Frequency-Based Object Orientation and Scaling Determination

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Abstract — A major disadvantage of image retrieval systems is their lack of objects orientation and scaling matching. This paper addresses the issue of accurate, effective, computationally efficient, fast and fully-automated 2D object orientation and scaling determination. The approach relies on the objects frequency-based features. The frequency-based features used by the proposed technique, are extracted by a 2D physics-based deformable model that parameterizes the objects shape. The method was evaluated on synthetic and real images. The experimental results demonstrate the accuracy of the method, both in orientation and the scaling estimations.

I. INTRODUCTION

Large collections of digital images are being created and used in many areas, including commerce, government, academia, and hospitals. Many of these collections are the product of digitizing existing collections of analogue photographs, diagrams, drawings, paintings, and prints. Usually, the only way of searching these collections was by keyword indexing, or simply by browsing. Digital image databases however, open the way to content-based searching.

Content-based image retrieval (CBIR) concerns automatic or semi-automatic retrieval of image data from an imagery database based on semantic similarity between the imagery content. The semantic similarity is typically defined through a set of imagery features. These features are extracted from the image visual content which include e.g. shape, texture, or color properties defined in the imagery domain. The relevance between a query image and images in the database is ranked according to the similarity measure computed from the features. Due to its wide application potential, CBIR research has received intensive attention over the last few years. A number of overviews on image database systems, image retrieval, or semi-automatic retrieval of image data from an imagery database based on semantic similarity between the imagery content. To obtain the local visual descriptors, an image is divided into homogenous regions according to some criterion using region segmentation algorithms that have been extensively investigated in computer vision. A more complex way of dividing an image, is to undertake a complete object segmentation to obtain semantically meaningful objects (like ball, car, horse) [4].

Most of the CBIR systems exploiting object segmentation, do not consider the fact that the objects in query and target images may not have the same orientation and scaling, something that is a major difficulty involved in CBIR research.

This paper deals with the above mentioned problem. It presents a novel method that determines the difference in orientation and scaling factor among two objects, one considered to be in the query image and the other in the target image. The features used for object orientation and scaling determination, are extracted by a 2D finite element-based model parameterizing the object shape.

The proposed approach was motivated by the technique presented in [5], which determines correspondences between objects for recognition using eigen-decomposition analysis. However, the proposed method determines the orientation and scaling of an object, exploiting the frequency-based features, obtained by the free vibrations of an initial circular chain (physics-based modeling) [6], [7], which parameterize the contour of the object under consideration. Modal features using physics-based models assist our method to be robust to missing contour data (frequent due to segmentation methods used).

The remainder of the paper is organized as follows. The 2D physics-based deformable model used as the feature generator is presented in Section II. In Section III, the determination of object orientation and scaling is introduced. Experimental results are presented in Section IV and conclusions are drawn in Section V.

II. 2D PHYSICS-BASED DEFORMABLE MODELING

The contour of the object under consideration was parameterized by the amplitudes of the vibration modes of a physics-based deformable model [7]. The model consists of 2D points sampled on a circular structure, following a circular chain topology [Figure 1.a]. Each model node has a mass and is connected to its two neighbors with springs of stiffness $k$. The nodes coordinates are stacked in vector

$$X_0 = (x_1^0, y_1^0, \ldots, x_N^0, y_N^0)^T,$$  \hspace{1cm} (1)

where $N$ is the number of points of the chain. This physical model is characterized by its mass matrix $M$, its stiffness
matrix $K$ and its dumping matrix $C$, and its governing equation given by [8]:

$$M \ddot{U} + C \dot{U} + Ku = F,$$

(2)

where $U$ stands for the nodal displacements of the initial circular chain $X_0$. The image force vector $F$ is based on the Euclidean distance between the chain nodes and their nearest contour points.

Since equation (2) is of order $2N$, where $N$ is the total number of nodes, it is solved in a subspace corresponding to the $M$ truncated vibration modes of the deformable structure [7]. The number of vibration modes retained in the object description, was chosen so as to obtain a compact but adequately accurate representation. A typical $a$ priori value for $M$ covering many types of standard deformations is the quarter of the number of degrees of freedom of the system (i.e. 25% of the modes were kept).

