AN ONTOLOGY OF STRUCTURAL EQUATION MODELS WITH APPLICATION TO COMPUTER SELF-EFFICACY

Completed Research Paper

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Abstract

Structural equation models are widely used in IS research as they can accommodate both latent and manifest variables. Recent discussion has focused on the relationships between these latent and manifest variables in structural equation models. Despite repeated attempts to clarify the relationships and provide guidelines on their interpretation, there remains much confusion, as evidenced by the ongoing debate about the use of formative measures or formative items. In this paper, we address the topic from a fresh perspective, by establishing a clear separation of theoretical and statistical models, and by introducing the notions of system inputs and outputs. The paper explores the consequences of these principles for the relationships between latent and manifest variables using the computer self-efficacy (CSE) construct as an illustrative example. The novel ideas in modeling allow a fresh perspective on the CSE construct and a resolution of the ongoing debate about the nature of the construct.

Keywords: Structural equation models, latent variables, manifest variables, causal indicators, reflective indicators, Ontology, Cognitive system, Computer Self-Efficacy
Introduction

Theories are sets of causal statements that relate constructs to one another (Bacharach, 1989). Realistic theories are complex and can include multiple exogenous, endogenous, mediator, and moderator constructs. Many of the theoretical constructs in IS theories cannot be directly observed, e.g. (self-perceived) computer self-efficacy, perceived ease of use, task-technology fit, etc. For these constructs a set of proxy measurement items (indicators) must be found. The relationship between the constructs and their measurement items has been the focus of much recent debate both in the IS literature as well as in the reference psychometrics literature, focusing on the notion of “formative items” or “formative measures”. At the heart of the issue is the question whether constructs cause their measurement or measurements cause their constructs. Frequently advanced exemplars for the former are psychological beliefs or perceptions, while social-economic status or house prices are often proposed as exemplars for the latter. Formative measurement has both strong proponents (e.g. Diamantopoulos and Winklhofer, 2001; Diamantopoulos et al, 2008) and strong opponents (e.g. Howell et al, 2007a; 2007b; Wilcox et al, 2008). Despite more than 20 years of argumentation in the psychometrics literature and more recently in the IS literature, including a number of recommendations on their use (e.g. Jarvis et al, 2003; Petter et al, 2007; Cenfetelli and Bassellier, 2009) there appears to be little consensus on whether and how such variables should be modeled or what they might represent.

The main purpose of this paper is to offer an ontology of structural equation models based on two principles. First, we highlight the distinction between theoretical and statistical models and the necessity of an explicit translation between the two. Statistical models represent theories; elements of statistical models need to be interpreted in theoretical terms, and results of statistical modelling need to be translated back to theoretical statements. This representation relation admits many “design decisions” on the part of the researcher. Second, we propose using the concepts of system and system inputs and outputs to clarify the meaning of the theoretical constructs represented by SEM models and the manifest variables in a SEM model. Using this ontology, we discuss the nature and measurement of Computer Self-Efficacy (CSE), we show that many of the competing claims of the nature of CSE are due to different assumptions, and we show how the precise terminology of our ontology can help resolve some of the competing claims.

The paper begins by briefly reviewing the debate about formative measurement. We then introduce our ontology of structural equation models, including basic terminology, the notions of system and system inputs and outputs, and the mapping from theoretical to statistical model. We then apply this ontology to investigate the nature of the Computer Self-Efficacy construct by exploring the consequences of different translations from theory to statistical model for models of CSE, first for systems with outputs only (typically found in survey research) and then for systems with both inputs and outputs. We close with a brief discussion and conclusion.

The Debate about Formative Variables

The debate about formative variables has been characterized primarily by statistical considerations. An early article by Bollen and Lennox (1991) provided guidelines on what to expect in terms of correlations between variables, internal consistency and latent-variable correlations. While Edwards and Bagozzi (2000) recognize that causality is the primary consideration when specifying SEM models, the remainder of their work focuses on the statistical properties of different models with formative variables. Diamantopoulos and Winkelhofer (2001) introduced the debate to management research, providing guidelines for index construction. Similar guidelines were developed by Jarvis et al. (2003) who also provide an illustration of parameter bias when models are mis-specified. Differences in the process of identifying a suitable set of manifest variables are demonstrated by Diamantopoulos and Siguaw (2006). Recent work in the psychometrics literature (Howell et al., 2007) also noted that prior work focused on the statistical properties and instead asks whether formative measurement should be used at all. Howell et al.’s primary issue with such variables is that of interpretational confounding. Their argument has been critiqued in responses by Bagozzi (2007) and Bollen (2007). More recent articles by Bollen and Davis (2009a, 2009b) are concerned with identification of formative models using covariance-based estimation. In Information Systems, the debate on formative measurement received considerable attention with Marakas et al. (2007) and the subsequent responses and rebuttals. At the same time, Petter et al. (2007)
provided guidelines for specifying formative structural equation models, primarily informed by the work of Jarvis et al. (2003). More recent work by Cenfetelli and Bassellier (2009) on the interpretation of formative measurement focused again on statistical issues such as multicollinearity, indicator weights, and interpretational confounding. The latter issue was also discussed by Kim et al. (2010).

