Nonlinear inferential multi-rate control of Kappa number at multiple locations in a continuous pulp digester

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Abstract

The continuous pulp digester represents a large-scale, distributed parameter system. Control of the spatial profile of degree of cooking, characterized by the Kappa number, rather than its endpoint value can effectively control properties that are dependent on the history of cooking. However, profile control of such large-scale distributed parameter systems throws up new challenges in estimation and control. We design a nonlinear model predictive controller using a multi-rate extended Kalman filter to infer and control discrete points along the Kappa number profile. Both, the plant and controller models are based on first principles. The design is tested for significant mismatches in parameters, initial state errors, and stochastic disturbances in the entering wood composition.

Keywords: Continuous pulp digester; Reaction profile control; Nonlinear model predictive control

1. Introduction

Pulping mills convert wood chips to pulp suitable for paper making by displacing lignin from cellulose fibers. In the Kraft process, this conversion is achieved through a combination of processes involving thermal and chemical degradation. The continuous Kraft process uses a large, vertical tubular reactor called a digester where the wood chips react with an aqueous solution of sodium hydroxide and sodium sulfide, known as white liquor, at elevated temperatures. While multiple variations of the digester configuration exist, we focus on a dual vessel digester, a schematic of which is shown in Fig. 1. Pre-steamed chips and the white liquor enter the impregnation vessel where the white liquor impregnates the pores of the wet chips.

The primary purpose of the impregnation vessel is to provide sufficient residence time for the chemicals of the white liquor to penetrate the chips. Subsequently, the wood–liquor mixture is transported to the digester vessel by the bottom circulation flow, after being heated to the reaction temperature by the bottom circulation heater. The digester vessel consists of three zones, namely the cook zone, modified continuous cook (mcc) zone and the extended mcc (emcc) zone. A significant amount of delignification occurs in the cook zone at the end of which the spent liquor is drawn out at the extraction screen. The wood chips continue their downward journey to the mcc and emcc zones where they encounter a countercurrent flow of cold diluted liquor that effectively quenches the delignification reactions. Additional heaters are provided at the mcc and emcc zones. As shown in Fig. 1, trim liquor may be injected in the cook, mcc and emcc zones to further manipulate the rates of delignification. Kraft pulping in the digester has a significant impact on key quality indicators such as
consistency, Kappa number, pulp viscosity and pulp yield. We focus our attention exclusively on the Kappa number which measures the extent of delignification.

Continuous digesters present several challenges that hinder efficient control. Pulping in the continuous digester represents a multiphase reactive flow with significant coupling between the mass, energy and momentum effects. For example, softening of the chips as cooking proceeds causes them to compact more densely, which in turn affects the chip velocity profiles [1]. On other hand, large transport delays, complex nonlinear dynamics particularly at low Kappa numbers, biological feedstock variability and process integration make control applications difficult. Implementation of control strategies is further hampered by infrequent measurements of the Kappa number. These difficulties combined with the capital-intensive nature of the pulp and paper industry mandate integration of operational strategies with modern tools such as process modeling, use of soft sensors for infrequently measured process variables, fault diagnosis methodologies and model based computer control. In this work, we explore the use of a multi-rate, inferential nonlinear model predictive control method for pulp quality control.

Several applications of model predictive control to both batch and continuous digesters have been presented in the recent literature. Development of online liquor analysis measurements for dissolved solids and dissolved lignin enabled estimation of Kappa number and has led to enhancements in batch digester control strategies [2,3]. Lee and Datta [4] exploited the availability of these new measurements to design an extended Kalman filter to infer the transient Kappa number during pulping in a batch digester. A nonlinear model predictive control (MPC) strategy was then developed that used successive linearization of the batch digester model for control of the inferred Kappa number. The recently reported liquor measurements based on NIR spectroscopy [5] will further aid developments in strategies to control the digester. Several solutions of the continuous digester control problem have also been reported in literature. Michaelsen et al. [6] implemented an MPC algorithm using a mechanistic model with an optimal state estimator on a Kamyr digester in Sweden. Amirthalingam and Lee [7] designed a multi-rate Kalman filter based linear MPC for endpoint Kappa number control. A least squares regression technique was employed to identify the optimal set of liquor measurements, followed by model building using subspace identification. Wisnewski and Doyle [8] studied reduction of a large-scale digester model using a Hankel-norm approximation. Subsequently, they used the reduced model in a linear MPC framework for control of endpoint Kappa number and compared the results with NMPC using successive linearization. The same authors [9] employed robust control analysis tools to identify a robust control structure for linear MPC of endpoint or emcc Kappa number using a first principles model as the plant. The input/measurement pairings were selected based on robust closed-loop performance for the rejection of both deterministic and stochastic unmeasured disturbances. They assumed that the Kappa number was available every 30 min with a measurement delay of 20 min. Wisnewski and Doyle [10] also compared the performances of two linear MPC designs (in one case, the linear model was generated by subspace identification techniques, and in the other case, by linearization of the fundamental model) with a structurally reduced fundamental model based NMPC using successive linearization. Their results indicate that NMPC is a superior strategy relative to linear MPC in effectively maintaining the Kappa number on target in the presence of a large number of stochastic disturbances. In the above process control applications, the pulp quality was characterized by the endpoint Kappa number at the end of the digester. Castro and Doyle [11] included control of effective alkali (EA) in the upper and lower extract streams to avoid depletion of the alkali and hence ensuring adequate delignification. An alternate control objective involves maximization of pulp yield. Andersson et al. [12] used a comprehensive softwood model to determine operating conditions for a circulation batch digester to maximize both, the pulp yield and viscosity at a constant Kappa number.

