An economic framework for spectrum allocation and service pricing with competitive wireless service providers

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***Abstract*—In the future, we can expect to see more dynamic service offerings and profiles, as users move from long-term service provider agreements to more opportunistic service models. Moreover, when the radio spectrum is itself traded in a market-based scenario, wireless service providers (WSPs) will likely require new strategies to deploy services, define service profiles, and price them. Currently, there is little understanding on how such a dynamic trading system will operate so as to make the system feasible under economic terms. From an economic point of view, we analyze two main components of this overall trading system: i) spectrum allocation to WSPs and ii) interaction of end users with the WSPs.**

For this two-tier trading system, we present a winner determining sealed-bid knapsack auction mechanism that dynamically allocates spectrum to the WSPs based on their bids. We propose a dynamic pricing strategy based on game theory to capture the conflict of interest between WSPs and end users, both of whom try to maximize their respective net utilities. We show that even in such a greedy and non-cooperative behavioral game model, it is in the best interest of the WSPs to adhere to a price threshold which is a consequence of a price equilibrium in an oligopoly situation. Through simulation results, we show that the proposed auction entices the WSPs to participate in the auction, makes optimal use of the common spectrum pool, and avoids collusion among WSPs. Moreover, numerical results demonstrate how pricing can be used as an effective tool for providing incentives to the WSPs to upgrade their network resources and offer better services.

I. Introduction

The presence of multiple wireless service providers in any geographic region together with the freedom of users in switching wireless service providers (WSPs) is forcing a competitive environment where each WSP is trying to maximize its profit. Essentially, a wireless service provider buys spectrum from the spectrum owner (for example, Federal Communications Commission in the United States of America) with a certain price and then sells the spectrum to the end users in the form of services (bandwidth). In such a scenario, the goal of each service provider is twofold: get a large share of users and the necessary spectrum to fulfill the demands of these users. As both the number of end users and capacity of spectrum band are finite, this gives birth to an interrelated two-tier competitive behavior, where wireless service providers compete among themselves to acquire a large portion of the spectrum and also attract as many users as possible. Though the resource allocation strategies of competing WSPs have been investigated in [11], to the best of our knowledge, this research is the first attempt to analyze the economic aspects that arise due to the interactions between spectrum owner, wireless service providers, and users.

A. Dynamic spectrum allocation

In most countries, chunks of spectrum are statically allocated to the WSPs [8]. Spectrum usage being both space and time dependent, a static allocation often leads to low spectrum utilization as reported in [26]. Static allocation also results in fragmentation of the spectrum creating “white space” (unused bands) that cannot be allocated to licensed/unlicensed services.

In order to break away from the inflexibility and inefficiencies of static allocation, a new concept of *Dynamic Spectrum Allocation* (DSA) is recently being investigated by network and radio engineers, policy makers, and economists. In DSA, spectrum will be allocated dynamically depending on need of the service providers which in turn depends on end users' demands in a time and space variant manner [3]. Emerging wireless technologies such as cognitive radios [13] will make DSA a reality. In DSA, the spectrum owner will create a common pool of *open* spectrum. Though this common spectrum can be created by taking back all the previously (statically) allocated spectrum chunks, it is not an option because of monies already invested. However, parts of the spectrum band that are not allocated or are no longer used can be made open to the WSPs. These parts of the band, that are open to all are known as the coordinated access band (CAB) [4].

B. Economic paradigm shift

Currently, each provider gets a chunk of the spectrum and has a unique user pool that they cater to. In future, a paradigm shift as depicted in figure 1, is very likely to occur where each provider will get a part of the spectrum from the common spectrum pool as and when they need

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through a *spectrum broker*. It is also anticipated that the concept of *service broker*, technically known as Mobile Virtual Network Operators (MVNO) [24], will evolve that will act as an interface between the providers and the users [19]. The users will be able to select their service provider as per their requirements through the service broker. In light of these new developments, it is important to investigate the economic issues that has a profound impact on the service quality and the prices paid by the end users.

The most important factors that the WSPs need to consider are the *amount* of spectrum they need and the *price* are they willing to pay. In effect, estimation of the *demand for bandwidth* and *expected revenue* will drive the provider's strategies. Service pricing by the providers, in turn, will affect the demand for the services by the users, thus resulting in a cyclic dependency in a typical supply-demand scenario. As a result, the relationship between spectrum owner and WSP has a strong correlation with the relationship between WSPs and end-users and must be analyzed together unlike any other industry service model.

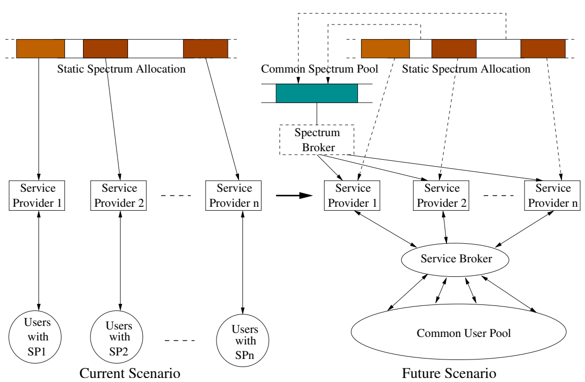

Fig. 1. The paradigm shift

In this paper, we devise a "winner determining sealed bid knapsack auction" mechanism for dynamic allocation of the spectrum from the coordinated access band (CAB). The WSPs bid to acquire *extra* chunks of spectrum in addition to the statically allocated band. We map the winner determination problem to the knapsack problem and use a sealed-bid mechanism to find the optimal allocation. Through a game-theoretic model, the conflicting objectives that are inherent in the WSP-user interactions are captured. We deviate from the notion of per-service static prices [12] and allow the providers to set the prices dynamically in order to maximize their profit and minimize resource wastage. Such a market mechanism is more flexible and realistic, as there does not exist any centralized authority to determine the price of a service. We prove the existence of an price (Nash) equilibrium, where no provider finds it beneficial to change its price unilaterally [15]. We also answer the following questions: i) how the spectrum is allocated from the coordinated access band (CAB) to the service providers, ii) how service providers determine the price of their services, and iii) how are the above two inter-related.

