Hough transform and thermo-vision for monitoring pantograph–catenary system

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Abstract: The pantograph-overhead contact wire system is investigated by using an infrared camera. As the pantograph has a vertical motion because of the non-uniform elasticity of the catenary, in order to detect the temperature along the strip from a sequence of infrared images, a segment-tracking algorithm, based on the Hough transformation, has been employed. An analysis of the stored images could help maintenance operations revealing, for example, overheating of the pantograph strip, bursts of arcing, or an irregular positioning of the contact line. Obtained results are relevant for monitoring the status of the quality transmission of the current and for a predictive maintenance of the pantograph and of the catenary system. Examples of analysis from experimental data are reported in the paper.

Keywords: catenary/pantograph system, Hough transform, high-speed trains, thermo-vision

1 INTRODUCTION

The development of innovative methods for improving efficiency, reliability, and safety in railway systems is a primary objective of the railway research. Especially in the case of high-speed trains, a regular current transmission in pantograph–catenary systems becomes a very difficult task to be satisfied. A regular current transmission in railway systems is more and more important if the European Standard 96/48/CE about interoperability is considered: Standard 96/48/CE defines the rules for an efficient transnational circulation of high-speed trains in Europe, i.e. a locomotive will be designed to freely operate inside European countries independently from the feeding voltage and frequency. It is well known that a high quality current collection is characterized by a continuous contact between the pantograph and the overhead contact wire. A poor contact produces various drawbacks, including break arcs. If break arcs have a long duration, the efficiency of the locomotive may be reduced, and moreover an excessive wear of the pantograph strips and of the contact wire may be produced. Besides the accelerated wear of materials and the electrical problems, the most harmful and dangerous effect due to a defective current collection is an unexpected sudden breaking of the contact wire: such event can put out of service lines and trains for hours or days and can cause accidents with possible threat to passengers’ health. Total heat quantity dissipated by the sliding contact during the current transmission is the sum of the heat by friction, contact resistance, and arc discharge, generated when the contact breaks. Temperature of arc plasma is known to reach 3500–4000 K, therefore, temperature of the surface region rises promptly around the spot where the arc is discharged. In case of bursts of arcing, electronic regulators (usually switching regulators) suffer anomalous stresses, consequently high-order harmonics on the line current further deteriorate the efficiency and reduce the quality of the traction torque. Furthermore, the presence of harmonics may be dangerous for possible interactions with signalling transmissions, with a critical increasing of electro magnetic emission and a dangerous reduction of safety. The risk of harmful interference phenomena is particularly relevant when also the signalling systems employ the tracks as a part of their circuitry, which was in the past a useful technical solution to reduce systems build-up costs.
Therefore, a reliable detection of the break arcs is very important for setting up controlled systems (e.g., active pantographs), for reducing the accelerate wear of the materials and for avoiding as much as possible any interference with communication networks. High speeds worsen these negative aspects: the higher speeds, the more critical the problems. Over 200 km/h, wires start oscillating and the pantograph head cannot maintain contact in the case of abrupt height variations. The presence of more than one pantograph on a train worsens the problem of oscillating wires for the rear pantograph. It encounters an overhead line already excited by the first one, so that the quality of the contact deteriorates and break arcs increases their frequency and duration. In order to improve the current collection at high speed and to reduce the deterioration due to break arcs, different solutions have been proposed. The traditional solution consists of increasing the uplift contact force between the pantograph and the contact wire, at the expense of a reduction of the life-time of the collector strips and particularly of the contact wire, because of erosive and abrasive wear and of temperature raising. Such a solution was adopted in the past, but it is today inadequate for high-speed trains. A primary objective in railway research is the reduction of contact forces. A first interesting solution is the design of innovative pantographs with reduced head mass and active control. All railway companies are well aware of the problems affecting the current collection: a relevant objective both for academic researchers and for railway companies is the development of innovative pantographs and the application of new sensors for a continuous monitoring of the sliding contact and for helping maintenance activities. Therefore, many universities have devoted their activities to deal problems relative to the rail transportation.

