Comptroller of the Currency

Administrator of National Banks

"Multivariate estimation for operational risk with judicious use of Extreme Value Theory"

Hulusi Inanoglu

Risk Analysis Division, OCC

Outline

- Part I: Statistical Framework
  - Unit of measure
  - Aggregation of Units
  - Modeling dependence structure
- Part II: Data
- Part III: Statistical Methods
  - Fitting the tail
  - Summary of Results
  - Conclusions

Motivation

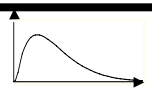
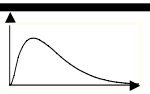
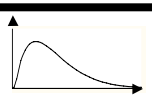
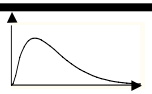
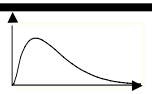
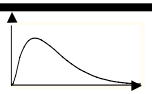
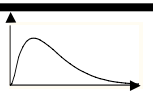
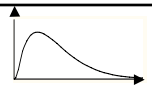
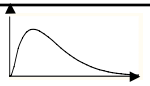
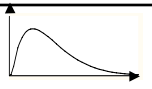
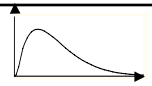
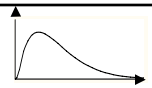
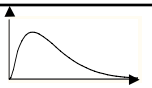
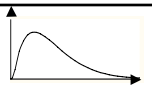
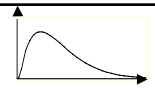
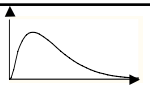
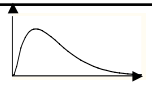
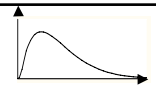
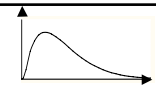
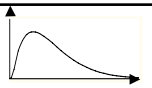
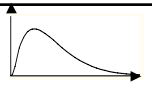
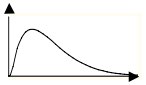
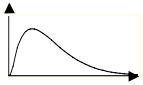
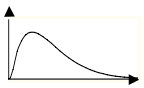
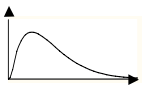
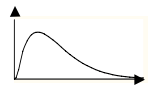
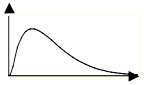
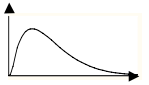
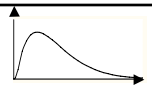
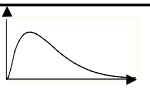
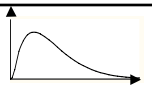
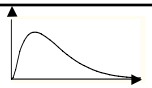
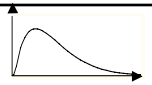
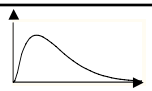
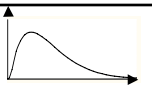
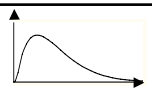
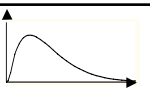
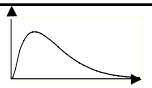
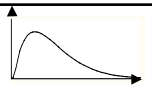
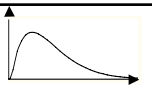
- Basel II recognizes diversification benefits within op risk.
  - Diversification benefits= (Simple Sum) - (Capital estimated with dependence structure)
- No convergence to date on how to estimate diversification benefits.
  - E.g., U.S. experience: diversification benefits range widely -- from 20% to 70%.
  - Most banks use *ad hoc* correlation factors among Event Types (ETs) /Business Lines (BLs) (10%, 20%) and then apply correlation matrix approach.
  - Copula approach.

Unit of Measure

"A bank's risk measurement system must be sufficiently 'granular' to capture the major drivers of operational risk affecting the shape of the tail of the loss estimates." (Basel II paragraph 669(c))

S27. The bank must employ a unit of measure that is appropriate for the bank's range of business activities and the variety of operational loss events to which it is exposed, and that does not combine business activities or operational loss events with different risk profiles within the same loss distribution.
(Proposed U.S. Supervisory Guidance on AMA, p219, February 2007)

Identifying the appropriate unit of measure: Standard Basel II Matrix

	Event Type / Business Line	ET 1 Internal Fraud	ET 2 External Fraud	ET 3 Employment Practices & Workplace Safety	ET 4 Clients, Products & Business Practices	ET 5 Damage to Physical Assets	ET 6 Business Disruptions & System Failures	ET 7 Execution Delivery & Process Management
BL1	Corporate Finance							
BL2	Trading & Sales							
BL3	Retail Banking							
BL4	Commercial Banking							
BL5	Payment & Settlement							
BL6	Agency Services							
BL7	Asset Management							
BL8	Retail Brokerage							