The rest of the paper is organized as follows. In section II, we discuss the relatively small body of work that relates to this research. We describe the proposed auction method in section III. The game models and equilibria conditions are presented in section IV. The demand for bandwidth is estimated in section V. The simulation model and results are presented in section VI. Conclusions are drawn in the last section.

II. Related Work

Auction theory and game theory have been used to analyze problems with conflicting objectives among interacting decision–makers. These theories have been extensively used in various industries including the competitive energy market, airlines industry, and Internet services. They have been proved to be very powerful tools to deal with problems in networking and communications from an economic point of view. This is because the service quality that each user receives in a competitive environment is often affected by the action of other users who also try to contend for the same pool of resources. A broad overview of game theory and its application to different problems in networking and communications can be found in [22] and the references therein. Network services, including pricing issues, have been studied with the help of auctions in [9], [18].

Auction theory has been used to understand markets, especially to model auction participants who bid to win and maximize profit [16]. A large number of Internet auction sites have been set up to process both consumer–oriented and business–oriented transactions. Currently, most auction sites (e.g., eBay [25]) support a basic bidding strategy through a proxy service for a single-unit auction where bidding continues till a winner evolves. In a single unit auction, Vickrey proved that "English" and "Dutch" type auctions yield the same expected revenue under the assumptions of risk neutral participants and privately known value drawn from a common distribution [20]. Vickrey's result is embodied in the "Revenue Equivalence Theorem" (RET) [10]. However, with emerging markets like electricity and spectrum band, single unit auctions are falling short to address the issues where bidders bid for multi-units and multiple winners emerge [1]. As bidders compete for a part of the available resource and are willing to pay a price for *that* part only, this auction model needed must be more generalized and is being currently investigated [2].

As far as game theory is concerned, there is an emerging body of work that deal with decision making in a multi–provider setting. In [5], a market in the form of a 'bazaar' was introduced where infrastructure–based wide area wireless services are traded in a flexible manner and at any time scale. The mobile bazaar architecture allows fine–grained service through cooperative interactions based on user needs. The problem of dynamically selecting ISPs for forwarding and receiving packets has been studied in [21]. A multi–homed user, i.e., user with access to multiple ISPs, has the freedom to choose a subset of ISPs from the available ones. Zemlianov *et al.* assumed the existence of two orthogonal technologies that were overlaid [23]. In particular, cellular and WLANs are considered where users are vertically transferred from one network to another based on the load of each network. In [11], service admission control was done based on the outcome of a game and Nash equilibrium was reached using pure strategy. Users were offered differentiated services based on the price they paid and the service degradation they could tolerate. However, dynamic pricing was not explored in [11]. In [14], authors study the economic interests of a wireless access point owner and his paying client, and model their interaction as a dynamic game. In [7], authors presented a non–cooperative game for pricing Internet services but concluded with an unfair Nash equilibrium where future upgradation of the networks were discouraged.

III. Spectrum Allocation through Auctions

In this section, we analyze a part of the logical model presented in figure 1, i.e., the interaction between the spectrum broker and the service providers. Spectrum allocation from the coordinated access band (CAB) can be done in two ways: asynchronous or synchronous allocation. In asynchronous method, whenever a service provider has a need for spectrum, it makes a request to the spectrum broker. If available, the spectrum broker assigns a chunk of spectrum for the lease period, upon expiry of which, the assigned spectrum is taken back. On the other hand in synchronous allocation, spectrum allocations (and de-allocations) are done in a synchronous manner i.e., providers make requests synchronously. The lease periods can be assumed as discrete unit short span of intervals. Here in this work, we focus on the synchronous allocation.

Similarly, pricing can be done in two ways depending on the total demand of spectrum from the service providers. If the total demand of spectrum *does not* exceed the spectrum available in CAB, then any of the following two pricing models can be adopted. 1. *Service provider dominant strategy:* providers advertise the price they are willing to pay. 2. *Spectrum broker dominant strategy:* spectrum broker advertises a (unit) price and service providers respond by deciding on the amount of spectrum they can acquire.

On the other hand, if the total demand of spectrum *exceeds* the total spectrum available in the CAB (which will be very often the case and is thus the focus of our research), then one of the strategies for the spectrum broker is to put up the spectrum for bids and decide on the allocation based on the bids i.e., to adopt an auction model. The auction for spectrum can be conducted on a periodic basis and on a small time granularity so that wireless service providers will bid for additional spectrum from CAB synchronously as this would allow the spectrum broker to compare all the requests to maximize the revenue. The assumption in this model is that service providers generates spectrum requests periodically at the beginning of each interval.

Moreover, in this work, we focus on the additional spectrum from CAB with regard to the bandwidth and not with regard to the frequency. We assume that total spectrum available in CAB is homogeneous and thus no band is superior or inferior than any other band and thus demand for any band from the CAB is equal. Adding frequency constraint to the spectrum allocation problem will be rather focus of our future work.

A. Auction Issues

A good auction design is important for any type of successful auction and often varies depending on the item on which the auction is held. Unlike classical single-unit auctions, spectrum auctions are multi-unit where bidders bid for a part of the spectrum band, i.e., the bids are for different amounts of bandwidth. Also, multiple winners evolve constituting a winner set. Thus determination of winner set depends heavily on the auction strategy adopted. In our auction model, the spectrum broker is the seller who owns the coordinated access band and service providers are the buyers/bidders. For designing the auction, we consider three important issues i) how to maximize the revenue generated from bidders, ii) how to entice bidders by increasing their probability of winning, and iii) how to prevent collusion among providers.

B. Formulation of Auction Rules

Recall, the service providers already have some spectrum that was statically allocated. It is the additional spectrum that is sought from the CAB. Though the objective of the spectrum broker is to sell the CAB and earn revenue, it is not at all intended that only big companies with higher spectrum demand are given additional spectrum. The goal here is to increase competition and bring new ideas and services at the same time. As a result it is necessary to make the small companies, who also have a demand of spectrum, interested in taking part in the auction.