The debate of statistical and estimation issues in the literature is important and raises many valid points. However, other than this brief overview, we do not wish to address the issue of formative variable models from a statistical perspective as we believe that the choice of statistical model must be driven primarily by theoretical concerns. We wish to merely point out the terminological confusion and the confounding of theory and statistical model that is evident in the literature. We give two examples here, but many more can be found. Bollen and Lennox (1991) use a confusing mix of mathematical or functional terminology (“indicators depend on the latent variable”), causal terminology (“cause indicators”, “effect indicators”) and compositional terminology (“excluding an indicator changes the composition of the latent variable”). Similarly, in the management literature, Diamantopoulos and Winkelhofer (2001) use both causal terminology (“indicators could be viewed as causing rather than being caused by the latent variable”) as well as compositional language (“[SES] is formed as a combination of education, income ...”). We believe this imprecise language is an important factor in the debate, as it leads to misunderstandings. Note also that causal language with respect to random variables is an example of confounding theory or “real world” with statistics. Causality operates in the world (as represented by our theory), but statistics makes no claims about causality. The following section will introduce precise terminology and provide a clearer separation of the theory and statistics to avoid such confusion.

An Ontology of Structural Equation Models for Theories of Individuals

As there is no clearly agreed upon terminology and the use of different terms for the same concepts and vice versa is a source of continuing confusion, we begin with definitions for the purpose of this paper. Our ontology is limited to theories of individuals.

Basic Terminology

A psychological construct is a cognitive state of mind, such as a belief, a perception, and attitude. It is not directly observable. While psychological constructs are frequently confused with latent variables in statistical models, the two are distinct. The first is a theoretical entity; the latter is a statistical entity: A latent variable is a variable in a statistical model for which no data is available. While it may appear natural to represent psychological constructs by latent variables, we emphasize that this is not an identity but a representation relationship whose properties are the central focus of this paper.

Theoretical relationships may be causal or non-causal. Among the non-causal relationships, we distinguish between compositional relationships, such as “part of”, “aspect of”, “dimension of” (Edwards, 2001; Law et al., 1998) and functional relationships, such as “function of”, “sum of”, “differences between”, “fit between”. These are frequently found in IS research (Evermann and Tate, 2011; Klein et al., 2009), for example in service quality, which is defined as differences in perceptions (Parasuraman et al., 1985), or task-technology fit (Vessey, 1991).

An observation is a set of measurements obtained from individuals, e.g. as a result of a questionnaire item, an experimental measurement or similar. Observations are frequently confused with questionnaire items; the item is simply a verbal statement presented to an individual, the observation is the set of individual’s responses to that statement. Observations are also frequently confused with manifest variables; the first is a theoretical entity, the latter is a statistical entity: A manifest variable is a variable in a statistical model for which data is available. While it may appear natural to represent observations by manifest variables, we emphasize that this is not an identity but a representation relationship whose properties are the central focus of this paper.

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1 This is true even for fMRI studies that are increasingly popular in the field of NeuroIS. While the indicators may differ in these studies, the presence and level of a construct must still be inferred from the observed data.
Cognitive Systems and Causality

Psychological constructs form an abstract, unobservable, cognitive, causal system that has inputs and outputs (Bunge, 1983; Fishbein and Ajzen, 1975; Newell and Simon, 1972). The outputs of the system occur in the form of observable performances, e.g. performances on experimental tasks, responses on a survey instrument, etc. The inputs to the system typically take the form of perceptions of experimental stimuli or survey items.

The outputs of the system form the end of any causal chain within the system. Thus, “causal force” rests only with cognitive constructs in the system and system inputs. Specifically, we assume that (1) cognitive constructs may cause other cognitive constructs or system outputs, and (2) that system inputs may only cause cognitive constructs. Specifically, system inputs cannot directly cause system outputs.

