MODELING UNDESIRABLE FACTORS IN EFFICIENCY MEASUREMENT USING DATA ENVELOPMENT ANALYSIS: A REVIEW

NOOR ASIAH RAMLI¹,²* AND SUSILA MUNISAMY¹

¹Faculty of Economics and Administration, University of Malaya, Kuala Lumpur, Malaysia. ²Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Shah Alam Selangor, Malaysia.

*Corresponding author: asiahramli@yahoo.com

Abstract: This paper presents a review on the development of efficiency measurement approaches incorporating undesirable outputs in the Data Envelopment Analysis (DEA) framework. The review is based on the literature analysis in DEA with the classification of the literature in two orientations – a methodological orientation and an empirical orientation. For the methodological orientation, two categories of direct and indirect approaches have been discussed extensively. As for the empirical orientation, the discussion brings up some empirical studies relating to environmental efficiency measurement.

KEYWORDS: Data envelopment analysis, environmental efficiency, undesirable outputs

Introduction

Conventional production theory views an organization as a production system in which input is the resource utilized by the organization and transformed into output. Economic efficiency based on production theory, implies that organizations should structure their outputs to achieve the lowest possible use of inputs. Pareto-Koopman’s description of efficiency provides explanation that “the performance of a decision making unit (DMUs) is efficient if and only if it is not possible to improve any input or output without worsening any other input or output” (Cooper et al., 2006). To estimate efficiency, the Data Envelopment Analysis (DEA) approach can be applied. DEA is a technique to measure the relative efficiency of a set of DMUs in their use of multiple inputs to produce multiple outputs.

The conventional formulation for efficiency measurement in DEA is based on the isotonicity property that is, increased input may reduce the efficiency while increased output may increase the efficiency (Dyson et al., 2001; Golany and Roll, 1989). In real cases, the circumstances are more complicated where increased output may also reduce the efficiency while increased input may also increase the efficiency. For instance, an increase in the emission of pollutant which is an undesirable output in production activities will decrease the production efficiency. Efficiency measurement that includes undesirable outputs such as emissions is referred to as environmental efficiency. Advanced DEA techniques enable the measurement of the efficiency while incorporating the undesirable output in the analysis.

This paper aims to provide a concise review on the development of efficiency measurement, specifically in the DEA framework, focusing on undesirable factors i.e. pollutants. Environmental pollution in production activities has drawn much attention as more and more researchers are concentrating on the issue of environmental sustainability in their studies. The methodological plan is to discuss the literature from two perspectives - the theoretical development and the progress in the empirical research. The discussion starts with detailing the various environmental efficiency measurement techniques incorporating undesirable outputs.

1 In the case of undesirable output that do not impact the environment, the efficiency is considered to be operational or technical efficiency.
in the DEA framework. Next, the discussion moves on to describing the empirical studies on environmental efficiency measurement. The conclusions are drawn in the final section. This review contributes to the literature by summarizing the current state of knowledge in this area as well as integrating the methodological orientation and the empirical orientation in the earlier literature.

Theoretical Orientation

In DEA efficiency measurement, there are several approaches to handle undesirable outputs. An evaluation pertaining to this topic has been discussed previously by several researchers. See for example; Hua and Bian (2007), Sueyoshi and Goto (2011a) and Bian (2008) for a review on this. However, their reviews are more on a comparison of the selected efficiency measurement technique rather than a discussion on the development of the various approaches. Since numerous approaches have been proposed in recent times, there is a need to gather and review the development of these approaches.

Before proceeding, it is important to understand the DEA efficiency measurement method. DEA is based on linear programming introduced by Charnes et al. (1978). DEA identifies a subset of efficient ‘best practice’ DMUs and, for the remaining DMUs, their efficiency level is derived from the comparison with a frontier constructed from the ‘best practice’ DMUs. The mathematical formulation for DEA is as follows:

Max $\theta_m$

Subject to

$\sum_{n=1}^{N} z_n x_{in} \leq x_{im}; \quad i = 1, 2, ..., l$

$\sum_{n=1}^{N} z_n y_{jn} \geq \theta_m y_{jm}; \quad j = 1, 2, ..., J$

$z_n \geq 0; \quad n = 1, 2, ..., N$

(1)

Where $z_n$ = intensity variables, $x_{in}$ = $i^{th}$ input of the $n^{th}$ DMU, $y_{jn}$ = $j^{th}$ desirable output of the $n^{th}$ DMU, $x_{im}$ = $i^{th}$ input of the $m^{th}$ DMU, $y_{jm}$ = $j^{th}$ desirable output of the $m^{th}$ DMU and DMUs. The DEA output oriented envelopment model seeks a set of $z$ values which maximize the $\theta_m$ and identifies a point within the production possibilities set whereby output levels of DMU $m$ can be increased as high as possible proportion while input remain at current level (Charnes et al., 1978).