The problem described here has a very close connection to the classical knapsack problem, where the goal is to fill a sack of finite capacity with several items such that the total valuation of the items in the sack is maximized. Here, the sack represents the finite capacity of spectrum in the CAB that is to be allocated to the WSPs in such a manner that the revenue generated from these WSPs is maximized. In this regard, we propose the "Winner Determining Sealed Bid Knapsack Auction".

We consider L WSPs (bidders) who compete for a total spectrum W. All the service providers submit their demands at the same time in a sealed bid manner. We follow sealed bid auction strategy, because sealed bid auction has shown to perform well in all–at–a–time bidding and has a tendency to prevent collusion [17]. Each service provider has knowledge about its own bidding quantity and bidding price but do not have knowledge about other's quantity and price.

We formulate the auction as follows. We denote the strategy adopted by service provider i by a tuple $q_i = \{w_i, x_i\}$ where, w_i denotes the amount of spectrum requested and x_i denotes the corresponding price that the service provider is willing to pay. If the sum of the bidding quantities do not exceed the spectrum available, W, then the requested quantities are allocated. Otherwise, auction is initiated when,

$$\sum_{i=1}^{L} w_i > W \tag{1}$$

Our goal is to solve the winner-determination problem in such a way so that the spectrum broker maximizes revenue by choosing a bundle of bidders (q_i), subject to condition that the total spectrum allocated does not exceed W, i.e.,

$$maximize \sum_i x_i \quad \text{such that,} \quad \sum_i w_i \leq W \tag{2}$$

Note that, a more generic approach would have been a multiple-choice knapsack formulation with each provider (bidder) submitting a complete demand curve, i.e., a vector of bandwidths requested along with their corresponding prices. Although it is theoretically possible for the providers to submit a demand curve , the solution will become computationally intractable and will also suffer from scalability issues.

C. Bidders' Strategies

We investigate bidders' strategies for both first and second price bidding schemes under knapsack model. In first price auction, bidder(s) with the winning bid(s) pay their winning bid(s) while in second price, bidder(s) with the winning bid(s) do not pay their winning bid but pay the second highest bid.

Let each bidder i submit its demand tuple q_i. Then the optimal allocation of spectrum is done by considering all the demand tuples. We denote this optimal allocation as M, where M incorporates all the winning demand tuples q_i and is subject to condition given in equation 2. Without loss of generality, we assume bids can take only integer values (as bids in dollar values are always expressed as integer) and number of bidders (providers) is typically of the order of 10. If the number of bidders is large, we use the scaling heuristic. Thus, we are able to solve the winner determination problem through dynamic programming with reasonably low computation. The aggregate bid can be obtained by summing all the bids from bidder,

$$\sum_{i \in M} x_i. \tag{3}$$

Let us consider a particular bidder j who was allocated spectrum and thus belongs to M. Then the aggregate bid generated from the optimal allocation M minus the bid of bidder j is given by

$$\sum_{i \neq j, i \in M, j \in M} x_i \tag{4}$$

Now consider that bidder j does not exist and the auction is among the remaining $L-1$ bidders. Let the optimal allocation be denoted by M^*. The aggregate bid generated in this case is

$$\sum_{i \neq j, i \in M^*, j \notin M^*} x_i \tag{5}$$

Therefore, minimum winning bid of bidder j must be at least greater than

$$X_j = \sum_{i \neq j, i \in M^* j \notin M^*} x_i - \sum_{i \neq j, i \in M j \in M} x_i \tag{6}$$

Thus, bidder j's request is granted if $x_j > X_j$ and not granted if $x_j < X_j$. If $x_j = X_j$, bidder j is indifferent between winning and loosing. Note that, the model under consideration is a non-uniform-price auction and X_j is not generally the same for all bidders.

Though equation (6) gives the winning bid for bidder j, it is not necessary that bidder j will be able to afford it. There exists a price threshold (bidder's reservation price) beyond which a bidder is simply unwilling to pay.

Bidder's Reservation Price: Bidder's reservation price is defined as the most a bidder would be willing to pay. When a service provider buys spectrum from the spectrum broker, the service provider needs to sell that spectrum in form of services to the end users who pay for these services. The revenue thus generated helps the provider to pay for the fixed (static) cost for the statically assigned spectrum and the extra spectrum that the provider might need from the CAB. If the total revenue generated from the users is R and R_{static} goes towards the fixed cost, then the difference, $R_{dynamic}$, is the maximum amount that the provider can afford for the extra spectrum from CAB i.e.,

$$R_{dynamic} = R - R_{static} \tag{7}$$

Note, $R_{dynamic}$ is *not* the bidder's reservation price but is a prime factor that governs this reservation price.

Lemma 1: *In second price knapsack auction, dominant strategy of the bidder is to bid bidder's reservation price.*

Proof: Let us assume jth bidder has the demand tuple $q_j = \{w_j, x_j\}$ and its reservation price for that amount of spectrum requested be r_j. Now, as shown above in equation 6, jth bidder's request will be granted and consequently belong to optimal allocation M, only if bid generated by jth bidder is at least X_j. Then according to the second price bidding policy, jth bidder will pay the second price which is X_j in this case. Then the payoff obtained by jth bidder is,

$$E_j = r_j - X_j \tag{8}$$

Through proof by contradiction, we show that jth bidder's true bid is its reservation price r_j.

We assume that jth bidder does not bid its true evaluation of the spectrum requested, i.e., $x_j \neq r_j$. Accordingly bidder j has two options of choosing x_j.

Option 1: Bid is less than the reservation price, i.e., $x_j < r_j$. The values of x_j, r_j and X_j are such that,

- $r_j > x_j > X_j$, then bidder j falls inside the optimal allocation M and its request is granted. The expected payoff obtained by jth bidder is still given by: $(r_j - X_j)$.
- $r_j > X_j > x_j$, then bidder j loses and its request is not granted. Accordingly, the expected payoff becomes 0.
- $X_j > r_j > x_j$, bidder j still loses and the expected payoff is again 0.

