Cigarette Money*

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Abstract

We study the circumstances under which commodities emerge endogenously as media of exchange – the way cigarettes apparently did, for example, in POW camps – both when there is fiat money available and when there is not. We characterize how specialization, the degree of trading frictions, intrinsic properties of commodities, and the amount of fiat money available determine whether a commodity serves as money and its exchange value. In some equilibria, the exchange value of commodity money is pinned down by its consumption value; in others, it is not. The value of fiat money may or may not be pinned down by that of commodity money, depending on circumstances. We also allow commodities to come in heterogeneous qualities and discuss the implications for Gresham's Law.

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1 Introduction

In this paper we study the circumstances under which certain commodities emerge endogenously as media of exchange, the way cigarettes did in prisoner-of-war camps, for example, as described by Radford (1945). Radford's POW camp is an ideal laboratory in which to study the trading process and commodity money, for several reasons. As is clear from Radford's discussion, the POW economy was relatively uncomplicated, while at the same time of considerable importance to the prisoner. Also, there is no question that cigarettes did indeed serve as money: "Between individuals there was active trading in all consumer goods and in some services. Most trading was for food against cigarettes or other food stuffs, but cigarettes rose from the status of a normal commodity to that of currency. ... With this development everyone, including non-smokers, was willing to sell for cigarettes, using them to buy at another time and place. Cigarettes became the normal currency, though, of course, barter was never extinguished." (Radford, pp. 190-191).[1]

Our objective is to develop a theoretical model that captures the phenomena described above, to help us understand when some individuals decide

[1] The use of commodities like cigarettes as money is not exclusive to POW camps. Another example is given by Friedman's (1992, pp.12-13) discussion of the situation in post-war Germany: "After World War II the Allied occupational authorities exercised sufficiently rigid control over monetary matters, in the course of trying to enforce price and wage controls, that it was difficult to use foreign currency. Nonetheless, the pressure for a substitute currency was so great that cigarettes and cognac emerged as substitute currencies and attained an economic value far in excess of their value purely as goods to be consumed. ... Foreigners often expressed surprise that Germans were so addicted to American cigarettes that they would pay a fantastic price for them. The usual reply was 'Those aren't for smoking; they're for trading'." See Neale (1975) for other related examples of commodity money.

that cigarettes are better for trading rather than smoking, and hopefully to generate some insight into exchange institutions more generally. We proceed using a search-based model of the trading process because this allows one to endogenously determine the equilibrium pattern of exchange and, hence, to determine which objects are used as money. The model includes general goods that everyone consumes (cigarettes) as well as specialized goods that only certain individuals consume but that yield a lot of utility for those who do (Brits love tea while the French prefer coffee). Specialization generates gains from trade, but can also make trade difficult. This leads to a natural role for a medium of exchange, and can lead some individuals to stop consuming and start trading generally desired goods. We will show how the extent to which this happens depends on things like the nature of the trading process, the extent of the double coincidence problem, and the properties of general and special goods. We also show how it depends on the existence of fiat currency.

The paper differs from previous analyses of commodity money that used the random matching or search framework in several ways. Most significantly, in the present model the key economic decision facing an agent is whether to consume a general good now or, alternatively, store it in an attempt to trade for a preferred special good later. By contrast, in the model in Kiyotaki and Wright (1989) and its extensions, an agent always consumes any good that he desires once he gets it, and his only decision is whether to trade one good that he does not desire for another good that he does not desire in an attempt to acquire a desired good more efficiently. There is no possibility that agents ever stop consuming a good. Here the central issue is to determine when people stop smoking cigarettes, either partially or completely, and begin to trade them. There are also several technical differences from earlier models that are motivated by our thinking about cigarette money.[2] While these differences in specification may seem minor, they actually can change the analysis and conclusions in interesting ways. If we had to focus on one key element in the current model that is missing from the related literature: here the money supply – i.e., the quantity of cigarette and/or fiuat money in circulation – is endogenous.

The rest of the paper and some results can be summarized as follows. We begin in Section 2 by describing a very simple version of the model, where there is no fiat money and where consumption goods are indivisible. We show that there is always a unique equilibrium where, depending on parameters, either no agents, some agents, or all agents stop consuming the generally desired consumption good and start using it as money. Relevant parameters include the degree to which goods and tastes are specialized, the relative supply of general and special goods and their intrinsic properties, the rates of depreciation and time preference, and the nature of the trading frictions. In Section 3 we incorporate divisible goods and let agents bargain over the terms of trade. We generalize the previous results, and generate the following key new result: as long as some general goods are used for consumption their value in exchange will be tied down by their value in consumption; but if parameter values are such the general goods are used only as money and never consumed they will trade at a premium over their consumption value.[3]

[2] Two obvious ones are the following. First, this paper allows for general commodities that are consumed by everyone, while in previous work all commodities were symmetric, at least in the sense that they were consumed by some agents and not others. Second, in this paper, for the potential commodity money to generate utility it must be "used up" in consumption (like cigarettes), while in previous models a commodity money could generate utility while in storage without depreciating (like gold jewelry, perhaps).

[3] Somewhat related results can be derived in other models (like overlapping generations

Although the indivisible goods model had a unique equilibrium, the bargaining model can have multiple equilibria. This is reminescent of models with fiat money, even though cigarette money is clearly commodity money. In Section 4 we incorporate genuine fiat currency, and show that this may add the possibility of new multiplicities. More interestingly, we prove the following new result: As long as we do not introduce too much fiat money, it does not affect the exchange process at all – each unit of fiat currency simply crowds out a unit of commodity money, and the economy proceeds as before (except for a one-time welfare gain due to the fact that someone can now consume the cigarettes that were previously circulating as money). However, if we introduce too much fiat money, we can drive commodity money from circulation completely and this will have real effects on the exchange process. One way to interpret this finding is that the economy may be quite resilient in monetary matters: an endogenous means of payment will emerge from the private sector if and only if an adequate supply of fiat money is not provided by the public sector.

In Section 5 we further extend the model to allow the general commodity to come in heterogenous qualities in an attempt to capture some of Radford's observations about good and bad cigarettes in the POW camp, and to discuss issues related to Gresham's Law more generally. Perhaps surprisingly, we find that whether bad cigarettes end up circulating as money while good cigarettes are consumed, or vice-versa, actually depends on parameter values. Intuitively, while good cigarettes are better for smoking they are also better for trading, given that relative prices are endogenous and reflect the intrinsic properties of the different commodities. We also analyze a version

models) of money, but the issue has not been considered previously in a search-and-bargaining framework like the one in this paper.

of the model where private information implies that relative prices do not reflect true intrinsic differences in quality, and show that, under these assumptions, good money is necessarily driven out of circulation before bad, in conformance to the predictions of Gresham.[4]

2 The Basic Model

Time is continuous. The economy is populated by a continuum of agents who act as if the horizon is infinite (or is at least random) and discount at rate $r > 0$. There are many goods, all of which are assumed to be costlessly storable and for now indivisible. One of them is called the *general good*, to be thought of as cigarettes, and the others are called *special goods*. All agents derive utility u_g from consuming one unit of the general good. Every agent derives utility u_s from consuming one unit of a particular type of special good and can not consume other special goods. We assume $u_s > u_g$. General goods are subject to depreciation, while specialized goods are not, but this is purely for simplicity. Thus, according to a Poisson process with parameter d, a general good spoils, or simply disappears. New goods enter the economy in the following way: after an agent consumes a general good, or after a general good depreciates, he always produces a special good other than the one that he likes to consume; but after he consumes his special good, he produces a special good with probability σ and a general good with probability $1 - \sigma$.

