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Abstract

In this paper we propose successive interference cancellers (IC) for variable data rate direct-sequence code division multiple access (DS/CDMA) systems. The performance under Rayleigh fading is analysed for single modulation systems and different systems for handling multiple data rates. One alternative for handling multi-rate systems is to use mixed modulation, which imply that each user chooses modulation format according to need. Therefore, we first analyse an interference cancellation (IC) scheme for M-level rectangular QAM. Another approach is to use parallel channels. Then each user transmits over one or several channels, which are mutually synchronous. We show that the successive IC, for mixed modulation or parallel channel systems, has a performance close to the single BPSK user bound and, consequently, it gives a considerable increase in performance and flexibility compared to both single modulation systems and mixed modulation or parallel channel systems employing a conventional detector.

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1 Introduction

In the future users will demand mobile telephone systems to be able to handle many more services than speech, like e.g. facsimile, Hi-Fi audio and fast transmission of computer data, which is not possible today. To achieve this we need to use a multiple-access method which is flexible and has the prospect of capacity increases and being able to handle variable data rates. Recently Code Division Multiple Access (CDMA) has been suggested to be a multiple-access method able to stand these requirements. The first company to propose this technique for a practical system is Qualcomm Inc in the USA, who are working on a CDMA digital cellular radio communication system.

A Direct-Sequence Code Division Multiple Access (DS/CDMA) system has several unique features. Some of them are spectrum sharing, rejection of multipath signal components or possibility to utilizing them for recombining [1] and frequency reuse factor of one in a cellular scenario [2]. These features are highly desirable, though a CDMA system employing a conventional detector is interference limited, which directly determines the system capacity. The conventional detector is composed of a bank of matched filters, each filter matched to a signature sequence of a user. The filter bank is then succeeded by a symbol-rate sampler and a decision device. This detector is optimal in a single-user channel corrupted only by additive white Gaussian noise (AWGN). The presence of a number of users in the system introduces multiple-access interference (MAI), since the signature sequences used are not perfectly orthogonal, which leads to an irreducible error probability. The performance of the conventional detector in a multiuser system is acceptable if the cross-correlations between the signature sequences are low and the energies of the received signals are nearly equal. In a mobile radio scenario the transmitters move in relation to the receiver and the energies of the received signals are to be neither equal nor constant. In this situation the conventional detector fails to demodulate weak users, even when the cross-correlation between the signals is relatively low. This is known as the near/far problem. One way to combat this problem is to use stringent power control [2]. Another approach would be to use more sophisticated receivers which are near/far resistant. Recently a lot of attention has been directed to the area of multi-user detectors, which has the prospect of mitigating the near/far problem and by cancelling the MAI also increasing the total system capacity.

The research in this area was trigged by Verdu [3], who related the multiple-access channel to a periodically time-varying, single-user, intersymbol interference (ISI) channel and worked on the optimal multi-user detector. The complexity of this detector increases exponentially with the number of users and that has initiated further research in the area of sub-optimal, lower complexity, detectors [4]-[8]. The problem with these sub-optimal detectors is that they are still far too complex, especially schemes using parallel detection [4]-[8]. An alternative to parallel detection, and therefore also parallel interference cancellation, is serial or successive interference cancellation (IC) [9]-[13].

The motivation for our work is then to evaluate an efficient detector for a multi-user and multi-rate DS/CDMA system. In this case, a method to handle multiple data rates would be to let different users use different forms of modulation [14]. A user could, depending on the need, choose between using e.g. BPSK, QPSK or 16-QAM modulation. Therefore, an IC scheme for M-ary QAM is analysed in this paper, based on the IC scheme for coherent BPSK modulation derived by Patel and Holtzman [9]-[11]. The analysis is extended to cover systems where the users employ different QAM formats. Another approach to handle multiple data rates would be to let each user transmit over one or several parallel channels according to requirements [14]. This can of course also be used in combination with different modulation formats.

The operation of the IC scheme is made in the following manner. The signal amplitude of each user is estimated from the output of a linear correlator, where we correlate the received signal with each user's signature sequence. It should be noted that this will not give a perfect estimate of the amplitude though the scheme will be simpler. We then rank the users in decreasing order of their received powers, which are obtained using these correlator outputs. This will give good enough accuracy for ranking the users [11]. The user's signals are decoded and cancelled from the received signal successively starting with the strongest user. In an uplink, where we are interested in all received signals, we will use a scheme where all the users are decoded and cancelled. On the other hand, in a downlink we would perform the successive IC scheme on all the interfering users before decoding the desired signal.

We will consider the coherent case of demodulation and frequency-nonselective fading. The performance measure used is average bit error probability. We assume perfect knowledge of the phase and the time delay. We also assume perfect ranking in the performance analysis, which implies knowledge of the channel gain for each signal. This knowledge is however not used in the IC scheme itself.

2 System Model and Decoder Structure

We consider a system model for a system utilizing square lattice QAM, where the received signal, for a K user system, is

$$r(t) = \sum_{k=1}^{K} \alpha_k \sqrt{\frac{2E_0}{T}} d_k^I(t-\tau_k)\, c_k^I(t-\tau_k) \cos(\omega_c t + \phi_k) + \alpha_k \sqrt{\frac{2E_0}{T}} d_k^Q(t-\tau_k)\, c_k^Q(t-\tau_k) \sin(\omega_c t + \phi_k) + n(t) \quad (1)$$

which is the sum of all the transmitted signals embedded in AWGN. $d_k^{I/Q}(t)$ is a sequence of $A_{k,l}^{I/Q}$ amplitude, rectangular pulses of duration T defined as follows

$$d_k^{I/Q}(t) = \sum_l A_{k,l}^{I/Q} p_T(t-lT) \quad (2)$$

where $p_T(t)$ is the pulse shape

$$p_T(t) = \begin{cases} 1 & 0 \le t < T \\ 0 & \text{elsewhere} \end{cases} \quad (3)$$

T is the inverse of the symbol rate, assumed to be equal for all users. $A_{k,l}^I$ and $A_{k,l}^Q$ are the information-bearing signal amplitudes of the quadrature carriers for the k^{th} user's l^{th} symbol element, which together generate M equiprobable and independent symbols. They take the discrete values

$$A_{k,l}^{I/Q} \in \{-\sqrt{M}+1, -\sqrt{M}+3, \ldots, \sqrt{M}-1\} \quad (4)$$

since it requires $\sqrt{M}$ signal amplitude levels for the In-phase (I) and Quadrature (Q) components to form a signal constellation for M-ary QAM. $2E_0$ is then the energy of the signal with lowest amplitude, see Figure 1. $c_k^{I/Q}(t)$ is the k^{th} user's signature sequence, which is used for spreading the signal in the in-phase or the quadrature branch. It consists of a sequence of antipodal, unit amplitude, rectangular pulses of duration T_c defined as follows

$$c_k^{I/Q}(t) = \sum_l C_{k,l}^{I/Q} p_{T_c}(t-lT_c) \quad (5)$$

where $C_{k,l}^{I/Q} \in \{-1, 1\}$. The period of all the users' signature sequences are $N = T/T_c$, so there is one period per data symbol. We assume without loss of generality that the signature

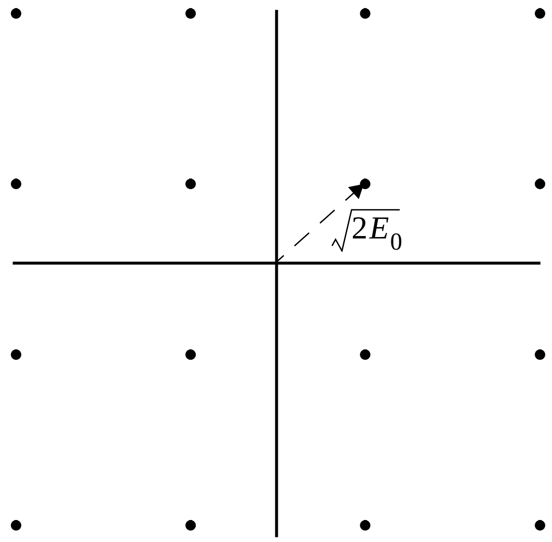

Figure 1. Signal constellation for M-ary QAM for M=16

sequences have power equal to one and therefore

$$\frac{1}{T}\int_0^T [c_k^{I/Q}(t)]^2 dt = 1 \tag{6}$$

τ_k is the time delay and ϕ_k is the phase of the k^{th} user. In the asynchronous, though symbol-synchronous, case they are i.i.d. uniform random variables over $[0, T)$ and $[0, 2\pi)$, respectively. Both parameters are assumed to be known in the analysis. ω_c represents the common centre frequency, α_k represents the channel gain and $n(t)$ is the AWGN with two-sided power spectral density of $N_0/2$.

Figure 2 shows the structure of the i^{th} user's receiver. At the output of the low pass filter of the I- and Q-branch, we get

$$\begin{aligned}\delta^I(t) &= \text{LPF}\{r(t)\cos\omega_c t\} = \\ &= \sum_{k=1}^{K} \alpha_k\sqrt{\frac{2E_0}{T}} d_k^I(t-\tau_k)\, c_k^I(t-\tau_k)\,\frac{\cos\phi_k}{2} + \\ &\qquad \alpha_k\sqrt{\frac{2E_0}{T}} d_k^Q(t-\tau_k)\, c_k^Q(t-\tau_k)\,\frac{\sin\phi_k}{2} + \frac{n_c(t)}{2} \\ &= \sum_{k=1}^{K}\sum_{l} s_{k,l}^I(t-\tau_k) + \frac{n_c(t)}{2}\end{aligned} \tag{7}$$

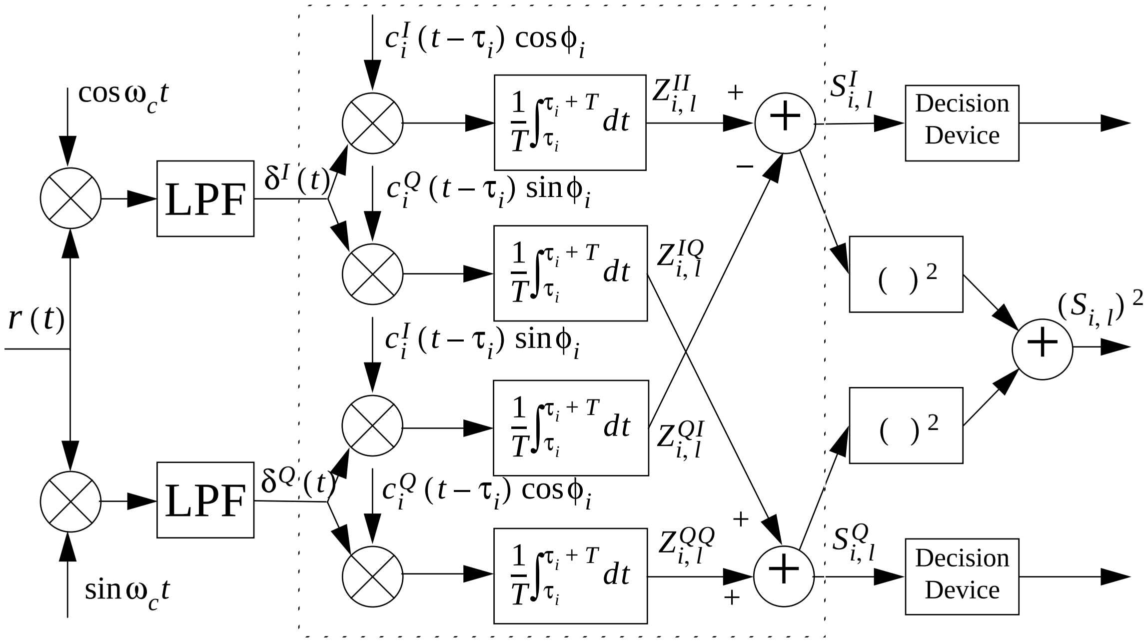

Figure 2. M-ary QAM DS/CDMA receiver

and

$$\begin{aligned}
\delta^Q(t) &= \text{LPF}\{r(t)\sin\omega_c t\} = \\
&= \sum_{k=1}^{K} \alpha_k \sqrt{\frac{2E_0}{T}} d_k^I(t-\tau_k)\, c_k^I(t-\tau_k)\left(-\frac{\sin\phi_k}{2}\right) + \\
&\qquad \alpha_k \sqrt{\frac{2E_0}{T}} d_k^Q(t-\tau_k)\, c_k^Q(t-\tau_k)\,\frac{\cos\phi_k}{2} + \frac{n_s(t)}{2} \\
&= \sum_{k=1}^{K}\sum_{l} s_{k,l}^Q(t-\tau_k) + \frac{n_s(t)}{2}
\end{aligned} \tag{8}$$

where $s_{k,l}^{I/Q}(t-\tau_k)$ is the received baseband signal for the l^{th} symbol element of the k^{th} user and $n_c(t)$ and $n_s(t)$ are the in-phase and quadrature components of the low pass filtered Gaussian noise $n(t)$. $n(t)$ can be represented by $n_c(t) + jn_s(t)$ after low-pass filtering.