Option 2: Bid is more than the reservation price, i.e., $x_j > r_j$. The values of x_j, r_j and X_j are such that,

- $x_j > r_j > X_j$, then bidder j falls inside the optimal allocation M and its request is granted. The expected payoff

obtained by jth bidder is still given by: $(r_j - X_j)$.
• $x_j > X_j > r_j$, though bidder j wins but the expected payoff becomes negative in this case. The expected payoff obtained by jth bidder is given by: $(r_j - X_j) < 0$. Bidder j will not be interested in this scenario.
• $X_j > x_j > r_j$, bidder j loses and the expected payoff is again 0.

It is evident that if bidder j wins, then the maximum expected payoff is given by $E_j = r_j - X_j$ and bidding any other price (higher or lower) than its reservation price r_j will not increase payoff. Thus, the dominant strategy of a bidder in second price bidding under knapsack model is to bid its reservation price. ■

Comments: Our result corroborates with the result shown in other contexts in the economics literature, e.g., in Clarke's tax [6]. Thus it is clear that bidders have no option of manipulating this auction. Note that, we focus on the above auction model only if the total demand of additional spectrum exceeds the capacity of CAB, otherwise, auction might not be the best solution for this problem.

Lemma 2: *In first price bidding, reservation price is the upper bidding threshold.*

Proof: Contrary to the Lemma 1, in first price bidding, the expected payoff obtained by jth bidder can be given by, $E_j = r_j - x_j$, as the actual price paid by the bidder is the same as the bid. Then, to increase the expected payoff, i.e., to keep $E_j > 0$, x_j must be less than r_j.

Again at the same time, to win, bid x_j must be greater than X_j (equation 6). Thus the dominant strategy for the bidder in first price auction is to bid less than the reservation price. ■

IV. Service Provisioning using Games

In this section, we consider the most generic abstraction of "always greedy and profit seeking" model that exists between WSPs and end-users. The WSPs compete among themselves to provide service to a common pool of users. The resource for the WSPs are spectrum chunks that have been statically allocated and the additional spectrum that they buy as discussed in section III. Users on the other hand select service providers depending on the benefit they obtain for the prices they pay. Let us discuss the conflict that arises between the WSPs and the users.

A. Conflict model

We consider the model as shown in the lower half of figure 1, where any user can access any WSP. The users are the potential buyers who buy services from the WSPs. The selection of a WSP is done on a dynamic basis i.e., a user compares the offerings both in terms of QoS and price for a particular service. Once a service is completed, the user relinquishes the radio resources. As the prices offered are not static, the users do not have any information about other users' strategies i.e., demand for resources or price willingness to pay. In such an incomplete information scenario, the benefit of a user depends not only on its own strategy but also on what others do. Since we assume that every user is selfish, the problem is modeled as a non–cooperative game.

Service providers, very much like the users, also act in their self-interest. As a seller of the services, they determine the price for its services depending on the amount of spectrum acquired and the price paid. Similar to the non-cooperative incomplete information game among the users, the service providers also do not have any information about other providers' strategies, such as, price assigned for services, alloted resource, remaining resource, existing load, etc. Based on this conflict model, we need to define the decisions that we need to make. First, let us state the assumptions.

Assumptions: The devices carried by the users have the capability of connecting to any wireless service providers. The WSPs are selected on a session by session basis. For every session, a user chooses one of multiple service providers that has the capability of providing the resource (bandwidth) demanded by the application.

B. Decision Model

As a user, the decision problem is to select the best service provider for the session requested. Now the question arises, how to select the best service provider or rather what criteria determines the best. The quality of service perceived by a user in a network must be considered in this regard. As quality of service depends on the traffic load and the pricing strategies, we must therefore perform a cost benefit analysis to find the best service provider. A natural question that arises in such settings is the existence of an equilibrium where no user will find it beneficial to change the strategy unilaterally. This by definition is known as Nash equilibrium [15].

As a service provider, the decision problem is to advertise a price for a service without knowing what prices are being advertised by its competitors. The optimization is to find a price such that the provider is able to sustain profit in spite of offering a low price i.e., is there any price threshold to reach Nash equilibrium? For finding the existence of Nash equilibrium, we define the preference of the providers and users – given by their utility functions.

C. Utility Function

An utility function is a mathematical characterization that represents the benefits and cost incurred. Here, we are defining the utility functions for both WSP and users.

We consider L service providers that cater to a common pool of $\mathcal{N}$ users. Let the price per unit of resource advertised by the service provider j, $1 \leq j \leq L$, at time t be $p_j(t)$. Let $b_{ij}(t)$ be the resource consumed by user i, $1 \leq i \leq \mathcal{N}$, served by provider j. We further assume that the total resource (capacity) of provider j is C_j.

The utility obtained by user i under the provider j can be given by [22],

$$u_{ij}(t) = a_{ij} log(1 + b_{ij}(t)) \tag{9}$$

where, the coefficient a_{ij} is a positive parameter that indicates the relative importance of benefit and acts as a weightage factor.

Note that, we could have chosen any other form for the utility that increases with $b_{ij}(t)$. But we chose the log function because the benefit increases quickly from zero as the total throughput increases from zero and then increases slowly. This reflects the intuition that the initial increase in the perceived throughput is more important to a user. Moreover, log function is analytically convenient, increasing, strictly concave and continuously differentiable.

Next, we consider the cost components incurred by user. The first cost component is the direct cost paid to the provider for obtaining $b_{ij}(t)$ amount of resource. If $p_j(t)$ is the price per unit of resource, then the direct cost paid to the jth provider is given by,

$$p_j(t)b_{ij}(t) \tag{10}$$

This direct cost component decreases user i's utility. Note that in expression (10), both price per unit resource and the resource amount requested are variables.

The second cost component incurred by the user is the perceived quality of service, one of the manifestations of which is the queuing delay which again depends on the resources consumed by the other users. We assume the queuing process to be $M/M/1$ at the links. Thus, the delay cost component can be written as

$$\begin{cases} \xi(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}) & if \sum_i^{N_j} b_{ij}(t) < C_j \\ \infty & if \sum_i^{N_j} b_{ij}(t) \geq C_j \end{cases} \tag{11}$$

where N_j is the number of users currently served by provider j and $\xi(\cdot)$ is a mapping cost function of delay.