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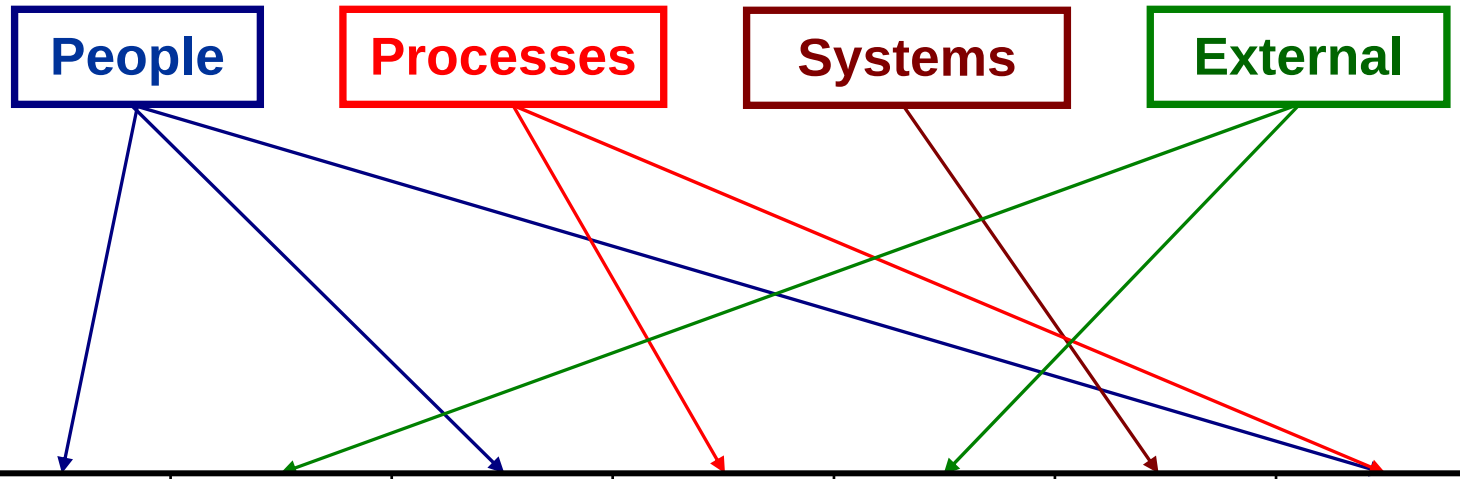
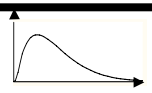
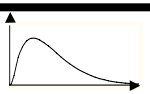
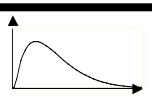
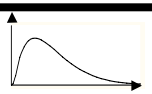
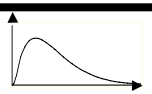
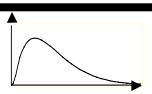
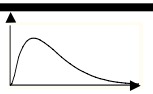
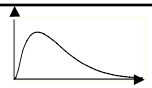
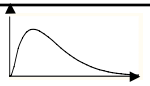
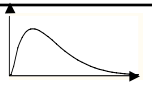
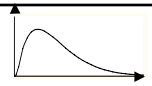
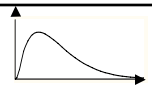
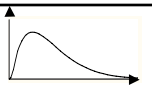
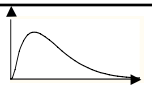
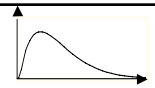
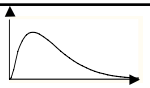
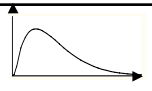
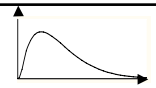
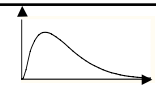
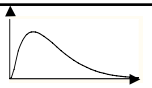
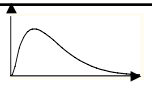
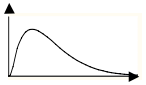
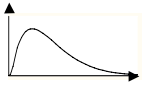
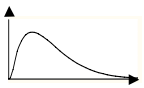
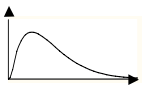
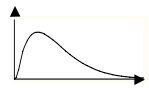
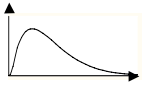
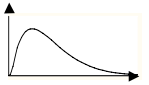
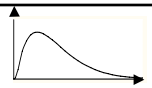
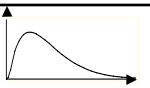
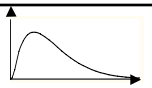
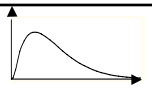
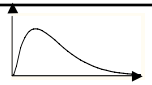
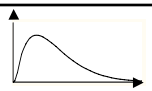
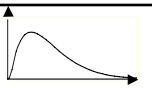
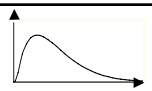
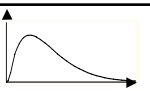
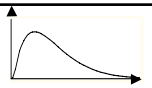
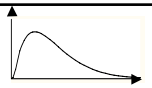
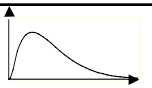
Identifying the appropriate unit of measure



ET / BL	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management
BL_1							
BL_2							
⋮							
BL_n							

El-Gamal, Inanoglu and Stengel 2007

Identifying the appropriate unit of measure: Standard Basel II Matrix

	Event Type / Business Line	ET 1 Internal Fraud	ET 2 External Fraud	ET 3 Employment Practices & Workplace Safety	ET 4 Clients, Products & Business Practices	ET 5 Damage to Physical Assets	ET 6 Business Disruptions & System Failures	ET 7 Execution Delivery & Process Management
BL1	Corporate Finance							
BL2	Trading & Sales							
BL3	Retail Banking							
BL4	Commercial Banking							
BL5	Payment & Settlement							
BL6	Agency Services							
BL7	Asset Management							
BL8	Retail Brokerage							