We note three important points. First, there may be constructs that form the end of a causal chain within the system. However, given the absence of subsequent outputs, we can in principle never know about the existence or the properties of such constructs and it makes little sense to include these in our theories. Second, it is possible that the output of one system may be the input of another cognitive system and both cognitive systems are included in the scope of the theoretical model. Only rarely is the output of a system also the input to the same system. Such a situation might occur in studies that involve time-dependent phenomena, e.g. where subjects produce an output and are later presented again with this as input.

To forestall the argument that some cognitive constructs, such as certain beliefs or attitudes, are formed only in the act of recording them with a survey instrument, we note that asking a survey question or, more precisely, its perception by a human, is itself an input to the system. When the researcher believes that the construct exists a-priori, this input may be omitted from the theoretical model. In contrast, when the researcher believes that the construct is formed as a response to a measurement stimulus, this input should be an explicit part of the theoretical model. We give examples of different models in a later section.

Finally, we emphasize that the notions of system input and output should not be confused with the notions of antecedents and consequences. The former are defined with respect to a cognitive system (i.e. “a human”), whereas the latter are elements of causal language defined with respect to our theories about that causal system.

Mapping between Theory and Statistical Model

The previous section established the entities found at the theoretical level, i.e. psychological constructs, system inputs and outputs. At the statistical level, structural equation models consist of regression relationships between latent variables (LV) and manifest variables (MV). Latent variables are variables for which data are not available, while manifest variables are variables for which data are available. This data may be observed (as from a survey) or fixed/manipulated (as from an experimental manipulation). The distinction means that the meaning of the term manifest variable is not identical to the meaning of the term observed variable, as manifest variables may be fixed or manipulated, rather than observed.

We already foreshadowed the existence of a representation relationship or mapping between theoretical entities and statistical entities. Table 1 summarizes the entities between which a mapping relation must be established. The mapping relation is used to translate the theoretical entities to the statistical entities that represent them, in order to apply statistical estimation techniques. Similarly, the mapping relation is used to translate the statistical estimation results back to statements about theoretical entities.

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2 At this point, we make no difference between the main types of statistical models in use in IS, that of covariance analysis and that of composite estimation (partial least squares). While our terminology is closer to the first, for the purpose of statistical model described here, we view the difference primarily as one of estimation method.
### Table 1: Theoretical and statistical entities

<table>
<thead>
<tr>
<th>Theoretical Entities</th>
<th>Statistical Entities</th>
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<tbody>
<tr>
<td>Psychological Constructs</td>
<td>Latent variables (LV)</td>
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<tr>
<td>Observations (&quot;System output&quot;)</td>
<td>Manifest variables (MV)</td>
</tr>
<tr>
<td>Manipulations (&quot;System input&quot;)</td>
<td>Regression relationships</td>
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<tr>
<td>Causal relationships</td>
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<tr>
<td>Non-causal relationships</td>
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<tr>
<td>• Compositional relationships</td>
<td></td>
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<tr>
<td>• Functional relationships</td>
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</table>

Some aspects of the mapping from theory to statistical model are unproblematic. For example, it makes little sense to represent relationships by latent variables. Typically, theoretical constructs are represented by LV, theoretical relationships are represented by regression relationships, and observations and manipulations are represented by manifest variables. However, even with these basic constraints on the mapping, there are a number of open questions, e.g.:

- Must every LV represent a construct?
- Can a construct be represented by more than one LV?
- Must every construct be represented by an LV?
- Can an LV represent more than one construct?

These questions show that the mapping of constructs to LV is non-trivial. Any mapping between two sets is technically a mathematical relation between the two sets. Therefore, this mapping can, like any mathematical relation, be characterized by its uniqueness and totality properties (Klip et al., 2000). Uniqueness concerns the question of how many elements of one set are mapped to how many elements of the other, while totality concerns the question of whether all elements of a set are mapped. Both, uniqueness and totality can be viewed from the perspective of either set, yielding the notions of left-uniqueness, right-uniqueness, left-total, and right-total. A sensible constraint on the mapping for it to be unambiguous is to require both left-uniqueness and right-uniqueness. In other words, no construct is represented by more than one LV, and no LV represents more than one construct. Hence, we focus on the totality properties, yielding four possible combinations of left- and right-totality (Table 2, Figures 1a-1d).

We make the assumptions that the mapping from MV to the union of system inputs and outputs is both right-total, right-unique, left-total and left-unique. In other words, every MV is mapped to exactly one system input or system output, and every system input and system output is mapped to exactly one MV.