The I-branch as well as the Q-branch is correlated with both the I and Q signature sequences of the i^{th} user to form four different $Z_{i,l}$ factors, which are the outputs at each instant of *T*. These factors contain all information about the amplitudes and they are used to form the decision variables, $S_{i,l}^I$ and $S_{i,l}^Q$.

Hence, we get the following outputs from user 1’s correlators

$$Z^{II}_{1,0} = \frac{1}{T}\int_{\tau_1}^{\tau_1+T} \delta^{I}(t)\, c^{I}_{1}(t-\tau_1)\cos\phi_1 dt$$

$$= \sqrt{\frac{E_0}{2T}}\left[\alpha_1 A^{I}_{1,0}\cos^2\phi_1 + \frac{1}{T}\int_{\tau_1}^{\tau_1+T}\alpha_1 A^{Q}_{1,0} c^{I}_{1}(t-\tau_1)\, c^{Q}_{1}(t-\tau_1)\sin\phi_1\cos\phi_1 dt + \sum_{k=2}^{K} I^{II}_{k,1}(\boldsymbol{A}^{I}_{k}, \boldsymbol{A}^{Q}_{k}, \tau_{k,1}, \phi_k)\cos\phi_1\right] + \frac{1}{2}n^{II}_{1} \tag{9}$$

where $\boldsymbol{A}^{I}_{k} = [A^{I}_{k,0}, A^{I}_{k,1}]$, $\boldsymbol{A}^{Q}_{k} = [A^{Q}_{k,0}, A^{Q}_{k,1}]$. The noise component for the first user is considered to be an uncorrelated Gaussian random variable stated as follow

$$n^{II}_{1} = \frac{1}{T}\int_{\tau_1}^{\tau_1+T} n_c(t)\, c^{I}_{1}(t-\tau_1)\cos\phi_1 dt \tag{10}$$

The sum of $I^{II}_{k,1}$ in Eq. (9) is the interference due to the resulting K-1 users and the component caused by the k^{th} user in the expression above can be evaluated in the following way.

$$I^{II}_{k,1}(\boldsymbol{A}^{I}_{k}, \boldsymbol{A}^{Q}_{k}, \tau_{k,1}, \phi_k) = \frac{\alpha_k}{T}\int_{0}^{T}\sum_{l=0}^{1} A^{I}_{k,l}\, p_T(t-\tau_{k,1}-lT)\, c^{I}_{k}(t-\tau_{k,1})\, c^{I}_{1}(t)\cos\phi_k + \sum_{l=0}^{1}\boldsymbol{A}^{Q}_{k,l}\, p_T(t-\tau_{k,1}-lT)\, c^{Q}_{k}(t-\tau_{k,1})\, c^{I}_{1}(t)\sin\phi_k dt \tag{11}$$

where $\tau_{k,1} = \tau_k - \tau_1$, and the delay τ_k is assumed to be shorter than τ_1. The reason for this is explained later. For convenience we evaluate the expression above further and state Eq. (11) as

$$I^{II}_{k,1}(\boldsymbol{A}^{I}_{k}, \boldsymbol{A}^{Q}_{k}, \tau_{k,1}, \phi_k) = \Gamma^{II}_{k,1}(\boldsymbol{A}^{I}_{k}, \tau_{k,1})\cos\phi_k + \Gamma^{QI}_{k,1}(\boldsymbol{A}^{Q}_{k}, \tau_{k,1})\sin\phi_k \tag{12}$$

The other $Z_{1,0}$ components are derived in the same manner as follows

$$Z_{1,0}^{QI} = \frac{1}{T}\int_{\tau_1}^{\tau_1+T} \delta^{Q}(t)\, c_1^{I}(t-\tau_1)\sin\phi_1 dt =$$

$$= \sqrt{\frac{E_0}{2T}}\Bigg[\alpha_1 A_{1,0}^{I}(-\sin^2\phi_1) + \frac{1}{T}\int_{\tau_1}^{\tau_1+T}\alpha_1 A_{1,0}^{Q} c_1^{I}(t-\tau_1)\, c_1^{Q}(t-\tau_1)\cos\phi_1\sin\phi_1 dt + \sum_{k=2}^{K} I_{k,1}^{QI}(\boldsymbol{A}_k^{I}, \boldsymbol{A}_k^{Q}, \tau_{k,1}, \phi_k)\sin\phi_1\Bigg] + \frac{1}{2}n_1^{QI} \tag{13}$$

where the interference component caused by the k^{th} user is

$$I_{k,1}^{QI}(\boldsymbol{A}_k^{I}, \boldsymbol{A}_k^{Q}, \tau_{k,1}, \phi_k) = \Gamma_{k,1}^{II}(\boldsymbol{A}_k^{I}, \tau_{k,1})(-\sin\phi_k) + \Gamma_{k,1}^{QI}(\boldsymbol{A}_k^{Q}, \tau_{k,1})\cos\phi_k \tag{14}$$

$$Z_{1,0}^{QQ} = \frac{1}{T}\int_{\tau_1}^{\tau_1+T} \delta^{Q}(t)\, c_1^{Q}(t-\tau_1)\cos\phi_1 dt =$$

$$= \sqrt{\frac{E_0}{2T}}\Bigg[\alpha_1 A_{1,0}^{Q}\cos^2\phi_1 + \frac{1}{T}\int_{\tau_1}^{\tau_1+T}\alpha_1 A_{1,0}^{I} c_1^{I}(t-\tau_1)\, c_1^{Q}(t-\tau_1)(-\sin\phi_1)\cos\phi_1 dt + \sum_{k=2}^{K} I_{k,1}^{QQ}(\boldsymbol{A}_k^{I}, \boldsymbol{A}_k^{Q}, \tau_{k,1}, \phi_k)\cos\phi_1\Bigg] + \frac{1}{2}n_1^{QQ} \tag{15}$$

where the interference component caused by the k^{th} user is

$$I_{k,1}^{QQ}(\boldsymbol{A}_k^{I}, \boldsymbol{A}_k^{Q}, \tau_{k,1}, \phi_k) = \Gamma_{k,1}^{IQ}(\boldsymbol{A}_k^{I}, \tau_{k,1})(-\sin\phi_k) + \Gamma_{k,1}^{QQ}(\boldsymbol{A}_k^{Q}, \tau_{k,1})\cos\phi_k \tag{16}$$

and

$$Z_{1,0}^{IQ} = \frac{1}{T}\int_{\tau_1}^{\tau_1+T} \delta^{I}(t)\, c_1^{Q}(t-\tau_1)\sin\phi_1 dt =$$

$$= \sqrt{\frac{E_0}{2T}}\Bigg[\alpha_1 A_{1,0}^{Q}\sin^2\phi_1 + \frac{1}{T}\int_{\tau_1}^{\tau_1+T}\alpha_1 A_{1,0}^{I} c_1^{I}(t-\tau_1)\, c_1^{Q}(t-\tau_1)\cos\phi_1\sin\phi_1 dt + \sum_{k=2}^{K} I_{k,1}^{IQ}(\boldsymbol{A}_k^{I}, \boldsymbol{A}_k^{Q}, \tau_{k,1}, \phi_k)\sin\phi_1\Bigg] + \frac{1}{2}n_1^{IQ} \tag{17}$$

where the interference component caused by the k^{th} user is

$$I_{k,1}^{IQ}(\boldsymbol{A}_k^I, \boldsymbol{A}_k^Q, \tau_{k,1}, \phi_k) = \Gamma_{k,1}^{IQ}(\boldsymbol{A}_k^I, \tau_{k,1})\cos\phi_k + \Gamma_{k,1}^{QQ}(\boldsymbol{A}_k^Q, \tau_{k,1})\sin\phi_k \tag{18}$$

We use the decision variables $S_{1,0}^I$ and $S_{1,0}^Q$ to estimate user 1's baseband signal for symbol element 0 and cancel it from the composite signal. Using the derived expressions above for the output of the correlators, we get

$$\begin{aligned} S_{1,0}^I &= (Z_{1,0}^{II} - Z_{1,0}^{QI}) = \sqrt{\frac{E_0}{2T}}\alpha_1 \boldsymbol{A}_{1,0}^I + N_1^I \\ S_{1,0}^Q &= (Z_{1,0}^{QQ} + Z_{1,0}^{IQ}) = \sqrt{\frac{E_0}{2T}}\alpha_1 \boldsymbol{A}_{1,0}^Q + N_1^Q \end{aligned} \tag{19}$$

where $N_1^{I/Q}$ is the noise term of the decision variable including both Gaussian noise and noise caused by interference. For convenience and reasons explained later in the sequel we will write $N_k^{I/Q}$ instead of $N_{k,l}^{I/Q}$ for the noise term of the decision variable of the zeroth symbol. It is easily shown with the help of trigonometric functions that $N_1^{I/Q}$ is defined as below

$$\begin{cases} N_1^I = \sqrt{\dfrac{E_0}{2T}} \displaystyle\sum_{k=2}^{K} I_{k,1}^{II}(\boldsymbol{A}_k^I, \boldsymbol{A}_k^Q, \tau_{k,1}, \phi_{k,1}) + \dfrac{1}{2}[n_1^{II} - n_1^{QI}] \\ N_1^Q = \sqrt{\dfrac{E_0}{2T}} \displaystyle\sum_{k=2}^{K} I_{k,1}^{QQ}(\boldsymbol{A}_k^I, \boldsymbol{A}_k^Q, \tau_{k,1}, \phi_{k,1}) + \dfrac{1}{2}[n_1^{QQ} - n_1^{IQ}] \end{cases} \tag{20}$$

where $\phi_{k,1} = \phi_k - \phi_1$ and $I_{k,1}^{II}$ and $I_{k,1}^{QQ}$ are defined by Eq. (12) and Eq. (16) where ϕ_k is exchanged by $\phi_{k,1}$.

3 Successive Interference Cancellation Scheme with M-ary QAM

A receiver with IC is shown in Figure 3. Each detector is a matched filter for M-ary QAM and the outputs are the decision variables, which are used both for deciding which user is the strongest and in the cancellation of that users signal. The strongest user, ideally the one with the largest α_k, is decoded first and cancelled at baseband from the received signal. The detector is a coherent detector and we assume optimum decision boundaries. Subsequently all the users are decoded and cancelled in decreasing order of their powers. Let us assume we have the means of deciding which user is the strongest and therefore also the one most likely to be decoded correctly.