The main result of our past research [1, 2] was the study and implementation of sensors (UV and current sensors) for detecting break arcs. A correlation of measured repetitive break arcs with the kilometric progression of the railway line Rome–Florence revealed its validity for an automatic checking of the status of the contact overhead line. These results validated from a strict cooperation with Trenitalia (Italian Railways) proved to be a useful tool for helping maintenance activities both for the overhead line and for the pantograph.

The new research line proposed is based on the study of the quality of the contact pantograph–catenary with infrared techniques. The research based on thermography is totally new for pantograph studies and its results are foreseen to be meaningful for testing new materials for collector strips and new typologies of pantographs and catenary. Thermographic images are more informative than standard camera images, because of their insensitivity to different weather conditions, to the daily-nightly runs or to the presence of tunnels. Hot spots on the overhead line and on the pantograph strips are easy to detect and a careful analysis of the stored images could help maintenance operations in the case of irregular positioning of the line (e.g., the wire should be staggered to even the wear on the train’s pantograph: in the case of a defective layout of the line, the pantograph strip dramatically overheats and such effect should be easily revealed).

2 THERMOGRAPHY FOR MONITORING PANTOGRAPH–CATENARY INTERACTIONS

Thermo-cameras have not been used extensively in railway research. In reference [3] preliminary results based on the analysis of infrared camera acquisitions were proposed. The quality of the current collection was evaluated by monitoring the infrared emission at the contact point. Measurements carried out during high-speed test runs and a post-processing of the data collected have revealed the effectiveness of the proposed method for the detection of the losses of contact, in order to evaluate and test the performance of running pantographs and of the contact wires.

Main advantages offered by thermo-vision are as follows.

1. It is a non-contact and non-destructive technique.
2. It is suited to monitor devices operating under high voltage or carrying high currents.

A critical aspect of thermo-vision is that the exact temperature of the body under test cannot be directly revealed. Nevertheless infrared imaging is an excellent method for extracting a qualitative map of the superficial temperature. As a matter of fact, thermal analysis gives relative information on the temperature, quoting [4]: ‘all objects at temperatures above absolute zero emit electromagnetic radiation. Radiation thermometry makes use of this fact to estimate the temperatures of objects by measuring the radiated energy from selected regions’.

Each physical process characterized by an increase or decrease in surface temperature is detectable with infrared thermography. The intensity of the emitted radiation depends on two factors: the body temperature and the factor emissivity of the surface, i.e., the ability of the object to radiate, defined by the Stefan–Boltzmann equation

$$E = \varepsilon \sigma T^4$$  \hspace{1cm} (1)

The emissivity factor $\varepsilon$ lies between zero and the unity, depending on the material nature and on the superficial roughness. Infrared imaging systems
convert infrared heat emissions into a picture showing the relative temperature differences in a range of grey tones, or in a series of colours, in such a way that the desired temperature information is easily interpreted by the user. An analysis of the acquired images leads to very interesting results: an IR imaging is more informative than a standard camera image.

In Fig. 1(a), a typical infrared image of pantograph–catenary interaction is shown (the train speed was 200 km/h in this case). The maximum infrared emission is in the neighbourhood of the contact surface between strip and wire. The interaction region between the overhead feeder and the strip shows a high thermal gradient. A continuous monitoring of the contact region gives interesting information on the quality of the current transmission: a continuously increasing temperature with slow time constant denotes that friction is increasing. On the other hand, fast phenomena produce bursts of arcing, i.e. losses of contact between catenary and pantograph (Fig. 1(b)).

A first question for applying thermo-camera images to pantograph–catenary interaction is: how to follow the contact point? In fact the wire is staggered relatively to the centre-line of the track, to even the wear on the train’s pantograph as it runs underneath. Moreover, the height of the contact wire may change (e.g. entering a tunnel). Therefore, a relevant displacement of the contact point may complicate the image processing, unless the moving spatial coordinates of the contact area are considered as known. A first step for an automatic image processing analysis is to follow frame-by-frame the position of some critical features (pantograph strips, overhead electric line). All these elements can be characterized by linear contours. This observation allows using standard algorithms for detecting straight lines inside each image under examination.

