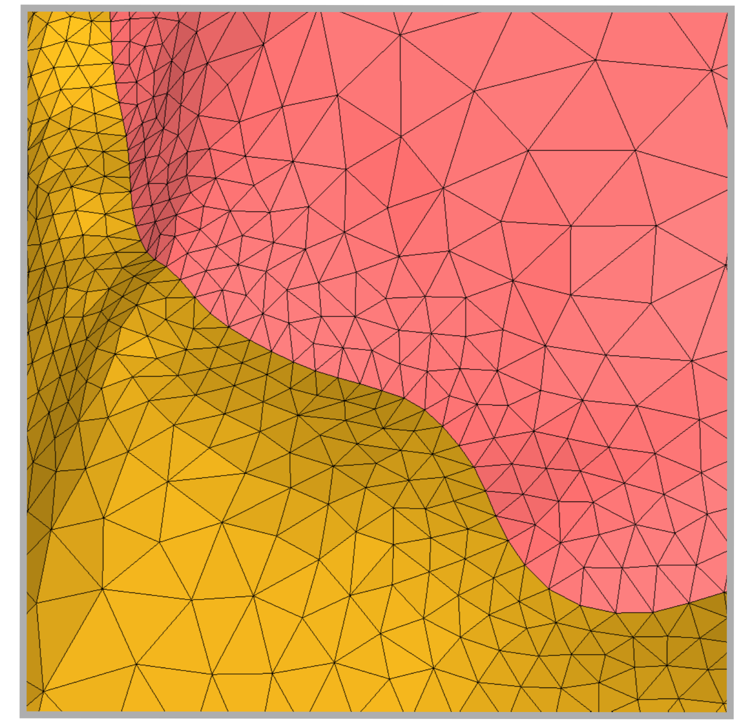
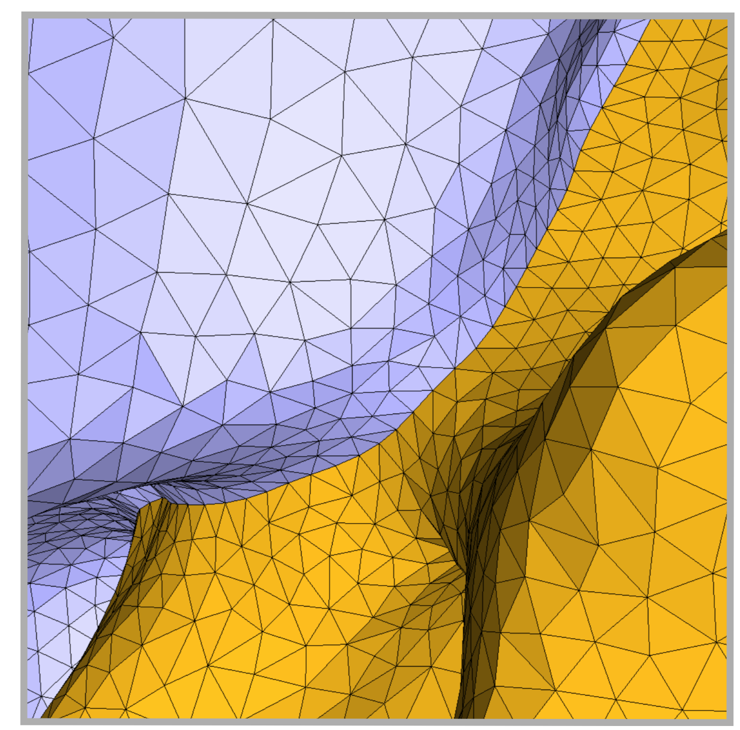
min rr: 0.39
ave rr: 0.94
num tris: 3050
min rr: 0.51
ave rr: 0.92