Without loss of generality we consider the decision of $A_{k,0}^{I/Q}$ at time $\tau_k + T$ corresponding to symbol element 0. All the symbol elements prior to the zeroth have already been decoded. We assume also without loss of generality that user 1 has the strongest signal and, accordingly, it is decoded first and cancelled from the composite signal. Assume that we have a situation where the strongest user has a time delay τ_1 which is shorter than any other user's time delay as e.g. user 2's in Figure 4. Then it would not be enough to cancel symbol element 0 of user 1 from the composite signal to reduce the noise caused by interference since the decision variable of user 2's zeroth symbol will also include interference from user 1's first symbol. To overcome

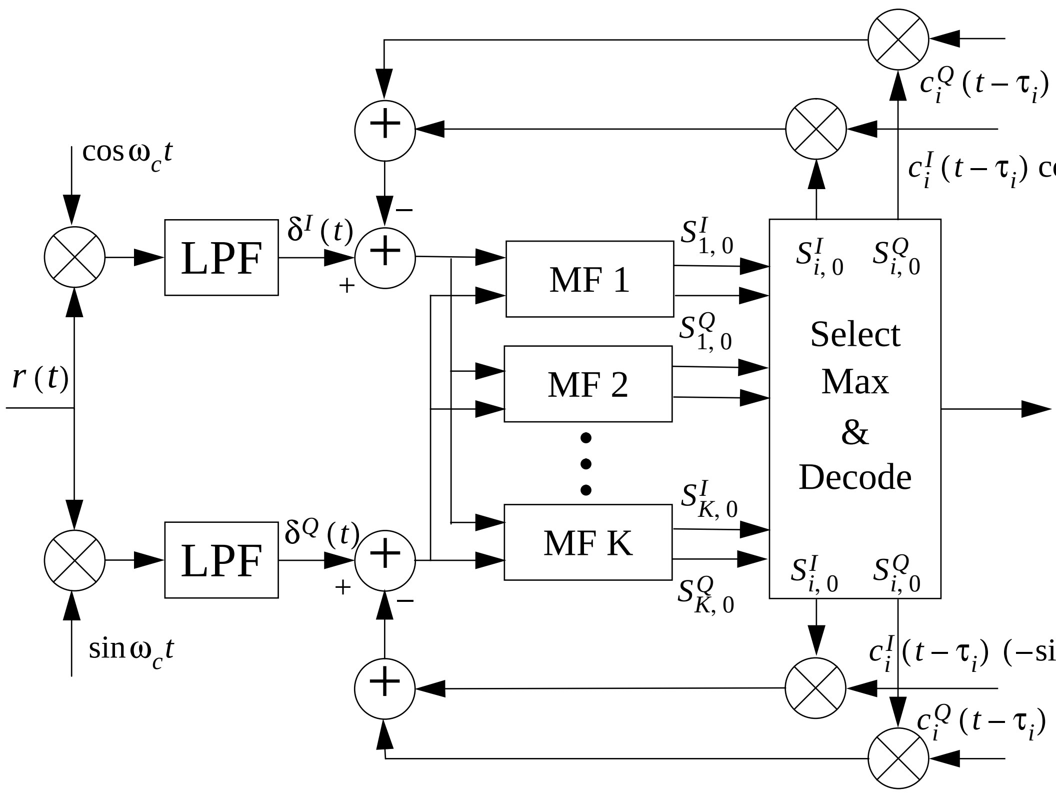

Figure 3. M-ary QAM receiver with Interference Cancellation

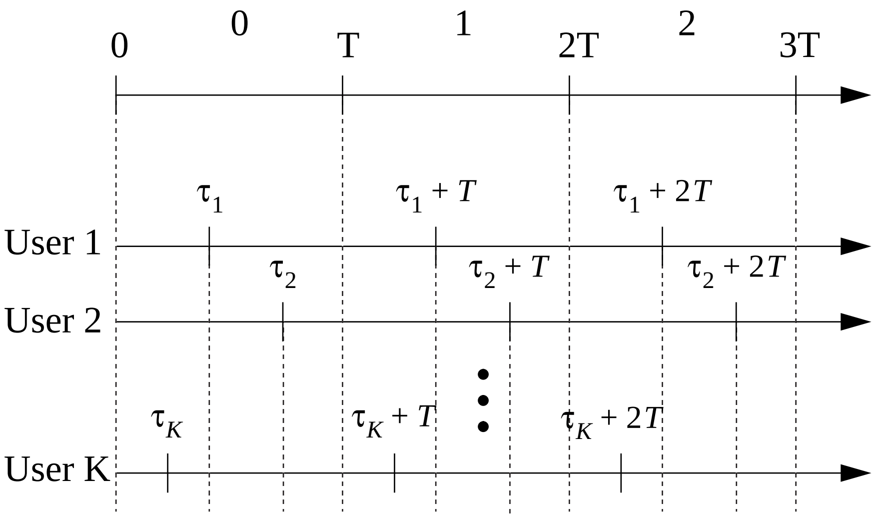

Figure 4. Asynchronous system

this we decode and cancel user 1's first symbol element too before we continue the IC scheme for the second strongest user. Therefore, without loss of generality, we assume in the analysis $\tau_1 > \tau_2 > \ldots > \tau_K$. In this symbol-synchronous case we decode symbol 0 and since we assume $\tau_k < \tau_1$, the interference from the k^{th} user in $[\tau_1, \tau_1 + T)$ originate from symbol element 0 in $\tau_1, \tau_k + T$ and symbol element 1 in $\tau_k + T, \tau_1 + T$.

We use the decision variables $S^I_{1,0}$ and $S^Q_{1,0}$ in Eq. (19) to estimate user 1's baseband signal for symbol element 0 and cancel it from the composite signal. This cancellation is not perfect since the signature sequences are not orthogonal and we do not know the channel gain of user 1. Furthermore the correlator output contains Gaussian noise. Let user 2 be the second strongest user and also the user with the second longest time delay. Now there are only K-2 interfering users left since the strongest user has been cancelled from the composite signal. Proceeding in the same manner, we will get the following expression for the decision variables before the h^{th} cancellation

$$
\begin{aligned}
S^I_{h,0} &= (Z^{II}_{h,0} - Z^{QI}_{h,0}) = \sqrt{\frac{E_0}{2T}}\alpha_h \boldsymbol{A}^I_{h,0} + N^I_h \\
S^Q_{h,0} &= (Z^{QQ}_{h,0} + Z^{IQ}_{h,0}) = \sqrt{\frac{E_0}{2T}}\alpha_h \boldsymbol{A}^Q_{h,0} + N^Q_h
\end{aligned}
\tag{21}
$$

Hence, there are now h-1 cancelled and K-h+1 remaining symbol elements 0. Considering the general case, the resulting baseband signal for the I channel after the h^{th} cancellation will be as follows

$$\delta^I_{h,0}(t) = \delta^I_{h-1,0}(t) - S^I_{h,0}p_T(t-\tau_h)c^I_h(t-\tau_h)\cos\phi_h - S^Q_{h,0}p_T(t-\tau_h)c^Q_h(t-\tau_h)\sin\phi_h \tag{22}$$

When h is 1 the term $\delta^I_{0,0}(t)$ corresponds to the remaining baseband signal after cancellation of all symbol elements prior to the zeroth, and consequently we will get $\delta^I_{1,0}(t)$ after cancelling user 1's zeroth symbol. If we rewrite the expression we can write it in the following way

$$\begin{aligned}\delta^I_{h,0}(t) = {} & \sum_{k=1}^{K}\sum_{l\ge 0} s^I_{k,l}(t-\tau_k) - \sum_{k=1}^{K}\sum_{l<0}\Lambda^I_{k,l} - \\ & \sum_{i=1}^{h}\alpha_i\sqrt{\frac{E_0}{2T}}A^I_{i,0}p_T(t-\tau_i)c^I_i(t-\tau_i)\cos\phi_i - \\ & \sum_{i=1}^{h}\alpha_i\sqrt{\frac{E_0}{2T}}A^Q_{i,0}p_T(t-\tau_i)c^Q_i(t-\tau_i)\sin\phi_i + \frac{n_c(t)}{2} - \\ & \sum_{i=1}^{h}p_T(t-\tau_i)\left(N^I_i c^I_i(t-\tau_i)\cos\phi_i - N^Q_i c^Q_i(t-\tau_i)\sin\phi_i\right)\end{aligned} \tag{23}$$

where the function $p_T(t)$ is defined as in Eq. (3). The first two terms is the remaining baseband signal after cancellation of all symbol elements prior to the zeroth and additional noise caused by imperfect cancellation defined as follows

$$\Lambda^I_{k,l} = p_T(t-\tau_k)\left(N^I_{k,l}c^I_k(t-\tau_k)\cos\phi_k - N^Q_{k,l}c^Q_k(t-\tau_k)\sin\phi_k\right) \tag{24}$$

where $N^{I/Q}_{k,l}$ are noise terms that varies from symbol to symbol, though slowly. The proceeding two sums of Eq. (23) are the cancelled baseband signals corresponding to the zeroth symbol elements of the h strongest users. Then we have the in-phase Gaussian noise and the remaining sum is additional noise components caused by imperfect cancellation of the h users' zeroth symbol elements.

Following the same procedure for the Q-branch as for the I-branch, the resulting baseband signal after the first cancellation becomes

$$\delta^Q_{h,0}(t) = \delta^Q_{h-1,0}(t) - S^I_{h,0}p_T(t-\tau_h)c^I_h(t-\tau_h)(-\sin\phi_h) - S^Q_{h,0}p_T(t-\tau_h)c^Q_h(t-\tau_h)\cos\phi_h \tag{25}$$

Rewriting Eq. (25) we get

$$\begin{aligned}\delta_{h,0}^{Q}(t) = {} & \sum_{k=1}^{K}\sum_{l\geq 0} s_{k,l}^{Q}(t-\tau_k) - \sum_{k=1}^{K}\sum_{l<0}\Lambda_{k,l}^{Q} - \\ & \sum_{i=1}^{h}\alpha_i\sqrt{\frac{E_0}{2T}}A_{i,0}^{I}p_T(t-\tau_i)\,c_i^{I}(t-\tau_i)\,(-\sin\phi_i) - \\ & \sum_{i=1}^{h}\alpha_i\sqrt{\frac{E_0}{2T}}A_{i,0}^{Q}p_T(t-\tau_i)\,c_i^{Q}(t-\tau_i)\cos\phi_i + \frac{n_s(t)}{2} - \\ & \sum_{i=1}^{h}p_T(t-\tau_i)\,(N_i^{I}c_i^{I}(t-\tau_i)\,(-\sin\phi_i) - N_i^{Q}c_i^{Q}(t-\tau_i)\cos\phi_i)\end{aligned} \tag{26}$$

where

$$\Lambda_{k,l}^{Q} = p_T(t-\tau_k)\,(N_{k,l}^{I}c_k^{I}(t-\tau_k)\,(-\sin\phi_k) - N_{k,l}^{Q}c_k^{Q}(t-\tau_k)\cos\phi_k) \tag{27}$$

Therefore, the total noise components for the h^{th} user in Eq. (21) are

$$\begin{aligned}N_h^{I} = {} & \sqrt{\frac{E_0}{2T}}\sum_{k=h+1}^{K} I_{k,h}^{II}(\boldsymbol{A}_k^{I},\boldsymbol{A}_k^{Q},\tau_{k,h},\phi_{k,h}) + \frac{1}{2}[n_h^{II}-n_h^{QI}] - \\ & -\sum_{j=1}^{h-1} N_j^{I}J_{j,h}^{II}(\tau_{j,h},\phi_{j,h}) + N_j^{Q}J_{j,h}^{QI}(\tau_{j,h},\phi_{j,h})\end{aligned} \tag{28}$$

in the I channel and

$$\begin{aligned}N_h^{Q} = {} & \sqrt{\frac{E_0}{2T}}\sum_{k=h+1}^{K} I_{k,h}^{QQ}(\boldsymbol{A}_k^{I},\boldsymbol{A}_k^{Q},\tau_{k,h},\phi_{k,h}) + \frac{1}{2}[n_h^{QQ}-n_h^{IQ}] - \\ & -\sum_{j=1}^{h-1} N_j^{I}J_{j,h}^{IQ}(\tau_{j,h},\phi_{j,h}) + N_j^{Q}J_{j,h}^{QQ}(\tau_{j,h},\phi_{j,h})\end{aligned} \tag{29}$$

in the Q channel, where the first sum consists of noise caused by the remaining interfering users, the second term is white Gaussian noise and the last sum is the resulting noise caused by imperfect cancellations. The correlation terms, $J_{j,h}^{II}$ and $J_{j,h}^{QI}$, in Eq. (28) above are defined as

$$\begin{aligned}J_{j,h}^{II}(\tau_{j,h},\phi_{j,h}) &= \frac{1}{T}\int_0^T c_j^{I}(t-\tau_{j,h})\,c_h^{I}(t)\cos(\phi_{j,h})\,dt \\ J_{j,h}^{QI}(\tau_{j,h},\phi_{j,h}) &= \frac{1}{T}\int_0^T c_j^{Q}(t-\tau_{j,h})\,c_h^{I}(t)\sin(\phi_{j,h})\,dt\end{aligned} \tag{30}$$

and the correlation terms, $J^{IQ}_{j,h}$ and $J^{QQ}_{j,h}$, in Eq. (29) above are defined as

$$\begin{aligned} J^{IQ}_{j,h}(\tau_{j,h}, \phi_{j,h}) &= \frac{1}{T}\int_0^T c^I_j(t-\tau_{j,h})\, c^Q_h(t)\, (-\sin(\phi_{j,h}))\, dt \\ J^{QQ}_{j,h}(\tau_{j,h}, \phi_{j,h}) &= \frac{1}{T}\int_0^T c^Q_j(t-\tau_{j,h})\, c^Q_h(t) \cos(\phi_{j,h})\, dt \end{aligned} \tag{31}$$

In these expressions the correlation of $c^{I/Q}_h(t)$ is over the noise caused by imperfect cancellation of symbol element -1 and 0 of the first user, since $\tau_h < \tau_j$. See Figure 5, where shaded lines indicate cancelled symbol elements.