[4] Although Gresham's Law has been analyzed in search models before (e.g., Velde, Weber and Wright [forthcoming] or Renero [forthcoming]), some of our results differ in ineresting ways from anything in the literature. For instance, we have not previously seen an equilibrium like the one we construct in which good commodities circulate as money while bad commodities are used as consumption goods.

Agents can also freely dispose of inventories whenever they like, but they do not receive a new good if they do.[5]

There are gains from trade due to specialized tastes and goods. Trade here does not occur through a centralized market, however, but between individuals. We assume that agents meet bilaterally according to an anonymous random matching process, and a pair trades if and only if this makes both agents strictly better off. Let the Poisson arrival rate of potential trading partners be denoted α. On meeting someone with a special good, let $x \leq 1$ be the probability (common to everyone) that his special good is the type you desire. Given this event, and given that you also have a special good, let $y < 1$ be the conditional probability that your special good is also the type he desires. Notice that y measures the extent of the double coincidence problem with direct barter and it is this, rather then random matching *per se*, that delivers a potential role for a medium of exchange; that is, we can assume that agents always know where to find the sellers of the good they desire (which amounts to setting $x = 1$), but as long as they cannot be sure of having goods desired by these sellers (which means $y < 1$) there will still be a role for a medium of exchange.[6]

[5] This way of modeling production is meant to capture, more or less, the way the endowments of various objects arrived in a POW camp. As Radford (1945) reports, "Our supplies consisted of rations provided by the detaining power and (principally) the contents of Red Cross food parcels – tinned milk, jam, buscuits, bully, chocolate, sugar, etc., and cigarettes. ... Private parcels of clothing, toilet requisites and cigarettes were also received" (p.190). We tried various other assumptions concerning exactly how general and special goods arrive, such as allowing either type to be produced at random after a general good is consumed (as well as after a special good is consumed), and the results were basically the same.

[6] According to Radford (1945), the actual process of exchange depended on circum-

It is always rational for an agent to trade whatever he has for the special good that he desires for consumption and to consume it immediately. Also, at least in any symmetric equilibria – and we only consider symmetric equilibria here – agents will never accept special goods unless they consume them (see Kiyotaki and Wright [1993] for details). It is always rational to trade a special good for the general good since, at the very least, you can consume it and produce another special good costlessly. What needs to be decided is whether to consume the general good when one gets it, or store it to facilitate a future trade for one's favorite special good. To be precise, we will say that a fraction θ of the population choose to *always* consume the general good when they get it, while the remaining $1-\theta$ choose to *never* consume it. We can also say that each agent makes the choice of consuming or storing the general good each time he gets it; this amounts to exactly the same thing because, in equilibrium, $\theta = 1$ when agents prefer consuming the general good, $\theta = 0$ when they prefer storing the general good, and $\theta \in (0,1)$ only when they are indifferent.

Given that θ individuals consume the general good while $1-\theta$ store it, and given the probability of producing the general good and its rate of depreciation, let G and $S = 1-G$ denote the steady state proportions of the population holding general and special goods. The steady state condition

stances. In the early stages, or in temporary camps, people more less literally searched at random, "wandering though the bungalows calling their offers – 'cheese for seven' (cigarettes)." In permanent camps, eventually a Exchange and Mart board went up where notices could be posted listing "name," "room number," "wanted" and "offered," and in some permanent camps public shops were set up where you could leave goods to be sold. Although more organized, these institutions still imply that there is a random time until someone will show up, want your good, and also have what you want, as the search model assumes.

(derived in the Appendix) is:

$$(1-S)[d+\alpha x S\sigma] = [\alpha(1-S)x + \alpha Sxy(1-\sigma)](S-\theta). \tag{1}$$

For any θ this implies a unique S. Alternatively, we can solve for the value of θ that generates a particular S,

$$\theta = \frac{Q(S)}{1-S+y(1-\sigma)S}, \tag{2}$$

where $Q(S) = -(1-\sigma)(1-y)S^2 + (1-\sigma+d/\alpha x)S - d/\alpha x$. For any $S \in [\widehat{S}, 1]$ there is a unique fraction of general good consumers $\theta \in [0,1]$ such that θ implies S will be the fraction of special good holders in steady state.[7]

We now describe steady state payoffs. Let V_s and V_g be the value functions of agents with special goods and general goods in inventory. Consider first an agent with a special good. In principle, there are two types of such agents: those who always consume general goods and those who always store general goods. When someone who consumes the general good acquires it he consumes and produces a special good for a total payoff of $u_g + V_s$. When he acquires the special good he desires, he consumes, and with probability σ a new special good is produced and stored, while with probability $1-\sigma$ a general good is produced and consumed, followed by a special good; hence, the payoff to acquiring the special good is $u_s + (1-\sigma)u_g + V_s$. Consider next someone with a special good who never consumes the general good. When he gets the general good his payoff is simply V_g. When he gets the type of special good he desires, he consumes and stores whatever he produces, for a total payoff of $u_s + \sigma V_s + (1-\sigma)V_g$.

[7] This follows because θ is strictly increasing in S, $\theta = 1$ if $S = 1$, and $\theta = 0$ if $S = \widehat{S}$, where $\widehat{S}$ solves $Q(\widehat{S}) = 0$ and satisfies $\widehat{S} \in [0,1)$ with $\widehat{S} = 0$ if and only if $d = 0$.

Based on these observations, Bellman's equation in flow terms for a special good holder is

$$\begin{aligned} rV_s &= \alpha Gx[\theta u_g + (1-\theta)(V_g - V_s)] \\ &\quad + \alpha Sxy[u_s + (1-\sigma)\theta u_g + (1-\sigma)(1-\theta)(V_g - V_s)], \end{aligned} \tag{3}$$

where one should interpret θ here as a dummy variable indicating that the agent chooses to consume or store the general good in order to maximize expected life time utility. Intuitively, (3) sets the flow return rV_s equal to the sum of two terms. The first term is the rate at which a special good holder meets general a good holder, αG, times the probability the latter desires the special good he holds, x, times his gain from trade, which is the net payoff from consuming the general good with probability θ and storing it with probability $1-\theta$. The second term is the rate at which he meets a special good holder, αS, times the probability they desire each other's special goods, xy, times his gain from trade, which in this case is the utility of consuming the special good, plus the probability he produces the general good, $1-\sigma$, times the net payoff to consuming it with probability θ and storing it with probability $1-\theta$.