### Table 2: Combinations of totality properties of a mapping

<table>
<thead>
<tr>
<th>Totality</th>
<th>Abbreviation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>Left-total</td>
<td>Right-total</td>
<td>L+R+</td>
</tr>
<tr>
<td>Not left-total</td>
<td>Right-total</td>
<td>L-R+</td>
</tr>
<tr>
<td>Left-total</td>
<td>Not right-total</td>
<td>L+R-</td>
</tr>
<tr>
<td>Not left-total</td>
<td>Not right-total</td>
<td>L-R-</td>
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</table>

We note that Latent Growth Curve models (Bollen and Curran, 2006) do in fact do this, but this type of longitudinal model is extremely rare in the IS area.
Figures 1a through 1d show the different combinations of totality properties of the mapping between constructs and latent variables in diagrammatic forms between the two sets. These mapping totality properties are similar to the well-known mapping properties in the conceptual modeling literature (Wand and Weber, 1993) where they are called “Ontological Completeness” (L+R+), “Construct Deficit” (L-R+) and “Construct Excess” (L+R-). In contrast to our focus on the totality of the mapping, Wand and Weber (1993) also incorporate notions of uniqueness into their discussion. Wand and Weber (1993) refer to a not left-unique mapping as “Construct Redundancy” and to a not right-unique mapping as “Construct Overload”.

The separation of theoretical and statistical entities implies a distinction between system inputs and exogeneity. The former is a concept at the theoretical level; the latter is a concept at the level of the statistical model and expresses the fact that a variable has no predictors in a particular model. The following situations can be distinguished:

- Exogenous, but not input - Constructs such as knowledge or experience are exogenous to many models of substantive theories, but are not system inputs. For example, in a survey study, experience may be exogenous in that the model is not concerned with its predictors, but it is not a system input, as experience is not manipulated or provided to subjects in the study. Exogenous variables that are not system inputs are latent variables, as they represent unobservable constructs in the cognitive system.
Exogenous and system input – Experimental manipulations are typically both exogenous as well as input to the cognitive system. For example, a theory about task difficulty on performance may not be concerned with the antecedents of task difficulty and a corresponding experimental study might manipulate task difficulty in the experimental stimuli provided to subjects. System inputs that are exogenous are manifest variables.

System input, but not exogenous – In this situation, the system input is also the output of either a different or the same system considered in the same theoretical model. For example, in a theory about pair-programming in system development, code complexity may be the output of one cognitive system (the first programmer) and serve as the input to another system (the second programmer). In this case, the theory is concerned with the antecedents of code complexity, yet, it is input to a cognitive system. System inputs that are not exogenous are manifest variables.

Neither exogenous nor system input – this is the case for system outputs (observations) and endogenous psychological constructs. The former are represented by manifest variables, the latter by latent variables.

We emphasize that there is not one correct way for a researcher to make the mapping. The four types of mappings in Table 2 are design decisions that the researcher can make when translating his or her theory to a statistical model for estimation and testing. However, each of these types of mappings has consequences for the statistical models that can be expressed, as well as the meaning and interpretation of the elements of that statistical model. We believe that the confusion in the literature about the status of latent and manifest variables, especially the issue of formative manifest variables, stems from the fact that these design decisions, the mapping and its properties are not made explicit.

An Application to Computer Self-Efficacy

In the IS literature, the debate about formative variables has begun with articles by Marakas et al. (2007) on the computer self-efficacy (CSE) construct. The CSE construct is defined as “an individual's perception of efficacy in performing specific computer-related tasks within the domain of general computing” (Marakas et al., 2007, pg. 16). For this discussion, we focus on the general CSE; the arguments for specific CSE constructs are analogous. The items used by Marakas et al. (2007) are shown in Table 3. Marakas et al. (2007) argue that CSE should be specified formatively, as they express independent questions and therefore need not covary. In their response, Hardin et al. (2008a) argue, also correctly, that since CSE is a psychological construct, changes in the construct precede changes in the measurement and thus the CSE items are reflective. We believe that the resolution of this debate requires a more detailed examination of the theoretical model and its translation into the statistical model. Neither Marakas et al. (2007) nor Hardin et al. (2008a) clearly separate the theoretical or substantive models from the statistical model.