To solve equation (2) in the subspace corresponding to the truncated vibration modes, the following change of basis is applied:

$$U = \Phi \tilde{U} = \sum_{i=1}^{M} \tilde{u}_i \phi_i,$$

(3)

where $\Phi$ is a matrix and $\tilde{U}$ is a vector, $\phi_i$ is the $i^{th}$ column of $\Phi$, $\tilde{u}_i$ is the $i^{th}$ scalar component of $\tilde{U}$ and $M$ is the truncated number of degrees of freedom. By choosing $\Phi$ as the matrix whose columns are the eigenvectors of the eigenproblem:

$$K \phi_i = \omega_i^2 M \phi_i,$$

(4)

and using the standard Rayleigh hypothesis [7], matrices $K$, $M$ and $C$ are simultaneously diagonalized:

$$\begin{cases} 
\Phi^T M \Phi = I \\
\Phi^T K \Phi = \Omega^2
\end{cases},$$

(5)

where $\Omega^2$ is a diagonal matrix whose elements are the eigenvalues $\omega_i^2$ and $I$ is the identity matrix.

An important advantage of this formulation is that the eigenvectors and the eigenvalues of a chain with circular topology have an explicit expression [7] and they do not have to be computed by slow eigendecomposition techniques (due to the dimensions of matrices $K$ and $M$). The eigenvalues are given by:

$$\omega_i^2 = \frac{4k}{m} \sin^2 \left( \frac{p\pi N}{N} \right),$$

(6)

and the eigenvectors are obtained by:

$$\phi_i = \left[ \ldots, \cos \frac{2p\pi n}{N}, \ldots \right]^T,$$

(7)

where $n \in \{ 1, 2, \ldots, N \}$.

Substituting (3) into (2) and premultiplying by $\Phi^T$ yields:

$$\tilde{U} + \tilde{C} \dot{U} + \Omega^2 \tilde{U} = \tilde{F},$$

(8)

where $\tilde{C} = \Phi^T C \Phi$ and $\tilde{F} = \Phi^T F$.

In many computer vision applications [7], when the initial and the final state are known, it is assumed that a constant load $F$ is applied to the body. Thus, equation (2) is called the equilibrium governing equation and corresponds to the static problem:

$$K U = F.$$

(9)

In the new basis, equation (9) is thus simplified to $2N$ scalar equations:

$$\omega_i^2 \tilde{u}_i = \tilde{f}_i,$$

(10)

In equation (10), $\omega_i$ designates the $i^{th}$ eigenvalue and the scalar $\tilde{u}_i$ is the amplitude of the corresponding vibration mode (corresponding to eigenvector $\phi_i$). Equation (10), indicates that instead of computing the displacements vector $U$ from equation (9), we can compute its decomposition in terms of the vibration modes of the original chain.

Figure 1 illustrates the vibration modes based parameterization of the 2D slices of a tooth germ volume. The circular chain is initialized around each slice [Fig. 1(a)] and the vibration amplitudes are explicitly computed by equation (10), where rigid body modes ($\omega_i = 0$) are discarded and the nodal displacement may be recovered using equation (3). The physical representation $X(\tilde{U})$ is finally given by applying the deformations to the initial circular chain:

$$X(\tilde{U}) = X_0 + \Phi \tilde{U}.$$  

(11)

Thus, the objects of the query and target images are described in terms of vibrations of an initial chain [Fig. 1(b)].

**III. OBJECT ORIENTATION AND SCALING DETERMINATION**

Having approached the object contours using the physics-based deformable model (eq. 11) described in the previous Section, the next step is to determine the orientation and the scaling factor of the object under consideration.

To determine the orientation and the scaling factor of the object, we use vector $\tilde{U}$, derived by (10), which expressed as:

$$\tilde{U} = [ \tilde{x}_1, \tilde{y}_1, \tilde{x}_2, \tilde{y}_2, \ldots, \tilde{x}_N, \tilde{y}_N ]^T,$$

(12)

where $N$ denotes the nodes number of the deformable model used and $\delta_i = [ \tilde{x}_i, \tilde{y}_i ]$ describes the $i^{th}$ coordinate of the
vector \( \mathbf{U} \), which describes the frequency-based properties of the deformation [7].