Now consider an agent with a general good. Bellman's equation is given by

$$rV_g = \alpha Sx\left[u_s + \sigma\left(V_s - V_g\right)\right] + d\left(V_s - V_g\right). \tag{4}$$

This sets the flow return rV_g equal to the rate at which he meets a special good holder with the desired special good, αSx, times the gain from trade, plus the rate at which the general good depreciates, d, times the capital loss $V_s - V_g$.[8]

[8] One can also write the term in square brackets in (4) as $u_s + \sigma\left(V_s - V_g\right) + (1-\sigma)\theta(u_g +$

The net gain from consuming the general good and storing the special good that one produces, rather than storing the general good, is given by $\Delta \equiv u_g + V_s - V_g$. Then individual optimization with respect to θ entails the best response condition:

$$\begin{aligned} &\Delta > 0 \Rightarrow \theta = 1; \\ &\Delta < 0 \Rightarrow \theta = 0; \\ &\Delta = 0 \Rightarrow \theta \in [0, 1]. \end{aligned} \tag{5}$$

Using (3) and (4), one can solve explicitly for

$$\Delta = A[1 - S + \tfrac{r}{\alpha x} + (1-\sigma)Sy + S\sigma + \tfrac{d}{\alpha x}]u_g - AS(1-y)u_s, \tag{6}$$

where A is a positive constant.

A *steady state equilibrium* may now be defined as a list $(S, V_s, V_g, \theta, \Delta)$ satisfying (2)-(6), subject to $V_g \geq 0$, $V_s \geq 0$, $0 \leq S \leq 1$, $0 \leq \theta \leq 1$. In practice, we simply look for pairs (S, θ) satisfying the steady state condition and the best response condition. There are three types of possible outcomes, depending on whether the general good is always consumed ($\theta = 1$), sometimes consumed ($0 < \theta < 1$), or never consumed ($\theta = 0$). If $\theta < 1$ the general good circulates as a commodity money (indeed, they are a universally acceptable commodity money, since all agents accept cigarettes, even though some may accept them for smoking while others accept them for retrading).

$V_s - V_g$), to indicate that the agent chooses to consume or store a general good after it is produced, but here we have used the fact that any agent with a general good in inventory has already chosen to be a general good non-consumer and so $\theta = 0$. This is merely a notational issue and has no effect on any results. The way we solve the agent's problem is by conjecturing a strategy and then checking that there is no profitable deviation, and since we only look at steady state equilibria, it does not matter whether we check, for example, that a general good non-consumer deviates by consuming a general good once, or deviates by always consuming general goods.

The following Proposition establishes when each type of equilibrium exists. To reduce notation, from now on we let $\rho = r/\alpha x$ and $\delta = d/\alpha x$.

Proposition 1 *There exists $y_1 < 1$ and $y_2 < y_1$ such that: (i) if $y \geq y_1$ then $\theta = 1$ and $S = 1$; (ii) if $y \in (y_2, y_1)$ then $\theta \in (0,1)$ and $S \in (\widehat{S}, 1)$; and (iii) if $y \leq y_2$ then $\theta = 0$ and $S = \widehat{S}$. These are all of the (steady state) equilibria.*

Proof: First consider an equilibrium with $\theta = 1$ and $S = 1$, which requires $\Delta \geq 0$. Setting $\theta = 1$ and $S = 1$ in (6), it is immediate that $\Delta \geq 0$ if and only if $y \geq y_1$, where

$$y_1 = \frac{u_s - (\rho + \sigma + \delta)u_g}{u_s + (1-\sigma)u_g}. \tag{7}$$

We conclude that an equilibrium with $\theta = 1$ and $S = 1$ exists if and only if $y \geq y_1$.

Next consider the case where $0 < \theta < 1$. We need to find $(S, \theta) \in (0,1)^2$ such that the steady state condition is satisfied and $\Delta = 0$. The method we pursue is to find a value of S, call it S_Δ, for which $\Delta = 0$, and then identify conditions under which $S_\Delta \in (\widehat{S}, 1)$, as this is equivalent to $(S, \theta) \in (0,1)^2$ by virtue of (2). Using (6) we have

$$S_\Delta = \frac{(1 + \rho + \delta)u_g}{(1-y)\left[u_s + (1-\sigma)\, u_g\right]}.$$

Notice that $\partial S_\Delta / \partial y > 0$ and $S_\Delta = 1$ if $y = y_1$, and so $S_\Delta < 1$ if and only if $y < y_1$. Using the function $Q(S)$ defined after (2), one shows that $S_\Delta > \widehat{S}$ if and only if $y > y_2$ where

$$y_2 = \frac{\left[u_s - (\delta + \rho + \sigma)u_g\right]\left[\delta u_s - (1+\rho)(1-\sigma)u_g\right]}{\delta \left[u_s + (1-\sigma)\, u_g\right]^2}.$$

We conclude that an equilibrium with $(S, \theta) \in (0,1)^2$ exists if and only if $y_2 < y < y_1$.

Finally, consider $\theta = 0$, which implies $S = \widehat{S}$, and requires $\Delta \leq 0$. We first note that Δ is increasing in y (this is easily verified by showing $\partial\Delta/\partial y > 0$, $\partial\Delta/\partial S < 0$ and $\partial\widehat{S}/\partial y < 0$). Then, since $\Delta = 0$ when $y = y_2$ and $S = \widehat{S}$, it follows that this equilibrium exists if and only if $y \leq y_2$. This completes the proof. ■

Since $\theta < 1$ if and only if $y < y_1$, Proposition 1 tells us that general goods are more likely to be used as money when y is small, which means that barter is difficult because goods and tastes are highly specialized, or when y_1 is big. And y_1 is big when: general goods are not very desirable relative to special goods (u_g/u_s is low); people are patient (r is low); general goods do not depreciate very quickly (d is low); search frictions are not too severe (αx is big); or special goods are produced infrequently (σ is low). Figure 1 shows where the different equilibria exist in various regions of parameter space. Figure 2 shows θ, as well as the amount of commodity money in circulation, G, as functions of parameters: the top panel varies y, which measures the difficulty of barter; the second panel varies α, which inversely measures the search frictions; and the third panel varies x and y together since the case $y = x$ is one that is often analyzed in the literature.[9]

[9] The bounds y_2 and y_1 defined in the proof of Proposition 1 satisfy $y_2 < y_1 < 1$. Notice, however, it is possible for y_1 or y_2 to be negative. For example, if $d = 0$ and general goods do not depreciate at all, then $y_2 < 0$ and we cannot have $y \leq y_2$. That is, if $d < 0$ then we cannot have an equilibrium with $\theta = 0$. Intuitively, $d = \theta = 0$ implies that in steady state everyone has cigarettes in inventory, which means that it is pointless trying to trade a cigarette for one's special good, and so it cannot be a best response to store rather than smoke cigarettes.