Combining all the components obtained in equations (9), (10), and (11), we get the net utility as

$$U_{ij}(t) = u_{ij}(t) - p_j(t)b_{ij}(t) - \xi(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}) \tag{12}$$

We also obtain the utility as obtained by the service providers. The utility of service provider j at time t is,

$$V_j(t) = p_j(t) \sum_i^{N_j} b_{ij}(t) - K_j \tag{13}$$

where, K_j is the cost incurred to provider j for maintaining network resources. For the sake of simplicity, we assume this cost to be constant.

D. Price Threshold

Now, we investigate the price constraint from the users' and providers' point of view to study the existence of Nash equilibrium. Consider user i has a certain resource demand and wants to connect to a provider at time t. All the providers advertise their price per unit of resource amount and the existing load. As user i wants to maximize his net utility (potential benefit minus cost incurred), he computes the resource vector that would maximize utilities from all the providers and the corresponding maximized utility vector.

User i would then connect to provider j if $U_{ij}(t)$ gives the maximum value in the maximized utility vector, $\{U_{i1}(t), U_{i2}(t), \cdots, U_{iL}(t)\}$, and $b_{ij}(t)$ is the requested resource amount from the optimal resource vector, $\{b_{i1}(t), b_{i2}(t), \cdots, b_{iL}(t)\}$.

Let us investigate if there exists any optimal resource amount for the users and any pricing bound from the providers that will maximize the users net utility. To do so we need to find whether the net utility given in equation (12) can be maximized with respect to the resource amount. If so, then a unique maximization point exists for $U_{ij}(t)$ with respect to $b_{ij}(t)$. Differentiating equation (12) with respect to $b_{ij}(t)$,

$$U'_{ij}(t) = \frac{a_{ij}}{1 + b_{ij}(t)} - p_j(t) - \xi'(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}) \tag{14}$$

Similarly, the second derivative is

$$U''_{ij}(t) = -\frac{a_{ij}}{(1 + b_{ij}(t))^2} - \xi''(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}) \tag{15}$$

If we assume delay and congestion component, such that, $\xi''(\frac{1}{C_j - \sum_i^{N_j} b_{ij}(t)}) > 0$, then, $U''_{ij}(t) < 0$ and it is clear that $U_{ij}(t)$ is strictly concave in the region bounded by $\sum_i^{N_j} b_{ij}(t) = C_j$; and $U_{ij}(t) \to -\infty$ as $\sum_i^{N_j} b_{ij}(t) \to C_j$. Moreover, it can be inferred from equation (15) that as $U''_{ij}(t) < 0$, $U_{ij}(t)$ contains a unique maximization point. Thus, equating equation (14) to 0, and solving for $b_{ij}(t)$ gives the optimal amount of resources needed by the users for a certain price $p_j(t)$ and this resource amount will maximize the utility of the user. From the reverse point of view, it is also clear from the above equation (14) that there exists a maximum threshold for the price $p_j(t)$.

As the users are homogeneous, to maximize users' utility, first derivative of all the users can be equated to zero,

$$U'_{1j}(t) = U'_{2j}(t) = \cdots = U'_{N_j j}(t) = 0 \tag{16}$$

Recall, N_j is the number of users currently served by provider j. Thus equation (16) reduces to,

$$\frac{a_{1j}}{1 + b_{1j}(t)} = \frac{a_{2j}}{1 + b_{2j}(t)} = \cdots = \frac{a_{N_j j}}{1 + b_{N_j j}(t)} \tag{17}$$

If $1 + b_{ij}(t) = m_{ij}(t)$ and with the help of identity, we get,

$$\frac{a_{ij}}{m_{ij}(t)} = \frac{\sum_i^{N_j} a_{ij}}{\sum_i^{N_j} m_{ij}(t)} \tag{18}$$

For notational simplicity, we represent $a_{Ij} = \sum_i^{N_j} a_{ij}$ and $m_{Ij}(t) = \sum_i^{N_j} m_{ij}(t)$. Thus, equation (18) can be written as

$$\frac{a_{ij}}{m_{ij}(t)} = \frac{a_{Ij}}{m_{Ij}(t)} \tag{19}$$

Putting the above form into equation (14), we get

$$U'_{ij}(t) = \frac{a_{Ij}}{m_{Ij}(t)} - p_j(t) - \xi'(\frac{1}{C_j + N_j - m_{Ij}(t)}) \tag{20}$$

Note $U'_{ij}(t)$ is strictly decreasing with the values of $m_{Ij}(t)$ lying in the interval $(C_j, C_j + N_j)$. Then for achieving the

Nash equilibrium by the providers, the pricing constraint $p_j(t)$ is upper bounded by,

$$\frac{a_{Ij}}{m_{Ij}(t)} - \xi'(\frac{1}{C_j + N_j - m_{Ij}(t)}) \tag{21}$$

This pricing upper bound helps the provider to reach the Nash equilibrium. If all of the other providers and users keep their strategies unchanged, and a provider changes its strategy unilaterally and decides not to maintain its pricing upper bound, then that provider will not be able to maximize its users' utility and thus users will not connect to this provider decreasing provider's revenue.

V. Estimating the Demand for Bandwidth

The amount of extra (dynamic) spectrum that a provider needs, depends on the demand for services by the users it supports. Therefore it is essential to estimate the resources consumed by the users and the price that is recovered from them. These estimates will help a provider determine the tuple $q_i = \{w_i, x_i\}$.