<table>
<thead>
<tr>
<th>Table 3: Survey items for General Computer Self Efficacy (Marakas et al., 2007)</th>
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<tbody>
<tr>
<td>CSE1</td>
</tr>
<tr>
<td>CSE2</td>
</tr>
<tr>
<td>CSE3</td>
</tr>
<tr>
<td>CSE4</td>
</tr>
<tr>
<td>CSE5</td>
</tr>
<tr>
<td>CSE6</td>
</tr>
</tbody>
</table>

Models of Computer Self-Efficacy with System Outputs

In this section, we focus on models of Computer Self-Efficacy, and especially the relationship between LV and MV, when the cognitive system to be represented has only outputs. This is typical in survey research. This means that every MV represents one observation (system output), and every observation is represented by one MV. Further, the MV, because they represent system outputs rather than inputs, have
no causal force; causal force rests only with psychological constructs. We next discuss the four types of mappings of constructs to latent variables.

**Left-total and Right-total Mapping (L+R+)**

Right- and left-totality of the mapping between constructs and latent variables means that (1) every construct is represented as exactly one latent variable, and (2) that every latent variable represents exactly one construct. From (1) it follows that CSE cannot be split across multiple latent variables and must be uni-dimensional in this model. From (2) it follows that there are no latent variables that represent anything but a psychological construct. With an assumption that every regression represents a causal relationship and observations (representing system outputs) do not have causal force within the system, it follows that the latent variables can only be regressor with respect to manifest variables (“reflective MV”) and cannot be regressand with respect to manifest variables (“no formative MV”) (although it may play neither role with respect to manifest variables and either role with respect to other LV). For CSE, this means that a model such as that in Figure 2 is not allowed.

![Figure 2: Formative model of CSE, not allowed under L+R+ assumption](image1)

![Figure 3: Formative model of CSE re-specified under L+R+ assumption](image2)

However, formative manifest variables may be re-presented as a model with additional latent variables and single reflective manifest variable, as in Figure 3. These additional latent variables must correspond to theoretical constructs, forcing the researcher to make theories explicit. Here, variables CSE1 through CSE6 in Figure 3 represent the belief in the ability to describe how a computer, the belief in the ability to install new software, etc. These separate beliefs cause the observations. Because the regression relationships represent causal relationships, the overall belief of computer self-efficacy is caused by, not composed of or a function of, the individual beliefs CSE1 to CSE6.

As CSE in Figures 2 and 3 represents an endogenous construct without consequences, it is in principle impossible to infer its existence or properties, as we can infer constructs only by observing their consequences. To estimate overall CSE, one might ask questions such as those in Table 4 and record their answers as system outputs. Representing these system outputs as additional manifest variables yields the statistical MIMIC (multiple indicators, multiple causes) model of CSE in Figure 4. By the same reasoning as above, this model needs to be re-presented as shown in Figure 5.
**Table 4: Additional survey questions to identify general computer self-efficacy**

<table>
<thead>
<tr>
<th>CSE-A</th>
<th>I believe I have the ability to use a computer effectively</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSE-B</td>
<td>I believe I have the ability to get a computer to do what I want it to</td>
</tr>
<tr>
<td>CSE-C</td>
<td>I believe I have the ability make a computer work the way I need it do</td>
</tr>
</tbody>
</table>

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**Left-total but not Right-total Mapping (L+R-)**

This type of mapping between constructs and latent variables means that (1) every construct is represented as exactly one latent variable, and (2) that some, but not all latent variables represent exactly one construct. The implications from (1) are as above. From (2) we now allow latent variables which need not represent cognitive constructs. These form part of the model for other reasons, e.g. sum-scores or indexes for data reduction purposes. These latent variables may therefore have formative manifest variables because there are no restrictive assumptions about them. However, because these latent variables do not represent constructs, they have no causal force w.r.t. other constructs and must not be regressors to other latent or manifest variables where that regression represents a causal relationship. In short, non-constructs cannot cause constructs. Further, the regressions between formative manifest variables and the latent variables themselves also cannot represent causal relationships. Thus, they must represent either compositional or other functional relationships. This is compatible with the interpretation of formative variables as sum-scores or indexes that are composed of observational data.

The model in Figure 6 consists of latent variables that are weighted sums or indices composed of observed CSE data. This composition is a non-causal relationship, which we indicated on the corresponding regression paths. We have also labeled the latent variables “Composite CSE-A”, “Composite CSE-B” and “Composite CSE” to indicate that they do not represent the cognitive construct CSE but a data composition. Such data composition might be useful for purposes of ranking different individuals by their composite CSE, for comparing composite CSE of the same individual over time, or some other purpose that requires data to be aggregated to a single datum, e.g. in the context of organizational performance evaluations.
Figure 6: Constructing composition hierarchies of CSE item observations. Composite CSE-A, Composite CSE-B and Composite CSE do not represent cognitive constructs.