3 Prices

Up to this point we have determined when general goods like cigarettes will be used as money, but we did not try to determine their purchasing power as all goods were assumed to be indivisible. Following Shi (1995) and Trejos and Wright (1995), we now endogenize prices by assuming that special goods are perfectly divisible and allowing agents to bargain over the amount of a special good that they trade for an indivisible unit of the general good. It will still be the case in this model that all agents always have either 1 unit or 0 units of the general good. While this is obviously an abstraction – cigarettes, or at least packages or cartons of cigarettes, are in fact divisible and multiple units can be accumulated – modeling price formation in this way turns out to be an order of magnitude more tractable than proceeding under the assumption that everything is perfectly divisible (see, e.g., Molico 1997).

Agents who receive q units of their special good in trade enjoy utility $u_s = U_s(q)$, while agents who produce q units suffer disutility $c(q)$. We assume that $U_s(0) = 0$, $U_s'(q) > 0$, $U_s''(q) < 0$, and that there is a $\hat{q} \in [0, 1]$ such that $U_s(\hat{q}) = \hat{q}$. With no loss in generality, we normalize $c(q) = q$. Note that the interpretation from now on is that with probability σ agents get an *opportunity* to a produce a special good later, rather than an actual unit of output; i.e., they do not actually produce until they trade, since we do not want the cost to be a sunk cost at the time of bargaining. Denote by Ω the expected net utility from bartering one special good for another (to be determined below), and by q the amount of special good that one gets for a general good. Recall that $\rho = r/\alpha x$ and $\delta = d/\alpha x$. Then, for any q and Ω, Bellman's equations are given by the following generalization of (3) and (4):

$$\begin{aligned} \rho V_s &= (1-S)\left[-q+\theta u_g+(1-\theta)(V_g-V_s)\right] \\ &\quad +Sy\left[\Omega+(1-\sigma)\theta u_g+(1-\sigma)(1-\theta)(V_g-V_s)\right] \qquad (8) \\ \rho V_g &= S\left[U_s(q)+\sigma(V_s-V_g)\right]+\delta(V_s-V_g). \end{aligned}$$

For direct barter transactions between two special good holders we adopt the symmetric Nash bargaining solution, which implies that both agents produce q^* units where q^* satisfies $U_s'(q^*) = c'(q^*) = 1$, and therefore $\Omega = U_s(q^*) - q^*$.[10] For trades of general goods for special goods, it turns out to simplify the analysis significantly to assume that the agent with the general good gets to make a take-it-or-leave-it offer. This implies that

$$q = \theta u_g + (1-\theta)(V_g - V_s), \tag{9}$$

as this is the greatest q that a special good holder would be willing to produce in exchange for a general good. Notice that (9) implies that the first term in the Bellman equation for V_s vanishes, which is natural because special goods holders do not get any of the gains from trade with general goods holders.

An equilibrium is defined as before, except that we make q endogenous and add the bargaining solution as an equilibrium condition. Thus, we now look for combinations (S, θ, q) satisfying (2), (5) and (9). Proposition 2 will show that at long as at least some general goods are used for consumption purposes (i.e., as long as $\theta > 0$) their value in exchange is pegged to their intrinsic value in consumption, $q = u_g$. However, when $\theta = 0$ general goods

[10] Nothing really depends on this assumption, however. All that happens when we change the bargaining solution in a barter opportunity is that Ω changes, and we allow Ω to take on any value here.

will trade at a premium over their intrinsic value, $q > u_g$, as Friedman (1992) observed happening when cigarettes were used as money in post-war Germany. Further, we show that there is a range of parameter values such that are multiple equilibrium values of θ, and there can also be more than one equilibrium value of q for a given equilibrium value of θ.

Proposition 2 *There exist $\bar{y}_1$ and $\bar{y}_2 < \bar{y}_1$ such that: (i) if $y \geq \bar{y}_1$ then there is an equilibrium with $\theta = 1$ and $q = u_g$; (ii) if $y \in (\bar{y}_2, \bar{y}_1)$ then there is an equilibrium with $\theta \in (0,1)$ and $q = u_g$; and (iii) if $y < \bar{y}_2$ then there is an equilibrium with $\theta = 0$ and $q > u_g$. Moreover, there is a $z > 0$ such that: if $u_g > z$ then these are the only equilibria; but if $u_g < z$ then $\bar{y}_2 > 0$ and there exists $\bar{y}_3 > \bar{y}_2$ such that when $y \in (\bar{y}_2, \bar{y}_3)$ there are two other equilibria, both having $\theta = 0$ but different values of $q > u_g$. These are all of the (steady state) equilibria.*

Proof: First consider equilibrium with $\theta = 1$, which implies $q = u_g$ by (9), and

$$\Delta = \frac{[\rho + \sigma + \delta + (1-\sigma)y]\, u_g + y\Omega - U_s(u_g)}{\rho + \sigma + \delta}.$$

This equilibrium exists if and only if $\Delta \geq 0$, which holds if and only if $y \geq \bar{y}_1$ where

$$\bar{y}_1 = \frac{U_s(u_g) - (\rho + \sigma + \delta)u_g}{\Omega + (1-\sigma)u_g}. \tag{10}$$

Next consider $\theta \in (0,1)$, which implies $u_g = V_g - V_s$, and therefore again implies $q = u_g$ by (9). As in Proposition 1, we solve $\Delta = 0$ for $S = \overline{S}_\Delta$, where

$$\overline{S}_\Delta = \frac{(\rho + \delta)u_g}{U_s(u_g) - y\Omega - [\sigma + (1-\sigma)y]\, u_g}, \tag{11}$$

and then check when $\overline{S}_\Delta$ is in $(\widehat{S}, 1)$. One can show $\overline{S}_\Delta < 1$ if and only if $y < \bar{y}_1$. Also, notice that $\overline{S}_\Delta$ is increasing in y while $\widehat{S}$ is decreasing in y.

Furthermore, there is a value $\bar{y}_2$ such that $\overline{S}_\Delta = \widehat{S}$. Hence, $\overline{S}_\Delta > \widehat{S}$ if and only if $y > \bar{y}_2$.

Now consider equilibria where $\theta = 0$, which implies $S = \widehat{S}$, and requires $\Delta \leq 0$. We can combine (8) and (9) into the single condition $T(q) = 0$, where

$$T(q) = \widehat{S}u_s(q) - \widehat{S}\Omega y - \left[\rho + \delta + \sigma\widehat{S} + (1-\sigma)\, y\widehat{S}\right] q.$$

A solution to $T(q) = 0$ is an equilibrium if and only if it satisfies $q \geq u_g$, since this is equivalent $\Delta \leq 0$. Note that $T'(0) > 0$, $T''(q) < 0$ for all q, and $T(q) < 0$ for large q, as shown in Figure 3. Also, if $y = 0$ then $T(q)$ has two roots, $q = 0$ and $q > 0$. As y increases, one can show that $T(q)$ shifts down, and therefore has two positive roots, until we reach a point $y = \bar{y}_3$ where $T(q)$ is tangent to the horizontal axis. Consequently, if $y > \bar{y}_3$ there are no solutions to $T(q) = 0$. Let z denote the value of q at which $T(q)$ is tangent to the axis when $y = \bar{y}_3$.