num tris: 1.5M
min rr: 0.31
ave rr: 0.93

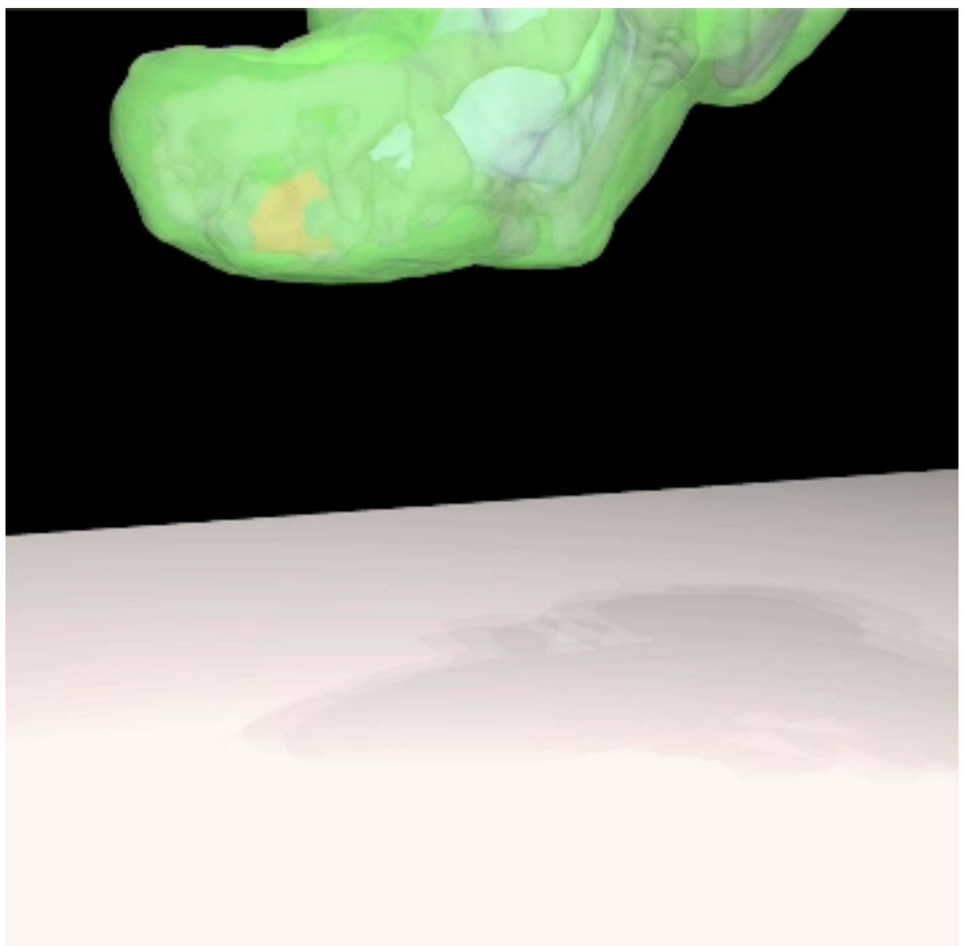




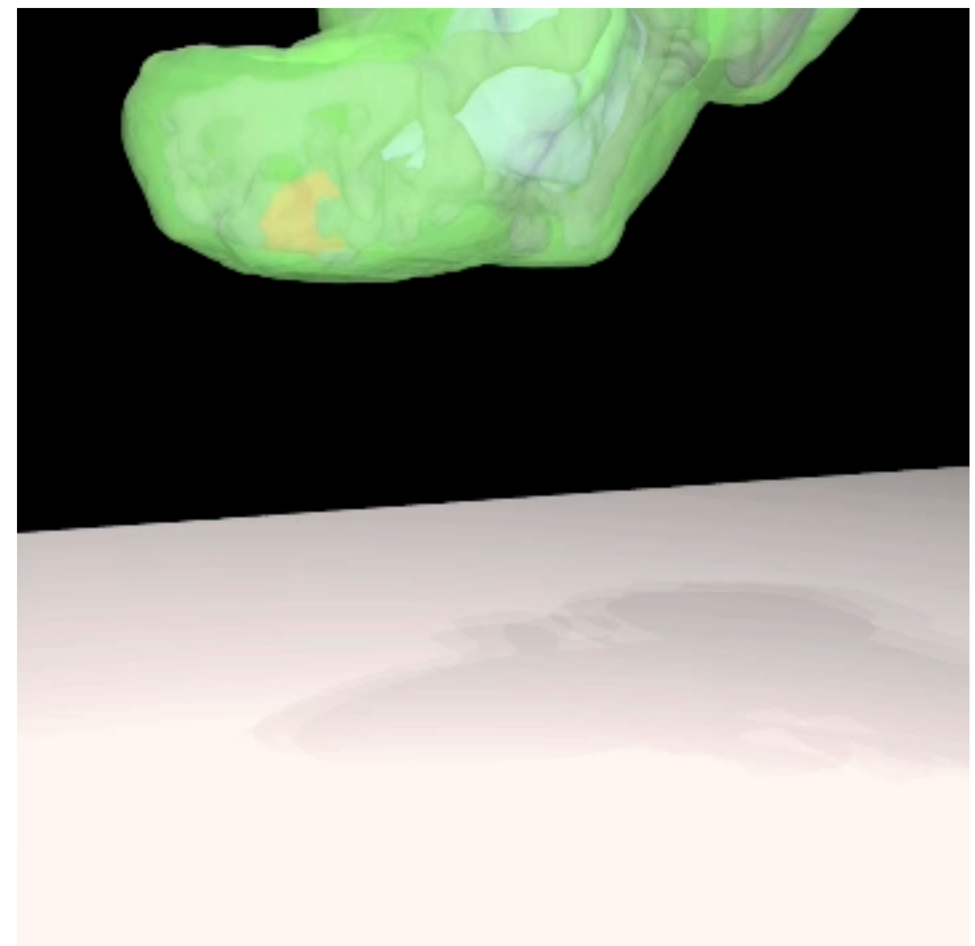


num tris: 618K
min rr: 0.30
ave rr: 0.94

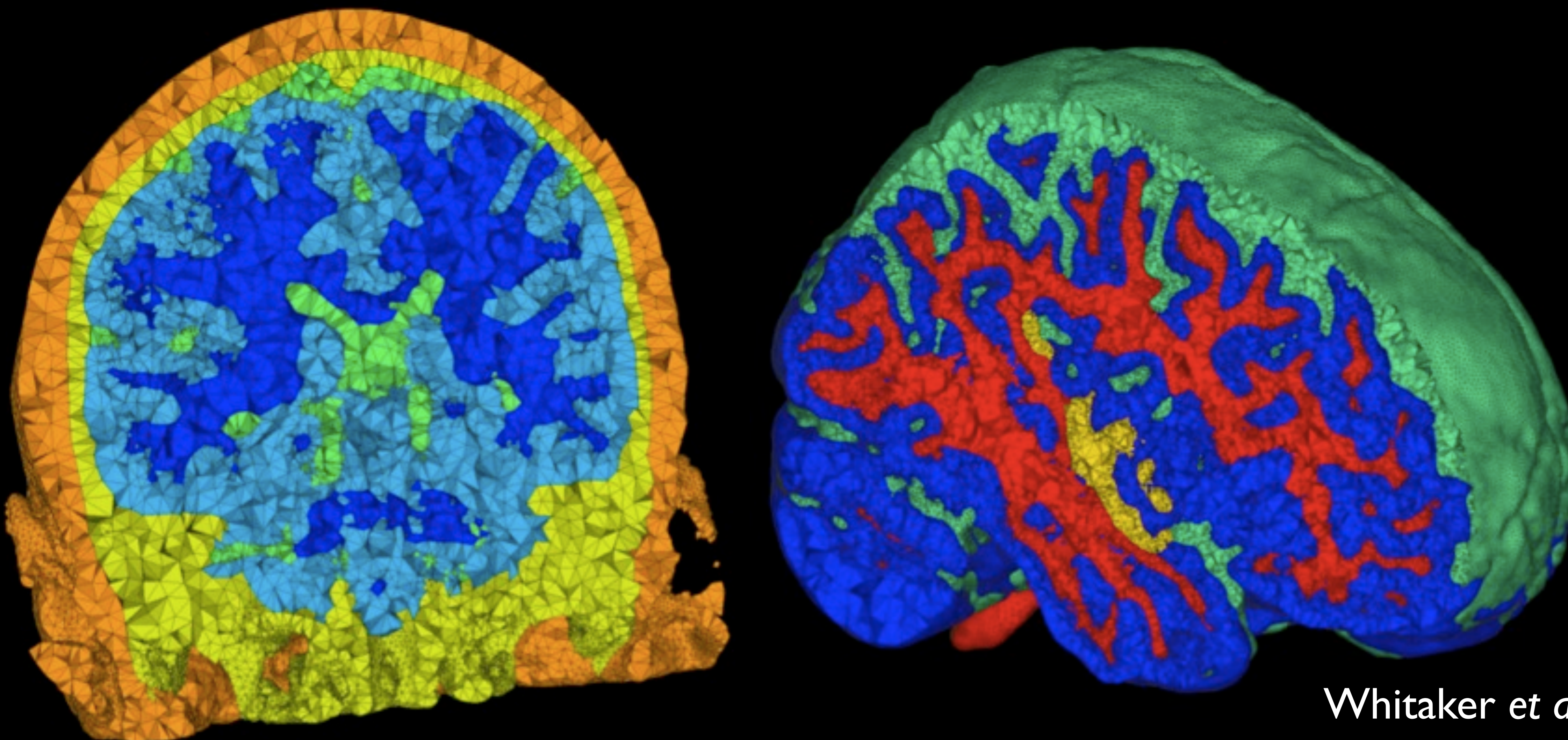
Single Material



Multiple Materials



Current & Future Work

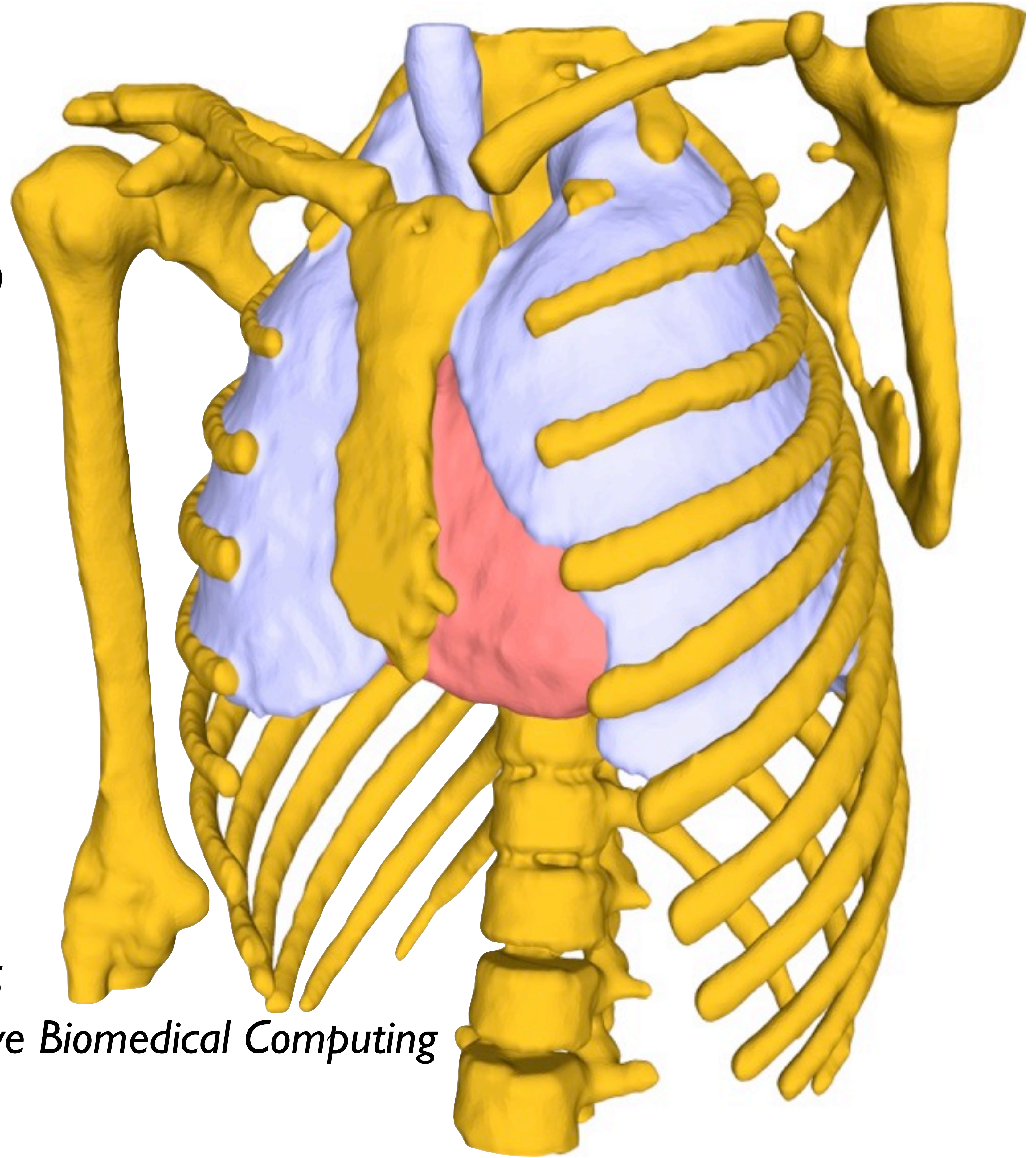


Whitaker *et al.* 2008

Investigate tetrahedralization algorithms

Optimize and parallelize pipeline

Questions?



Acknowledgements:

Claudio Silva
Matthew Jolley
Tolga Tasdizen

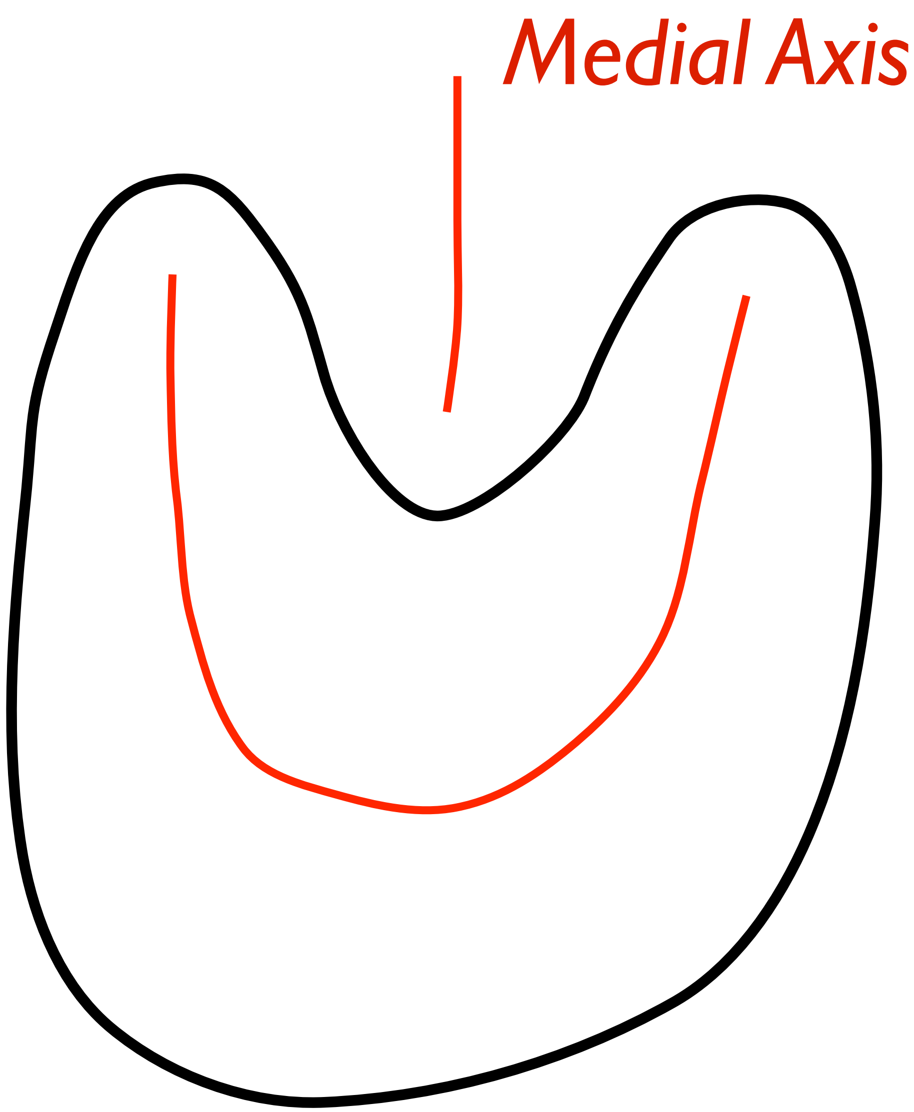
ARO grant W911NF-05-1-0395
NIH/NCRR Center for Integrative Biomedical Computing
P41-RR12533-07
NSF CNS 0551724

Dense sampling of points → Topologically equivalent surface mesh

Given a smooth surface $F \subset \Re^3$, a sufficiently dense sampling P is one such that for any point $s \in F$ the Euclidean distance between s and the closest sample point $p \in P$ is no greater than ε times the local feature size at s.