We are also considering a slowly fading channel which implicate that the interference power can be regarded as equal for two proceeding symbol elements. Therefore we do not distinguish between two subsequent noise terms as e.g. $N^I_{h,-1}$ and $N^I_{h,0}$ in Eq. (28) and (29).

3.1 Ranking of the users

Throughout the analysis we assume perfect ranking of the users. Though in the detector the decision of which user is the strongest is made using the sum of the squared decision variables for each user, i.e.

$$(S_{i,l})^2 = (S^I_{i,l})^2 + (S^Q_{i,l})^2 \tag{32}$$

See Figure 6. In an asynchronous system it is not obvious which bits are compared with what other bits, since each user has its own time delay, τ_k. One way to decide which user is the strongest is to define a frame of n bits for each user [11]. We have a symbol-synchronous system (τ_k takes values on $[0, T)$) so the maximum time between the first and last bit would be $(n+1) \cdot T$ as shown in Figure 6. After we have received the whole frame we can decide

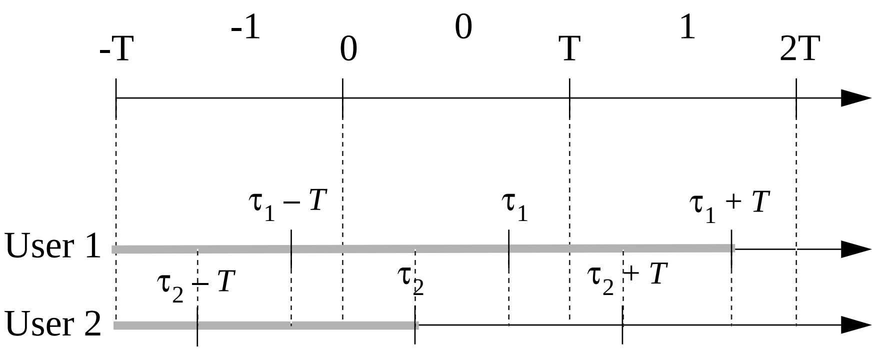

Figure 5. Cross-correlation between users

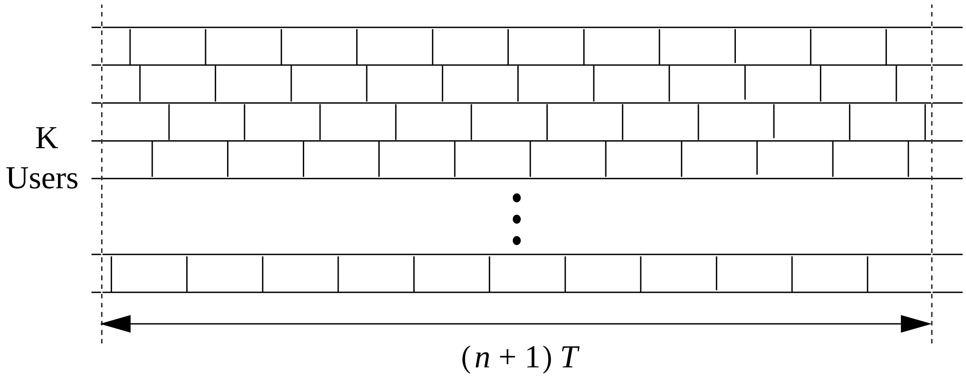

Figure 6. Definition of a frame in an asynchronous system

which user is the strongest by comparing the average $(S_i)^2$ for the n bits for all the users. The channel parameters change slowly, thus during a relatively short frame we may consider the channel gain, α_k, constant. In the case of e.g. 16-QAM this means that we consider the average received power, not the instantaneous power. Therefore a 16-QAM user with less instantaneous power may be decoded and cancelled before a slightly stronger QPSK user.

4 Performance Analysis on a Stationary Channel

Evaluating the performance of the IC scheme will be done by applying Gaussian approximation [9]-[11]. The noise components, caused by interference from the other users in the system are therefore modelled as independent Gaussian noise. We have chosen to use the Gaussian approximation partly since it is commonly used and partly because we have not found any other practical way to evaluate the performance. By using a Gaussian approximation an increase in noise and interference variance immediately leads to an increase in error probability. It is likely that this will occur also for the true distribution and we believe therefore that relative performance will not change significantly. Absolute performance is likely to be too optimistic though [19]. Additionally, since we do not consider any fading in this section, the channel gain, α_k, will be constant. We also assume that power control is used for distance and shadow fading and consequently, in this case, all the α_k's will be equal.

4.1 Performance Analysis of QAM IC Scheme

We start with calculating the variance of the I-channel decision variable conditioned on α_k, i.e.

$$\eta_h^I = \mathrm{Var}\left[N_h^I \middle| \alpha_h\right] \tag{33}$$

It is easy to show that all the random variables in N_i^I and N_i^Q are independent and with zero mean. Consequently we can model N_i^I and N_i^Q as independent Gaussian random variables with zero mean and variance of η_i^I and η_i^Q, respectively. Rewriting Eq. (33) we get

$$\begin{aligned}\eta_h^I = \frac{E_0}{2T} \sum_{k=h+1}^{K} & \mathrm{Var}\left[I_{k,h}^{II}\,(\boldsymbol{A}_k^I, \boldsymbol{A}_k^Q, \tau_{k,h}, \phi_{k,h}) \middle| \alpha_k\right] + \\ & \mathrm{Var}\left[\frac{1}{2}\,(n_h^{II} + n_h^{QI})\right] + \\ & \sum_{j=1}^{h-1} \eta_j^I \mathrm{Var}\,[J_{j,h}^{II}\,(\tau_{j,h}, \phi_{j,h})] + \eta_j^Q \mathrm{Var}\,[J_{j,h}^{QI}\,(\tau_{j,h}, \phi_{j,h})]\end{aligned} \tag{34}$$

Analysing the expression above yields the results below. For a detailed analysis, see Appendix A.

The variance of the noise term caused by the remaining interfering users for deterministic signature sequences, with rectangular shaped pulses of length T_c, is

$$\mathrm{Var}\left[I^{II}_{k,h}\,(\boldsymbol{A}^{I}_{k},\boldsymbol{A}^{Q}_{k},\tau_{k,h},\phi_{k,h})\;\middle|\;\alpha_k\right] = \frac{M-1}{3}\cdot\frac{\alpha_k^2}{6N^3}\,(r^{II}_{k,h}+r^{QI}_{k,h}) \tag{35}$$

where $\frac{M-1}{3}$ is the normalized average transmitted power in each branch, $T = NT_c$ and $r_{k,h}$ is the average interference [15]. (See also Appendix A)

For random signature sequences, the variance stated in Eq. (35) is [15]

$$\mathrm{Var}\left[I^{II}_{k,h}\,(\boldsymbol{A}^{I}_{k},\boldsymbol{A}^{Q}_{k},\tau_{k,h},\phi_{k,h})\;\middle|\;\alpha_k\right] = \frac{M-1}{3}\cdot\frac{\alpha_k^2}{6N^3}\,(2N^2+2N^2) = \frac{M-1}{3}\cdot\frac{2\alpha_k^2}{3N} \tag{36}$$

The variance of one of the cross-correlation terms, where $\tau_{j,h}$ and $\phi_{j,h}$ are uniformly distributed over $[0, T\ )$ and $[0, 2\pi\ )$ respectively, is [15]

$$\mathrm{Var}\,[J^{II}_{j,h}\,(\tau_{j,h},\phi_{j,h})] = \frac{r^{II}_{j,h}}{6N^3} \tag{37}$$

for deterministic sequences. The other cross-correlation term will give the same result with minor changes in indices. For random sequences all the terms will give the same result, which is stated below [15].

$$\mathrm{Var}\,[J^{II}_{j,h}\,(\tau_{j,h},\phi_{j,h})] = \frac{1}{3N} \tag{38}$$

Finally, the variance of the thermal noise components is

$$\mathrm{Var}\left[\frac{1}{2}\,(n^{II}_{h}+n^{QI}_{h})\right] = \frac{N_0}{4T} \tag{39}$$

Simplifying Eq. (34) for deterministic sequences yields

$$\begin{aligned}\eta^{I}_{h} = {} & \frac{N_0}{4T}+\left(\frac{M-1}{3}\cdot\frac{E_0}{12N^3T}\right)\sum_{k=h+1}^{K}\alpha_k^2\,(r^{II}_{k,h}+r^{QI}_{k,h}) + \\ & \frac{1}{6N^3}\sum_{j=1}^{h-1}\eta^{I}_{j}r^{II}_{j,h}+\eta^{Q}_{j}r^{QI}_{j,h}\end{aligned} \tag{40}$$

and for random sequences we get

$$\eta_h^I = \frac{N_0}{4T} + \left(\frac{M-1}{3} \cdot \frac{E_0}{3NT}\right) \sum_{k=h+1}^{K} \alpha_k^2 + \frac{2}{3N} \sum_{j=1}^{h-1} \eta_j^I \tag{41}$$

where we have used the equality $\eta_j^I = \eta_j^Q$.

It is easy to obtain the probability of error from the theory of single transmission of QAM signals over a AWGN channel [1], when using the Gaussian approximation. We use the variance given in Eq. (40) or Eq. (41) depending on which kind of sequences are used, and define a signal-to-noise value for the h^{th} user in the ideal coherent case as

$$\rho_h^I = \frac{\sqrt{\frac{E_0}{2T}}\alpha_h}{\sqrt{\eta_h^I}} \tag{42}$$

for the I channel and subsequently the probability of error for the transmission over the I channel will be [1]

$$Pe_h^I = 2\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right) Q(\rho_h^I) \tag{43}$$

where the Q-function defines the standardized Gaussian tail[1]. Pe_h^Q is obtained similarly. The symbol error rate can then be expressed as

$$Pe_h = 1 - (1 - Pe_h^I)(1 - Pe_h^Q) \tag{44}$$

We then obtain the average probability of symbol error if we take the average of all symbol error rates (Eq. (44)) calculated for each stage of cancellation, i.e.

$$Pe = \frac{1}{K} \sum_{k=1}^{K} Pe_k \tag{45}$$

4.2 Numerical Examples

The ranking of the users is completely random since all the α_k's are equal. Therefore the order of cancellation will change continuously and we obtain the average probability of symbol error

1. $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{\left(-\frac{x^2}{2}\right)} dx$

for each user if we take the average of all SER calculated for each stage of cancellation.