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Range of U.S. Banks' Unit of Measure

- Single Unit (Pooling all data in one unit)
  - ➢ Allocation Issue

  (See paragraph Basel II 666(b))

- 1 dimensional
  - ➢ Either by BL or ET

  e.g. 5 ETs (Excluding BSDF and DPA)

- 2 dimensional BL/ET

Aggregation of units:

A.K.A. Correlation/Dependence

"Risk measures for different operational risk estimates must be added for purposes of calculating the regulatory minimum capital requirement. However, the bank may be permitted to use internally determined correlations in operational risk losses across individual operational risk estimates, provided it can demonstrate to the satisfaction of the national supervisor…"

(Basel II paragraph 669(d))

Aggregation: A brief detour on correlation…

- Recall that in market risk, correlations are described as "equal-time correlations".
- E.g. for two assets:

TIME	ASSET 1	ASSET 2
t_1	r_{11}	r_{21}
t_2	r_{12}	r_{22}
t_3	r_{13}	r_{23}
.	.	.
.	.	.
.	.	.
t_n	r_{1n}	r_{2n}

- Correlation of op risk loss severities in the market-risk manner is "ill-defined".

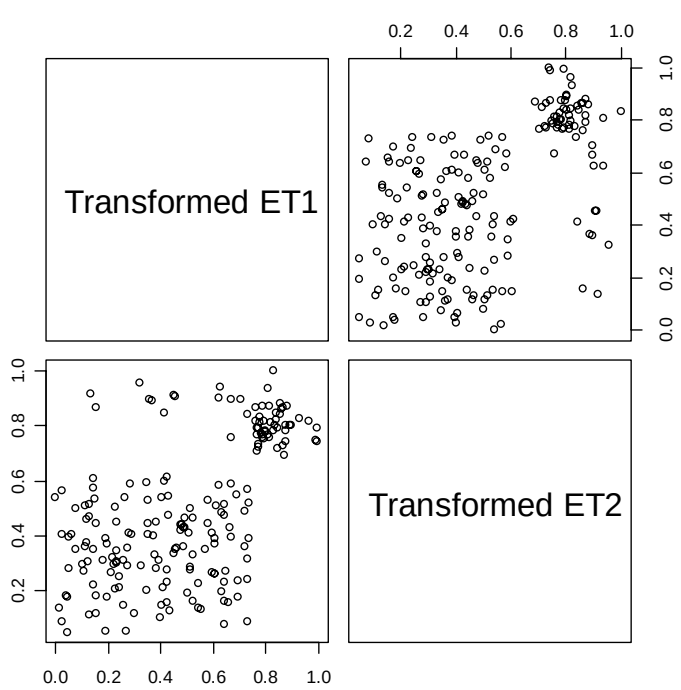
TIME	ET4	ET7
t_1	N/A	$L7_1$
t_2	N/A	$L7_{2,1}, L7_{2,2}$
t_3	N/A	$L7_3$
.	.	.
t_k	$L4_k$	$L7_k$
.	.	.
t_n	N/A	$L7_n$

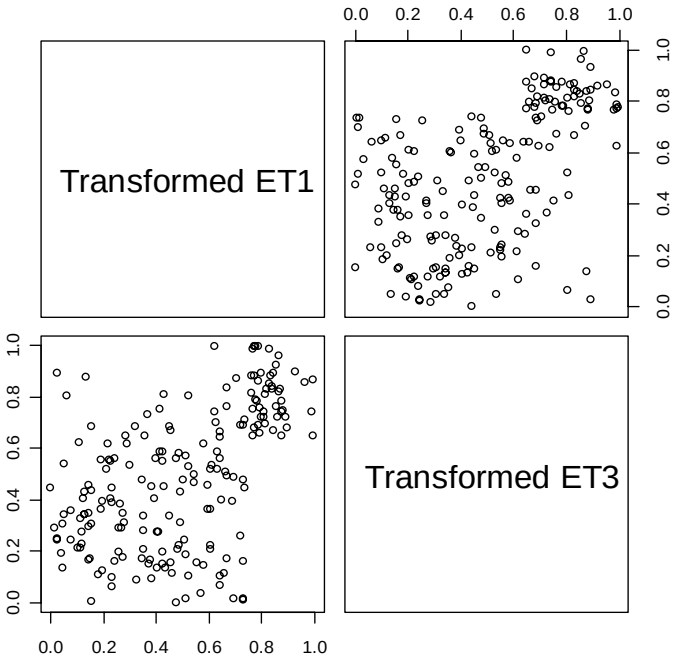
Dependence Structure: A technical problem

- In market risk, correlation boils down to correlating the returns of assets.
- In op risk, the modeler has to decide on what to correlate, i.e.;
  - Frequencies (minimal effect on capital)
  - Severities (ill-defined)
  - Aggregate Losses
- Some previous work which touched base with this fact:
  - Powojowski, and et al (2002)
  - Frachot, and et al (2005)