Under the L+R- assumption it is possible to have models with LV that represent either constructs or non-constructs. Thus, we could add the cognitive CSE construct, represented by another LV to Figure 6, as shown in Figure 7. In this model, some LV represent constructs, others do not. Some regressions represent causal relationships, others represent compositional relationships. While the latent variables “CSE” and “Composite CSE” do not represent the same theoretical entity, one would expect that they will be correlated. This however should be an empirical question, rather than a research assumption. The important point in Figure 7 is that it shows that the formative latent variable “CSE” and the reflective latent variable “Composite CSE” are not identical and must not be confounded.

Further, Figure 7 includes additional system outputs as consequences of the psychological construct “CSE”, modeled as manifest variables “cse-A”, “cse-B” and “cse-C”. The inclusion of “CSE” without these makes little sense, as we cannot in principle infer any properties or even the existence of a construct without any consequences.

Figure 7: Adding the CSE cognitive constructs to Figure 6
Figure 8 below represents an alternative to Figure 7. This model also consists of LV that represent the CSE construct (“CSE”) and a composite of observed data (“Composite CSE”). This model makes explicit the assumption that the composite score of the observations is closely related to the CSE. Instead of explaining the causes of the individual observations “cse1” through “cse6” as in Figure 7, the model in Figure 8 explains their weighted sum-score or index “Composite CSE”. A regression of “CSE” on “Composite CSE” representing a causal relationship cannot be modeled because constructs can only be caused by other constructs or system inputs, whereas “Composite CSE” represents neither of these.

Models that represent both psychological constructs and related composites, such as the models in Figures 7 and 8, are useful to evaluate the relationship between a cognitive construct and the related composite. For example, using the model in Figure 7 or 8, a researcher can establish to what extent the composite CSE can be used as a valid proxy or aggregate for the psychological “CSE” construct, and what the optimal values for the weights of the individual manifest variables are.

**Not Left-total and Right-total Mapping (L-R+)**

This type of mapping from constructs to latent variables means that (1) some (but not all) constructs are represented as exactly one latent variables and (2) every latent variable represents exactly one construct. The consequences from (2) are as for the L+R+ correspondence above. The consequences from (1) means that there are some constructs that are not represented by latent variables. One situation in which this may occur is when researchers omit well-established mediators, e.g. because of concerns about questionnaire length, or because such questionnaire items might lead to questionnaire effects. Another situation is where such constructs do not have consequences that are within scope of the theory or study.

**Not Left-total and Not Right-total Mapping (L-R-)**

This type of mapping between constructs and latent variables means that (1) some (but not all) constructs are represented by exactly one latent variable and (2) some (but not all) latent variables represent exactly one construct. The implications of (1) are as from (1) for the L-R+ mapping correspondence and as from (2) for the L+R- mapping.
Models of Computer Self-Efficacy with System Inputs and Outputs

Systems with both inputs and outputs are typically examined in experimental work, but can also be useful for survey research. We discuss these in turn in the following two subsections.

System Inputs in Experimental Research

Recall that manifest variables may represent system inputs (e.g. experimental manipulations) that have causal force. Hence, for the systems of Computer Self-Efficacy we discuss here, formative manifest variables, i.e. MV that are regressors with respect to LV, are admissible not only in the L-R- and L+R- conditions, but also in the L-R+ and L+R+ conditions described above. However, they must represent system inputs, not outputs.

While CSE as defined by Marakas et al. (2007) is a belief, their operationalization lends itself immediately to experimental work. Instead of asking subjects for their beliefs about their ability to perform certain tasks with a computer, one could devise an experiment to measure their ability to perform that task. Each of the six CSE survey questions (“cse1” through “cse6”, Table 1) thus has a corresponding experimental task. In such a situation, the cognitive system has both system inputs and outputs. The presentation of an experimental stimulus, manipulated at different levels, is a system input. The performance on the experimental task represents system output, which is caused by the perception of the stimulus as well as the actual ability. A manipulation check that measures how a subject perceives the task is also a system output that is represented as a manifest variable. Figure 9 shows such a model for experimentally assessing what we have termed here “Ability1” through “Ability6”. In this model we have assumed that the specific abilities corresponding to the six items of the (perceived) CSE construct are caused by a common cause, labeled “Actual CSE”, though another plausible model would represent the set of abilities as the causes of actual CSE. This is a substantive question for empirical inquiry.