3 APPLICATION OF THE HOUGH TRANSFORM FOR MONITORING PANTOGRAPH–CATENARY INTERACTION

The task of the proposed research is the application of tracking algorithm of predefined objects (e.g. the contact point on the pantograph strip) over a sequence of images acquired from thermo-camera, in order to monitoring variables of interest (e.g. temperature at the contact point). The Hough transform is well suited for detecting straight-lines in the real image: its application can be extremely useful for monitoring the contact point, characterized from the intersection of a segment representing the strip and a straight line representing the overhead line contact. Furthermore, it can identify different rectilinear structures, such as the poles along the railway line, helping in finding a more precise correlation between the overheating of the contact point (or a burst of arcing) and the position of the train. A detailed description of the Hough [5] transform is reported in Appendix 2.

The analysis presented in this paper considers images recorded along the Florence–Rome railway line. Suppose to know approximately the position and the possible movement of the features to be monitored, so that two observation windows into the image are locked: a first one includes the pantograph structure and the second one reveals the passage of the support structures.

Independently from the electronic format of the recorded file (outputs are considered in .bmp, .mpg or .avi formats), single frames are extracted from the image sequence. Each image is first stored in a

![Fig. 1](a) Infrared image of pantograph–catenary interaction and (b) thermal image in case of a burst of arcing
matrix, then converted from true colour (RGB) to grey scale.

Each image is suitably filtered for reducing noises and sharpens objects, then edges are extracted and image is then converted from a grey scale to a binary image. Edge detection was performed using the Canny algorithm [6] with a preliminary noise reduction.

After this step, each frame is represented by a binary image with edges evidenced, making the identification of the pantograph, the overhead contact lines, and the poles, an easy task.

Object locations in each frame is characterized from an analysis of edges and a detection of reference straight lines.

All straight lines can be localized using the Hough transform: to increase algorithm efficiency and to reduce computational burden, the straight line search can be limited inside suitable rectangular regions of the image. Sizes of the windows and their positions relative to the full image are chosen inside the sequence: the hypothesis is that they have constant dimensions inside the sequence.

A relevant advantage of reducing the analysis to a window inside the full image, is that computational time may be greatly reduced and a good practical choice is to impose simple rules to define the positions of the windows as soon as possible.

An object recognizing procedure allows a fast detection of the object position and a repetitive algorithm allows following the trajectory of an object inside successive frames. After a first detection of the trajectory, it is a good strategy to eliminate frames with discontinuous trends of the object and to filter the image for reducing noise and to sharpen the image.

Following this logic, a software tool was created that allows the user to automatically detect:

(a) pantograph position;
(b) catenary wires;
(c) position of the poles.

This procedure can be repeated for each frame of the sequence. As the pantograph moves slowly with respect to the frame rate of the camera, the position of the strip detected is used in the last image to window next image appropriately. Therefore, the computation time can be greatly reduced.

3.1 Position of the pantograph inside an image: detection logic

The pantograph is the key object of the recorded scene: it appears at each frame and its position moves slowly. Small displacements both in vertical and in horizontal direction are allowed: they are due to the movements of the main frame and of the contact arms.

In thermo-images, two different pantograph heads and two different strips are evidenced: the goal of the following study is to follow only the strip nearest the IR camera. To be clearer, the task is to detect and follow the nearest pantograph head, for monitoring variations in the temperature of the upper edge, i.e. in the contact strip.

By considering the application of the Canny algorithm for edge detection (Figs 2(a) and (b)), some almost horizontal segments are detected (evidenced inside a red ellipse in Figs 2(c) and (d) in the following study they will be considered as reference lines).

Contact analysis is applied to the squared window of Fig. 3(a), named reference box: its size and position inside the image will remain unchanged for the limited displacement of the pantograph head. Several segments are easily recognized by Hough transform (Fig. 3(a)) only the upper one represents the contact line for monitoring (Fig. 3(b)).

Reference box allows following the pantograph displacements along a vertical direction. For a correct analysis, the horizontal displacement also has to be measured. Lateral displacements can then be estimated from an analysis of the two lateral arms sustaining the pantograph heads, (Fig. 3(c)).

The two segments representing the external profile of these lateral supports can be identified by repeating the Hough analysis inside two new reference boxes, as shown in Fig. 3(d)).