Modeling Dependence Structure

- Limitations of linear correlation
- Interest in “joint extreme values”
- Facilitate multivariate modeling





Scatter Plots

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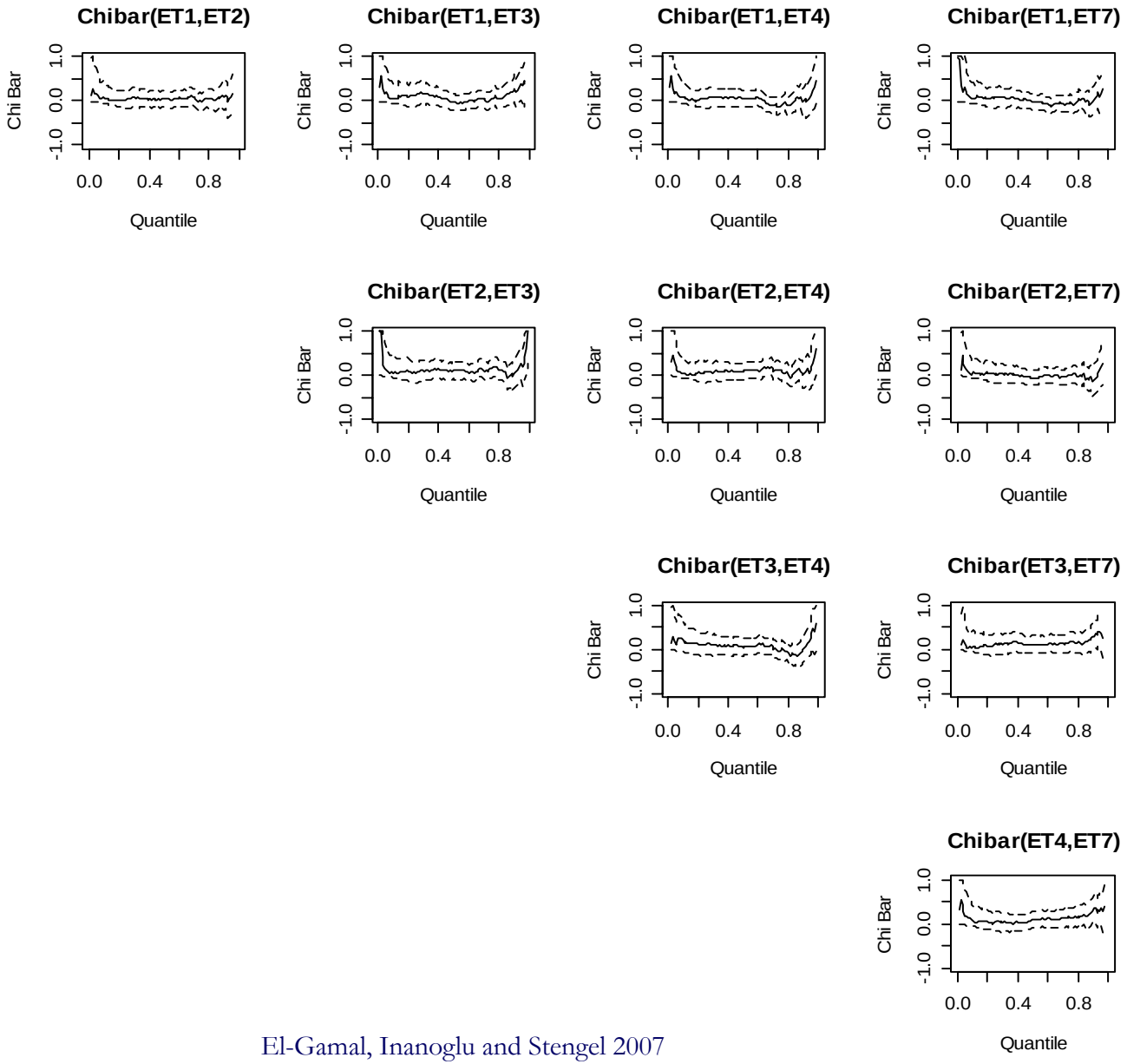
Dependence Structure at Various Quantiles



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Modeling Dependence Structure: Copula Approach

- Sklar's Theorem:

$$F(x) = C_F(F_1(x_1), ..., F_d(x_d)), \; x \in \mathbb{R}^d$$

Multivariate Distribution Function

Copula Function

Marginal Distribution Functions

- Choice of copula: t-copula

$$C_F = t_{\nu,P}(t_\nu^{-1}(u_1), ..., t_\nu^{-1}(u_d)), \; \mathrm{u} \in [0,1]^d$$