Amenta, Bern, and Eppstein 1999

Dense sampling of points → Topologically equivalent surface mesh



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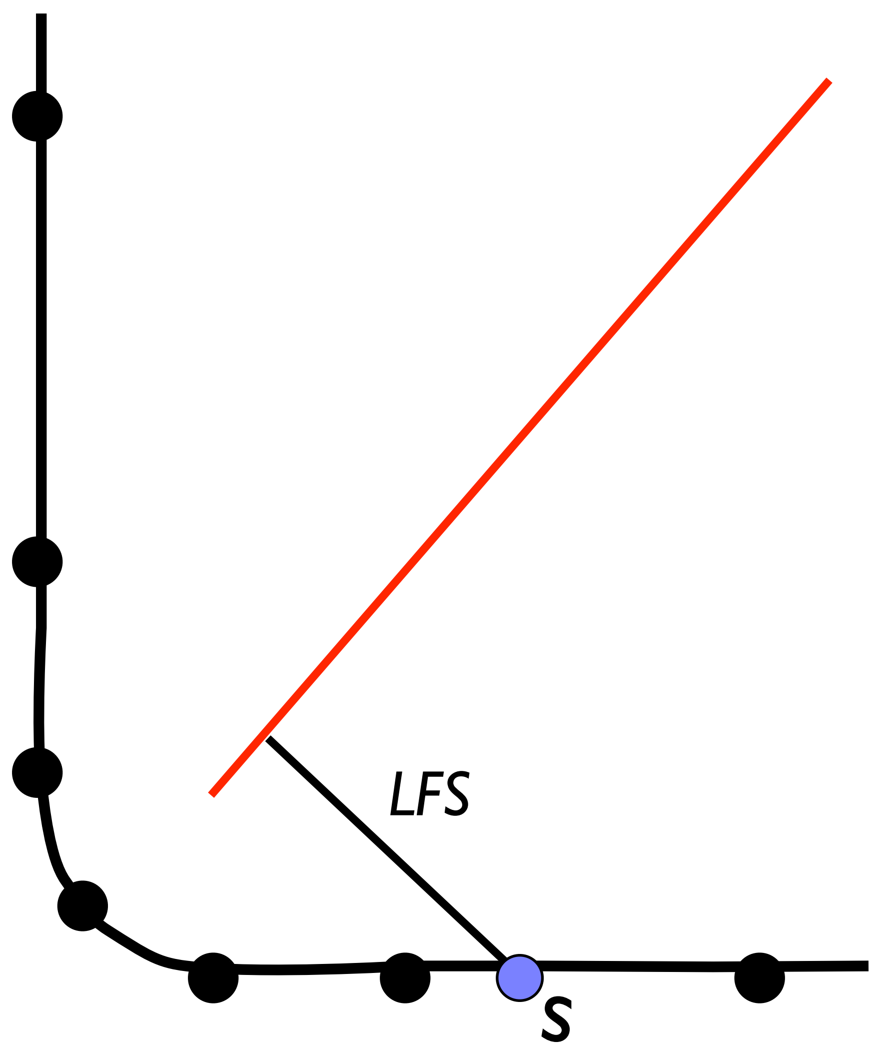

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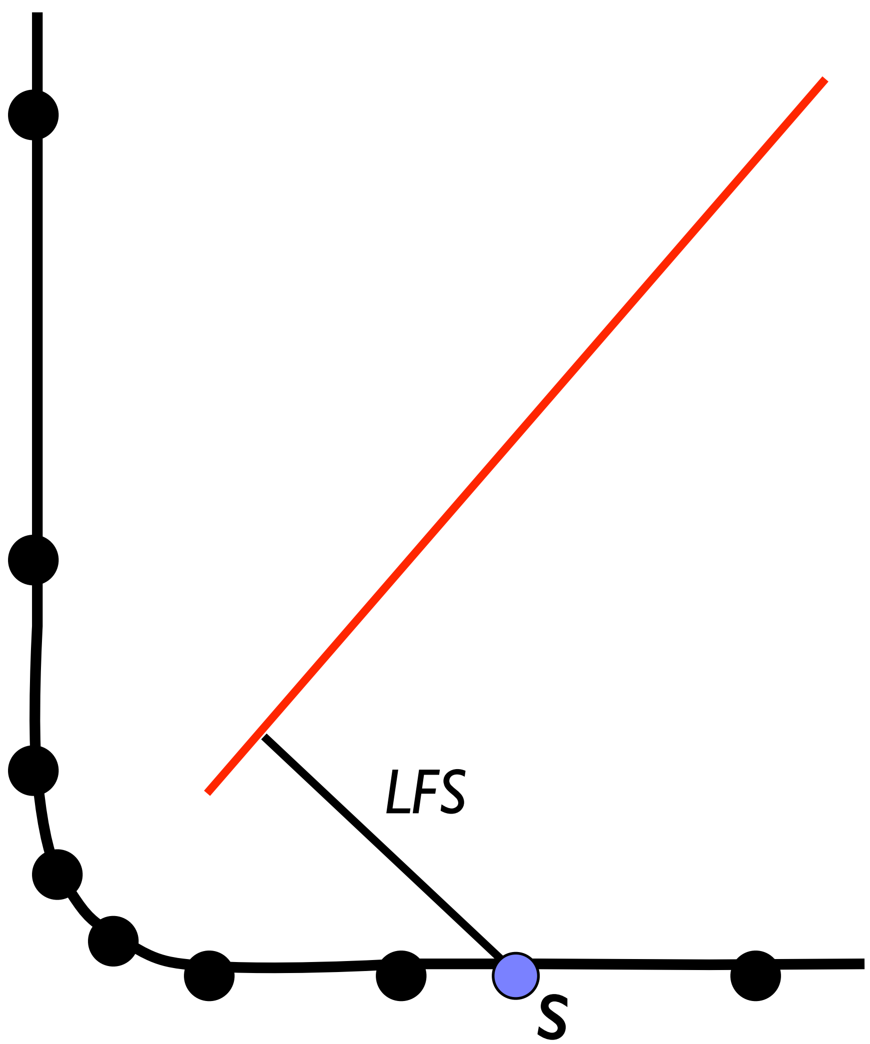
Sampling Requirements



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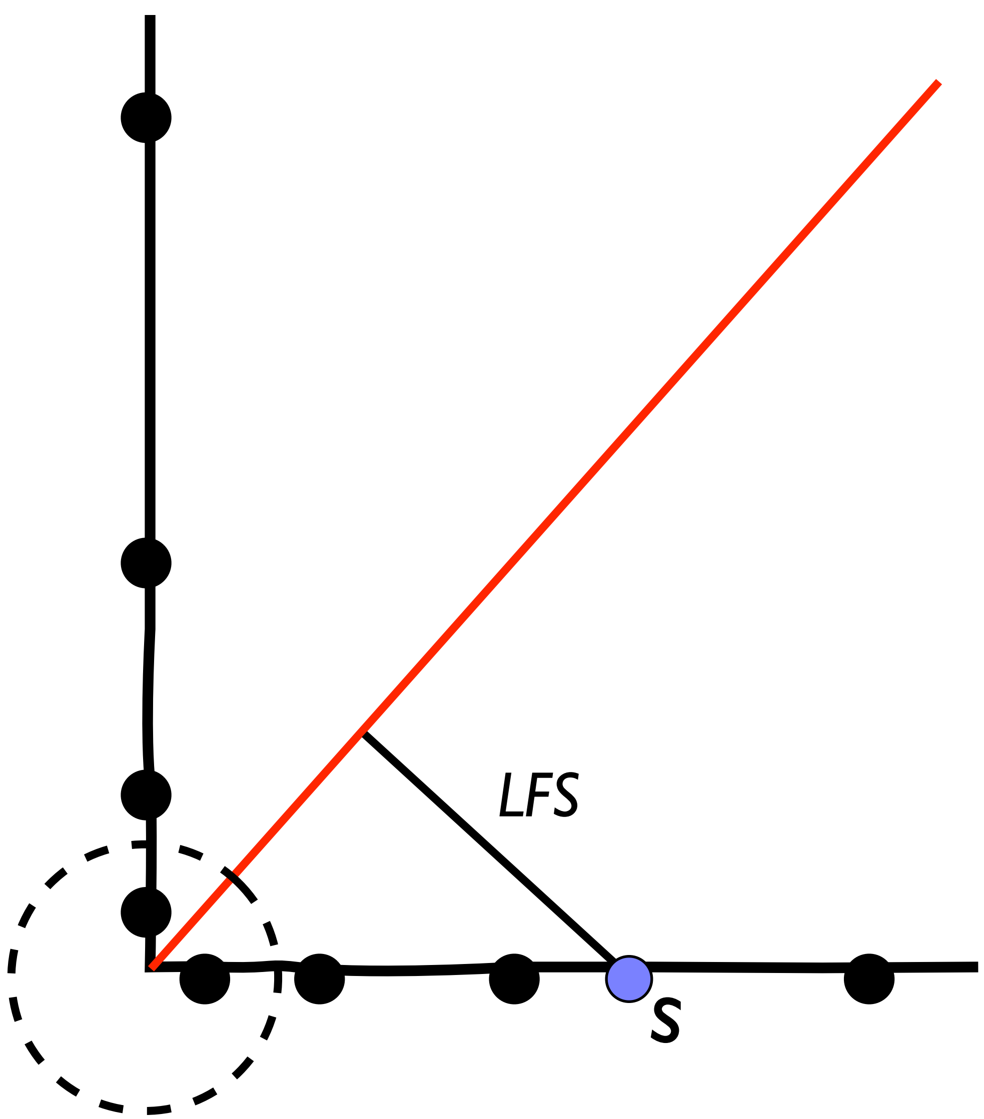