Various Approaches in DEA Efficiency Measurement with Undesirable Output

The approaches for incorporating undesirable output into the DEA model can be divided into two categories which are indirect and direct approaches (Scheel, 2001). The indirect approach means that the data of the undesirable output variables are transformed into the desirable output. This approach manipulates the undesirable output value so that they can be included in the standard DEA model with the desirable output. In contrast, the direct approach means that the undesirable output data are applied directly into the modification of the DEA model in order to treat the undesirable output appropriately. In DEA efficiency measurement, there are two types of measures, namely, radial and non-radial. According to Zhu (1996), radial measures are the models that adjust all inputs, or alternatively all outputs of a DMU by the same proportion, whereas, a non-radial DEA measure allows for non-proportional reductions in each positive input, or augmentations in each positive output (i.e. Russell measure). The general structure of the efficiency measurement approach with undesirable output is illustrated in Figure 1. The classification of various approach are discussed in detail below.

Indirect Approaches

The first indirect approach is by transforming undesirable outputs with the additive inverse method whereby the value of undesirable outputs is multiplied by -1 (Koopmans, 1951). The second indirect approach considers undesirable outputs as inputs. The undesirable outputs variable are moved from the output side to the input side of the model (Tyteca, 1997). However, treating the undesirable outputs as input opposes the physical laws and standard production theory. It also leads to conceptual
confusion and will not reflect the true production process in DEA result (Seiford and Zhu, 2002). The next approach is the translation invariance (Iqbal Ali and Seiford, 1990) where a large scalar is added to each of the undesirable output values, such that the resulting output values are positive. The drawback of this approach is that it moves the zero to a different position and the choice of the scalar can alter the efficient frontier. Another approach is the multiplicative inverse whereby each undesirable output is incorporated as desirable output using their reciprocal (Golany and Roll, 1989). The drawback of this approach is that it destroys the ratio of the interval scales of the original data.

Scheel (2001) then suggested that the undesirable outputs to be incorporated with the desirable outputs with a negative sign differentiating the undesirable outputs. However, in this “non-separating” outputs approach, the efficiency scores and ranking are different from the measurement of the efficiency score with separation on the output approach. This approach is limited to a situation where there is only one negative output or undesirable output. Another alternative approach is by employing a linear monotone decreasing transformation (LMDT) (Seiford and Zhu, 2002), whereby all the undesirable outputs are multiplied by –1 and added with a proper translation vector for the undesirable outputs. This translation will transform negative data to non-negative data. In many real-life applications, the transformation of the data may no longer make sense. This motivates researchers to explore direct approaches where no transformation is done to the data set.

**Direct Approaches**

*Hyperbolic Efficiency (HE) Model*

This direct approach attempts to modify the multilateral productivity using the HE measure, which treats desirable and undesirable output differently (Färe *et al.*, 1989). The concept of this approach is to increase the desirable output which is strongly disposable (i.e., the waste can be released without cost) and decrease the undesirable output which is weakly disposable (i.e., the waste needs to be released with cost).

The model is formulated as below:

$$
\text{Max } \theta_m \\
\text{Subject to}
\sum_{n=1}^{N} z_n x_{in} \leq \theta_m^{-1} x_{im} ; \quad i = 1,2,...,I
\sum_{n=1}^{N} z_n y_{jm} \geq \theta_m y_{jm} ; \quad j = 1,2,...,J
\sum_{n=1}^{N} z_n b_{kn} \geq \theta_m^{-1} b_{km} ; \quad k = 1,2,...,K
z_n \geq 0 ; \quad n = 1,2,...,N
$$

(2)
Where \( b_{kn} \) = \( k \)th undesirable output of the \( n \)th DMU and \( b_{km} \) = \( k \)th undesirable output of the \( m \)th DMU. The optimal value of \( \theta_m \) measures the same proportion of increasing the desirable outputs and decreasing the undesirable output simultaneously for each DMU \( m \). Since this alternative is based on non-linear programming, the solution to the model is complex.