Notice that when $y = \bar{y}_2$, where $\bar{y}_2$ was defined above by $\overline{S}_\Delta = \widehat{S}$, one solution to $T(q) = 0$ is always given by $q = u_g$ (which implies that $\bar{y}_3 > \bar{y}_2$). There are two possible cases. The first case is $u_g > z$. This implies that when $y = \bar{y}_2$, $q = u_g$ is the higher root of $T(q) = 0$. Then for all $y \in (0, \bar{y}_2)$ the higher root of $T(q) = 0$ is the unique solution such that $q > u_g$, and for $y > \bar{y}_2$ there is no solution to $T(q) = 0$ such that $q \geq u_g$. The second case is $u_g < z$. This implies that $\bar{y}_2 > 0$, because $T(u_g) > 0$ when $y = 0$. It also implies that when $y = \bar{y}_2$, $q = u_g$ is the lower root of $T(q) = 0$. Then for all $y \in (0, \bar{y}_2)$ the higher root of $T(q) = 0$ is the unique solution such that $q > u_g$, for $y \in (\bar{y}_2, \bar{y}_3)$ both roots of $T(q) = 0$ satisfy $q > u_g$, and for $y > \bar{y}_3$ there is no solution to $T(q) = 0$.

We conclude the following. On the one hand, for $u_g > z$, for all $y \in (0, \bar{y}_2)$ there is a unique equilibrium with $\theta = 0$, and for $y > \bar{y}_2$ there are no equilibria with $\theta = 0$. On the other hand, for $u_g < z$, for all $y \in (0, \bar{y}_2)$ there is a unique equilibrium q with $\theta = 0$, for $y \in (\bar{y}_2, \bar{y}_3)$ there are two equilibria with $\theta = 0$ and different values of q, and for $y > \bar{y}_3$ there are no equilibria with $\theta = 0$. This completes the proof. ■

Figure 4 shows the regions in (y, u_g) space where the different equilibria exist. Notice in particular that there are multiple equilibria when $u_g < z$ and $y \in (y_2, y_3)$: there are two equilibria with $\theta = 0$ and different values of q, both greater than u_g; and also an equilibrium with $q = u_g$ and either $\theta = 1$ or $\theta \in (0, 1)$, depending on whether $y > y_1$ or $y < y_1$. We know that q can never be less than u_g, since otherwise no one would use the general good as money. What is interesting here is that q can be greater than u_g (recall footnote 1). It is tempting to say that when $\theta = 0$ the general good acts a lot like a fiat money, at least in the sense that it's exchange value is not pinned down by it's intrinsic value, even though it is clearly a commodity money in the sense that $u_g > 0$. In the next section we introduce genuine fiat money.

4 Fiat Money

We now introduce a second potential money in the form of fiat currency.[11] We assume that fiat currency can neither be produced nor consumed by an individual, and an exogenous fraction M of the population are simply endowed with it. Following the method in the previous section, we assume that both fiat money and general goods are indivisible, and determine relative prices by letting special goods be divisible. Also, we restrict attention here to the case where general goods do not depreciate ($d = 0$). This implies that there can be no equilibrium with $\theta = 0$, and so without fiat money the equilibrium would be unique and satisfy $q = u_g$. Any multiplicities that occur in this section are therefore due to the existence of the fiat object. Moreover, due to its intrinsic uselessness, it is clear that there is always an equilibrium in which the value of fiat money is $V_m = 0$. In this case the

[11] Radford (1945) reports that there was some official fiat money (RMk.s) in the camp, but it “had no circulation save for gambling debts” (p.190). However, around D-Day, during relatively good economic times, the camp introduced paper currency. This money was backed 100 percent by food at the shop and restraunt – hence its name, the “Bully Mark” – and so it was not exactly fiat money: “Originally one BMk. was worth one cigarette and for a short time both circulated freely inside and outside the restraunt.” However, “The BMk. was tied to food, but not to cigarettes: as it was issued against food, say 45 for a tin of milk and so on, any reduction in the BMk. prices of food would have meant that there were unbacked BMk.s in circulation.” (p.197). Hence, even though BMk.s were partially backed, it still seems interesting to consider fiat money in the model. For the record, “In August parcels and cigarettes were halved and the camp was bombed. The Restraunt closed for a short while and sales of food became difficult. ... The BMk. fell to four-fifths of a cigarette and eventually farther still, and it became unacceptable save in the restraunt. There was a flight from the BMk., no longer converttible into cigarettes or popular foods. The cigarette reestablished itself.”

model reduces to the one analyzed above, and so from now on we focus on equilibria where $V_m > 0$.

Let the steady state proportions of the population holding special goods, general goods, and money be given by S, G and M. A generalized version of the steady condition from the model without fiat money can be used to show that S varies monotonically between 0 and $1 - M$ as θ varies between 0 and 1. Let q_g and q_m be the amount of special good one can get for a unit of general good and a unit of fiat money, respectively. Then Bellman's equations are

$$\begin{aligned}
\rho V_s &= Sy\left[\Omega + (1-\sigma)\theta u_g + (1-\sigma)(1-\theta)(V_g - V_s)\right] \\
&\quad + G\left[-q_g + \theta u_g + (1-\theta)(V_g - V_s)\right] + M\left[-q_m + V_m - V_s\right] \\
\rho V_g &= S\left[U_s(q_g) + \sigma(V_s - V_g)\right] \qquad (12) \\
\rho V_m &= S[U_s(q_m) + \sigma(V_s - V_m) \\
&\quad + (1-\sigma)\theta(u_g + V_s) + (1-\sigma)(1-\theta)V_g - V_m].
\end{aligned}$$

Assuming that agents with either general goods or fiat money get to make take-it-or-leave-it offers, bargaining implies

$$q_g = \theta u_g + (1-\theta)(V_g - V_s) \qquad (13)$$

$$q_m = V_m - V_s. \qquad (14)$$

Notice that (13) and (14) imply that both the second and third terms in Bellman's equation for V_s vanish.

An equilibrium is now defined as the obvious generalization of the previous section: we look for combinations (S, θ, q_g, q_m) satisfying the relevant conditions. We will show below that as long as M is not too large there is an equilibrium where fiat money simply crowds out commodity money one-for-one and $q_m = q_g = u_g$. Hence, the introduction of a small amount of fiat money can have no effect at all (except for a one-time seigniorage gain that occurs because someone can now consume the general goods that were serving as money before we introduced the fiat object). Once M is sufficiently big so as to drive general goods completely from circulation, however, the introduction of more fiat money has real effects on the economy. We also find that the economy with fiat money may display multiple equilibria even when M is small (e.g., when fiat and commodity money coexist they can trade at either the same value or at different values); and that there are parameter values for which there cannot be any commodity money in circulation (because everyone immediately consumes the general good) but there can still be valued fiat money.