Multivariate Student t

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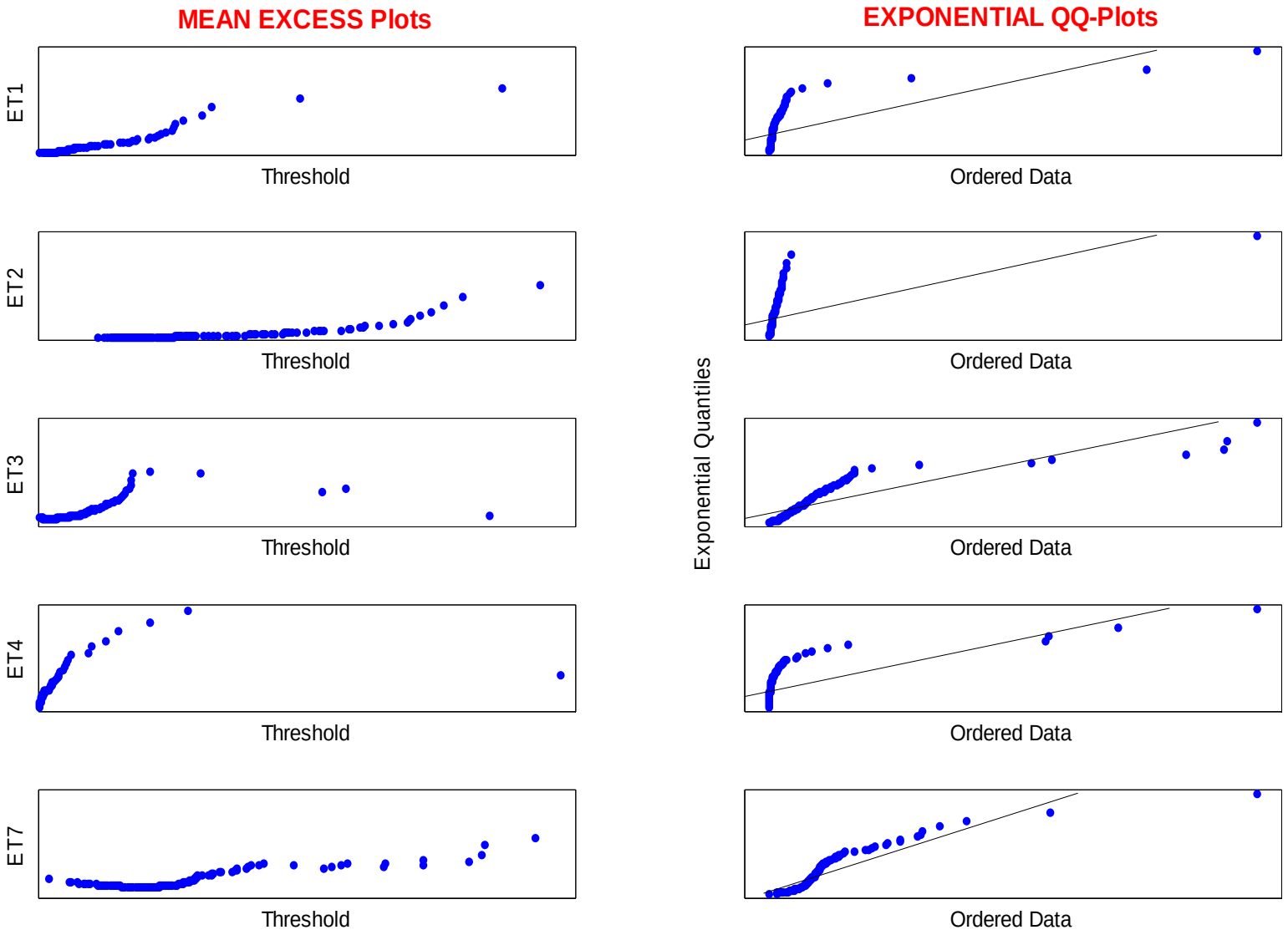
Our Data

- Of the banks that participated in the 2004 LDCE, we will focus on one institution only.

- Data Range:
  - Aggregated Weekly Data (2001-1 to 2004-9) for losses
  - 196 weekly observations for each of 5 Event Types

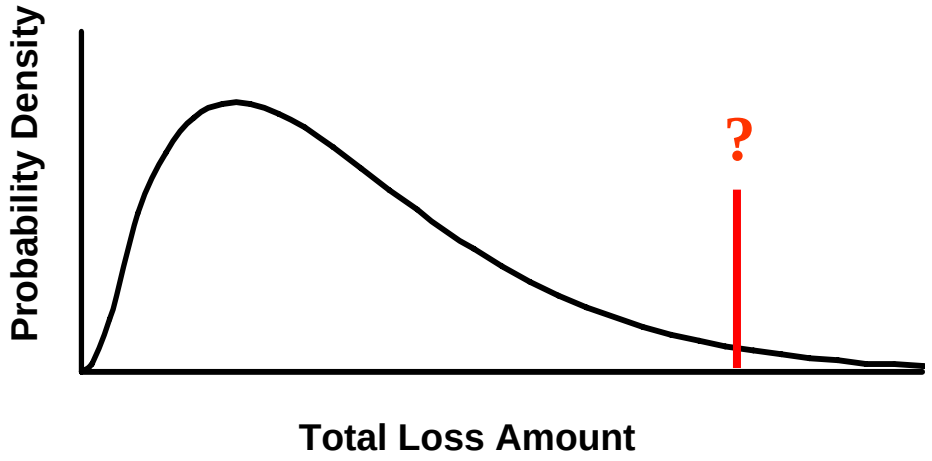
Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Execution, Delivery & Process Management