A normalization preprocessing step, is necessary, in order to
determine the orientation and the scaling factor of an object.
Thus, we subtract the average value \( \mu = \frac{1}{N} \sum_{i=1}^{N} \mathbf{U}_i \) of vector
\( \mathbf{U} \) from each value of the same vector, in order the normalized vector \( \mathbf{U}_n = \mathbf{U} - \mu \) to have a zero average value.

Now, our method is ready to proceed to the main procedure of
the object orientation and scaling determination. We first
form the array I:

\[
I = \left[ \frac{\sum_{i=1}^{N} \tilde{x}_i^2}{\sum_{i=1}^{N} \tilde{x}_i \tilde{y}_i}, \frac{\sum_{i=1}^{N} \tilde{x}_i \tilde{y}_i}{\sum_{i=1}^{N} \tilde{y}_i^2} \right],
\]

(13)

where \( \tilde{x}_i, \tilde{y}_i \) are the coordinates of the normalized vector
\( \mathbf{U}_n \).

In order to determine the object orientation, we calculate the
eigenvalues and eigenvectors of the array I. We discard the
eigenvector corresponds to the smallest eigenvalue, keeping
the second value:

\[
\cos(\theta) = \mathbf{Vec}(1),
\]

(14)

\[
\sin(\theta) = \mathbf{Vec}(2).
\]

(15)

Combining equations (14) and (15) the calculation of the
object orientation \( \theta \) is now a trivial issue.

The object scaling factor is estimated using information of both query \( \mathbf{U}_n^{query} \) and target \( \mathbf{U}_n^{target} \) image (object). Firstly,
the proposed method construct vector \( \mathbf{d}_n^{obj} \):

\[
d_i^{obj} = \sqrt{\left(\frac{x_i^{obj}}{y_i^{obj}}\right)^2 + \left(\frac{y_i^{obj}}{y_i^{obj}}\right)^2},
\]

(16)

where \( obj \) determines the object (in target or in query image)
under consideration and \( x_i^{obj}, y_i^{obj} \) are the coordinates
of the corresponding normalized vector \( \mathbf{U}_n^{obj} \). Afterwards,
our method divides each element of vector \( \mathbf{d}_n^{query} \) with its
corresponding element in vector \( \mathbf{d}_n^{target} \):

\[
s_i = \frac{d_i^{target}}{d_i^{query}}.
\]

(17)

The median of vector \( \mathbf{S} = [s_1, s_2, \ldots, s_N]^T \) defined in (17) is
the difference scaling factor among the query and the target
image (object).

**IV. EXPERIMENTAL RESULTS**

To evaluate our method, we applied the proposed
algorithm to determine the orientation of an artificially rotated
image (Figure 2). The original 512 × 512 artificial image
was transformed using rotations varying from −90° to +90°
degrees with no scaling. The transformations in each case
were uniformly, which means that all integer rotation angles
between −90° and +90° were tested in order not to privilege
any angle [Figs. 2(b)–2(d)]. Table I presents statistics of the
rotation and scaling recovery errors. As it can be seen, median
and mean scaling and rotation errors are much less than 0.05
and 0.50° degrees respectively. Also maximum errors are
less than 0.10 and 1° degree respectively, showing the robustness
of the proposed technique.

Furthermore, the proposed method was tested using the
same image, but in this kind of experiment the rotation angle
was set equal to 0° and the scaling factor have taken values
varying from 0.25 to 4. Table II presents statistics of the
rotation and scaling recovery errors. As it can be seen, median
and mean scaling and rotation errors are much less than 0.05
and 0.20° degrees respectively. Also maximum errors are
less than 0.10 and 0.50° degrees respectively, showing the
effectiveness of the proposed technique. Both the above

<table>
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<th>( \Delta )</th>
<th>( \Delta s )</th>
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<td>median</td>
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<tr>
<td>maximum</td>
<td>0.42°</td>
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<tr>
<td>mean ± s. dev</td>
<td>0.11° ± 0.11°</td>
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**TABLE I**

AN ARTIFICIAL IMAGE WAS ARTIFICIALLY ROTATED USING ANGLES
VARYING FROM −90° TO +90° DEGREES WITH NO SCALING. DIFFERENT
STATISTICS ON THE ERRORS ARE PRESENTED.