Our objective is to maximize provider's net utility, $V_j(t)$ subject to the constraint given by equation (21). Replacing $\sum_i^{N_j} b_{ij}(t)$ by $m_{Ij}(t) - N_j$, we get,

$$V_j(t) = \Big(\frac{a_{Ij}}{m_{Ij}(t)} - \xi'(\frac{1}{C_j + N_j - m_{Ij}(t)})\Big)(m_{Ij}(t) - N_j) - K_j \tag{22}$$

Differentiating equation (22) with respect to $m_{Ij}(t)$, we get,

$$V_j'(t) = \Big(\frac{a_{Ij}}{m_{Ij}(t)} - \xi'(\frac{1}{C_j + N_j - m_{Ij}(t)})\Big) + \Big(-\frac{a_{Ij}}{(m_{Ij}(t))^2} - \xi''(\frac{1}{C_j + N_j - m_{Ij}(t)})\Big)(m_{Ij}(t) - N_j) \tag{23}$$

Differentiating again, and studying the expression for $V_j''(t)$, we get, $V_j''(t) < 0$; which implies that utility for the providers has a maximization point obeying the pricing bound.

For finding the maxima, we equate equation (23) to 0, which gives the optimal value of $m_{Ij}(t)$. Equation (23) is not in closed form because the exact nature of $\xi(\cdot)$ is not known. We assume the solution of the above equation to be $m_{Ij(opt)}(t)$. Of course, for a given $\xi(\cdot)$, the value of $m_{Ij(opt)}(t)$ can always be obtained.

The optimal price that will maximize provider j's utility can be obtained by substituting $m_{Ij(opt)}(t)$ in equation (21). Thus, we get the optimal price as

$$p_{j(opt)}(t) = \frac{a_{Ij}}{m_{Ij(opt)}(t)} - \xi'(\frac{1}{C_j + N_j - m_{Ij(opt)}(t)}) \tag{24}$$

Note that, $p_{j(opt)}(t)$ is clearly dependent on N_j.

To have a better insight into the analysis, we assume a simple closed form of $\xi(\frac{1}{C_j+N_j-m_{Ij}(t)})$ as $\frac{1}{(C_j+N_j-m_{Ij}(t))^\alpha}$, where α is a power coefficient in the delay and congestion component. While taking an exact form of $\xi(\cdot)$, we made sure that it satisfies the constraint of its 1st, 2nd and 3rd derivatives to be positive. Any other form of $\xi(\cdot)$ would also suffice if the derivatives are positive. Rewriting equation (23), we get

$$V_j'(t) = \Big(\frac{a_{Ij}}{m_{Ij}(t)} - \frac{\alpha}{(C_j + N_j - m_{Ij}(t))^{\alpha+1}}\Big) + \Big(-\frac{a_{Ij}}{(m_{Ij}(t))^2} - \frac{\alpha(\alpha+1)}{(C_j + N_j - m_{Ij}(t))^{\alpha+2}}\Big)(m_{Ij}(t) - N_j) \tag{25}$$

Equating equation (25) to 0, we can find the solution of $m_{Ij}(t)$ for finding the maxima. It can be seen that the equation is not in its closed form. Thus to solve the equation, we consider a special case.

Special case: We assume $\alpha = 1$ and $N_j = C_j$ and equate equation (25) to 0 to obtain

$$(2C_j - m_{Ij}(t))\sqrt[3]{a_{Ij}C_j} = m_{Ij}(t) \tag{26}$$

Solving the above equation for optimal $m_{Ij}(t)$, we get, $m_{Ij(opt)}(t) = \frac{2C_j\theta}{1+\theta}$, where, $\theta = \sqrt[3]{a_{Ij}C_j}$.

Using the optimal value of $m_{Ij}(t)$, we get the optimal value of $p_j(t)$ as

$$p_{j(opt)}(t) = \frac{a_{Ij}}{2C_j}\Big(1 + \frac{1}{\theta}\Big) - \Big(\frac{1+\theta}{2C_j}\Big)^2 \tag{27}$$

Thus, we see that the providers can achieve Nash equilibrium under the given pricing constraint and at the same time they can maximize their utility if the price is set as given by equation (27). Next, we use this pricing strategy as an incentive for the providers to upgrade their resources and users to improve their utility.

A. Pricing as an Incentive

With the strategies to determine the prices and the expected profit known, let us investigate if there is any incentive for the providers to upgrade their radio/network resources, and if this additional resource provides any incentive to the users too.

Substituting $m_{Ij(opt)}(t)$ in equation (19), we get

$$m_{ij(opt)}(t) = \frac{a_{ij}}{a_{Ij}}\Big(\frac{2C_j\theta}{1+\theta}\Big) \tag{28}$$

We know, $m_{ij}(t) = 1 + b_{ij}(t)$; the optimal resource consumed by user i under provider j is given by

$$b_{ij(opt)}(t) = \frac{a_{ij}}{a_{Ij}}\Big(\frac{2C_j}{1+\frac{1}{\theta}}\Big) - 1 \tag{29}$$

Thus, the optimal amount of resources for a provider to be demanded from a spectrum broker in the equilibrium can be given by $\sum_i b_{ij(opt)}(t)$. Moreover, the utility of provider j can be written as $p_{j(opt)}(t)\sum_i b_{ij(opt)}(t)$.

Note, by using a proper transformation function (which is beyond the scope of this research), the total utility of provider j can be converted to a dollar value denoted by R (refer equation 7) – the total revenue obtained by provider j.

$$R = T\bigg(p_{j(opt)}(t)\sum_i b_{ij(opt)}(t) - K_j\bigg) \tag{30}$$

where $T(\cdot)$ is some transformation function.

Thus, provider's reservation price is governed by $T\bigg(p_{j(opt)}(t)\sum_i b_{ij(opt)}(t) - K_j\bigg) - R_{static}$.

VI. Numerical Results and Interpretation

We present our results in this section. In section VI-A, we simulate our auction model and show how the knapsack auction outperforms the classical highest bid auction models. Later we model the interaction between WSPs and users.

A. Spectrum Auctioning

The main factors that we consider for demonstrating the performance of the proposed knapsack auction are: revenue generated by spectrum broker, total spectrum usage, and probability of winning for bidders. For the simulation model we follow second price sealed-bid mechanism. We could have chosen the first price bidding policy; the only reason for choosing second price policy is that it has more properties than first price in terms of uncertainty [20]. We assume that all the bidders are present for all the auction rounds; bidders take feedback from previous rounds and generate the bid tuple for next round. The bid tuple q_i generated by bidder i consists of i) amount of spectrum requested, w_i and ii) the price the bidder is willing to pay, x_i.