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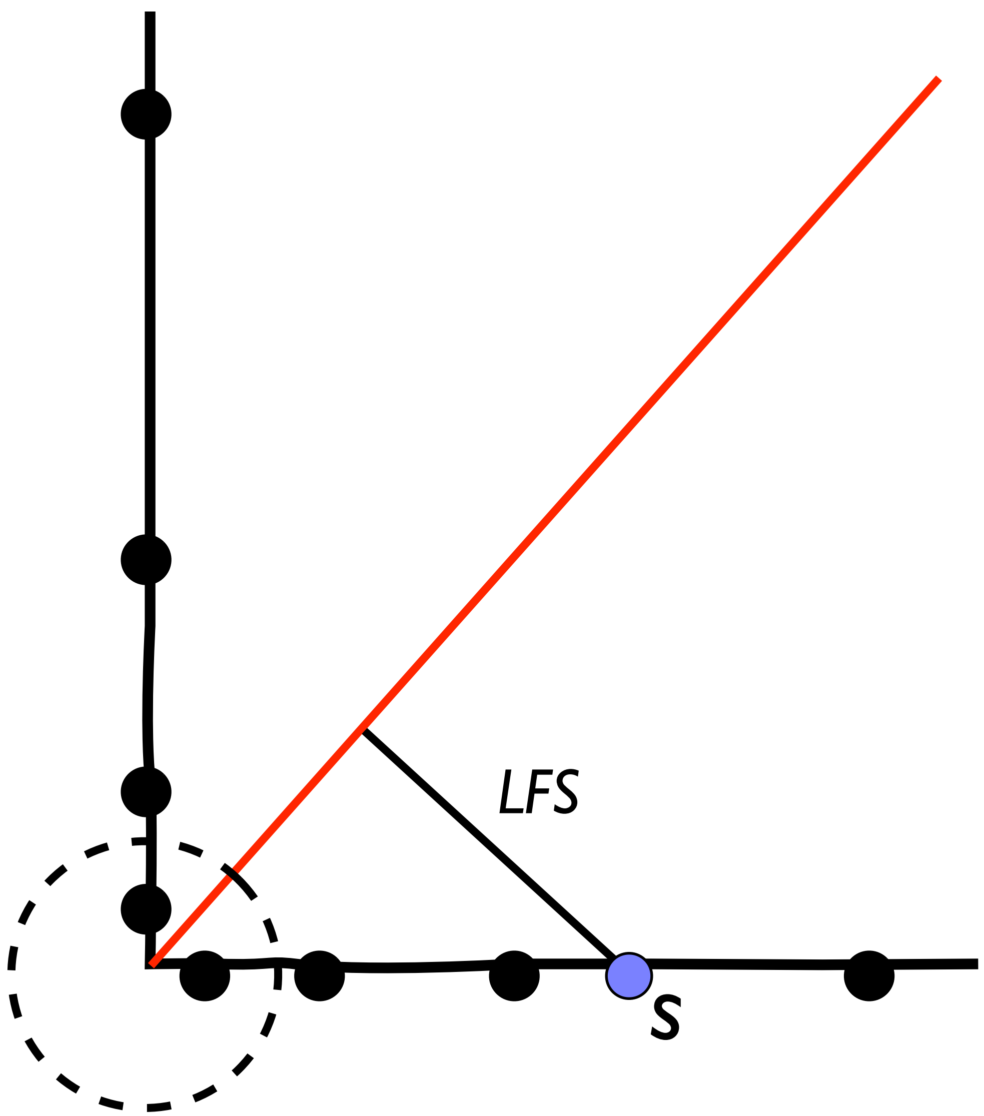

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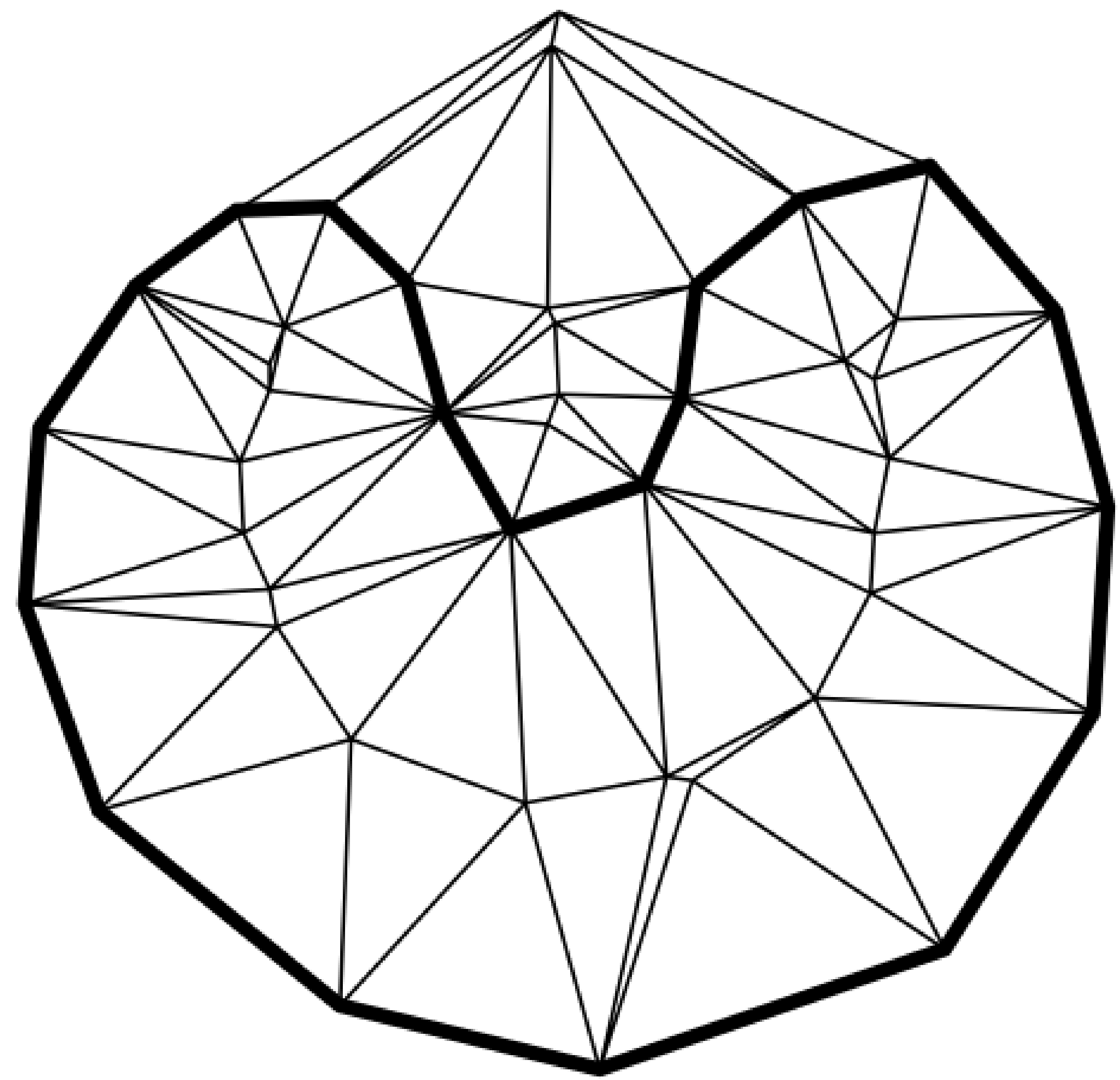
Amenta, Bern, and Eppstein 1999