The performance of pure BPSK, QPSK and 16-QAM systems over a Gaussian noise channel is shown in Figure 7. The performance is measured in average bit error rate (BER). When using BPSK modulation in AWGN the decision of the bit sign is $\hat{b}_i = \text{sign}[S_i]$, where S_i is the decision variable of the i^{th} user. The BER conditioned on α_k is then given by the well known equation

$$Pb_i^{\text{BPSK}} = \Pr\left[\hat{S}_i < 0 | b_i = 1\right] = Q(\rho_h^I) \tag{46}$$

To compare the results obtained for QAM users with BPSK users we need to calculate the bit error rate (BER). We assume a Gray-encoded version of M-ary QAM. When the probability of symbol error is sufficiently small, the probability of mistaking a symbol for the adjacent one vertically or horizontally is much greater than any other possible symbol error [16]. Then the BER is easily derived from the symbol error rate, stated in Eq. (44), through

$$Pb_h^{\text{QAM}} \approx \frac{1}{\log_2 M} Pe_h \tag{47}$$

The graph presents the performance as a function of E_b/N_0, where E_b is energy per bit. The approximation of the BER in Eq. (47) is very good for $E_b/N_0 \geq 10dB$ which is the region of interest. Three different groups of systems with the same total transmitted bit rate are presented

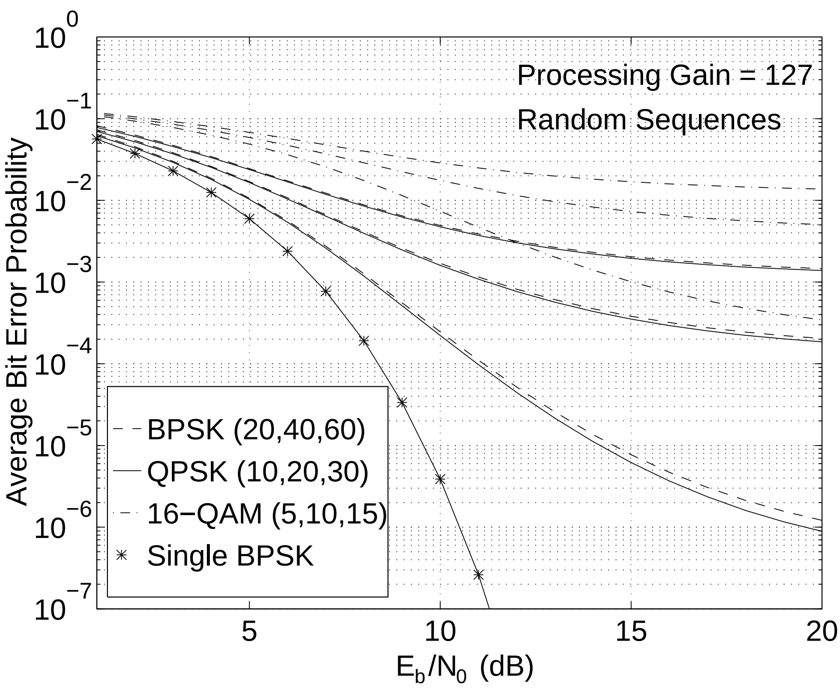

Figure 7. Performance of single modulation systems with IC under ideal PC.

together with the single BPSK user bound. One group of systems with the same throughput is for example: 20 BPSK, 10 QPSK and 5 16-QAM users. We can gather from the graph that the performance of 16-QAM systems on a stationary channel is considerably poorer than for BPSK and QPSK systems, especially for a low number of users.

Figure 8 shows the results of three single user systems with the same throughput both in the case of IC and when employing a conventional matched filter detector. The result is for a stationary channel and it shows a moderately large gain in performance for systems employing IC.

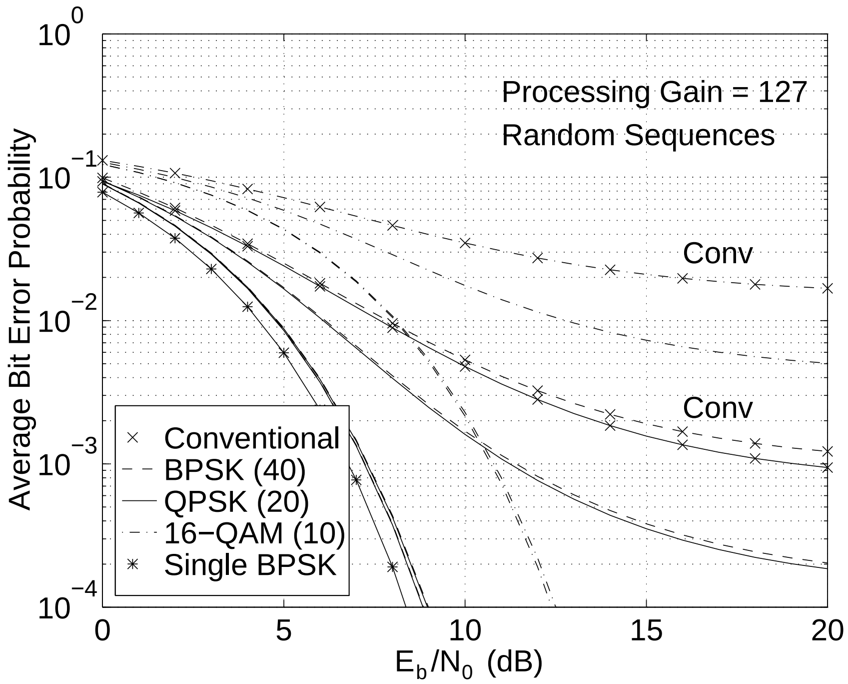

Figure 8. Performance of single modulation systems with and without IC under ideal PC.

5 Performance Analysis of QAM IC Scheme under fading

In the case of fading, we consider a system where the K users are received through independent, frequency-nonselective slowly fading channels. This model is suitable in areas with small delay spread and for mobiles with slow speed (small Doppler frequency). These conditions also make estimation of ϕ_k and α_k (needed for decision boundaries for M-ary QAM and power ranking) feasible. No instantaneous power control is applied, only average power control which takes care of shadowing and distance attenuation.

5.1 IC Scheme for QAM under Single-Path Rayleigh Fading

In this paper we will use the same method, for analysing the single-path Rayleigh fading channel, as was used in [11] for BPSK modulation. The equations for the noise variance and error probabilities obtained in the previous section were all conditioned on α_k. The amplitudes decide the order of cancellation, thus the order will change continuously with fading.

The distributions of the ordered amplitudes are obtained by using order statistics [18]. Then the unconditioned error probability for each stage of cancellation is easily obtained from the conditioned probability of error in Eq. (43). The amplitudes are assumed to be Rayleigh distributed with unit mean square value. That is, the average received power from all the users is equal. We then assume perfect power control for shadowing and long term fading. The amplitude distribution is

$$f(x) = 2xe^{-x^2} \qquad x \geq 0 \tag{48}$$

and zero elsewhere. The cumulative density function is

$$F(x) = 1 - e^{-x^2} \qquad x \geq 0 \tag{49}$$

and zero elsewhere. The mean square values of the ordered amplitudes, α_k, where $\alpha_1 > \alpha_2 > \ldots > \alpha_K$, are given by

$$E_{\alpha_k}[\alpha_k^2] = \int_0^{\infty} x^2 f_{\alpha_k}(x)\, dx \tag{50}$$

where

$$f_{\alpha_k}(x) = \frac{K!}{(K-k)!\,(k-1)!}F^{K-k}(x)\;[1-F(x)]^{k-1}f(x) \tag{51}$$

is the pdf of α_k. The amplitudes are ordered in non-decreasing order and K is the total number of users. The conditioned variance, stated in Eq. (41) for random sequences, is approximated as a sum of independent Gaussian random variables and the expected value with the respect to α_k is then

$$E_{\alpha_k}[\eta_h^I] = \frac{N_0}{4T} + \left(\frac{M-1}{3}\cdot\frac{E_0}{3NT}\right)\sum_{k=h+1}^{K} E_{\alpha_k}[\alpha_k^2] + \frac{2}{3N}\sum_{j=1}^{h-1} E_{\alpha_k}[\eta_j^I] \tag{52}$$

Using the expression above we here define ρ_h^I for the h^{th} user in the I channel in the same manner as in Eq. (42), i.e.

$$\rho_h^I = \frac{\sqrt{\frac{E_0}{2T}}\alpha_h}{\sqrt{E_{\alpha_k}[\eta_h^I]}} \tag{53}$$

The unconditioned probability of error is obtained, using the expression for the conditioned probability of error, Pe_h^I, given in Eq. (43), as follows

$$\hat{P}e_h^I = \int_0^{\infty} Pe_h^I f_{\alpha_h}(x)\,dx \tag{54}$$

It should be noted that this integral, as well as the integral in Eq. (50), is calculated numerically. For random sequences $\hat{P}e_h^I = \hat{P}e_h^Q$ and the unconditioned symbol error rate can be written as

$$\hat{P}e_h = 1-(1-\hat{P}e_h^I)^2 \tag{55}$$

Taking the average of all symbol error rates at all stages of cancellation yields the average probability of symbol error in the same way as given in Eq. (45).

5.2 Numerical Examples

The average BER of different single user systems, 40 BPSK, 20 QPSK and 10 16-QAM users, with the same throughput under Rayleigh fading is shown in Figure 9. The length of the random sequences are 127. The result is compared to the single user bound under fading for both

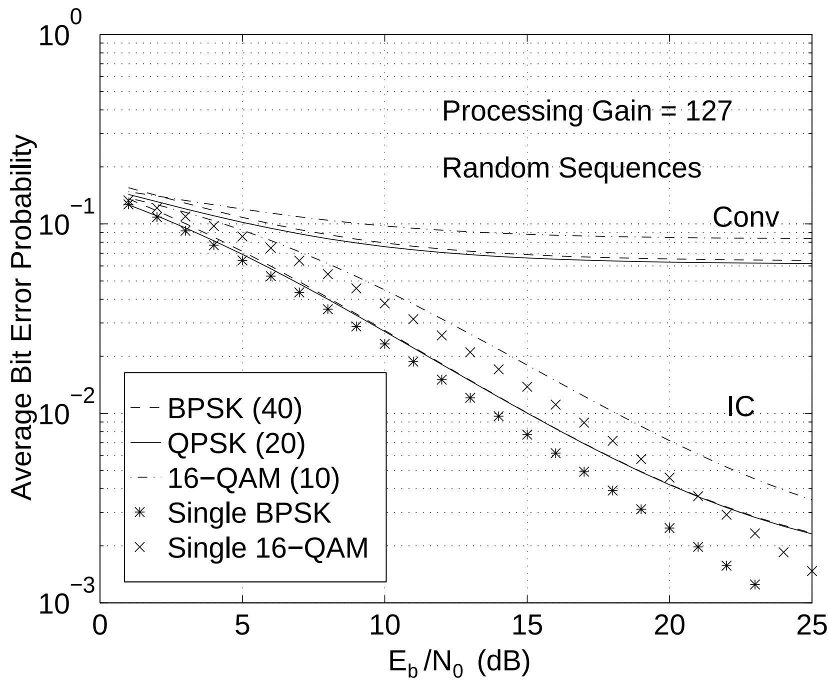

Figure 9. Performance of single modulation systems with and without IC under Rayleigh fading.

BPSK and 16-QAM users and corresponding systems employing a conventional detector. Overall, the performance for all the systems using IC is considerably better compared to the systems employing the conventional detector. The results also show that the QPSK and BPSK systems (partly overlapping) perform almost equally well as in the case of transmission of a single BPSK user and that the performance of the 16-QAM system is inferior but it is close to the single 16-QAM user bound.

6 Performance Analysis of Mixed Modulation Systems

One way to handle multi-rate systems would be to let each user choose a modulation format in correspondence with required transmitted data rate [14]. In the sequel we will evaluate the performance of a system where the users employ different forms of modulation, e.g. a combination of BPSK, QPSK and 16-QAM users.