**Directional Distance Function (DDF) Model**

The concept of the DDF model is to expand desirable outputs and reduce undesirable outputs simultaneously based on a given direction vector (Chung *et al.*, 1997). This approach, which is based on Luenberger’s shortage function (1992) is formulated as below:

Max \( \theta_m \)

Subject to

\[
\begin{align*}
\sum_{n=1}^{N} z_n x_{in} & \leq x_{im} ; \ i = 1,2,\ldots,l \\
\sum_{n=1}^{N} z_n y_{jn} & \geq y_{jm} + \theta_m y_{jm} ; \ j = 1,2,\ldots,l \\
\sum_{n=1}^{N} z_n b_{km} & = b_{km} - \theta_m b_{km} ; \ k = 1,2,\ldots,K \\
z_n & \geq 0 ; \ n = 1,2,\ldots,N
\end{align*}
\]

A proportion seeks to increase the desirable output and reduce the undesirable output simultaneously. However, there is a major drawback of using this model. The direction vector to the production boundary is fixed arbitrarily thus may not provide the best efficiency measures.

**Slack-based Measure (SBM) Model**

Based on the original slack-based measure (SBM) model proposed by Tone (2001), Zhou *et al.* (2006) extended the model so that it can incorporate undesirable output. This model minimize the ratio of the average undesirable output reduction to the average desirable output increment. In this model, are positive multipliers used to compute a linear combination of the DMU formulated as follows:

Max \( \sum_{i=1}^{l} s_i + \sum_{j=1}^{j} s_j + \sum_{k=1}^{K} s_k \)

Subject to

\[
\begin{align*}
\sum_{n=1}^{N} z_n x_{in} + s_i & = x_{im} ; \ i = 1,2,\ldots,l \\
\sum_{n=1}^{N} z_n y_{jn} - s_j & = y_{jm} ; \ j = 1,2,\ldots,l \\
\sum_{n=1}^{N} z_n b_{km} + s_k & = b_{km} ; \ k = 1,2,\ldots,K \\
z_{nj}, s_i, s_j, s_k & \geq 0 ; \ n = 1,2,\ldots,N
\end{align*}
\]

Where \( s_i \) and \( s_j \) are the slack values of the \( i \)th input and \( j \)th desirable output, respectively. This non-radial model is treated as a composite index for modeling economic environmental performance. Compared to radial efficiency measurement, this model provides a higher discriminating power in modeling environmental performance (Zhou *et al.*, 2006).

**Additive Model**

Bian (2008) extended the additive DEA model introduced by Charnes *et al.* (1985) by taking into account the slack variables. A basic additive model has to add another slack variables, which is to release \( k \) undesirable output \( (b_{1j}, b_{2j},\ldots,b_{tj}) \). The undesirable output slack \( () \) follows the same manner as input slack whereby the objective is to maximize the opportunity to improve efficiency in the amount of excesses produced by the DMU in the evaluation of the comparison with other DMUs. The proposed model is as below:

Max \( \sum_{i=1}^{l} s_i + \sum_{j=1}^{j} s_j + \sum_{k=1}^{K} s_k \)

Subject to

\[
\begin{align*}
\sum_{n=1}^{N} z_n x_{in} + s_i & = x_{im} ; \ i = 1,2,\ldots,l \\
\sum_{n=1}^{N} z_n y_{jn} - s_j & = y_{jm} ; \ j = 1,2,\ldots,l \\
\sum_{n=1}^{N} z_n b_{km} + s_k & = b_{km} ; \ k = 1,2,\ldots,K \\
z_{nj}, s_i, s_j, s_k & \geq 0 ; \ n = 1,2,\ldots,N
\end{align*}
\]

Where \( s_i \), and \( s_j \) are the slack values of the \( i \)th input, \( j \)th desirable output and \( k \)th undesirable output, respectively. The advantage of this model is that it does not require any data transformation or user specified direction vectors (Bian, 2008).
Range Adjusted Measure (RAM) Model