A researcher may also wish to form an index or composite of the actual experimental performance measures, e.g. for purpose of comparisons across time or subjects. For this, we require the relaxation of the L+R+ assumption and must assume L+R-. Under the latter assumption, the statistical model may contain a latent variable that does not represent a construct, shown in Figure 10 as “Measured CSE”. “Measured CSE” does not represent the same theoretical entity as “Actual CSE”, though they are of course related, similar to the relationship between belief and composite in Figures 7 and 8. The manifest variables “Performance 1” through “Performance 6” represent system outputs and have therefore no causal force. Thus, the regressions with “Measured CSE” do not represent causal relationships but compositional relationships. For reasons of clarity, the model in Figure 10 omits the manipulation checks from Figure 9.

Both models, Figure 9 and Figure 10, can of course be extended to include the CSE (belief) construct, which we have discussed in the previous section. We emphasize again that the (belief) CSE construct is different both from what we have termed “Actual CSE” and “Measured CSE” here.
System Inputs in Survey Research

Manifest variables representing system input may be employed in survey research to model questionnaire effects. This subsection shows possible models in the context of Computer Self-Efficacy.

The model in Figure 11 shows sequence effects, whereby the response to the first question is represented by manifest variable “cse1”, which then serves as input to some cognitive or perceptive construct Q1 representing the questionnaire effect. The response to the second question is represented by manifest variable “cse2”, which is influenced both by the construct under study X and the questionnaire effect from the first response Q1. Note that the model in Figure 12 suggests that the response, rather than the presentation of the question, affects the following responses. In other words, responses represent both system outputs and inputs, where respondents read/perceive their own outputs and react to these.

A different model of question order effects is shown in Figure 12. In that model, the manifest variables “pcse1” through “pcse6” represent the presentation of the questions (system inputs) in the CSE instrument (Table 3), while manifest variables “cse1” through “cse6” represent the corresponding responses on each question. The presentation of questions leads to some cognitive perceptions or beliefs that represent the questionnaire effects, modeled as latent variables Q1 through Q6. These, together with the construct under consideration, CSE, cause the responses. The researcher would usually hope that the effect of CSE on “cse1” through “cse6” is much stronger than that of Q1 through Q6. In this model, system
inputs and outputs are clearly separated, and the respondent reacts not to his/her own responses but to the presentation of the following questionnaire item.

**Figure 11:** Modelling questionnaire effects using MV that represent both output and input of the same system

**Figure 12:** Modelling questionnaire effects Q1 through Q6 with presentation of questions represented explicitly as “pcse1” through “pcse6”.

**Figure 13:** Questionnaire effects as autoregressive model
A third way in which question order effects might be represented is shown in Figure 13. This represents an autoregressive model in simplex form. The manifest variables “pcse1” through “pcse6” and “cse1” through “cse6” represent manipulations (questions) and observations (responses) as in the previous model. However, instead of assuming that the questions have no influence on CSE itself, this model assumes that asking questions about CSE now changes a subject’s belief about her own CSE, in effect leading to seven different CSE constructs over time.

We add that not only asking a question might be construed as an input, but that the questionnaire introduction or context framing can also be thought of as an input or experimental stimulus and be modeled explicitly if it was systematically manipulated.

**Conclusion**

We have presented an ontology to aid in representing theories in structural equation statistical models and have applied it to modeling the Computer Self-Efficacy construct. Our ontology of structural equation models is founded on (1) a clear separation of theory and statistics with an explicit representation mapping and (2) the notion of a cognitive system with inputs and outputs. While we have focused on Computer Self-Efficacy in this paper, the reasoning is applicable to other constructs in the Information Systems literature as well.

This paper makes two main contributions. First, we have shown that a clear separation of theory from statistical model can help clarify the meaning of variables in a statistical model. Despite similarities between theoretical and statistical models the two are distinct and this distinction requires a translation between the two: Theoretical models are represented by statistical models. We have shown that this representation relationship, despite some obvious constraints, admits multiple “design decisions” on the part of the researcher, especially with respect to the totality of the mapping from theoretical elements to statistical elements. We do not believe that there is a single correct translation or representation relationship, but instead encourage researchers to make this translation more explicit.

Second, we showed that conceptualizing a phenomenon in terms of a system with inputs and outputs is helpful in identifying the type of theoretical relationships and their statistical representation. Thus, we recommend that researchers specify the precise nature of their constructs and identify which of them are cognitive constructs. For example, it is important to differentiate perceptions of abilities from demonstrated abilities, as the former are constructs, while the latter are sum-scores of experimental observations. Further, researchers should specify the system boundary and the inputs and outputs at that boundary. The steps shown in Table 5 can help IS researchers to clearly identify and specify their theoretical models and to translate them to statistical models. The explicit specification of all theoretical entities and their representation in the statistical model allows an informed and nuanced debate about the appropriateness of the representation and comparison to other representations and avoids comparing the proverbial apples and oranges.