6.1 DS/CDMA Mixed System Model Description

We consider a system where we have K_1 BPSK, K_2 QPSK and K_3 16-QAM users. To make a comparison between different forms of modulation we let the transmitted bit energy, E_b, be equal for all users independent of the modulation format used. We then rewrite the energy E_0 as a function of E_b, i.e.

$$E_0 = E_b \frac{3\log^2 M}{2(M-1)} \tag{56}$$

The expression in Eq. (56) is derived from the fact that [1]

$$P_{av} = 2 \cdot \frac{M-1}{3} \cdot \frac{E_0}{T} = \frac{E_b \log^2 M}{T} \tag{57}$$

We rewrite the expressions for the variance of the h^{th} user (using random signature sequences), stated in Eq. (41) for QAM modulation and in [9] for BPSK modulation, for convenience.

$$\text{QAM:} \quad \eta_h^I = \frac{N_0}{4T} + \left(\frac{M-1}{3} \cdot \frac{E_0}{3NT}\right) \sum_{k=h+1}^{K} \alpha_k^2 + \frac{2}{3N} \sum_{j=1}^{h-1} \eta_j^I \tag{58}$$

and

$$\text{BPSK:} \quad \eta_i = \frac{N_0}{4T} + \frac{E_b}{6NT} \sum_{k=i+1}^{K} \alpha_k^2 + \frac{1}{3N} \sum_{j=1}^{i-1} \eta_j \tag{59}$$

If we define $M_1 = 4$ (QPSK) and $M_2 = 16$ (16-QAM) we can express the signal-to-noise value, ρ_h^I, conditioned on α_k, for the h^{th} QPSK user as follows

$$\rho_h^I = \frac{\sqrt{\frac{E_b\alpha_h^2}{2T}\cdot\frac{3\log^2 M_1}{2(M_1-1)}}}{\sqrt{\eta_i+\eta_h+\eta_m}} \tag{60}$$

where *i*, *h*-1 and *m* specifies the number of cancelled BPSK, QPSK and 16-QAM users, respectively. Rewriting Eq. (60) we get

$$\begin{aligned}\rho_h^I = \alpha_h\Bigg[&\frac{M_1-1}{3}\left(\frac{N_0}{E_b\log^2 M_1}+\frac{2}{3N}\sum_{k=h+1}^{K_2}\alpha_k^2\right)+\frac{2}{3N}\sum_{j=1}^{h-1}(\rho_j^I)^{-2}+\\ &\frac{2(M_1-1)}{3\log^2 M_1}\cdot\frac{1}{3N}\sum_{j=i+1}^{K_1}\alpha_k^2+\frac{1}{3N}\cdot\frac{2(M_1-1)}{3\log^2 M_1}\sum_{j=1}^{i}\rho_j^{-2}+\\ &\frac{2(M_1-1)\log^2 M_2}{3\log^2 M_1}\cdot\frac{1}{3N}\sum_{k=m+1}^{K_3}\alpha_k^2+\frac{(M_1-1)\log^2 M_2}{(M_2-1)\log^2 M_1}\cdot\frac{2}{3N}\sum_{j=1}^{m}(\rho_j^I)^{-2}\Bigg]^{-1/2}\end{aligned} \tag{61}$$

where we find the noise caused by interference from QPSK users on the first line, from BPSK users on the second line and from 16-QAM users on the last line of the equation. For 16-QAM users the expression will be similar to the one in Eq. (61) with some changes in indices and for BPSK users we get the following expressions

$$\begin{aligned}\rho_i^I = \alpha_i\Bigg[&\left(\frac{N_0}{2E_b}+\frac{1}{3N}\sum_{k=i+1}^{K_1}\alpha_k^2\right)+\frac{1}{3N}\sum_{j=1}^{i-1}\rho_j^{-2}+\\ &\log^2 M_1\cdot\frac{1}{3N}\sum_{j=h+1}^{K_2}\alpha_k^2+\frac{\log^2 M_1}{(M_1-1)N}\sum_{j=1}^{h}(\rho_j^I)^{-2}+\\ &\log^2 M_2\cdot\frac{1}{3N}\sum_{j=m+1}^{K_3}\alpha_k^2+\frac{\log^2 M_2}{(M_2-1)N}\sum_{j=1}^{m}(\rho_j^I)^{-2}+\Bigg]^{-1/2}\end{aligned} \tag{62}$$

where interference caused by BPSK users is found on the first line, from QPSK users on the second line and from 16-QAM users on the last line of the equation.

6.2 Performance Analysis on a Stationary Channel

To evaluate the performance of a mixed modulation system we again use the Gaussian approximation. To calculate the total BER for the system we first calculate the BER for each user as in Eq. (46) and (47). To get the total BER for the whole system we weight together each users' BER in the following way

$$Pb_{tot} = \frac{\sum_i Pb_i^{\mathrm{BPSK}} R_1 + \sum_h Pb_h^{\mathrm{QPSK}} R_2 + \sum_m Pb_m^{\mathrm{QAM}} R_3}{K_1 R_1 + K_2 R_2 + K_3 R_3} \quad (63)$$

where R_1, R_2 and R_3 are the data rates for the BPSK, QPSK and 16-QAM users respectively and the Pb terms are the individual BER's.

6.3 Mixed Modulation Systems under Rayleigh Fading

In the case of Rayleigh fading we follow the same procedure using order statistics as in "IC Scheme for QAM under Single-Path Rayleigh Fading" on page 24. The difference is that now we have a mixed system and the amplitudes are ordered independent of modulation. Using Eq. (43), (53) and (54) together with Eq. (61) we can calculate the performance for each QAM user of the system. Calculating the performance for BPSK users we use Eq. (46) and (62) instead of Eq. (43) and (61).

6.4 Numerical Examples

The average performance of a mixed system with 20 BPSK, 10 QPSK and 5 16-QAM users with and without IC is shown in Figure 10. The length of the random sequences are 127. The result is from an average of 100 mixtures, where a mixture indicates a certain outcome of the

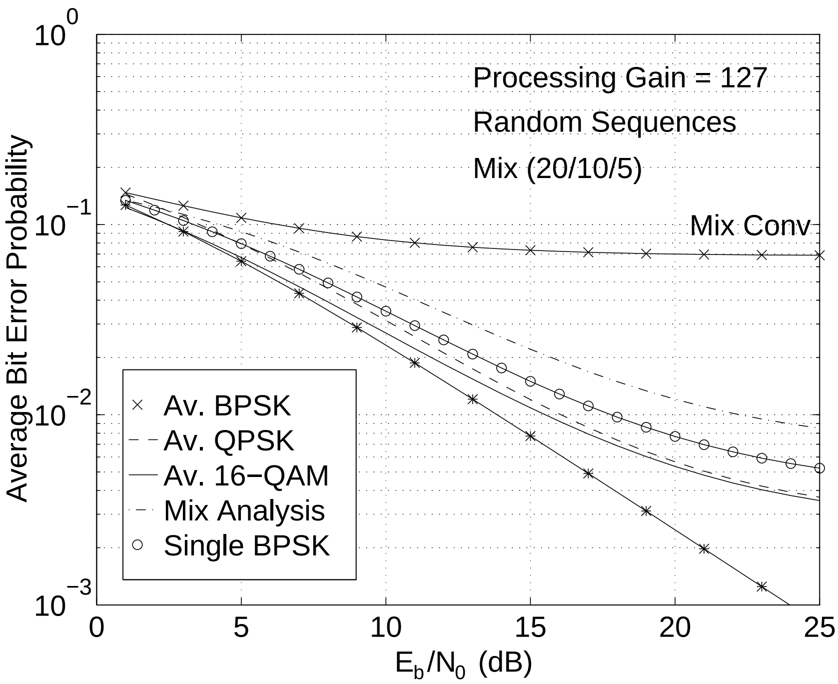

Figure 10. Performance of mixed modulation system with and without IC under Rayleigh fading.

ordered users. So for each mixture the order (according to signal strength) of the users are distributed randomly indepentent of modulation format. The average BER for each group of users is also shown in the graph together with the single user bound.

Figure 11 depicts the same mixed system as in Figure 10, but in this graph we compare the result with three single modulation systems with the same throughput (60 BPSK, 30 QPSK and 15 16-QAM). Inspecting the two graphs we can see that the average performance for the QPSK and BPSK users is slightly better in the case of 35 mixed users compared to their corresponding single modulation systems. On the other hand, the average performance of the 16-QAM users is better for the single modulation system than for mixed system. This can be explained by the fact that the number of users in the mixed system is lower compared to the BPSK and QPSK single modulation systems but higher for the 16-QAM systems. (This is if we count QAM users as two because of the I- and Q-branch.) Furthermore we have to contemplate the fact that the average is made over a finite number of mixtures, because of the very time consuming calculations, which may affect the result.

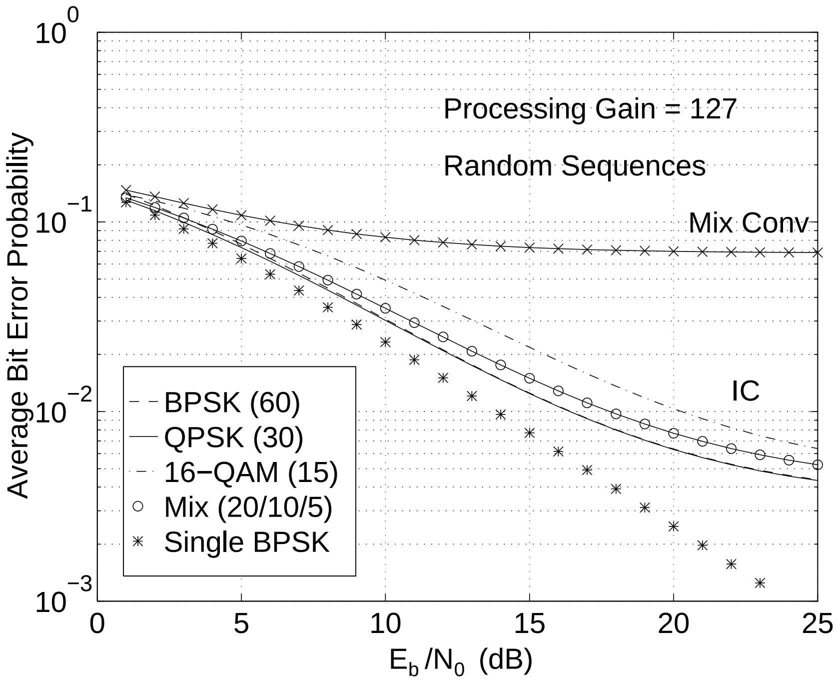

Figure 11. Performance of mixed modulation system and single modulation systems with IC under Rayleigh fading.

7 Unequal Powers of the Users within the System

The disadvantage of using mixed modulation as a method of handling multiple data rates is that the 16-QAM users have a higher average BER than the BPSK and QPSK users. See Figure 10. A possible way to reduce the BER of the 16-QAM users is to increase their transmitted power. The increase in power, however, should not be too large since this causes additional interference to the BPSK and QPSK users. Though, since we know that the 16-QAM users are the ones most sensitive to interference a small increase in power may endorse them with minor effect on the other users.

To evaluate the performance of a mixed system with unequal powers we use two different Rayleigh distributions for the 16-QAM users respectively the BPSK and QPSK users. Thus, we have a system with K_1 BPSK and K_2 QPSK users distributed with one power level and K_3 16-QAM users distributed with another. In this case the theory of order statistics can not directly be used. We have a distinct number of users from two distributions and therefore the distribution for each ordered user has to be calculated separately. See Appendix C. Figure 12 shows the average BER for a mixed system with 20 BPSK, 10 QPSK and 5 16-QAM users with the code length 127. The power of the 16-QAM users is increased in steps corresponding to 1 dB E_b/N_0 compared to the power of the other users, which is kept constant. The E_b/N_0 for the other users, BPSK and QPSK, is 18 dB. The performance is evaluated from an average

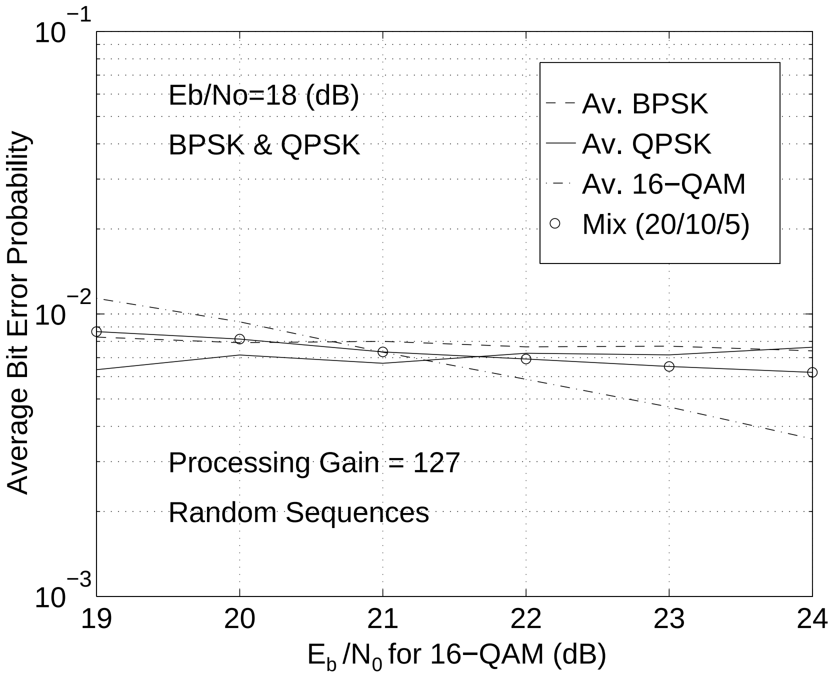

Figure 12. Performance of mixed modulation system under fading with users employing different power levels.

of 200 mixtures. The mixtures were formed as follows: $K_1 + K_2$ amplitudes were picked from a Rayleigh distribution with one variance and K_3 from a Rayliegh distribution with another variance. Then the order of the different users was determined from the order of the amplitudes. The graph shows that the average BER of the 16-QAM users may be decreased by increasing the power of these users without severe degradation of the performance of the BPSK and QPSK users. Hence, by increasing the power with an amount corresponding to about 2 dB E_b/N_0 the average BER of the 16-QAM users is just slightly higher than for the BPSK and QPSK users. Letting the 16-QAM users have a slightly higher BER is a fair price for a higher data rate.