<table>
<thead>
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<th>( \Delta )</th>
<th>( \Delta s )</th>
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<tbody>
<tr>
<td>median</td>
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<td>maximum</td>
<td>0.42°</td>
</tr>
<tr>
<td>mean ± s. dev</td>
<td>0.11° ± 0.11°</td>
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</table>

**TABLE II**

AN ARTIFICIAL IMAGE WAS UNIFORMLY SCALED USING VALUES VARYING
FROM 0.25 TO 4 WITH NO ROTATION. DIFFERENT STATISTICS ON THE
ERROS ARE PRESENTED.

![Fig. 2. Orientation and scaling recovery of an artificial image. (a) The initial image. The initial image after (b) +30° degree rotation, (c) +45° degree rotation, and (d) +60° degree rotation.](image-url)
mentioned experiments show no particular orientation and scaling value is privileged by the proposed approach. Moreover, the proposed technique was applied on a variety of other experiments using the specific image, where the rotation parameter have taken uniformly random angles varying from $-90^\circ$ to $+90^\circ$ degrees while the scaling parameter have taken uniformly random values as well varying in this case from 0.25 to 4. Table III presents statistics of the rotation and scaling recovery errors from those experiments. As it can be seen, median and mean scaling and rotation errors are much less than 0.05 and $0.50^\circ$ degrees respectively. Also maximum errors are less than 0.15 and $2^\circ$ degrees respectively, showing that the proposed algorithm can recover the initial orientation and scaling from an object very accurately.

Furthermore, the proposed algorithm was applied on a variety of other artificial and natural images, a sample of which is shown in Figure 3, using akin transformations as in the previous experiments. Table IV presents statistics of the rotation and scaling recovery errors from those images. As it can be seen, median and mean scaling and rotation errors are much less than 0.10 and $1^\circ$ degree respectively. Also maximum errors are about 0.20 and $12^\circ$ degrees respectively.

Finally, let us notice that the algorithm has a computational complexity $O(N)$, where $N$ is the number of nodes of the deformable model. It requires approximately 1 sec. to recover the initial orientation and scaling factor of a $512 \times 512$ image on a Pentium IV (3.0 GHz) workstation under Windows XP Professional without any particular code optimization.

### Table III

<table>
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<tr>
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<th>$\Delta \theta$</th>
<th>$\Delta s$</th>
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<td>$0.14^\circ$</td>
<td>0.03</td>
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<tr>
<td>maximum</td>
<td>$1.71^\circ$</td>
<td>0.15</td>
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<tr>
<td>mean ± s. dev</td>
<td>$0.19^\circ ± 0.21^\circ$</td>
<td>0.04 ± 0.03</td>
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![Fig. 3. Examples of other artificial and natural images applied on the proposed algorithm.](image)

Table IV presents statistics on the errors are presented.

<table>
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<tr>
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<th>$\Delta \theta$</th>
<th>$\Delta s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>median</td>
<td>$0.50^\circ$</td>
<td>0.05</td>
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<tr>
<td>maximum</td>
<td>$12.28^\circ$</td>
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<td>mean ± s. dev</td>
<td>$0.96^\circ ± 1.22^\circ$</td>
<td>0.06 ± 0.05</td>
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### V. Conclusions

A fast and robust algorithm for the determination of 2D objects orientation and scaling was presented. The contours of the objects under consideration were parameterized by a physics-based deformable model, which was used as a frequency-based object feature generator. The eigen-decomposition of the frequency-based features was exploited in order to determine the correct object orientation and scaling factor. The proposed technique is proven to produce very low orientation and scaling errors. It is a very fast and accurate technique.

Furthermore, no particular orientation and scaling value is privileged by the proposed approach. Also, the low frequency modal parameterization of the object contours makes the technique robust to missing data or outliers.

The low computation time and the good quality of the object orientation and scaling determination makes the method a promising tool for the CBIR systems.

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### References