Exploratory Data Analysis



Fitting the Tail

Threshold choice: Dividing the body from the tail



Previous Literature

- Hill Plot (Graphical Method)
- Optimization in terms of bias-variance trade-off
- Combined likelihood function (Cibele, et al. 2004)

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Combined Likelihood Function

$$L(x \mid \mu, \sigma, \lambda, \gamma, \tau) = \begin{cases} f_{LN}(x \mid \mu, \sigma) & \text{if } x \le T_\lambda \\ (1 - F_{LN}(T_\lambda \mid \mu, \sigma)) \times f_{GPD}(x - T_\lambda \mid \gamma, \tau) & \text{if } x \ge T_\lambda \end{cases}$$

- Treat threshold as a parameter.
- Estimate the threshold in addition to distribution parameters within the MLE process.

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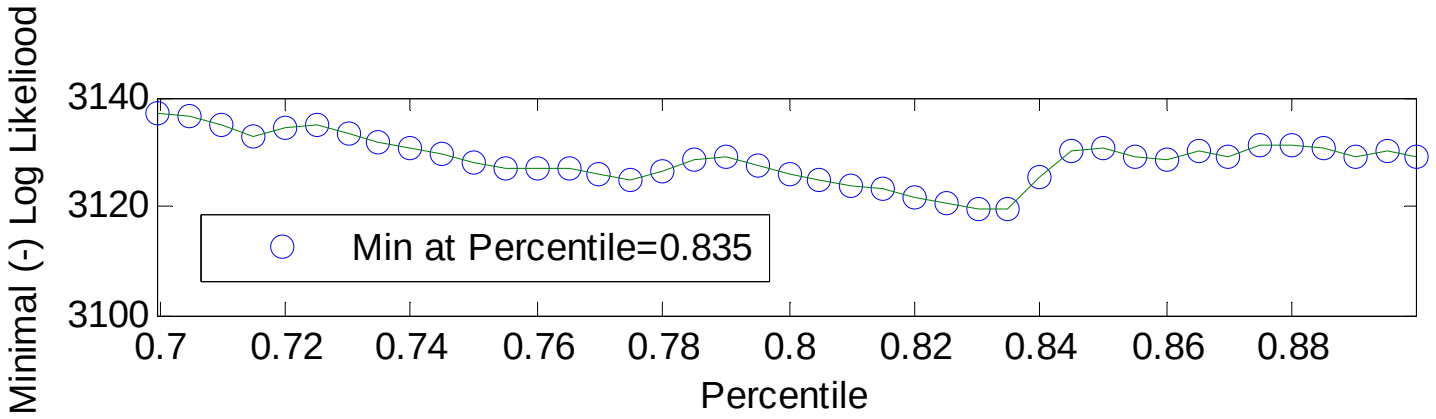
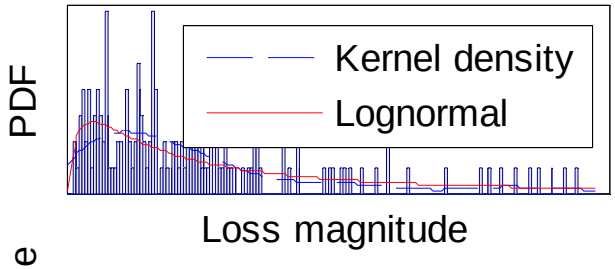
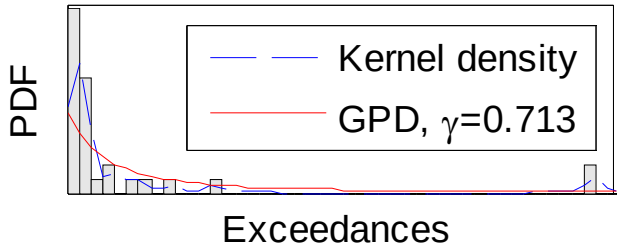
Fitting Severities (ET4)