can use **Crust** for angles > 45°

Crust Algorithm

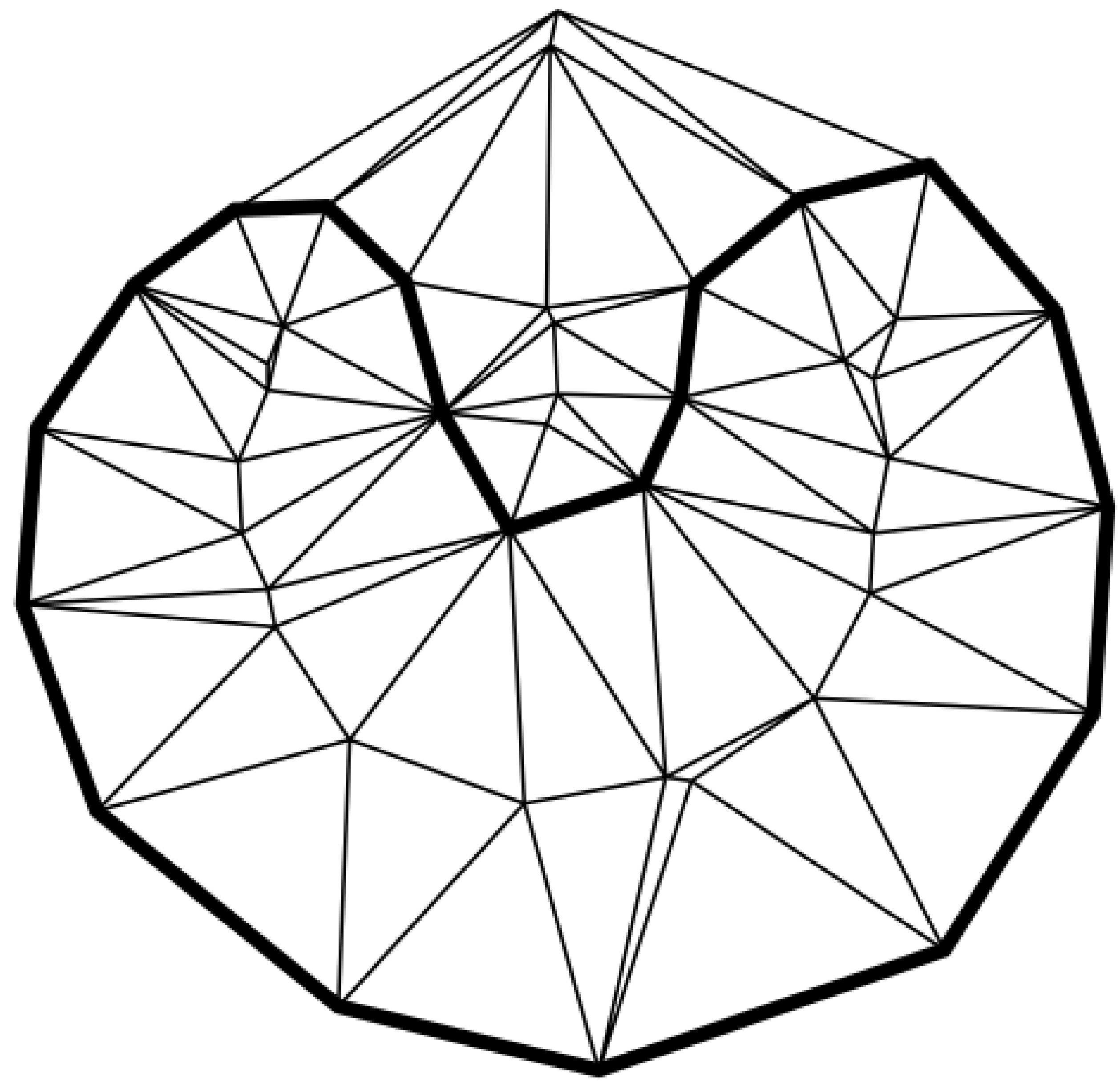
Amenta *et al.* 1998



Crust Algorithm

Amenta *et al.* 1998

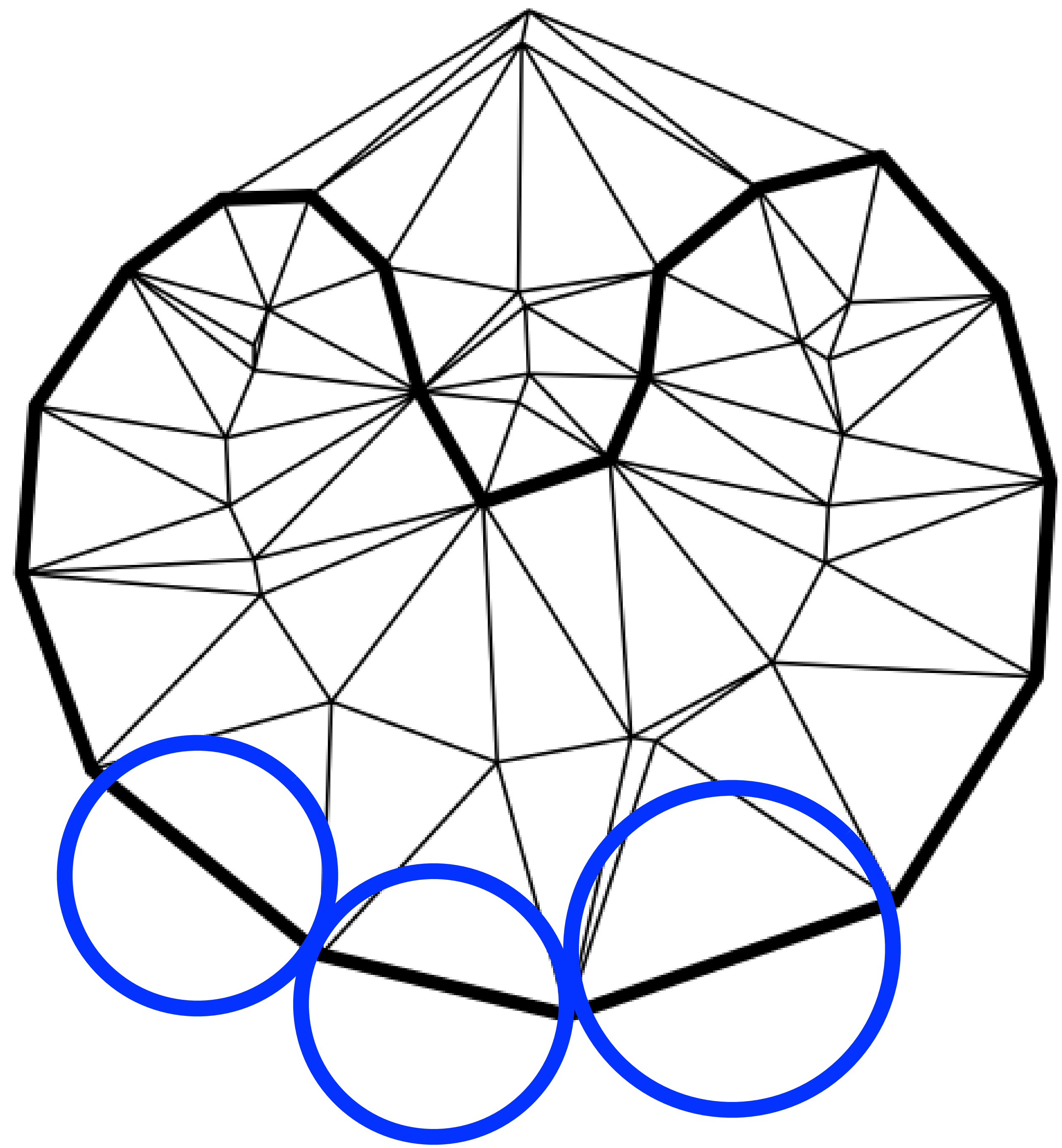
- Delaunay triangulation of P∪V



Crust Algorithm

Amenta *et al.* 1998

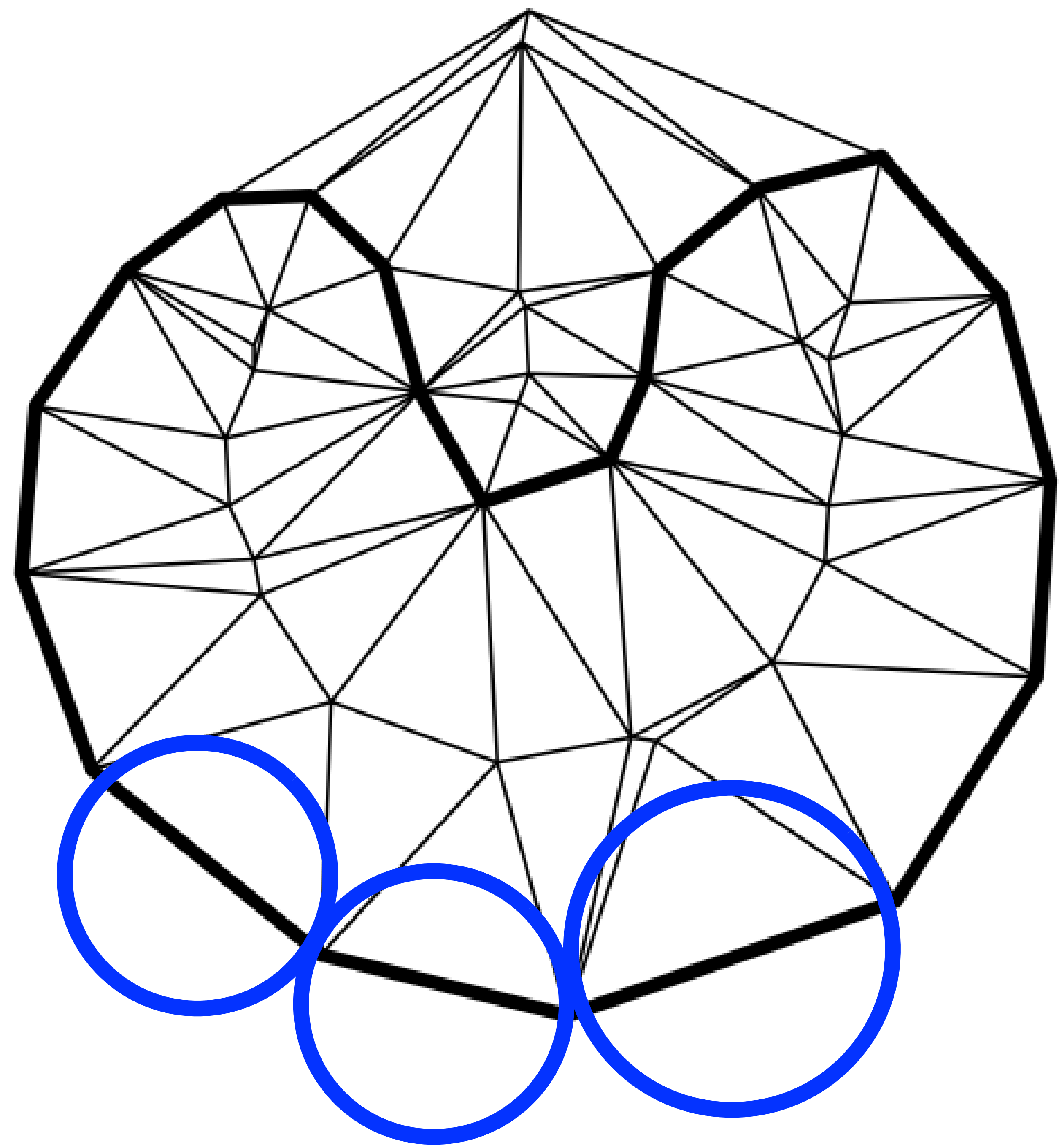
- Delaunay triangulation of P∪V
- Surface edges are Delaunay