These proverbial apples and oranges are evident in the debate about the CSE construct. We have shown that different assumptions about the mapping relationships admit different statistical models of Computer Self-Efficacy. Because researchers are free to define any mapping relationship, there is no single correct model of Computer Self-Efficacy. Different assumptions about the mapping and the inclusion of the system inputs also admit different notions of Computer Self-Efficacy, for example the traditional self-perception of CSE, the composite of self-perception observations, an experimental CSE, or an autoregressive model of CSE. It is important that these different notions of the construct be clearly separated in the literature to avoid the confusion that has plagued the debate of the construct to date. For theoretical purposes, they must be treated as different but related constructs, and their relationships to each other should be seen as an opportunity for substantive research. Figure 8 is an example of the possible relationship between a latent variable representing a composition of CSE measurements and a latent variable representing a psychological constructs. As we have noted in the discussion of that figure, the two are not identical and the relationship between them must be empirically examined. Recognizing different CSE constructs, which are based on different mapping assumptions, resolves many of the arguments about the nature of the construct. The question about the nature of CSE cannot be answered either “formative” or “reflective” but must be answered “both”, depending on which notion of CSE the researcher has in mind. The psychological construct “CSE” (belief) must be reflective, while, under some
mapping assumptions, a composite of the manifest variables is admissible, but must be clearly distinguished as not representing that psychological construct, and so is best termed “composite CSE”. In general, many other related latent variables can be formed that represent other types of CSE constructs, such as in Figures 9 and 10. Asking whether “CSE” is formative or reflective misses the point; the term “CSE” is ill-specified and vague and requires more precision before the question can be answered.

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<th>Table 5: Recommended steps for building structural equation models</th>
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Finally, we wish to emphasize that this work does not invalidate existing guidelines on formative measurement, e.g. by Cenfetelli and Bassellier (2009), Jarvis et al. (2003), or Petter et al. (2003). These guidelines are primarily related to statistical properties, such as model identification and interpretational confounding, and these remain valid concerns when formative manifest variables are used in any of the situations we discussed in this paper. Our appendix briefly discusses some of these issues. Our work also does not invalidate the existing guidelines and established process for constructing valid and reliable reflective measurement. Our paper has focused on how to represent theoretical entities in statistical models; how such theoretical entities, which include constructs, survey items, and experimental stimuli, are developed is beyond the scope of this work.

References


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Hardin, A.M., Chang, J.C.J. and Fuller, M.A. 2008 (b) “Clarifying the use of formative measurement in the IS discipline: The case of computer self-efficacy,” *Journal of the Association for Information Systems* (9:9), 544-546.


Appendix: A Note on Model Estimation

In IS, structural equation models are typically estimated using either partial least squares (PLS) or covariance-based (CB) techniques. While the statistical differences between PLS and CB-SEM are immaterial to our argument, the modelling capabilities of the two techniques differ. For example, while a CB-SEM model may contain a LV without any MV, or contain an MV as regressand of multiple LV, or contain an LV that is both regressor and regressand to MV, this is not possible in PLS.

In many of our models we use latent variables that are regressors of single manifest variables. In these cases, both the error variance of the manifest variable as well as the regression coefficient need to be constrained. An alternative to numerical constraints is the use of equality constraints. For example, it may be plausible to assume that the strength of causal influence from each of the abilities (LV) to the corresponding performance (MV) is the same for all six abilities and performances in Figures 9 and 10. These constraints retain the model structure and still allow estimation of theoretically interesting model parameters.

Models in Figures 6 to 8 contain endogenous latent variables that are not regressors to manifest variables. Such models are in principle unidentified; this reflects the theoretical impossibility to infer properties (or even the existence) of a phenomenon when the phenomenon has no observable consequences.

As the models in Figures 6 to 8 represent composition hierarchies of observed data, e.g. for purposes of indexing, it is natural that the regression coefficients of such indices be defined, rather than estimated. An example of an index for purposes of comparisons is the consumer price index (CPI). The CPI would not be very useful for comparison purposes if the weights of its components changed from one application to the next. In other words, the relative contributions of the price of eggs and the price of gasoline have to remain stable to allow comparisons. Thus, to allow regressions coefficients of such “formative” indices to be estimated defeats their use for comparison across time or objects. In contrast, we suggest that researchers define a set of coefficients for reference purposes.