Fig. 2 Edge detection: (a) Canny algorithm, (b) Canny algorithm with noise reduction, (c) pantograph edge detection, and (d) support infrastructure and overhead wires edge detection
The three reference boxes can be joined together for localizing a triangle as the reference shape representing the pantograph, as shown in Fig. 4. The upper side of the triangle is representing the line useful for monitoring the contact with the catenary wires. Note that the representation of the pantograph head with a straight line is only an approximation of the real scenario. Nevertheless, the maximum error accumulates in the lateral regions of the segment, where the pantograph head is not rectilinear: as a matter of fact in such regions an analysis of the contact is less relevant, as the local temperature is lower than in the middle zone of the segment.

3.2 Identification of the contact between pantograph and overhead contact wire

After the identification of the pantograph structure, the following step is devoted to the identification of the contact zone between pantograph and the

![Fig. 3](image3.png)

**Fig. 3** Application of the Hough transformation: (a) horizontal detected segments into the reference box of the strip, (b) upper edge of the nearest strip, (c) detected segments into the reference boxes of the lateral supports, and (d) detected lateral supports

![Fig. 4](image4.png)

**Fig. 4** Pantograph detection: (a) detected line by Hough transformation and (b) reference shape
overhead wire. In the case of the image sequences recorded along the railways from Florence to Rome, it can be observed that there are many straight lines, between them two overhead wires (Fig. 5(a)) are in close proximity to the contact region.

It must be highlighted that the contact points are not corresponding to the hottest points of the strip, usually in the middle of the contact strip. Such observation excludes to follow maxima temperatures for detecting the contact points and it requires considering the interaction between the upper side of the triangle representing the pantograph and the overhead contact wires.

Hough transform can be applied again to detect overhead wires, as shown in Fig. 5(b).

As shown in Fig. 5, many straight lines are detected. A selection of the correct contact wires was performed on the basis of the two following rules.

1. Contact wires are always adjacent and at the exterior of the set of the identified lines.
2. Contact wires are thicker than non-contact ones.

After the detection of the pencil of lines of Fig. 5(b), the contact points are uniquely located via the following steps.

1. Computing the intersection between the upper side of the triangle (pantograph head) and the two straight lines exterior to the pencil of lines (Fig. 5(c)).
2. Analysing the colour distribution of the pixels inside the image, near the two points selected at step 1. The thickest line of the two is selected, for detecting the contact zone, whose extension is predefined by a fixed number of pixels in the direction of the middle of the pantograph head (Fig. 5(d)).

In such a way, only a reduced box surrounding the contact points is recognized, instead of the two contact points; nevertheless, this result constitutes a good approximation both for monitoring the temperature in the contact region (e.g. its mean value inside the reduced box) and for checking a correct positioning of the contact wire in terms of its lateral movements, relatively to the centre-line of the track.

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Fig. 5 Identification of the contact between pantograph and overhead contact wire: (a) image under examination, (b) detected straight lines representing catenary wires, (c) detected boundaries, and (d) identified contact region
3.3 Passages under the portal structures of the railways

In the case of a railway line with portal structures, a similar method can be employed for detecting the poles. The horizontal structure is the easiest element to be recognized: it is the wider structure and it appears even if the poles are not included inside the image, as shown in Fig. 6(a).

Therefore, by counting the passages of horizontal portals, it is easy to localize the train position along the line: this task can be performed using an observation box positioned in a suitable region of the image. An easy solution is to locate this box in a region where only the horizontal portals are passing, e.g. in the upper left side of each frame (Fig. 6(b)).

The passages of horizontal portals must be detected avoiding any possible false condition. The Hough transform for straight lines recognizing can be applied inside the observation box and a reliable criterion of portal detection may be based on an accurate analysis of the accumulation matrix. Its maximum value is, very high in case of passages of portal structures: a threshold value is therefore, useful for discriminating noisy images or for single straight lines passages.

In Fig. 7, the values contained in the accumulator matrix for a sequence of 500 frames are shown: the differences of maxima in cases of passages of portal structures: a threshold value is therefore, useful for discriminating noisy images or for single straight lines passages.