For simulation purpose, the parameters considered are as follows. Total amount of spectrum in CAB is assumed as 100 units, whereas min. and max. amount of spectrum requested by each bidder is 11 and 50 units respectively. Min. bid per unit of spectrum is considered as 25 unit.

Revenue and spectrum usage: Figures 2 and 3 compare revenue and spectrum usage for two strategies for each auction round. The number of bidders considered is 10. Note that, both revenue and usage are low in the beginning and subsequently increases with rounds. In the initial rounds, bidders are dubious and make low bids. With increase in rounds, potential bidders emerged as expected and raised the generated revenue. We observe that the proposed auction generates 10-15% more revenue compared to the classical model and also reaches steady state faster.

Bidder participation: In figures 4 and 5, we look at our auction model from the bidders' perspective. Higher revenue requires high participation in number of bidders. However, classical auctions always favor bidders with high spectrum request and/or high bid, thus discouraging low potential bidders and giving the higher potential bidders a chance to control the auction. In order to evaluate the bidder participation, we consider two cases: a) bidder with the lowest spectrum request and b) bidder with the lowest bid. For these two cases, we compare the two strategies in terms of probabilities to win a bid. We observe that the proposed auction strategy has a significantly high probability of winning compared to classical strategy. Note that probability of winning in classical strategy almost reaches zero with increase in bidders.

Collusion prevention: The occurrence of collusion must be prevented in any good auction so that a subset of bidders can not control the auction that might decrease the spectrum broker's revenue. We consider two cases: i) when bidders collude and ii) when bidders do not collude. In our simulation model, we assume bidders randomly collude in pair in all possible combinations with others.

In figure 6, we show the average revenue generated by spectrum broker with increase in number of bidders both in presence and absence of collusion. Though at the beginning with less number of bidders, presence of collusion reduces the average revenue slightly, but with increase in number of bidders the effect due to collusion decreases. Thus with increase in number of bidders, i.e., with increase in (perfect) competition, revenue generated even in the presence of collusion reaches almost the same value as that of without collusion. Figure 7 presents the usage of spectrum in the presence and absence of collusion. The most interesting result from bidders' perspective is shown in figure 8. When the number of bidders is low (less than or equal to 4 in our case) collusion provides better probability of winning but as the number of bidders increases, probability of winning with the help of collusion decreases, discouraging bidders to collude.

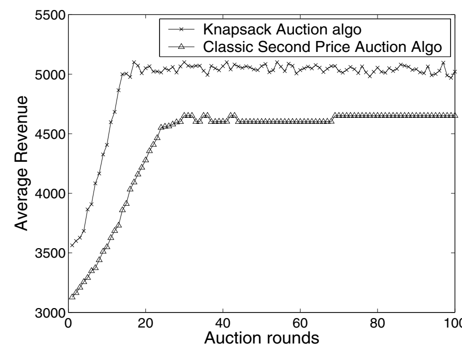

Fig. 2. Revenue Maximization with auction rounds (with 10 bidders)

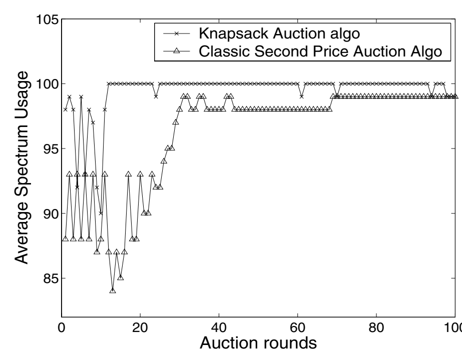

Fig. 3. Usage Maximization with auction rounds (with 10 bidders)