More recently, Sueyoshi et al. (2010) extend the basic original model of RAM introduced by Cooper et al. (1999) to integrate undesirable outputs. Sueyoshi et al. introduced two separated DEA models to estimate the operational and environmental performance. They then combined both performances to produce a unified efficiency score. The combined formulation becomes:

\[
\begin{align*}
\text{Max} & \quad \frac{1}{1 + \frac{1}{I} + \frac{1}{K}} \left( \sum_{i=1}^{I} s_{ix}^x + s_{ix}^x - x_{im} ; \ i = 1, 2, ..., I \right) \\
\text{Subject to} & \quad \sum_{m=1}^{N} z_{ni} x_{in} - s_{ix}^x + s_{ix}^x = x_{im} ; \ i = 1, 2, ..., I \\
& \quad \sum_{n=1}^{N} z_{nj} y_{jm} - s_{ij} y_{jm} = y_{jm} ; \ j = 1, 2, ..., J \\
& \quad \sum_{k=1}^{K} z_{nk} b_{kn} + s_{k} b_{km} = b_{km} ; \ k = 1, 2, ..., K \\
& \quad z_{ni}, s_{ix}^{x+}, s_{ix}^{x-}, s_{ij}, s_{k} \geq 0 ; \ n = 1, 2, ..., N 
\end{align*}
\] (6)

The slack variable \((s_{ix}^{x+})\) in the above equation is related to the \(i\)th input with the separation of positive and negative parts \((s_{ix}^{x+} + s_{ix}^{x-})\) while \(s_{ij}\) and \(s_{k}\) refer to the slack variables for desirable output and undesirable output, respectively. The range \((R)\) is calculated by the maximum and minimum value over all \(n\) for each variable. Some drawbacks have been identified by the authors themselves, whereby, its efficiency score is larger than those of the radial DEA models and the results of the efficiency scores obtained are close to unity. This model also fails to provide a valid ranking of performance and it is biased against large DMUs (Sueyoshi et al., 2010).

Due to the drawbacks of the individual efficiency measurement techniques incorporating undesirable outputs discussed above, the new paradigm is to use a combination of models in the DEA approach. Some of them are Portela et al. (2004) who propose the range directional model, Fukuyama and Weber (2010) propose the directional network slack based inefficiency model and Färe and Grosskopf (2010) propose the slack based measure model based on directional distance function.

**Empirical Orientation**

A number of empirical works have been carried out, taking into consideration the undesirable output in efficiency measurement using the DEA approach. A review by Zhou et al. (2008) presents a literature survey on the application of DEA to energy and environmental performance. Another paper by Tyteca (1996) reviewed an analysis of environmental inefficiencies from industrial activities. Inspired by these reviews, this section summarizes a number of papers applying the methods discussed in previous section besides the discussion on the effect of environmental regulation on the environmental efficiency as well as sources of pollution by different industries.

**Application of Various Approaches**

There are various papers evaluating efficiency with undesirable factors. Some of them which are from the indirect approaches such as Athanassopoulos and Thanassoulis (1995) employing multiplicative inverse and Lu and Lo (2007) employing LMDT approach. Further researches utilizing direct approaches include Hernandez-Sancho et al. (2000) and Taskin and Zaim (2001) applying the HE model; Boyd et al. (2002), Färe et al. (2006), Mandal and Madheswaran (2010), Murty and Kumar (2002), Picazo-Tadeo et al. (2005), Picazo-Tadeo et al. (2012), Zhou et al. (2012) applying the DDF model while Li and Hu (2012), Song et al. (2012) and Zhou et al. (2007) applying the SBM model. As for the RAM model, it includes Sueyoshi and Goto (2010; 2011a; 2011b). From all the studies, it is found that the DDF approach is a popular approach among researchers. The reason for this popularity might be because it is simple, intuitive and can be easily put into practice while expanding desirable output and contracting the undesirable output simultaneously.