Cases of different consecutive values overcoming the chosen threshold in adjacent frames are due to the finite time of the portals for crossing the observation box: a correct counting must take into account only the first of a set of consecutive frames. A variable time delay, depending on the speed of the train, can be incorporated for evaluating the passages underneath the portal.

To be clearer, consider an IR camera-fixed reference frame: the relative motion of the portal has an opposite direction with respect to the train, as depicted in Fig. 8(a). The grey triangle is representing the vision angle of the IR camera, the black square is the location of the portal at the instant when the pantograph is passing underneath. In such instant, the portal is outside the vision angle and only when the train moves, the portal is revealed from the camera image (Fig. 8(b)). The distance $s$ between the points A and B is constant and can be computed from the knowledge of the vision angle and on its slope with respect to the horizontal axis. If the train speed is acquired, it is an easy computation to obtain the time delay of the portal for moving from point A to point B, and it is possible to estimate the exact time of the passage of the pantograph under the portal structure from the knowledge of the instant when the portal is revealed from the camera.

3.4 Trajectory improvement

All the procedures previously described lead to a reliable recognizing of the pantograph and of the contact region included into the IR images: they are repeated for each frame of the overall sequence.

![Fig. 6](a) Horizontal bridge sustaining catenary wires, and (b) observation box for counting the passages of portals

![Fig. 7](Maxima contained in the accumulator matrix for a sequence of 500 frames)
The procedures work correctly, apart from a very restricted number of cases, where the algorithm fails or the results are not accurate. In Fig. 9, the trajectory of the contact point is shown for a sequence of 20 frames.

Corresponding to frames 11 and 13, two outlier points are detected: each abrupt variation of the trajectory is due to errors in the recognizing procedure because of the physical continuity of the contact point in adjacent frames. This problem is easily solved using a threshold for discriminating unacceptable variations of the position in adjacent frames and, in case of outlier points, replacing them with different values (e.g. the previous value can be saved and maintained).

To smoothen the scattering effect of the obtained points, a low-pass filtering action can be introduced, e.g. using a first-order filter of Bessel type (Fig. 9(b)). The cut-off frequency of the filtering action can be suitably chosen as a function of the velocity of displacement of the objects in the sequence.

Circles are the original values, straight lines represent the sequence after the outlier replacement, and dotted line is the result of the first-order filtering action: it is a realistic smooth approximation of the continuous displacement of the contact point.

4 EXPERIMENTAL RESULTS

An accurate choice of the infrared camera is a very important factor. Two categories of camera (IR) are commercially available: short wavelengths infrared cameras (SW_IR) 3–5 μm and long wavelengths infrared cameras (LW_IR) 8–13 μm. Infrared images have to look through atmosphere, but, unfortunately, the atmosphere does not transmit infrared energy well at all wavelengths. SW and LW are the spectral ranges with high atmospheric transmission factor. Short wave systems are not as good for outdoor use, because of greater interferences from natural sunlight. Long wave systems are also less distorted by the atmosphere: this is a basic motivation for employing them in our research. A preliminary laboratory test was carried out in order to yield the emissivity of the pantograph strip. In Italian railways (3 kV d.c.), the contact material strip is copper (or copper based
alloys). By using the IR camera software, the average emissivity of the strip was determined to be 0.2. The measurement equipment was then installed on board of an ETR500, a high-speed train of Italian railways. The block diagram of Fig. 10 illustrates the measurement chain.

Trial runs have been carried out travelling along the railway line connecting Florence to Rome and vice versa. IR camera (produced by the Flir Systems) was located on the top deck of the locomotive, in front of the pantograph.

### 4.1 Portal structure detection of a positioning of the overhead contact line

In Figs 11(a) and (b), two plots are shown, the first one consider a portal structure detection, and the second one reports the position of the contact point with respect to the centre of the pantograph head (a positive value means the contact point is located on the right w.r.t. the median position). They are referred to an analysis of 200 frames. Plots are functions of a position relative to the first frame analysed. Train speed is considered as constant that is equal to 200 km/h.

It can be noted that portal structures are positioned each 60 m about, in a good agreement with the expected layout. Each portal corresponds to an inversion of the direction of displacement of the contact point: it indicates a correct positioning of the overhead line.