8 Performance Analysis of Parallel Channel Systems

We have previously discussed the possibility to handle multi-rate systems by the means of employing higher modulation formats. Another approach would be to use parallel channels for transmission of information [14]. We simply let a user send simultaneously over as many channels as required for a specific data rate. In this kind of system there are going to be a larger number of interfering signals, though the synchronous signals corresponding to the same user will have considerably lower cross-correlation than asynchronous signals.

8.1 Parallel Channels

We consider first a system with purely synchronous transmission. We have Δ parallel channels with exactly the same channel parameters since they belong to the very same user. In other words, the relative time delay and phase between the channels are equal to zero. For a detailed analysis of the following steps see Appendix B .

Rewriting the expression for the total noise component given in Eq. (28) for M-ary QAM, in the case when $\tau_{i,h}$ and $\phi_{i,h}$ both are zero, yields

$$N_h^I = \frac{1}{2}[n_h^{II} - n_h^{QI}] + \sqrt{\frac{E_0}{2T}} \sum_{i=h+1}^{\Delta} \hat{\Gamma}_{i,h}^{II}(\boldsymbol{A}_i^I) - \sum_{j=1}^{h-1} N_j^I J_{j,h}^{II} \tag{64}$$

where the remaining terms all are equal to zero. Using Eq. (12) and Eq. (28) we define the function $\hat{\Gamma}_{i,h}^{II}$ by

$$\hat{\Gamma}_{i,h}^{II}(A_i^I) = \Gamma_{i,h}^{II}(A_i^I, 0) \tag{65}$$

where we have used that $\tau_{i,h}$ and $\phi_{i,h}$ are equal to zero. Taking the variance of the noise components we get

$$\eta_h^I = \frac{N_0}{4T} + \left(\frac{M-1}{3} \cdot \frac{E_0}{2TN^2}\right) \sum_{i=h+1}^{\Delta} \alpha_i^2 (\theta_{i,h}^{II}(0))^2 + \frac{1}{N^2} \sum_{j=1}^{h-1} \eta_j^I (\theta_{j,h}^{II}(0))^2 \tag{66}$$

where $\theta_{i,h}^{II}$ is the periodic cross-correlation function [15]. In the case of BPSK modulation the total noise component will be as follows

$$N_h = \frac{1}{2}[n_h^I - n_h^Q] + \sqrt{\frac{E_b}{2T}} \sum_{i=h+1}^{\Delta} \hat{\Gamma}_{i,h}(b_i) - \sum_{j=1}^{h-1} N_j J_{j,h} \tag{67}$$

where

$$\hat{\Gamma}_{i,h}(b_i) = \frac{\alpha_i}{T}\int_0^T b_i c_i(t)\, c_h(t)\, dt \tag{68}$$

and $b_i = \{\mp 1\}$. Taking the variance of the noise components we have

$$\eta_h = \frac{N_0}{4T} + \frac{E_b}{2TN^2}\sum_{i=h+1}^{\Delta}\alpha_i^2\theta_{i,h}^2(0) + \frac{1}{N^2}\sum_{j=1}^{h-1}\eta_j\theta_{j,h}^2(0) \tag{69}$$

The variance stated in Eq. (66) and Eq. (69) are for deterministic sequences. We obtain the following equations for random sequences using the fact [19] that $E[\theta_{i,h}^2(0)] = N$.

$$\text{QAM:}\quad \eta_h^I = \frac{N_0}{4T} + \left(\frac{M-1}{3}\cdot\frac{E_0}{2TN}\right)\sum_{i=h+1}^{\Delta}\alpha_i^2 + \frac{1}{N}\sum_{j=1}^{h-1}\eta_j^I \tag{70}$$

and [9]

$$\text{BPSK:}\quad \eta_h = \frac{N_0}{4T} + \frac{E_b}{2TN}\sum_{i=h+1}^{\Delta}\alpha_i^2 + \frac{1}{N}\sum_{j=1}^{h-1}\eta_j \tag{71}$$

8.2 Combination of Synchronous and Asynchronous Transmission

We consider now a system where we have *K* users and where each user, *k*, transmits over Δ_k channels. Thus, we will have a system with a total number of information-bearing channels equal to

$$\kappa = \sum_{k=1}^{K}\Delta_k \tag{72}$$

Accordingly, we have both synchronous and asynchronous interferers and if we use deterministic sequences the major part of the interference will come from the asynchronous users. Therefore cancellation of parallel signals is excluded and instead we consider a receiver where each user’s parallel signals are decoded and cancelled simultaneously. The complexity of the receiver will not increase considerably since the decision variables for all signals have to be generated anyway to determine which signal is the strongest. Futhermore the channel parameters, τ_h and ϕ_h, are equal for parallel channels.

Combining Eq. (40) and Eq. (66) we can write the noise variance for the g^{th} signal of the h^{th} user in a M-ary QAM system with both asynchronous and synchronous transmission as

$$\eta_{g,h}^{I} = \left(\frac{M-1}{3}\cdot\frac{E_0}{12N^3T}\right)\sum_{k=\kappa_h+1}^{\kappa}\alpha_k^2\,(r_{k,h}^{II}+r_{k,h}^{QI}) + \frac{1}{6N^3}\sum_{j=1}^{\kappa_{h-1}}\eta_j^I r_{j,h}^{II}+\eta_j^Q r_{j,h}^{QI} + \left(\frac{M-1}{3}\cdot\frac{E_0}{2TN^2}\right)\sum_{\substack{i=1\\ i\neq g}}^{\Delta_h}\alpha_i^2\,(\theta_{i,h}^{II}(0))^2+\frac{N_0}{4T} \tag{73}$$

where κ_{h-1} is the total number of interfering asynchronous signals, relative to the signals transmitted by user h, which have been cancelled, i.e.

$$\kappa_{h-1} = \sum_{k=1}^{h-1}\Delta_k \tag{74}$$

$\kappa-\kappa_h$ are the remaining interfering asynchronous signals.

8.3 Synchronous and Asynchronous Transmission under Rayleigh Fading

When we transmit information on parallel channels under Rayleigh fading the parallel channels will be affected by the same channel parameters. That is the delay, the phase shift and the amplitude will be the same. Since synchronous channels interfere less with each other we have not considered successive interference cancellation of parallel channels; instead they are all detected and cancelled simultaneously. The performance of a system with parallel channels has been analysed using the same method with order statistics as described earlier. The difference in this case is that we order the K user, each one with Δ_k parallel channels, and then assign the same pdf $f_{\alpha_k}(x)$ and mean square value $E_{\alpha_k}[\alpha_k^2]$ to all the Δ_k channels of user k.

8.4 Numerical results

The performance of systems employing parallel channels under Rayleigh fading are shown in Figure 13. Systems with K users and 2, 3 or 4 parallel channels per user are compared with asynchronous systems with $2\cdot K$, $3\cdot K$ and $4\cdot K$ users. K is in this case equal to 10. One user's all parallel signals are detected and cancelled simultaneously and the results show that the performance of the parallel channel systems employing Gold sequences is very close to the performance of asynchronous systems (the curves are almost not distinguishable). This is caused by the fact that the interference from the strongest interferer will be larger in the case of

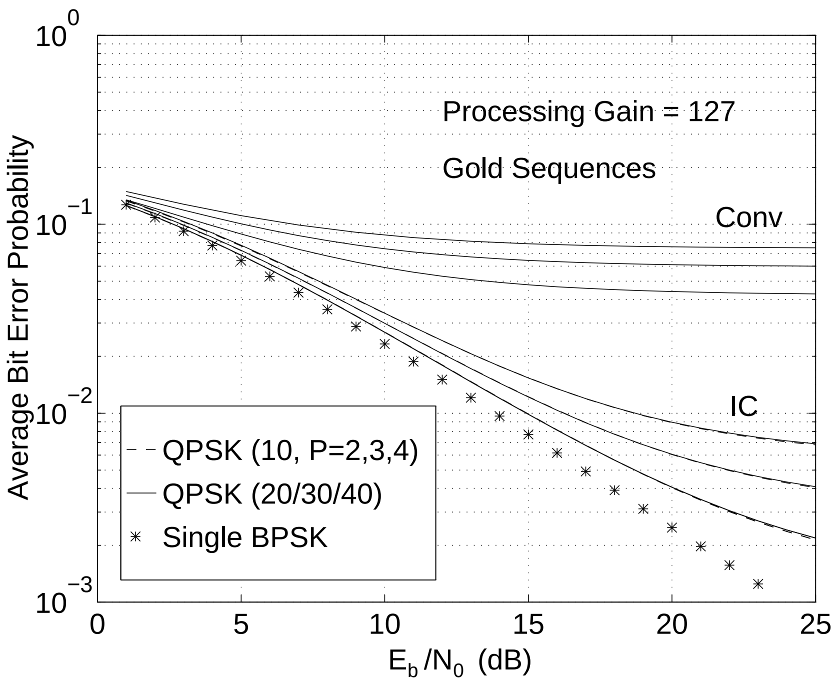

Figure 13. Performance of parallel channel systems employing Gold sequences.

parallel channels since that user will send over Δ_k channels instead of one. This will counter-balance the improvement caused by the good cross-correlation properties of Gold sequences in the synchronous case.

In Figure 14 we have compared the average performance of 15 QPSK users with two parallel

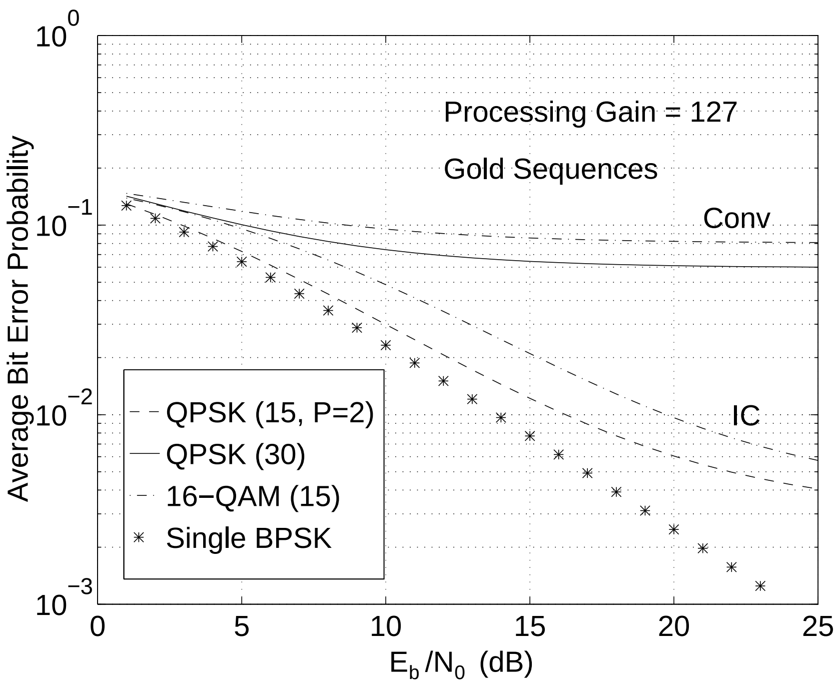

Figure 14. Comparison between a parallel channel QPSK system and asynchronous 16-QAM and QPSK systems.

channels each with 15 16-QAM users. The result show that QPSK users employing parallel channels perform better than a pure 16-QAM system.