Proposition 3 *There exists $\breve{y}_1 < 1$ such that: (i) if $y \geq \breve{y}_1$ then $\theta = 1$; and (ii) if $y < \breve{y}_1$ then $\theta \in (0, 1)$. In any equilibrium $q_g = u_g$, while q_m depends on parameter values. On the one hand, if $y < y_1$, there is a value $\breve{y}_A < 1$ such that, when $y < \breve{y}_A$ the unique equilibrium has $q_m = u_g$, and when $y > \breve{y}_A$ there are two equilibrium values of q_m, one equal to and one lower than u_g. On the other hand, if $y > y_1$, there is a value $\breve{y}_B \in (\breve{y}_A, 1)$ such that: when $y < \breve{y}_A$ there is a unique equilibrium $q_m < u_g$; when $y \in (\breve{y}_A, \breve{y}_B)$ there are two equilibrium values of q_m, both lower than u_g; and when $y > \breve{y}_B$ there are no equilibrium with $q_m > 0$. These are all of the (steady state) equilibria with valued fiat money.*

Proof: First note that there is no equilibrium with $\theta = 0$ and $S = 0$, since this implies $\Delta > 0$ and this contradicts $\theta = 0$. Now suppose $\theta = 1$ and

$S = 1 - M$. Then (13) implies $q_g = u_g$, and

$$\Delta = \frac{[\rho/(1-M) + \sigma + (1-\sigma)y]\, u_g + y\Omega - U_s(u_g)}{\rho/(1-M) + \sigma}.$$

This equilibrium requires $\Delta \geq 0$, which holds if and only if $y \geq \breve{y}_1$ where

$$\breve{y}_1 = \frac{U_s(u_g) - [\rho/(1-M) + \sigma]u_g}{\Omega + (1-\sigma)u_g}. \tag{15}$$

Now suppose $\theta \in (0,1)$. This requires $\Delta = 0$, which again implies $q_g = u_g$ by (13). We need to solve $\Delta = 0$ for S and check when $S \in (0, 1-M)$. The value of S that solves $\Delta = 0$ is given by the same $\bar{S}_\Delta$ that makes $\Delta = 0$ in the model without fiat money, given in (11) above, which is obviously independent of M. From this one can easily check that $\bar{S}_\Delta \in (0, 1-M)$ if and only if $y < \breve{y}_1$.

It remains to determine q_m. We can combine (14) and (12) into the single condition $q_m = \breve{T}(q_m)$, where

$$\breve{T}(q) = S\,[U_s(q) - y\Omega + (1-y)(1-\sigma)u_g] - (\rho + S)q,$$

and $S = 1 - M$ when $\theta = 1$ and $S = S_\Delta$ when $\theta < 1$. Note that $\breve{T}'(0) > 0$, $\breve{T}''(q) < 0$ for all q, and $\breve{T}(q) < 0$ for large q; hence $\breve{T}$ is qualitatively the same as the function T shown previously in Figure 3. The zeros of $\breve{T}$ depend on y.

First consider the case where $y > y_1$, and let

$$\breve{y}_A = \frac{(1-\sigma)u_g}{\Omega + (1-\sigma)u_g}.$$

Then $y < \breve{y}_A$ implies $\breve{T}(0) > 0$ and so there is a unique solution to $\breve{T}(q_m) = 0$. As y increases, $\breve{T}(q)$ shifts down, until we reach $\breve{y}_B > \breve{y}_A$ where $\breve{T}$ is tangent to the horizontal axis, given by

$$\breve{y}_B = \frac{U_s(\breve{q}) - (\rho + 1 - M)/(1-M)\breve{q} + (1-\sigma)u_g}{\Omega + (1-\sigma)u_g}$$

where $\breve{q}$ solves $(1-M)u'(\breve{q}) = \rho+1-M$. Then $y \in (\breve{y}_A, \breve{y}_B)$ implies there are 2 solutions to $q_m = \breve{T}(q_m)$ and $y > \breve{y}_B$ implies there is no solution. Moreover, one checks that $q_m < u_g$ for all $y > 0$ by showing $\breve{T}(u_g) \leq 0$ at $y = 0$, that $\breve{T}$ is decreasing in y.

Now consider $y < y_1$. In this case, one can easily check that one solution to $\breve{T}(q) = 0$ is given by $q = u_g$ for any y. For $y < \breve{y}_A$, since $\breve{T}(0) > 0$, $q = u_g$ is the only solution. For $y > \breve{y}_A$, since $\breve{T}(0) < 0$, there is also a solution with $q < u_g$. This completes the proof. ■

The set of equilibria in (y, u_g) is shown in Figure 5.[12] Notice that is more likely that $y < \breve{y}_1$, and hence it is more likely that an equilibrium with $\theta \in (0,1)$ exists, if M is small. However, $S = \bar{S}_\Delta$ is independent of M. This implies that as the quantity of fiat money M increases, the endogenous quantity of commodity money G decreases dollar for dollar, at least as long as we stay in an equilibrium with $\theta < 1$. Another way to say this is that, given a relatively small stock of fiat money, the private sector will respond by creating commodity money, and the total amount of money $\bar{G} + M$ will be independent of M. However, as M gets bigger, eventually we must have $y \geq \breve{y}_1$, which means that $\theta = 1$ and $G = 0$. Once everyone is consuming the general good, G cannot fall any further, and additional increases in M must decrease S. Moreover, for small M and small y the value of fiat money must be the same as that of commodity money, $q_m = q_g = u_g$. For higher values of y the value of fiat money may or may not be the same as that of commodity money. For higher values of M, the value of fiat money is necessarily less than u_g.

[12] The figure is drawn using the following easily verified results: $\breve{y}_1$, $\breve{y}_A$ and $\breve{y}_B$ are 0 when $u_g = 0$; $\breve{y}_1$ increases with u_g for small u_g, then decreases and becomes negative as u_g gets big; and $\breve{y}_A$ and $\breve{y}_B$ increase monotonically to 1 as u_g gets big.

5 Heterogeneous Goods

Radford describes how heterogeneous cigarettes traded at different prices in the POW camp. As one might expect from Gresham's Law, certain brands were more regularly used for consumption, while other presumably inferior cigarettes (including hand-rolled ones made from pipe tobacco) were used mainly as money. Moreover, the inferior cigarettes tended to circulate at a premium over their intrinsic value. To study this phenomena, suppose that general commodities come in two qualities: good cigarettes and bad cigarettes. The probability of producing a special good, a good cigarette, and a bad cigarette are given by σ, γ, and β, and the steady state proportions of agents holding these objects are S, G, and B. The probability of consuming a good cigarette upon acquiring it is θ_g and the probability of consuming a bad cigarette upon acquiring it is θ_b. The utilities of consuming a good cigarette or a bad cigarette are u_g or u_b. Cigarette holders make take-it-or-leave offers to special good holders, and the amount of the special good that a good or a bad cigarette commands is q_g or q_b. For simplicity, assume no depreciation and no fiat money.