### 4.2 Example of break arc detection

Consider now, 100 consecutive frames with a break arc corresponding to frame 50, shown in Fig. 12(a). The observation box is representative of the contact point.

The temperature profile along the strip surface for frame 50 is shown in Fig. 12(b). An analysis of the overall frames gives the trend of temperatures; in Fig. 13(a), the maximum temperature on the strip surface is shown.
surface is considered, in Fig. 13(b) the maximum temperature in a region surrounding the contact point. Both of them indicate a burst of arcing. Figure 14 shows the infrared map of the strip along the railway line connecting Florence to Rome between 60 and 63 km. Temperature peaks are correlated with arcing occurrences. This elaboration has been obtained by processing 2700 frame of the video sequence with a computation time of 540 s (using an AMD® Athlon processor, 1.2 GHz).

5 CONCLUSIONS AND FUTURE WORK

The use of an infrared camera for monitoring the pantograph–catenary status has been proposed and experimentally tested. The temperature profile of the pantograph strip was detected on infrared images by using the Hough transformation. An analysis of the stored images could help maintenance operations, revealing, for example an irregular positioning of the line. In overhead contact
lines, the wires should be staggered, relative to the centre-line of the track to even the wear: in the case of a defective layout of the line, the pantograph strip dramatically overheats and a thermal analysis easily reveals such effect.

Future work will consist of a comparative testing of new materials for collector strips. New typologies of pantographs and catenary layout can be thermically analysed.

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REFERENCES


APPENDIX 1

Notation

\[ E \] \quad \text{radiation in W/m}^2

\[ T \] \quad \text{temperature in Kelvin}

\[ \varepsilon \] \quad \text{emissivity}

\[ \sigma \] \quad \text{Boltzmann constant}

APPENDIX 2

The Hough transform

In order to detect automatically the temperature of the contact strip, a tracking algorithm must be employed. The problem is typical of the image analysis: it consists of detecting and tracking straight lines in a sequence of images. Several methods applied to two-dimensional binary images are well-known in the literature [4]: among them a powerful method for detecting a segment in a binary image is based on the Hough transform [5]. Therefore, the problem addressed can be summarized in the application of a suitable search algorithm for extracting edges from the binary image and for grouping pixels from edges, such to identify (within a tolerance bound) collinear pixels.

In the sequel, a short description of the Hough transform is presented.

Hough transform (1962) is a modification of the Radon transform, specialized to identify collinear points (lines) in digital images [6]. It was naturally derived for identifying straight lines in a two-dimensional space for parameters, although it can be extended to fit other regular shapes (circles, ellipses, squares, etc.) at the expense of an increasing dimensionality in the space of parameters, e.g. a circular Hough transform requires a three-dimensional space for parameters. A generalized Hough transform can be employed in the cases where a simple analytical description of shapes is not possible. The main feature of the Hough transform is that it is an algorithmic procedure based on a pixel image transformation into a new space, denoted as Hough or parameters’ space. After defining the line that needed to be searched into the image, every pixel of the edge indicates its contribution to the defined line: every point present in the real space image casts its votes into the Hough space for each of the lines that can possibly pass through it. To be clearer, consider the problem of straight line detection. The straight line connecting a sequence of pixels can be expressed in Cartesian coordinates as

\[ y = mx + b \]  

where the slope \( m \) and the intercept \( b \) are the two parameters describing any straight line.
Unfortunately, Cartesian description has the advantage that in an image \( m \) can become infinitely large for vertical lines. Therefore, the polar coordinates are usually considered

\[
\rho = x \cos (\theta) + y \sin (\theta)
\]  

(3)

where \((\rho, \theta)\) defines the shortest vector from origin to the line, as shown in Fig. 15(a). This vector is perpendicular to the line. Consider now a two-dimensional space defined by two parameters \( \rho \) and \( \theta \) (Hough space). Any line in the \( x-y \) plane plots to a point in this space (Fig. 15(b)).