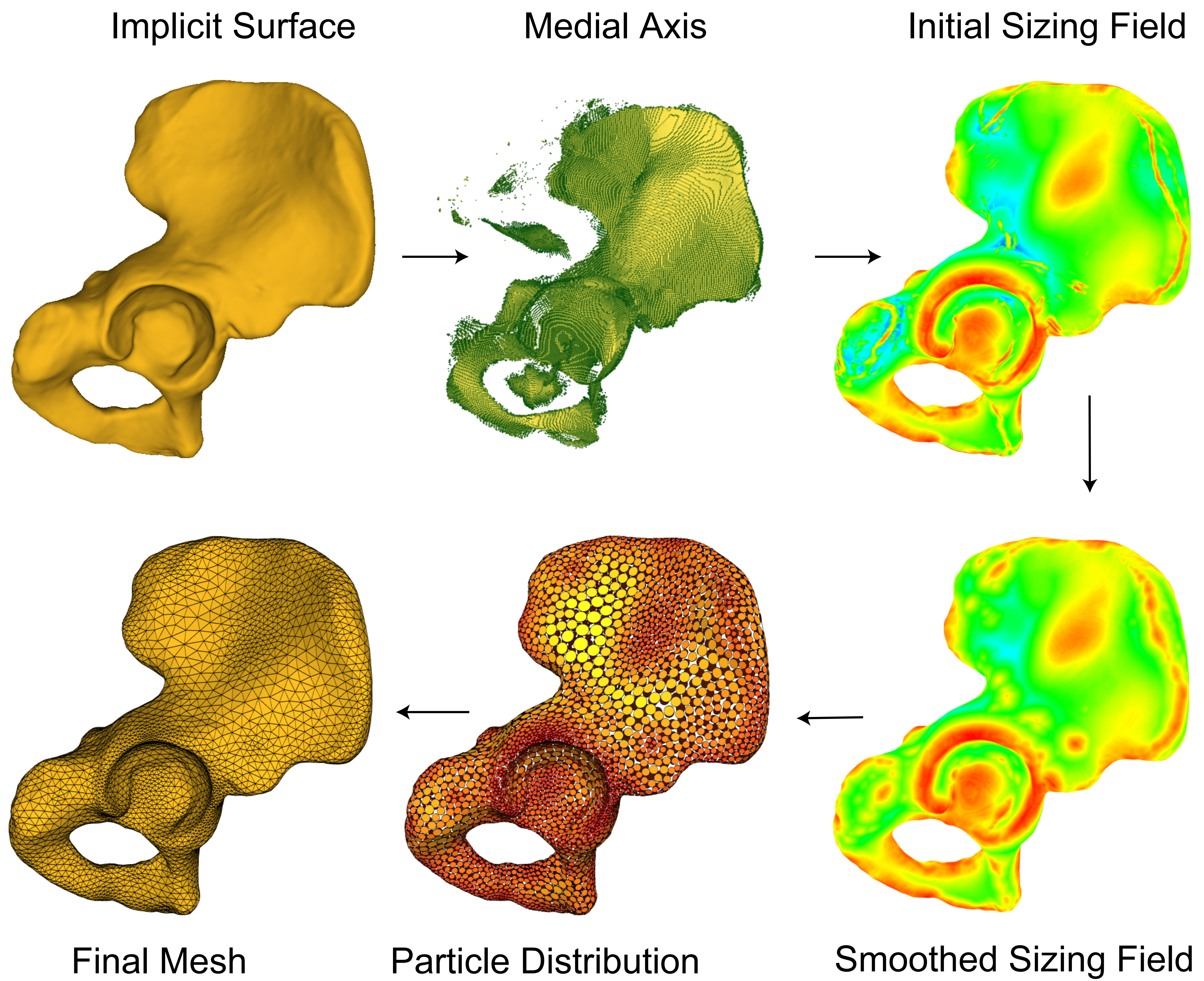


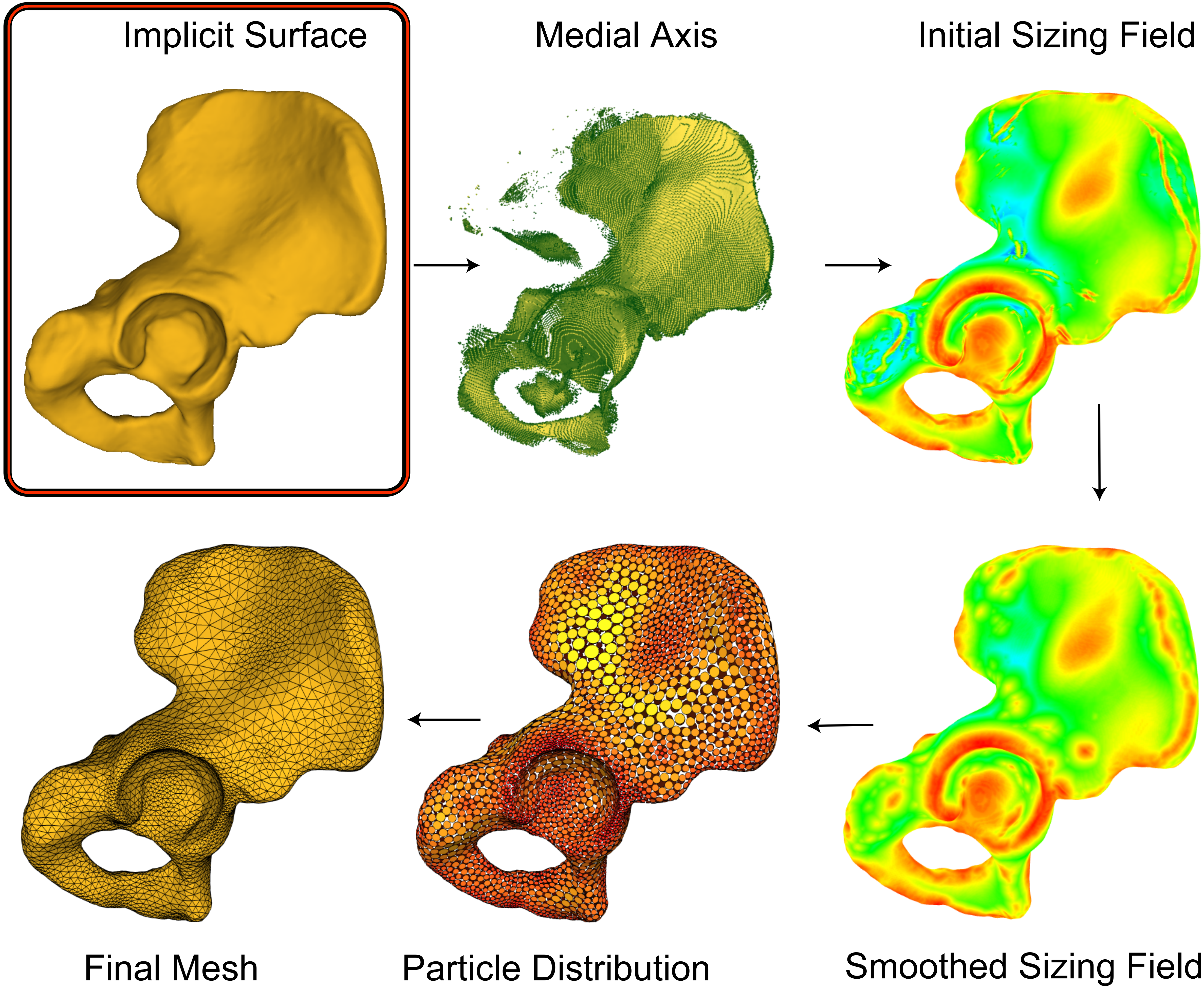
Crust Algorithm

Amenta *et al.* 1998

- Delaunay triangulation of P∪V
- Surface edges are Delaunay
- **Crust**: edges connecting surface samples




Implicit Surface
Medial Axis
Initial Sizing Field
Smoothed Sizing Field
Particle Distribution
Final Mesh