Bellman's equations are

$$\begin{aligned}
\rho V_s &= Sy\left[\Omega + \gamma\theta_g u_g + \gamma(1-\theta_g)(V_g - V_s) + \beta\theta_b u_b + \beta(1-\theta_b)(V_b - V_s)\right] \\
\rho V_g &= S\left[U_s(u_g) + \sigma(V_s - V_g) + \beta\theta_b(u_b + V_s - V_g) + \beta(1-\theta_b)(V_b - V_g)\right] \\
\rho V_b &= S\left[U_s(u_b) + \sigma(V_s - V_b) + \gamma\theta_g(u_g + V_s - V_b) + \gamma(1-\theta_g)(V_g - V_b)\right].
\end{aligned}$$

In writing these in this way we have used the fact that a special good holder gets no surplus from trading with cigarette holders, and that anyone holding a good cigarette must not be a good cigarette consumer, although he may be a bad cigarette consumer, and vice-versa. We have also inserted the bargaining solution $q_j = u_j$, which holds here for the same reason that it held in the previous sections.

There are steady state conditions generalizing those in the homogenous general good model that determine (S, G, B); but it turns out that we will not need to analyze them explicitly. The reason is as follows. There are exactly four possible types of equilibria corresponding to the four combinations of $\theta_j = 1$ or $\theta_j \in (0,1)$, for $j = g, b$ (since $\theta_j = 0$ cannot be an equilibrium with no depreciation, exactly as in the simpler model with homogenous general goods). We will see below that generically there is no equilibrium where θ_g and θ_b are both less than 1, and as long as one θ_j equals 1 there will be no type j general goods circulating in steady state, so we can use the steady state condition for the model with only one type of general good.

Consider the case where $\theta_g = \theta_b = 1$, which implies $S = 1$ and $B = G = 0$. Bellman's equations reduce in this case to

$$\begin{aligned}
\rho V_s &= y\left(\Omega + \gamma u_g + \beta u_b\right) \\
\rho V_g &= U_s(u_g) + \sigma(V_s - V_g) + \beta(u_b + V_s - V_g) \\
\rho V_b &= U_s(u_b) + \sigma(V_s - V_b) + \gamma(u_g + V_s - V_b).
\end{aligned}$$

The equilibrium condition corresponding to $\theta_j = 1$ is $\Delta_j \geq 0$, where Δ_j is proportional to $u_j + V_s - V_j$. Solving for the Δ_j, we obtain

$$\begin{aligned}
\Delta_g &= y\Omega + \left(\rho + \sigma + \beta + \gamma y\right) u_g - (1-y)\beta u_b - U_s(u_g) \\
\Delta_b &= y\Omega + \left(\rho + \sigma + \gamma + \beta y\right) u_b - (1-y)\gamma u_g - U_s(u_b).
\end{aligned}$$

It is easy to check that $\Delta_j \geq 0$ if and only if $y \geq y_j$, where

$$\begin{aligned}
y_g &= \Sigma\left[U_s(u_g) + \beta u_b + \gamma u_g - (1+\rho)u_g\right] \\
y_b &= \Sigma\left[U_s(u_b) + \beta u_b + \gamma u_g - (1+\rho)u_b\right]
\end{aligned}$$

and $\Sigma = 1/\left(\Omega + \beta u_b + \gamma u_g\right)$.

Hence, $\theta_g = \theta_b = 1$ is an equilibrium if and only if $y \geq \max\{y_g, y_b\}$. One might expect that $y_g < y_b$, so that the equilibrium condition for smoking good cigarettes holds automatically as long as the equilibrium condition for smoking bad cigarettes holds; this is not true in general. One can check that $y_g < y_b$ if and only if

$$\frac{U_s(u_g)}{1+\rho} - u_g < \frac{U_s(u_b)}{1+\rho} - u_b. \tag{16}$$

Notice that $U_s(u_j)/(1+\rho) - u_j$ can be interpreted as the gain from trading a type j cigarette next period rather than smoking it now, and $y_g < y_b$ if and only if this gain is bigger for bad quality cigarettes. The point is that good cigarettes are not only better for smoking, they are also better for trading, and so it is not unambiguous whether the incentive condition to smoke rather than trade a cigarette is more severe for good or bad cigarettes.

Consider now an equilibrium where $\theta_g = 1$ and $\theta_b < 1$. Since good cigarettes are always consumed, only bad cigarettes and special goods circulate, and we can use the conditions from the homogeneous general good model (with B replacing G) to find the steady state. To construct an equilibrium of this sort, one proceeds as follows. First solve for the $\Delta_j = u_j + V_s - V_j$ as functions of S; then solve $\Delta_b = 0$ for the steady state S and check that it is in $(0, 1)$; and finally substitute S into Δ_g and check that it is nonnegative (one can recover θ_b from S using the steady state condition, but it is not necessary to check anything else because $0 < \theta_b < 1$ if and only if $0 < S < 1$). What one finds is that $S \in (0, 1)$ if and only if $y < y_b$, where y_b was defined above, and $\Delta_g \geq 0$ if and only if $y \leq \hat{y}$, where

$$\hat{y} = \Sigma \left[\frac{u_g U_s(u_b) - u_b U_s(u_g)}{u_g - u_b} + \beta u_b + \gamma u_g \right].$$

Next, consider an equilibrium where $\theta_g < 1$ and $\theta_b = 1$. Similar to the previous case, one shows that $S \in (0,1)$ if and only if $y < y_g$, where y_g was defined above, and $\Delta_b \geq 0$ if and only if $y \geq \hat{y}$. Finally, consider the case where $\theta_g < 1$ and $\theta_b < 1$. This cannot be an equilibrium, except possibly for parameter values in a set of measure zero, because we would have to find a value of S that satisfies both $\Delta_g = 0$ and $\Delta_b = 0$. This completes the analysis of the different possible equilibria.

In terms of describing the results, we break things into two cases: $y_g < y_b$ and $y_g > y_b$, corresponding to whether condition (16) does or does not hold. When $y_g < y_b$, one can easily show $\hat{y} > y_b$, and so based on the above analysis we conclude the following: for $y > y_b$ the unique equilibrium is $\theta_g = 1$ and $\theta_b = 1$; and for $y < y_b$ the unique equilibrium is $\theta_g = 1$ and $\theta_b < 1$. When $y_g > y_b$, one can show $\hat{y} < y_b$, and so we conclude the following: for $y > y_g$ the unique equilibrium is $\theta_g = 1$ and $\theta_b = 1$; for $y \in (\hat{y}, y_g)$ the unique equilibrium is $\theta_g < 1$ and $\theta_b = 1$; and for $y < \hat{y}$ the unique equilibrium is $\theta_g = 1$ and $\theta_b < 1$. These results are all depicted in Figure 6.