If a point \((x_i, y_i)\) is considered in the real image (Cartesian plane), many possible lines are passing through this point, and each of these lines maps to a different point in the Hough \((\rho-\theta)\) space. If the point \((x_i, y_i)\) is known, although \( \rho \) and \( \theta \) are the unknown variables in equation (2), the locus of all such lines in the \( x-y \) space maps to cissoidal curves in the Hough parameter space. This point-to-curve transformation is the Hough transformation for straight lines.

If a set of collinear edge point \((x_i, y_i)\) with common parameters \( \rho_0 \) and \( \theta_0 \) are present (Fig. 15(c)), then each point maps to a different curve in the Hough space. However, all curves must intersect at the point \((\rho_0, \theta_0)\), as this is their common straight line (Fig. 15(d)).

A typical method for practically counting the intersection points \((\rho_0, \theta_0)\) is based on a discrete approach, called ‘voting’. The Hough transform is algorithmically implemented by quantizing the parameter space into accumulator arrays: in this sense, the Hough space is an accumulator space [7].

Consider the four points in the image shown in Fig. 16(a) Hough transform maps them into the four cissoidal curves of Fig. 16(b). The crossing points in the Hough plane are numbered and ‘voted’. For example, the intersecting point numbered as \( c \) in Fig. 16(b), corresponds to three intersections (three votes), i.e. the points B, C, and D are collinear in the real image, as in the inverse Hough transform shown in Fig. 16(c).

In such a way, the Hough transform defines an accumulator function: the point grouping problem is converted into a peak detection problem and the local maxima represent the more voted points, i.e. the lines that more likely belongs to the real image. The main advantages of the Hough transform are its effectiveness of detecting curves of any shape, even non-regular, and its robustness with respect to image gaps and to image noises. As a matter of fact, the local maxima in the accumulation function are poorly affected from gaps and image noises.

A possible algorithm able to detect straight lines into an image can follow the steps.

1. A binary image is obtained from the real image using an edge detector algorithm (e.g. the Canny algorithm [8]).

An edge detector algorithm produces a feature description into a real image. After this preliminary step, the Hough transform identifies the features (e.g. straight lines) and their quantity within
the edge description. For every pixel belonging to the edge \((x, y)\), consider the straight line in the Hough space. A straight line in the real image is characterized by a point \((\theta, \rho)\) in the parameters' space and a pencil of lines through \((x', y')\) can be represented by the curve \(2\) in the Hough space, with the constants \(x' e y'\), and the parameters \(\theta\) and \(\rho\) as unknowns. In such a way, the transformation maps each point of the real image into a unique curve in the Hough plane \(\theta, \rho\), and \(n\)-collinear pixels in the image plane are mapped into \(n\) curves intersecting in a point \((\bar{\theta}, \bar{\rho})\), whose coordinates are the parameters defining the straight line in the Cartesian plane.

After this preliminary step, the algorithm follows as:

2. Define an accumulator matrix such that its elements correspond to small regions of the Hough space (quantisation of the region).

The problem at this step is the arbitrary choice of the rows and columns to use in the matrix, i.e. the quantisation interval in terms of the subdivision in small regions of the Hough plane.

The accumulator matrix associates every element \(P[\theta, \rho]\) to a possible pair \((\theta, \rho)\), with \(\theta \in [\theta_{\min}, \theta_{\max}]\) and \(\rho \in [\rho_{\min}, \rho_{\max}]\). Interval limits must be selected to include all straight lines of interest in the real image.

3. Initialize to zero every element \(P[\theta, \rho]\) of the accumulator matrix.

4. For every pixel \((x, y)\) of the edge do

   \[
   \text{for } (\theta = \theta_{\min}, \theta \leq \theta_{\max}, \theta + +) \\
   \rho = x \cos \theta + y \sin \theta \\
   \text{round } \rho \text{ to a possible value} \\
   P[\theta, \rho] = P[\theta, \rho] + 1
   \]

   The problem at this step is to create for each point \((x, y)\) the curve \(\rho = \rho(\theta)\) and then to increment the elements (give votes) of the accumulator matrix.

5. Detect the largest elements of the accumulator matrix \(P[\theta, \rho]\), i.e. apply a peak location algorithm.

6. Apply the inverse Hough transform converting back the peaks to straight lines in the real image.