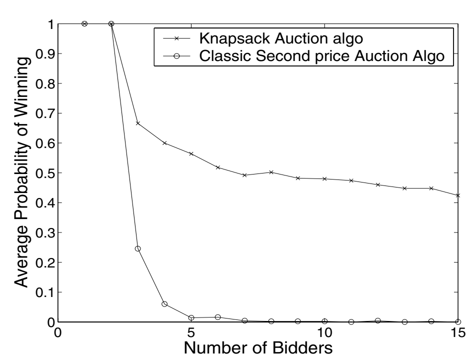

Fig. 4. Probability of winning with lowest spectrum request

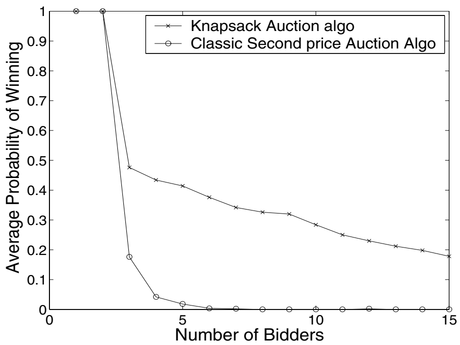

Fig. 5. Probability of winning with lowest value bid

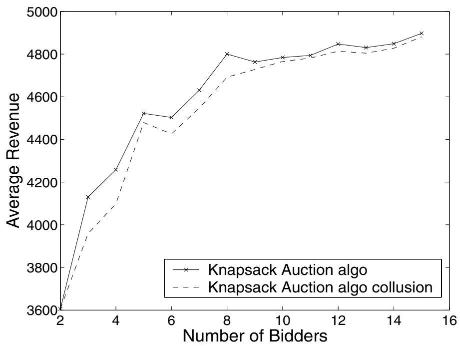

Fig. 6. Avg Revenue with and without collusion

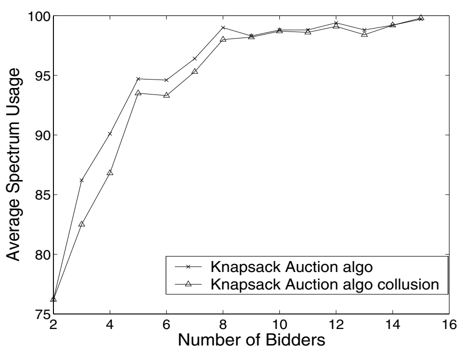

Fig. 7. Avg Usage with and without collusion

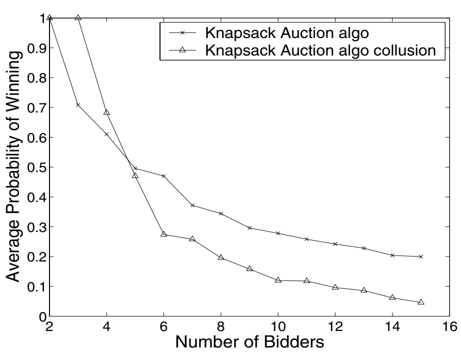

Fig. 8. Avg probability of winning with and without collusion

B. Pricing: Numerical results

Here, we provide some insights on how the pricing strategies proposed for WSP and end users interaction work as incentives for both. We consider two cases – fixed and increasing number of users.

• **Fixed number of users:** We keep the number of users fixed with the total resource of the provider increasing. Recapitulate from equations (27) and (29) that increasing resource implies increasing C_j and fixed number of users implies a fixed value of a_{Ij}. We consider all users have equal weightage factor $a_{ij} = 1.5$ and the value of C_j varies from 1 to 100 units. These values used for obtaining the numerical results are arbitrary and are merely for the sake of demonstration. Any other values of a_{ij} and C_j can be used as long as they satisfy the constraint that the price per unit resource is positive.

Figure 9 shows how the provider must decrease the price per unit of resource if the total amount of resources increases with the same user base. This decrease in unit price is necessary if resource utilization is to be maximized which also serves as an incentive for the users.

The total profit of the provider is presented in figure 10. With the number of users fixed, we observe that the total profit of the provider increases till a certain resource and then decreases. For a fixed number of users, this result allows us to estimate the amount of resource that the provider must have such that its profit is maximized.

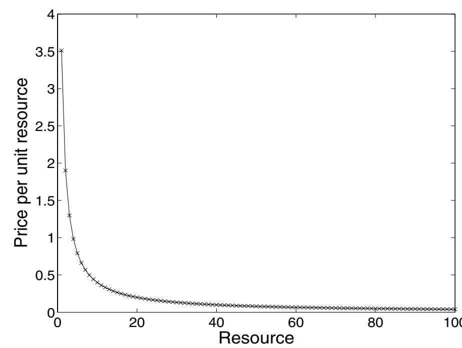

Fig. 9. Price per unit of resource vs. resource (with number of users fixed)

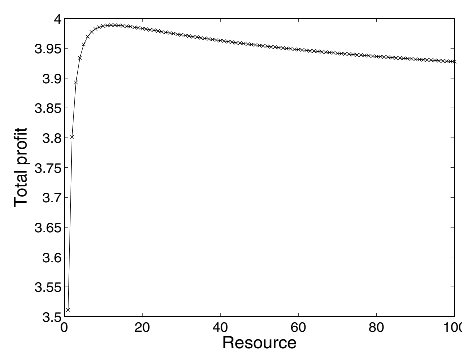

Fig. 10. Total profit of a provider vs. resource (with number of users fixed)

• **Increasing number of users:** The number of users (population base) is increasing in this case which is typical of any market. We start with 1 user under a provider. For fair comparison with the previous case (i.e., with fixed number of users), we increase resources from 1 to 100 units. Note that,

a_{Ij} is no longer fixed and increases with increasing number of users. For this simulation, we assume that the increase in number of users is such that the ratio of a_{Ij} and C_j is fixed.

In figure 11, the price per unit of resource is presented. As the initial number of users is very low, increasing resource necessitates an initial increase in price per unit of resource. But as the number of users increase, it is imperative that price per unit resource decreases providing incentive for the users.

In figure 12, we present the total profit of the provider. Unlike the previous case (figure 10), we see that with users increasing proportionally with resources, the total profit is always increasing which presents a better incentive for the providers than the case with fixed number of users. Note, the linear increase in profit is just due to the assumption: the ratio between the users and resources is fixed.

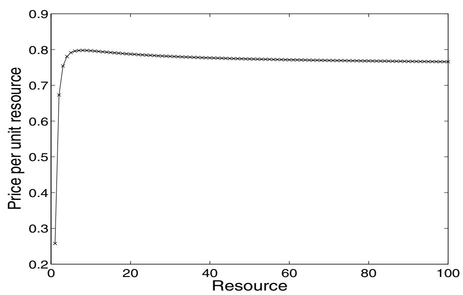

Fig. 11. Price per unit of resource vs. resource (with increasing number of users such that the ratio of a_{Ij} and C_j is fixed)

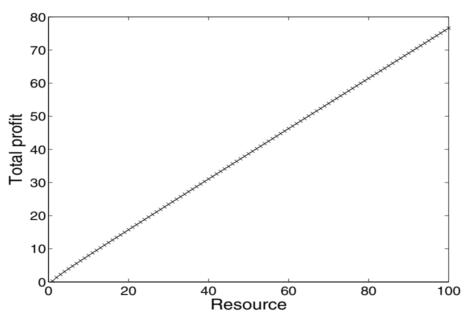

Fig. 12. Total profit of a provider vs. resource (with increasing number of users such that the ratio of a_{Ij} and C_j is fixed)

VII. Conclusions

Dynamic spectrum allocation coupled with fine granularity switching of services by end–users will engender a flexible and competitive environment for trading wireless services. In this research, we provide a framework based on auction and game theories that capture the interaction among spectrum broker, service providers, and end–users in a multi–provider setting. We propose a winner determining sealed bid knapsack auction that dynamically allocates spectrum from CAB and at the same time maximizes revenue generated, entices WSPs by increasing their probability of winning, and prevents collusion. Utility functions are formed for WSPs and users modeling their conflicts and existence of Nash equilibrium is shown under certain threshold conditions. We also show how proper pricing can provide incentives to providers to upgrade their resources and users to opt for better services.

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