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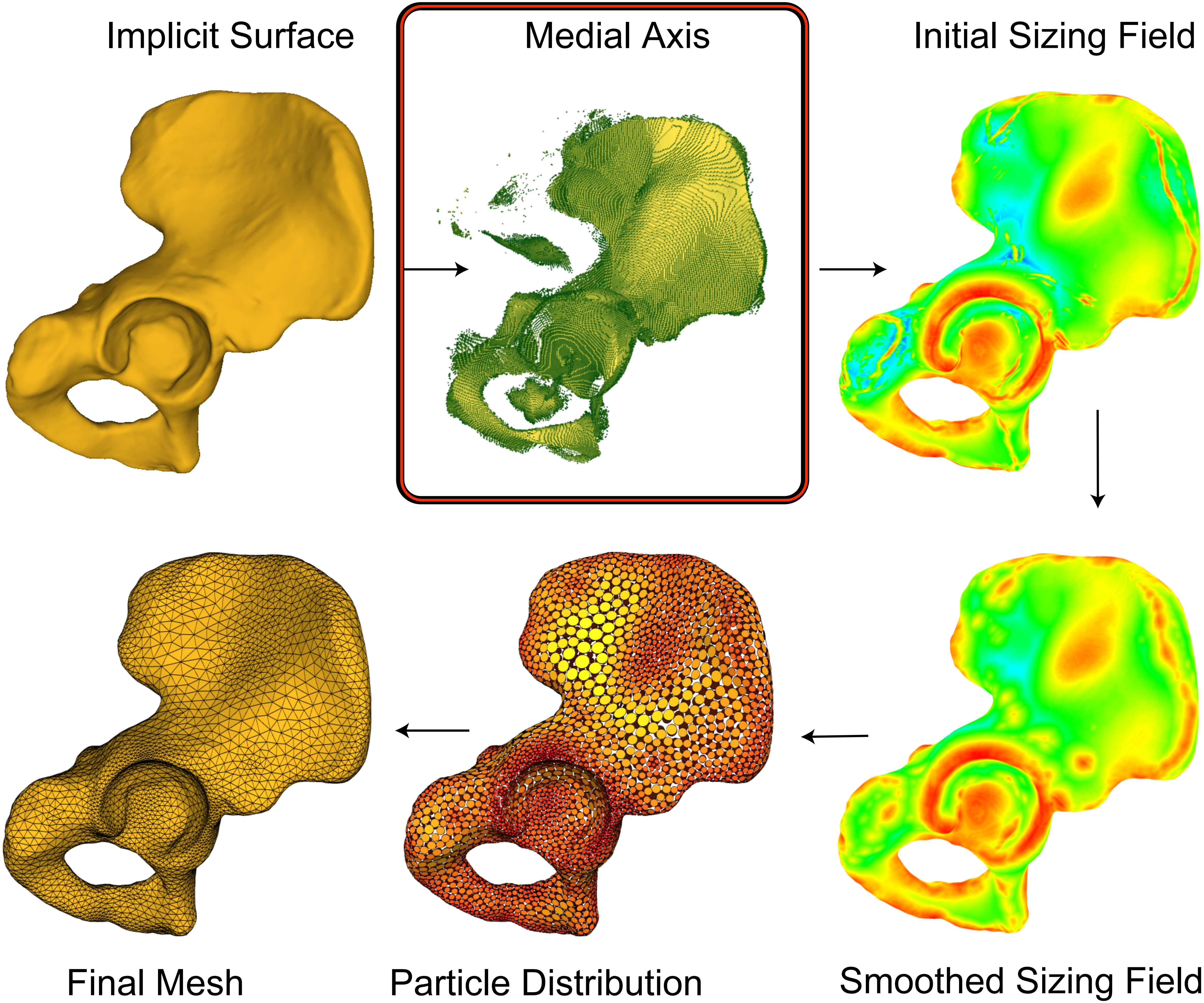
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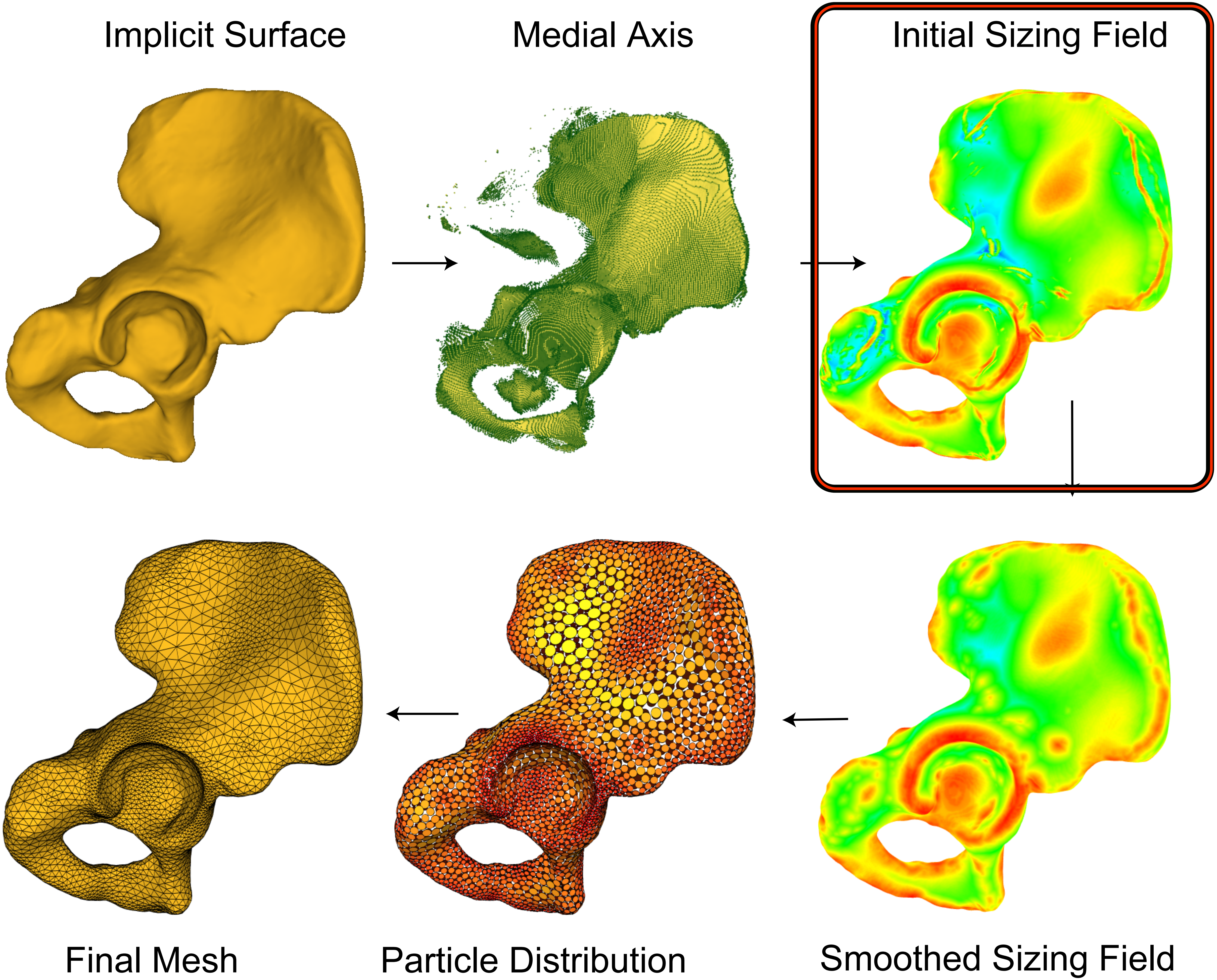

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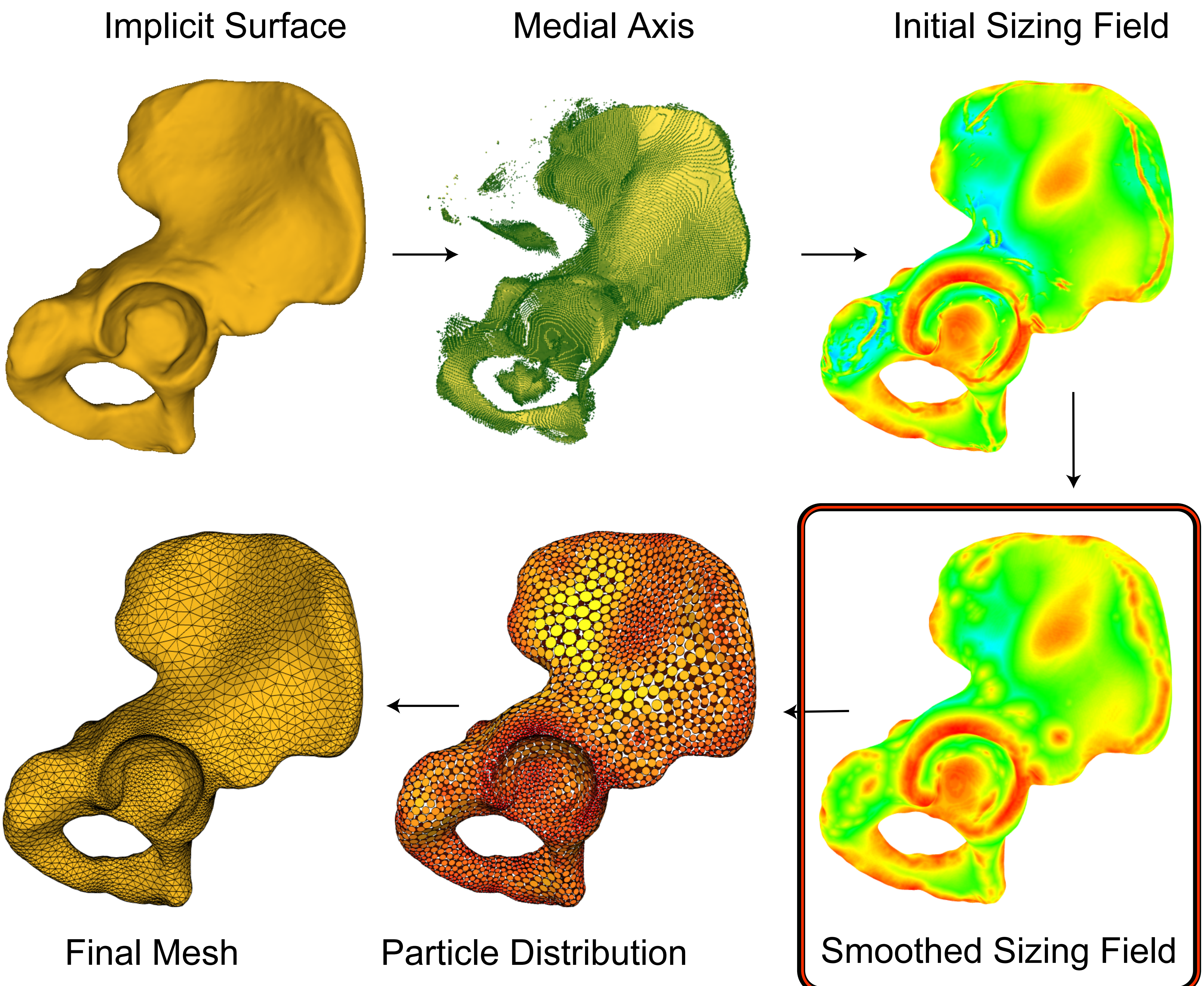