Doyle and Kayihan [13] took cognizance of the fact that the continuous digester represents a distributed parameter system and the same endpoint Kappa number may be obtained through various operating conditions. However, each distinct operating condition defines a unique reaction
They therefore suggested that a tightly managed Kappa number profile and the corresponding liquor concentration profile would improve the overall operation of the digester. Furthermore, critical fiber properties such as strength depend on the reaction path as well as the final conversion. The authors demonstrated that control using linear MPC of Kappa number profile (characterized by endpoint, cook and mcc Kappa numbers) rather than endpoint Kappa number alone results in lower variability of the endpoint Kappa number relative to a PI + feedforward controller. In the current work, we adopt the ideas of profile control strategy from Doyle and Kayihan [13]. To motivate the need for profile control, a simulated spatial–temporal Kappa number profile is shown in Fig. 2. The desired reaction profile is typically based on industrial experience to achieve certain pulp characteristics. Modified Kraft cooking schemes are essentially a set of operational guidelines to achieve a desired profile [12]. For example, severe cooking in the cook zone, characterized by a steep Kappa profile may result in short, low strength fiber in the pulp as well as low yield. Estimation and control of the entire reaction profile is difficult due to limited number of measurements and manipulated inputs. In the current work, we use a multi-rate extended Kalman filter (MR-EKF) to estimate the Kappa profile from a set of dual rate measurements and control the Kappa number at discrete points, namely end of cook, mcc, and emcc zones along with the effective alkali using nonlinear model predictive controller (NMPC) [14]. In particular, the MR-EKF uses a fundamental model of the continuous digester to infer the cook and mcc Kappa numbers for which no direct measurements are assumed. An input disturbance model is employed to account for plant and model mismatch. Extensive simulation studies reveal that the MR-EKF augmented with seven input disturbance states provides reasonable state estimates in the presence of a variety of mismatches between the plant and the model. However, large mismatch in the activation energies of the delignification reactions has a debilitating effect on the estimator performance. The inferential measurements of the three Kappa numbers mentioned above constitute the Kappa number profile that needs to be controlled.

The current work differs from the work of Doyle and Kayihan [13] in various aspects. First and foremost, we make use of a fundamental nonlinear model as the controller model. Secondly, we present a detailed design of an extended Kalman filter along with an input disturbance model and explore its potential in providing unbiased estimates of the Kappa number profile along the length of the digester. On the other hand, the study of Doyle and Kayihan [13] assumed measurements of cook and mcc Kappa or used the open-loop model to estimate the same. We also perform extensive simulation studies to identify sensitivity of estimator performance to parametric uncertainties and unmeasured process disturbances. Thirdly, we show NMPC performance in presence of severe disturbances, both stochastic and deterministic. The structure of the paper is as follows; Section 2 gives a brief overview of the model, Section 3 defines the estimation and control problem including the disturbance modeling. Finally, Section 4 documents the results.

2. Model for the continuous pulp digester

A significant effort has been directed to development of first principle models that describes the thermal-hydraulic degradation of the wood chips in the continuous pulp digester. Three broad categories of the kinetic model for path through the length of the digester. They therefore suggested that a tightly managed Kappa number profile and the corresponding liquor concentration profile would improve the overall operation of the digester. Furthermore, critical fiber properties such as strength depend on the reaction path as well as the final conversion. The authors demonstrated that control using linear MPC of Kappa number profile (characterized by endpoint, cook and mcc Kappa numbers) rather than endpoint Kappa number alone results in lower variability of the endpoint Kappa number relative to a PI + feedforward controller. In the current work, we adopt the ideas of profile control strategy from Doyle and Kayihan [13]. To motivate the need for profile control, a simulated spatial–temporal Kappa number profile is shown in Fig. 2. The desired reaction profile is typically based on industrial experience to achieve certain pulp characteristics. Modified Kraft cooking schemes are essentially a set of operational guidelines to achieve a desired profile [12]. For example, severe cooking in the cook zone, characterized by a steep Kappa profile may result in short, low strength fiber in the pulp as well as low yield. Estimation and control of the entire reaction profile is difficult due to limited number of measurements and manipulated inputs. In the current work, we use a multi-rate extended Kalman filter (MR-EKF) to estimate the Kappa profile from a set of dual rate measurements and control the Kappa number at discrete points, namely end of cook, mcc, and emcc zones along with the effective alkali using nonlinear model predictive controller (NMPC) [14]. In particular, the MR-EKF uses a fundamental model of the continuous digester to infer the cook and mcc Kappa numbers for which no direct measurements are assumed. An input disturbance model is employed to account for plant and model mismatch. Extensive simulation studies reveal that the MR-EKF augmented with seven input disturbance states provides reasonable state estimates in the presence of a variety of mismatches between the plant and the model. However, large mismatch in the activation energies of the delignification reactions has a debilitating effect on the estimator performance. The inferential measurements of the three Kappa numbers mentioned above constitute the Kappa number profile that needs to be controlled.