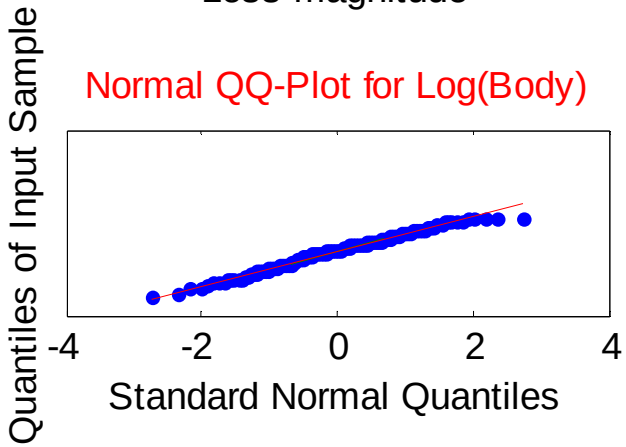
Body Histogram & Fitted Lognormal



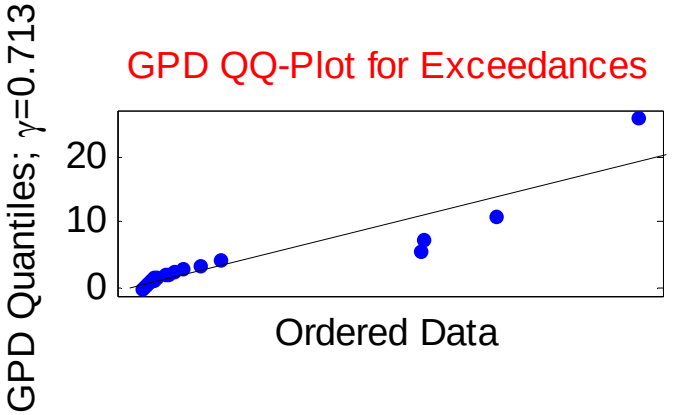
Tail Histogram & Fitted GPD



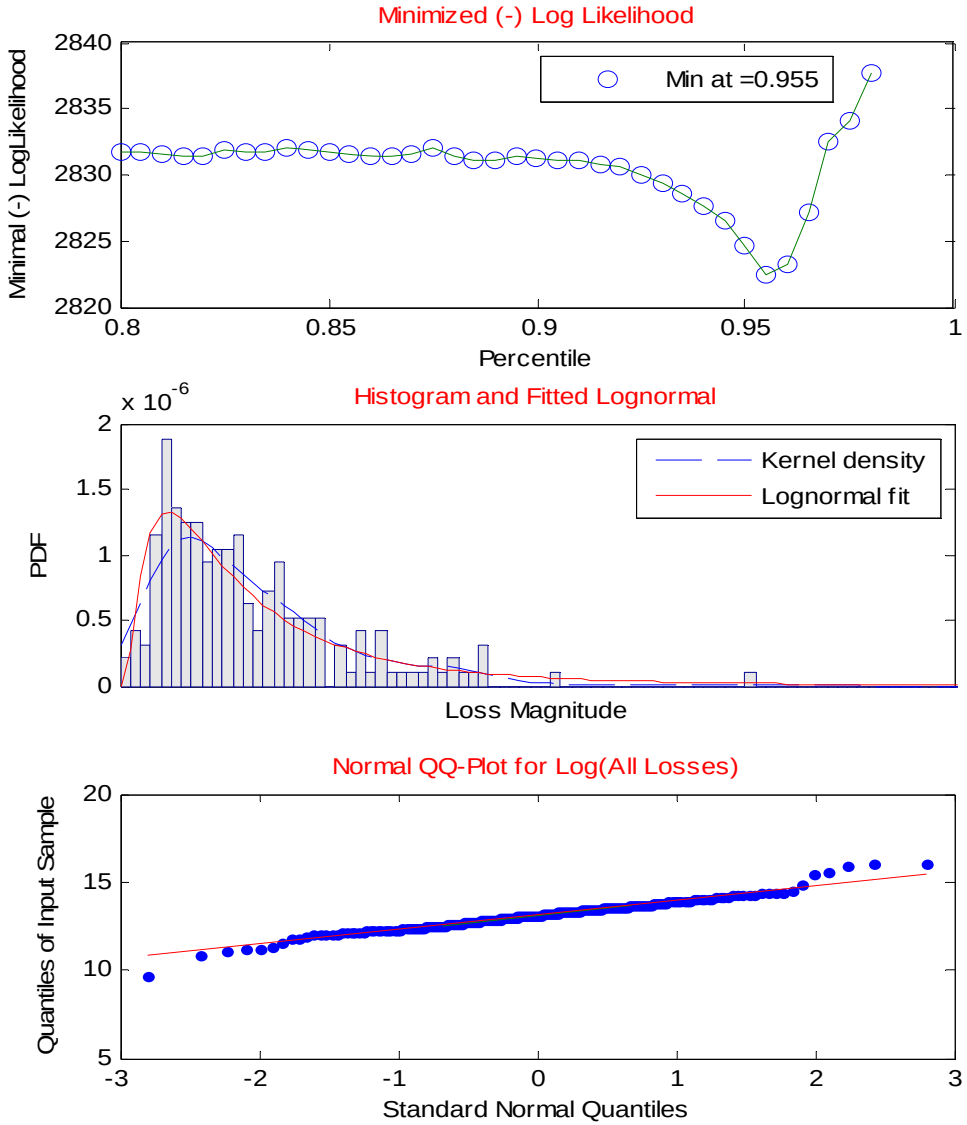
Normal QQ-Plot for Log(Body)



GPD QQ-Plot for Exceedances



Fitting Severities (ET3)



Results:

Value at Risks and Expected Shortfalls*

- **Aggregation of "units" using t-copula**

Model	$VaR_{95\%}$	$VaR_{99\%}$	$VaR_{99.9\%}$	$ES_{95\%}$	$ES_{99\%}$	$ES_{99.9\%}$
All data, lognormal only	0.160	0.208	0.327	0.192	0.259	0.424
All data, with EVT	2.795	6.561	28.407	7.478	21.855	119.330
Without 3 outliers, with EVT	0.481	0.731	1.721	0.678	1.148	2.869
Without 4 outliers, with EVT	0.283	0.354	0.551	0.335	0.442	0.763
Without 5 outliers, with EVT	0.230	0.271	0.367	0.258	0.312	0.461