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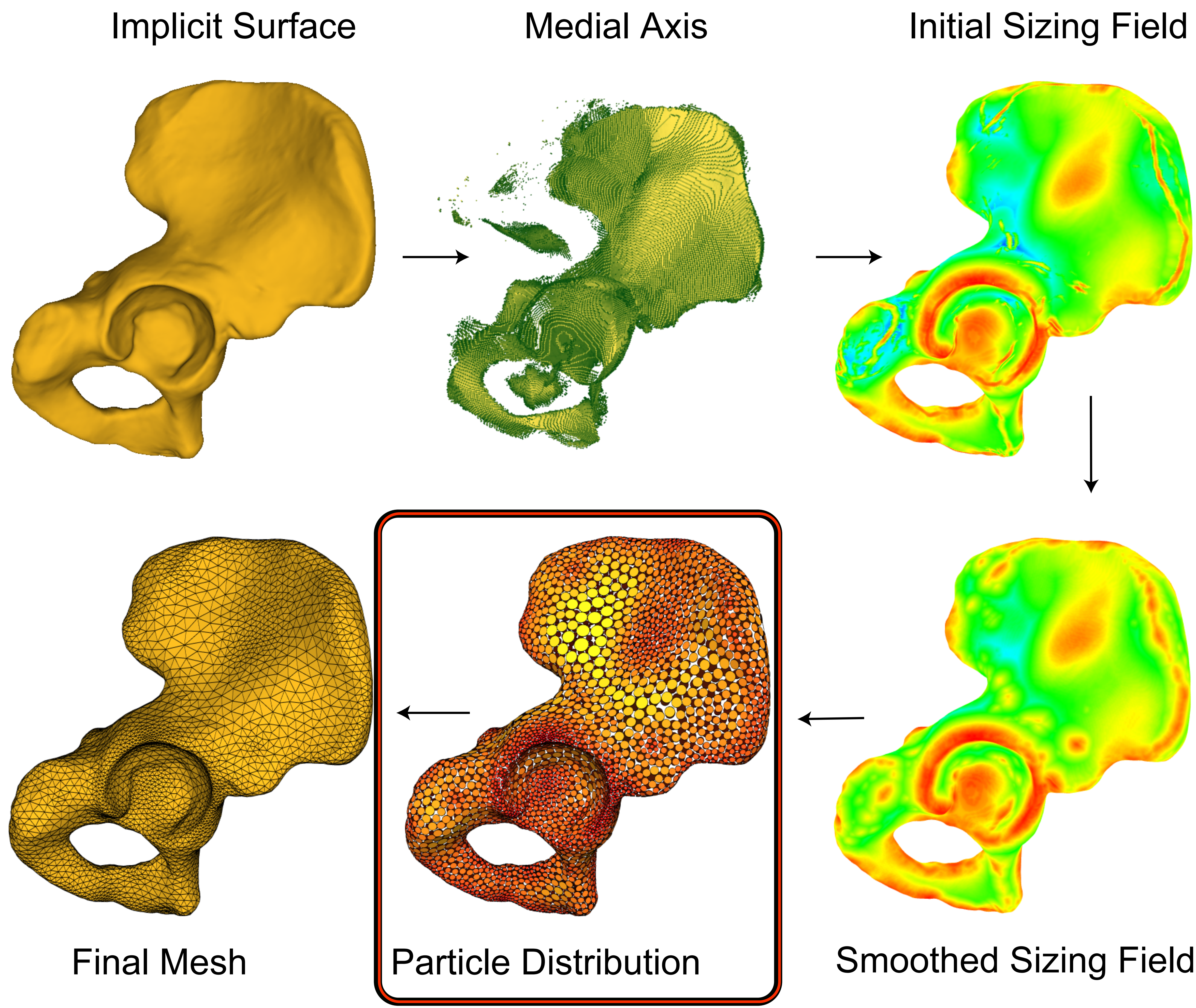
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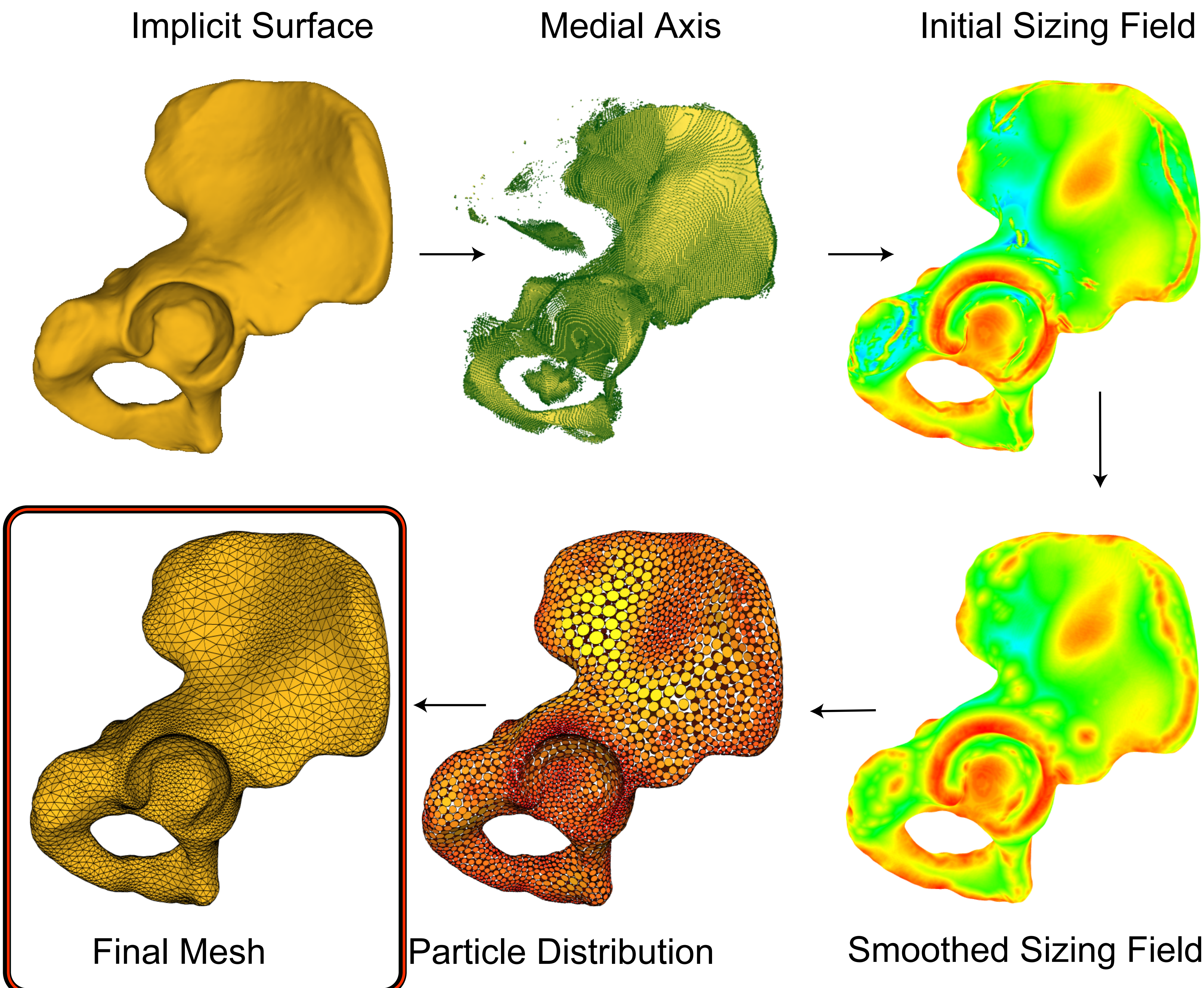
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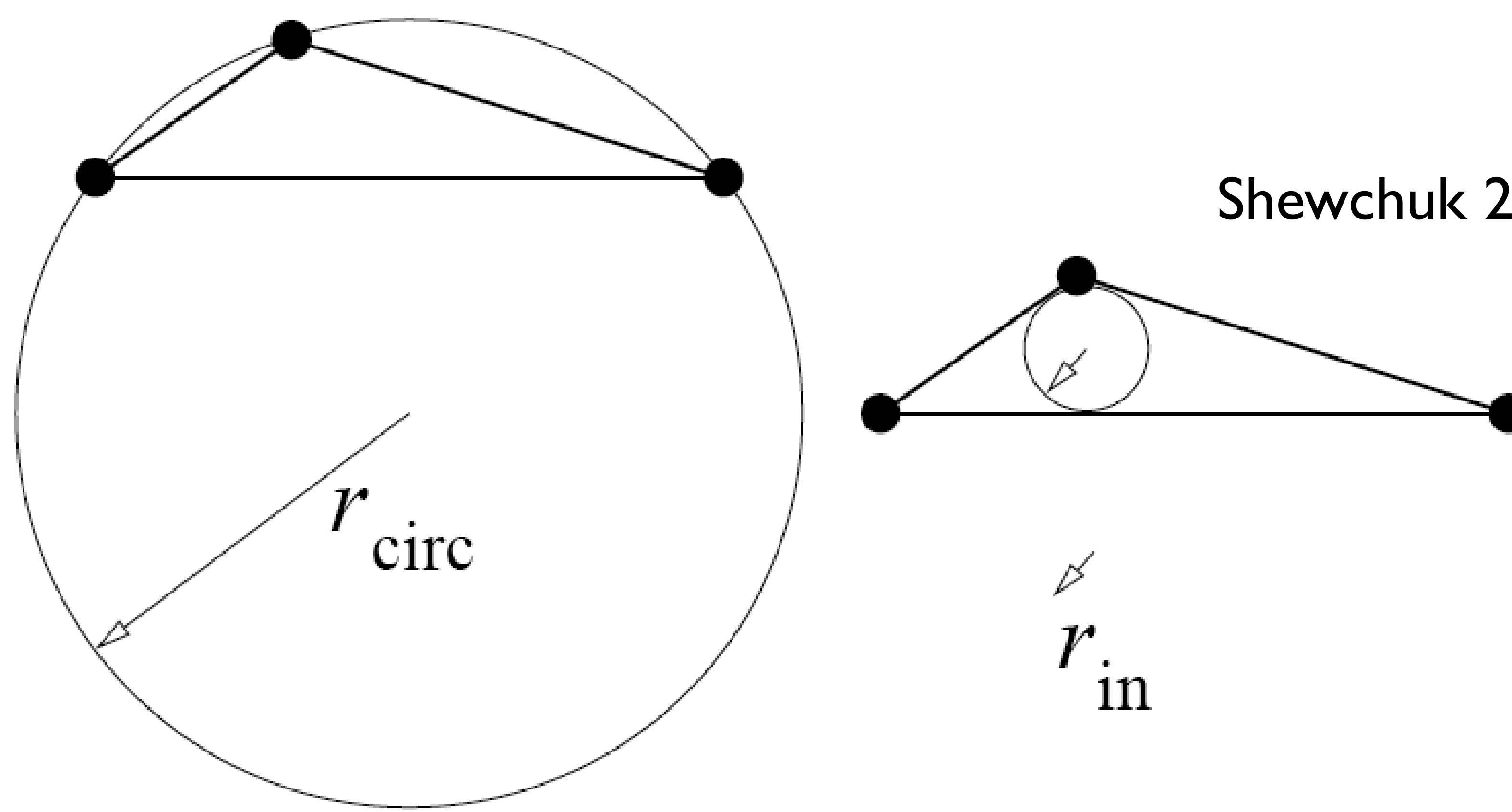
Quality Metric: Radius Ratio

$$2\frac{r_{in}}{r_{circ}}$$



Shewchuk 2002