9 Conclusions

We have demonstrated the use of M-level rectangular QAM with successive IC in single modulation systems and compared the average BER between systems with the same throughput. Two different methods, mixed modulation and parallel channels, for handling multiple data rates have been analysed. These systems have been studied in the case of Rayleigh fading and they have been compared with single modulation systems.

The conclusion is that the successive IC scheme has relatively low complexity even in the case of using M-ary QAM, the performance, even for large systems, is close to the single user bound and, consequently, it yields considerable increase in performance compared to conventional matched filter detection. Mixed modulation systems offer more flexibility at the cost of a slight decrease in average performance compared to pure asynchronous QPSK systems. The 16-QAM users in the mixed system, who are most sensitive to noise, will have the highest average BER. On the other hand, if we instead have a QPSK system where the users employ parallel channels the average performance is almost equal for all the users. (It would be equal if all the users in the system use the same number of parallel channels.) Though, to achieve good performance in a parallel channel system it is important to use signature sequences with good cross-correlation properties to make use of the advantages with synchronous channels. A minor drawback with parallel channels is that sooner or later we run out of signature sequences and in some cases it may be better to add a 16-QAM user, if the user can accept a small decrease in performance, instead of a QPSK user with two parallel channels. Thus, the greatest system flexibility is obtained by a combination of parallel channels and mixed modulation.

Future work within this project will be to study multi-stage IC schemes, where we decrease the interference further and increase the capacity by employing a modified version of the IC scheme in several stages. Multipath fading channels with RAKE combiner will also be included in the analysis, as will channel coding.

Two different IC schemes, resulted from work independent from ours, have been proposed recently in [20] and [21]. Though their work have only minor similarities to ours. In both papers they consider solely BPSK systems and Gaussian channels. Hence, no fading is taken into consideration. In [20] they evaluate the average BER for a multistage successive IC scheme under the assumption of unequal power control. Also in [21] they consider a multi-stage IC scheme, but in this case parallel detection is employed.

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Appendix A

A.1 Single modulation systems

A.1.1 Noise caused by interference

Using Eq. (11) and (20) we get

$$\begin{aligned} &\mathrm{Var}\left[I^{II}_{k,h}(\boldsymbol{A}^I_k, \boldsymbol{A}^Q_k, \tau_{k,h}, \phi_{k,h}) \middle| \alpha_k\right] = \\ &\mathrm{Var}\left[\frac{\alpha_k}{T}\Big(\{A^I_{k,0}R^{II}_{k,h}(\tau_{k,h}) + A^I_{k,1}\hat{R}^{II}_{k,h}(\tau_{k,h})\} \cos\phi_{k,h} + \right. \\ &\left. \{A^Q_{k,0}R^{QI}_{k,h}(\tau_{k,h}) + A^Q_{k,1}\hat{R}^{QI}_{k,h}(\tau_{k,h})\} \sin\phi_{k,h} \Big)\right] \end{aligned} \tag{75}$$

where

$$\begin{aligned} R^{II}_{k,h}(\tau) &= \int_0^{\tau} c^I_k(t-\tau)\, c^I_h(t)\, dt \\ \hat{R}^{II}_{k,h}(\tau) &= \int_{\tau}^{T} c^I_k(t-\tau)\, c^I_h(t)\, dt \end{aligned} \tag{76}$$

Evaluating Eq. (75), we get

$$\begin{aligned} &\mathrm{Var}\left[I^{II}_{k,h}(\boldsymbol{A}^I_k, \boldsymbol{A}^Q_k, \tau_{k,h}, \phi_{k,h}) \middle| \alpha_k\right] = \\ &= \frac{\alpha_k^2}{T^2}\left\{\frac{1}{T}\int_0^T \frac{1}{2\pi}\int_0^{2\pi} \{E[(A^I_{k,0})^2](R^{II}_{k,h}(\tau_{k,h}))^2 + E[(A^I_{k,1})^2]\Big(\hat{R}^{II}_{k,h}(\tau_{k,h})\Big)^2\} \cos^2\phi_{k,h} + \right. \\ &\{E[(A^Q_{k,0})^2](R^{QI}_{k,h}(\tau_{k,h}))^2 + E[(A^Q_{k,1})^2]\Big(\hat{R}^{QI}_{k,i}(\tau_{k,h})\Big)^2\} \sin^2\phi_{k,h} d\phi_{k,h} d\tau_{k,h} = \\ &= \frac{M-1}{3}\cdot\frac{\alpha_k^2}{2T^3}\int_0^T (R^{II}_{k,h}(\tau_{k,h}))^2 + \Big(\hat{R}^{II}_{k,h}(\tau_{k,h})\Big)^2 + (R^{QI}_{k,h}(\tau_{k,h}))^2 + \Big(\hat{R}^{QI}_{k,h}(\tau_{k,h})\Big)^2 d\tau_{k,h} = \\ &= \frac{M-1}{3}\cdot\frac{\alpha_k^2}{6N^3}(r^{II}_{k,h} + r^{QI}_{k,h}) \end{aligned} \tag{77}$$

where the last step is made according to [15]. $r^{II}_{k,h}$ can be written using the cross-correlation parameters as

$$r^{II}_{k,h} = 2\mu^{II}_{k,h}(0) + \mu^{II}_{k,h}(1) \tag{78}$$

where

$$\mu^{II}_{k,h}(n) = \sum_{l=1-N}^{N-1} C^{II}_{k,h}(l)\, C^{II}_{k,h}(l+n) \tag{79}$$

and

$$C^{II}_{k,h}(l) = \begin{cases} \sum_{j=0}^{N-1-l} c^{I}_{k,j} c^{I}_{h,j+l} & 0 \le l \le N-1 \\ \sum_{j=0}^{N-1+l} c^{I}_{k,j-l} c^{I}_{h,j} & 1-N \le l < 0 \\ 0 & |l| \ge N \end{cases} \tag{80}$$

$r^{QI}_{k,h}$ can be written in a similar way with a minor change in indices.

Evaluating the expression in Eq. (37) using Eq. (30) yields

$$\begin{aligned} \mathrm{Var}\,[J^{II}_{j,h}(\tau_{j,h}, \phi_{j,h})] &= \mathrm{Var}\left[\frac{1}{T}\int_0^T c^I_j(t-\tau_{j,h})\, c^I_h(t)\cos(\phi_{j,h})\,dt\right] = \\ &= \mathrm{Var}\left[\frac{1}{T}\left(R^{II}_{j,h}(\tau_{j,h}) + \hat{R}^{II}_{j,h}(\tau_{j,h})\right)\cos\phi_{j,h}\right] = \\ &= \frac{1}{T^2}\left[\frac{1}{T}\int_0^T (R^{II}_{j,h}(\tau_{j,h}))^2 + \left(\hat{R}^{II}_{j,h}(\tau_{j,h})\right)^2 d\tau_{j,h}\frac{1}{2\pi}\int_0^{2\pi}\cos^2\phi_{j,h}\,d\phi_{j,h}\right] = \frac{1}{6N^3} r^{II}_{j,h} \end{aligned} \tag{81}$$

A.1.2 Gaussian noise

The variance of one of the Gaussian noise components is

$$\begin{aligned} \mathrm{Var}\left[\frac{1}{2}n^{II}_h\right] &= \frac{1}{T^2}\mathrm{Var}\left[\int_0^T n(t)\, c^I_h(t-\tau_h)\cos wt \cos\phi_h\, dt\right] = \\ &= \left\{E[n(t)\,n(t')] = \frac{N_o}{2}\delta(t)\right\} = \frac{N_o}{8T} \end{aligned} \tag{82}$$

which leads to Eq. (39).

Appendix B

B.1 Parallel channels system

B.1.1 Noise caused by interference

Interference caused by not yet cancelled users is defined in Eq. (65) and the variance of this term will be [15]

$$\begin{aligned} \mathrm{Var}\left[\hat{\Gamma}^{II}_{i,h}(A^I_i)\,\middle|\,\alpha_i\right] &= \mathrm{Var}\left[\frac{\alpha_i}{T}\int_0^T A^I_{i,0}\,c^I_i(t)\,c^I_h(t)\,dt\right] = \\ &= \mathrm{Var}\left[\frac{\alpha_i}{T}A^I_{i,0}\hat{R}^{II}_{i,h}(0)\right] = \frac{M-1}{3}\cdot\frac{\alpha_i^2}{N^2}(\theta^{II}_{i,h}(0))^2 \end{aligned} \tag{83}$$

where $\hat{R}^{II}_{i,h}(0) = T_c\theta^{II}_{i,h}(0)$ and $\theta^{II}_{i,h}(l)$ is the periodic cross-correlation function defined by

$$\theta^{II}{}_{i,h}(l) = \sum_{j=0}^{N-1} c^I_{i,j}\,c^I_{h,j+l} \tag{84}$$

where $c^I_{i,j}$ is the j^{th} chip in user i's signature sequence.

Evaluating the noise term corresponding to interference caused by imperfect cancellation in Eq. (64) yields

$$\begin{aligned} \mathrm{Var}[N^I_j J^{II}_{j,h}] &= \mathrm{Var}\left[\frac{N^I_j}{T}\int_0^T c^I_j(t)\,c^I_h(t)\,dt\right] = \\ &= \frac{1}{T^2}\hat{R}^{II}_{j,h}(0)\,\mathrm{Var}[N^I_j] = \frac{1}{N^2}(\theta^{II}{}_{j,h}(0))^2\eta^I_j \end{aligned} \tag{85}$$

Appendix C

C.1 Unequal Power Levels

When we calculate the order statistics for a mixed modulation system with different power levels we have to consider a system with a distinct number of users in each group. This is because the distributions of the ordered users depend on how many users there are. In this case we consider two groups of users, BPSK and QPSK users and 16-QAM users. The former group have variance σ_1^2 and the latter group have variance σ_2^2.

We have studied the case of a total number of 35 users, where $n_1 = 30$ users employ BPSK or QPSK modulation and $n_2 = 5$ users employ 16-QAM. We let the BPSK and QPSK users have the amplitude distribution $f_1(x, \sigma_1)$ and the 16-QAM users $f_2(x, \sigma_2)$. Using order statistics and the mixing theorem [18] we can calculate the density function for the strongest user, user 1, as

$$\begin{aligned} f_{\alpha_1}(x) &= f_1(x, \text{BPSK or QPSK is strongest})\, P\{\text{all 16-QAM are weaker}\} + \\ &\quad f_2(x, \text{16-QAM is strongest})\, P\{\text{all BPSK and QPSK are weaker}\} = \\ &= n_1 F_1^{n_1-1}(x, \sigma_1)\, f_1(x, \sigma_1)\, F_2^{n_2}(x, \sigma_2) + \\ &\quad n_2 F_2^{n_2-1}(x, \sigma_2)\, f_2(x, \sigma_2)\, F_1^{n_1}(x, \sigma_1) \end{aligned} \tag{86}$$

where we have two possible outcomes. If we calculate the distribution for the next strongest user we have to consider four possible outcomes. The strongest user could be either one of the n_1 users or one of the n_2 users and the same for the next strongest user. Accordingly we get the density function for the next strongest user as follows

$$\begin{aligned} f_{\alpha_2}(x) &= \frac{n_1!}{(n_1-2)!} F_1^{n_1-2}(x, \sigma_1)\, [1 - F_1(x, \sigma_1)]\, f_1(x, \sigma_1)\, F_2^{n_2}(x, \sigma_2) + \\ &\quad n_1 F_1^{n_1-1}(x, \sigma_1)\, f_1(x, \sigma_1)\, n_2 F_2^{n_2-1}(x, \sigma_2)\, [1 - F_2(x, \sigma_2)] + \\ &\quad \frac{n_2!}{(n_2-2)!} F_2^{n_2-2}(x, \sigma_2)\, [1 - F_2(x, \sigma_2)]\, f_2(x, \sigma_2)\, F_1^{n_1}(x, \sigma_1) + \\ &\quad n_2 F_2^{n_2-1}(x, \sigma_2)\, f_2(x, \sigma_2)\, n_1 F_1^{n_1-1}(x, \sigma_1)\, [1 - F_1(x, \sigma_1)] \end{aligned} \tag{87}$$

Following the same procedure for each ordered user we will get all the ordered density functions and as long as $n_1 \gg n_2$ or $n_1 \ll n_2$ the possible outcomes will be limited to a small number and the calculations will be manageable.