**The Effect of Environmental Regulation on the Environmental Efficiency**

The reviews of empirical literature indicate a number of studies emphasizing on environmental efficiency measurement towards environmental
regulation. (See for example; Murty et al., 2006; Telle and Larsson, 2007; Hernandez-Sancho et al., 2000; Mandal and Madheswaran, 2010; Sueyoshi et al., 2010; Picazo-Tadeo et al., 2005). The studies by Hernandez-Sancho et al. (2000) and Picazo-Tadeo et al. (2005) evaluated the impact of environmental regulation on Spanish furnishing industry and Spanish ceramic tile industry, respectively. When environmental regulation is assumed, Hernandez-Sancho et al. (2000) found that firms would have to decrease some desirable outputs in order to reduce waste from input resource utilization. This finding is supported by Picazo-Tadeo et al. (2005) where when firms face environmental regulation the potential to increase desirable outputs drops. Another study by Murty et al. (2006) concluded that firms in the sugar industry in India also had to reduce production of sugar or incur additional input cost to reduce pollution according to the environmental regulations. Telle and Larsson (2007) attempt to determine the relationship between environmental regulation and productivity and found that environmental regulation hamper productivity growth.

Contrastingly there are studies arguing that environmental regulation can increase environmental efficiency. Banerjee (2007) discovered environmental efficiency gains during the initial phase of regulation implementation while Wang et al. (2011) and Yörük and Zaim (2008) reported that the regulations have positive affects to the environmental performance.

**A Source of Pollution by Different Industries**

Different industrial sectors emit different types of pollution. The studies by Burnett and Hansen (2008), Cuesta et al. (2009) and Tyteca (1997) used electric utilities for their application. The undesirable output for these analyses is sulphur dioxide (SO₂). Even though SO₂ is not an inclusive indicator of environmental performance, it is an extremely important pollutant in the electric utility industry (Burnett and Hansen, 2008). For cement industry, carbon dioxide (CO₂) has been employed as an undesirable output (Mandal and Madheswaran, 2010; Riccardi et al., 2012) while nitrogen oxides (NOₓ) has been utilized in the glass industry (Boyd et al., 2002; Boyd and Pang, 2000). CO₂ and NOₓ are among the preferred undesirable output variables since CO₂ is a major contributor to global warming while NOₓ reduces plant growth and contributes to acidification and the formation of ground-level ozone (Färe et al., 2004).

Besides element of air pollutant as an undesirable output, there are also examples considering element of water pollutant in the literature. Brännlund et al. (1998), Chung et al. (1997), Hua et al. (2007), Seiford and Zhu (2002) and Telle and Larsson (2007) analyzed the pulp and paper industry in their studies. According to Telle and Larsson (2007), the pulp and paper industry is among the most energy intensive manufacturing sectors and also a major contributor to emissions of water pollutants in Norway releasing biological oxygen demand (BOD) and total suspended solid. Brännlund et al. (1998) also agreed that the pulp and paper industry destroys the marine environment in Sweden. Picazo-Tadeo et al. (2005), Picazo-Tadeo and Prior (2009) and Domazlicky and Weber (2004) found another example of water pollutant which is watery mud released by the ceramic tile industry.

**Conclusion**

This paper provides a review on the development of efficiency measurement with the incorporation of undesirable output, specifically, in the DEA framework. Previous studies proposing new models and the advantages and/or limitations of those models have been discussed in the theoretical section earlier and a review of empirical literature has been conducted subsequently. This paper contributes to the area by reviewing what is already known and the limitations that exist in knowledge about this the area. This review may assist future researchers in choosing a more appropriate method that are in the literature.

One of the most significant findings that has been identified from this review is that the

---

*J. Sustain. Sci. Manage. Volume 8 (1) 2013: 126-135*
body of knowledge is lacking theoretical based studies compared to empirical based studies. Although empirical studies are important, it is hoped that this study can initiate future research, introducing a new outstanding technique while filling the weaknesses of previous models, with regards to efficiency measurement with undesirable output in DEA. New developments on comprehensive efficiency measurement would be a very significant contribution for future research in DEA framework.

In recent times, environmental sustainability has become a major issue regarding global warming and climate change. This phenomenon is reflected in the empirical literature, where the emphasis is on the measurement of environmental efficiency in a variety of industries, since the issue of environmental performance seems very relevant to the element of undesirable output in production activities. However, there are very limited studies with regards to environmental efficiency analysis in the Malaysia context. The topic of environmental efficiency can be considered as a new issue in Malaysia. Thus, this review may help other researchers to establish their study on environmental efficiency in Malaysian context and contribute to the discussion.

Despite the relevance and importance of this review on the theoretical and empirical literature in the area, the limitation of this paper is the complete reliance on previously published research and the availability of these studies. There is no supplemental primary data that is analysed in this paper.

References