In particular, when y is sufficiently big both good and bad cigarettes are smoked; when y is sufficiently small, good cigarettes are smoked and bad cigarettes are traded; and, as long as u_g is not too big relative to u_b, there is an intermediate range of y such that good cigarettes are traded while bad cigarettes are smoked. This last possibility – that for some parameter values good commodities circulate as money while bad commodities are consumed – seems to fly in the face of at least a naive version of Gresham's Law that says "bad money drives out good". The explanation here is that the relative price of good cigarettes makes them more desirable for trading, even though their intrinsic properties also make them more desirable for consumption, and the net outcome of this tension depends on the difficulty of trade and the relative intrinsic values of the commodities (y and u_g/u_b).

A more sophisticated version of Gresham's Law says that it applies "only when there is a fixed rate of exchange between the two [candidate monies]" (Friedman and Schwartz 1963, 27n). Suppose introduce private information, by assuming that agents being offered a cigarette cannot tell whether it is good or bad until after a trade for it has been completed. Then obviously we must have $q_g = q_b$. This implies that trading good and bad cigarettes has the same payoff: $V_g = V_b$. Hence, given that $u_g > u_b$, we have $\Delta_g = u_g + V_s - V_g > u_b + V_s - V_b = \Delta_b$, and therefore it is not possible to have $\theta_g < 1$ and $\theta_b = 1$ (since that would require $\Delta_g = 0 \leq \Delta_b$). In other words, anyone who indifferent between smoking and trading a bad cigarette now must strictly prefer smoking to trading a good cigarette. This implies that we cannot have good money circulating while bad money is being consumed.

Given the above argument, the only possible equilibria are $\theta_g = \theta_b = 1$ or $\theta_g = 1$ and $\theta_b \in (0, 1)$, and in either case we have $q = u_b$. One can easily verify that $\theta_b = 1$ if $y \geq y_b$ and $\theta_b \in (0, 1)$ if $y < y_b$. Hence, in Figure 6, all that matters now is whether y is above or below y_b. In the region where we formerly had $\theta_g \in (0, 1)$ and $\theta_b = 1$ with no private information – i.e., the region where the equilibrium flew in the face of Gresham's Law – good cigarettes are now driven from circulation because they are forced to trade at the same price as bad cigarettes. There are two different cases in this region: if $y \geq y_b$ then good cigarettes are driven out of circulation and replaced by direct barter; and if $y < y_b$ then good cigarettes are driven from circulation and replaced by bad cigarettes as money (although not one-for-one: it is easy to show that there are more bad cigarettes in circulation with private information that there were good cigarettes in circulation with full information).

We think that the above model provides an interesting perspective on Gresham's Law. However, recall that Radford claimed "certain brands were more popular than others as smokes, but for currency purposes a cigarette was a cigarette." Our model implies that if cigarettes are not homogeneous in consumption then they will not trade at the same price in trade unless quality is private information to the seller of the cigarette. If quality was not private information, it remains a puzzle why good and bad cigarettes trades at the same price. In any case, we summarize the main results of this section in the following proposition, the proof of which is already in the above discussion.

Proposition 4 *With full information, $q_b = u_b$ and $q_g = u_g$, and there exist y_b, y_g, and $\hat{y}$ defined in the text such that the following is true: if $y > \max\{y_b, y_g\}$ then $\theta_b = \theta_g = 1$; if $y < \min\{y_b, \hat{y}\}$ then $0 < \theta_b < 1$ and $\theta_g = 1$; and for $\theta \in (\hat{y}, y_g)$, which is nonempty if and only if u_g is not too big relative to u_b, then $\theta_b = 1$ and $0 < \theta_g < 1$. With private information, $q_g = q_b = u_b$ and $\theta_g = 1$ for all parameters, while $\theta_b = 1$ if and only if $y \geq y_b$. These are all of the (steady state) equilibria.*

6 Summary

This paper has analyzed when a generally desired commodity will be used as money, depending on parameters, including those that measure the degree of specialization, trading frictions, depreciation rates, and so on. We solved for amount of commodity money in circulation and its value. When the commodity money is sometimes consumed, its value in trade is pinned down by its utility value in consumption; when it is used exclusively as money, however, it trades at a premium over its consumption value. When commodity and fiat money both circulate, for some parameter values – in particular, when the stock of fiat money is small – the only equilibria is one where the two monies are treated as perfect substitutes. However, in other circumstances there is a multiplicity of equilibria, and fiat money may be less valuable than commodity money. When a small amount of fiat money is introduced, it crowds out commodity money one-for-one, and has no real net effect on the trading process. When too much fiat money is introduced, however, it drives commodity money from circulation and this has real effects. We also studied the circumstances in which Gresham's Law does or does not hold in a heterogeneous goods version of the model.

It seems that the model does fairly well at capturing some of the phenomena described by Radford (1945) concerning the exchange process in a primitive economy like a POW camp. There are several other interesting phenomena discussed by Radford that the model could in principle be used to address, but we have not considered them here (e.g., the effects of time-varying cigarette endowments). At a more general level, the model generates predictions about exchange institutions that we think may also apply to more complicated economies. For example, it predicts that the private sector can create institutions like money without public sector intervention in terms of the provision of fiat currency. There may be some advantages to introducing fiat money, including the seigniorage gain from freeing up real commodities, and there can also be disadvantages, especially if too much fiat money is introduced. We leave a more detailed exploration of policy and welfare along these lines to future research.

7 Appendix

Here we derive (1). Begin by considering the probability that a general good holder becomes a special good holder. First, the general good can depreciate, which occurs with probability d per unit time. Second, he can trade for his desired special good, which happens with probability αSx per unit time, after which he produces a special good with probability σ (note that if he produces a general good he stores it, given he was holding a general good in the first place). Hence, the probability per unit time that a general good holder becomes a special good holder is $P_{gs} = d + \alpha Sx\sigma$.

Next consider the probability that a special good holder becomes a general good holder. First he must get his hands on the general good, which can happen in two ways: by trading for the general good, which occurs with probability $\alpha(1-S)x$ per unit time; or by trading for his special good, which occurs with probability αSxy per unit time, and then producing the general good, which occurs with probability $1-\sigma$. In either case, we claim that *conditional* on having held the special good, he will store rather than consume the general good once he gets his hands on it with probability $1-\theta/S$ (which generally differs from the *unconditional* probability $1-\theta$). This can be seen as follows. First note that general good consumers never store the general good. Now let ω denote the probability that an agent consumes the general good given he currently has the special good. The total number of special good holders includes all the general good consumers, θ, plus the general good non-consumers who happen to hold the special good, $(1-\omega)S$. Hence, $S = \theta + (1-\omega)S$, which means $\omega = \theta/S$, which was our claim.

Therefore the probability per unit time that a special good holder becomes a general good holder is $P_{sg} = [\alpha(1-S)x + \alpha Sxy(1-\sigma)](S-\theta)/S$. Equating $SP_{sg} = (1-S)P_{gs}$, we get the steady state condition (1).

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