***in $ US billions,**

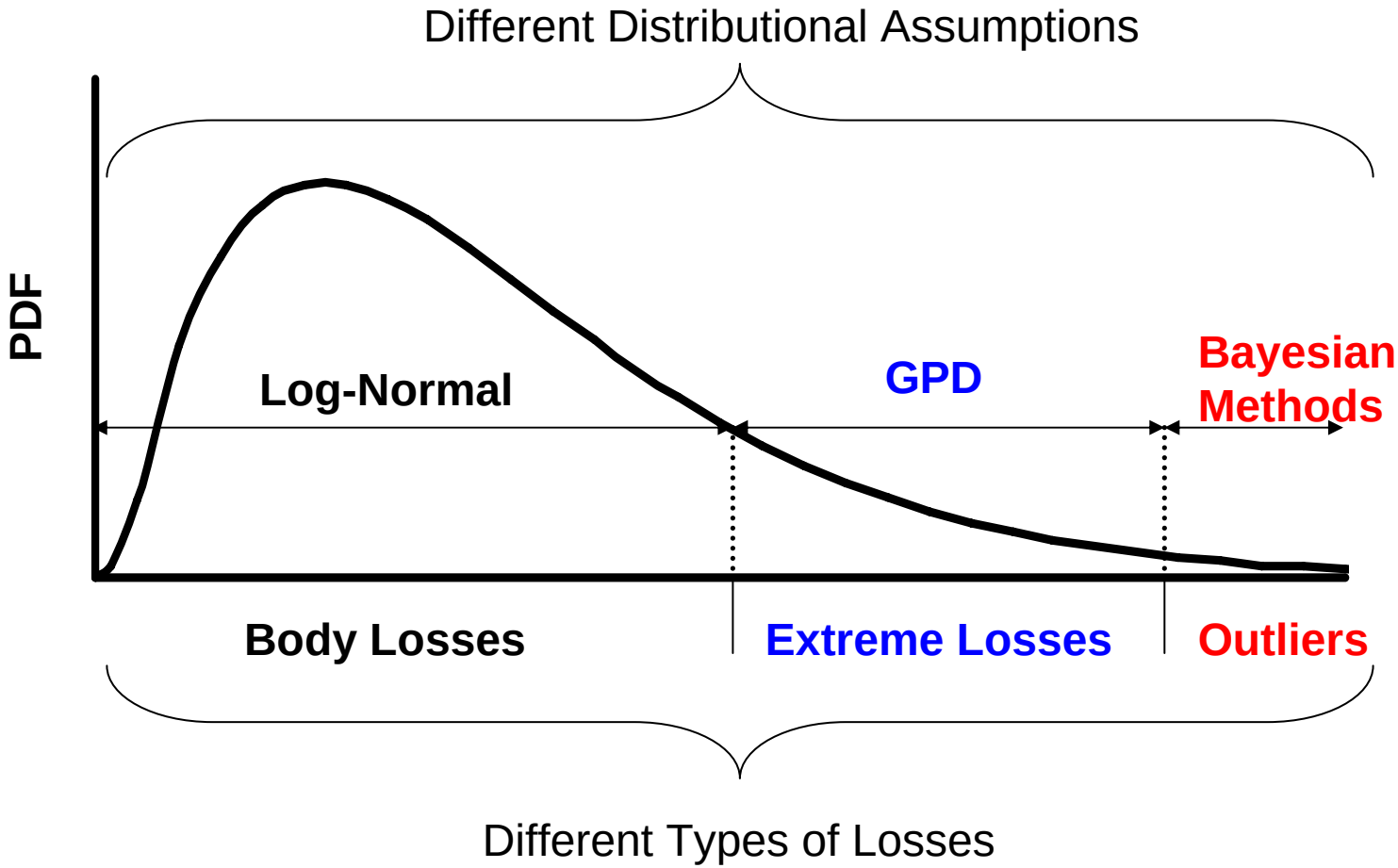
Results:

Diversification Effects

- Assuming perfect correlation for $VaR_{99.9\%}$ across "units of measure"

Model	t-copula $VaR_{99.9\%}$	"Simple Sum" $VaR_{99.9\%}$	Diversification Effect
All data, lognormal only	0.327	0.362	10.7%
All data, with EVT	28.407	28.535	0.5%
Without 3 outliers, with EVT	1.721	1.761	2.3%
Without 4 outliers, with EVT	0.551	0.590	7.1%
Without 5 outliers, with EVT	0.367	0.399	8.9%

Conclusions and Future Work:



- Use EVT judiciously.
- Complement the analysis with Bayesian Methods for outliers.

Conclusions continued:

- Use test

  "The bank's internal operational risk measurement system must be closely integrated into the day-to-day risk management processes of the bank. Its output must be an integral part of the process of monitoring and controlling the bank's operational risk profile…"

  (Basel II paragraph 666(b))