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2. Model for the continuous pulp digester

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Kraft pulping have appeared in literature, namely the Purdue model and its latter extensions [1,15–17], the three-stage model proposed by Gustafson et al. [18] and its extensions [19–21], and the recently proposed comprehensive kinetic model by Andersson et al. [22]. The Purdue model uses five wood components, namely high reactive lignin (s1), low reactive lignin (s2), cellulose (s3), araboxylan (s4), and galactoglucomman (s5) to describe the wood dynamics. The three-stage model uses only two components, namely cellulose and lignin. Also the delignification occurs in three phases in series, namely initial, bulk and residual with different kinetics in all the three stages. Andersson et al. [23] validated the Purdue model and the Gustafson model on a batch circulation digester using cook data. Their results show that for modern cooking schemes the three-stage model captures the endpoint lignin better while the Purdue model follows the transient trajectory better for the majority of the cook. Subsequently, they presented a comprehensive model for softwood cooking. The wood was assumed to have 12 components and the delignification kinetics were validated using data from a series of laboratory autoclaves and circulation cooks. They comprehensively demonstrated that the 12-component model outperforms both, the Purdue as well as the three-stage model.

Although each of the three kinetic models can potentially be used with their extensions (for the Purdue and the three-stage models) for online optimization and process control purposes, we make use of the Purdue kinetic model to describe the thermal degradation of the wood chips [17]. The resultant model represents a distributed parameter system and is converted to a lumped form by approximating it with continuous stirred tank reactors (CSTRs) in series. The impregnation and digester vessels are divided into 46 axial CSTRs of unequal lengths to approximate the axial and the temporal profiles of the energy and the mass transport. The digester dimensions and the flow configurations are representative of an industrial continuous digester. As in the Purdue model, each CSTR is assumed to contain three phases: solid phase, entrapped liquor phase, and free liquor phase. The entrapped liquor phase resides within the pores of the wood chips where it reacts with the solid substance. The solid phase consists of five components described above whereas the free and entrapped liquor consist of alkali, sodium sulfide, dissolved lignin and dissolved solids. A schematic of a CSTR is shown in Fig. 3. The model includes diffusion of chemicals from the free liquor phase to the entrapped liquor phase, before reacting with the solid phase. Temporal variations within each CSTR are described by mass and energy conservation equations resulting in 19 ODEs in each of the 46 CSTRs. The resulting 874 states were implemented in MATLAB and solved using the ode45 solver. For the purpose of brevity, the model equations have been summarized in Appendix A and Wisnewski et al. [17] is cited for the general scheme and the model parameter values. As discussed previously, the Kappa number is one of the key quality variables that are strongly influenced by the cooking process and is defined as

\[
\text{Kappa number} = \frac{\rho_{s1} + \rho_{s2}}{0.00153 \left( \sum_{i=1}^{5} \rho_{s_i} \right)}
\]

where, \( \rho_{s_i} \) (kg·m\(^{-3}\)) refers to the concentration of the \( i \)th component of the wood solid. However, only infrequent online measurements of Kappa number are available and are expensive to measure [24]. The sparse Kappa measurements therefore necessitate design of a MR-EKF that can integrate the multi-rate measurements in providing the state estimates. An added advantage of state estimation is that it provides inferential measurement of the upstream Kappa numbers that characterize the Kappa number profile. The following section outlines the estimation and control strategy employed in the current work.

3. Estimator and controller design

The continuous digester represents a nonlinear process where the nonlinearity is pronounced at lower Kappa numbers. Among the various nonlinear estimation techniques, the extended Kalman filter is the most popular. The main objective of the estimator design is to obtain state estimates that converge to the true plant states in presence of parametric mismatches, unmeasured disturbances and large errors in the initial state estimates. In this work, we assume that the key quality variable namely the emcc Kappa number is available at every 20 min with no measurement...
delay whereas, other non-Kappa number measurements are available at an interval of 5 min. Consistent with the reaction profile control scheme of Doyle and Kayihan [13], the estimator should provide inferential values of the internal Kappa numbers at the upstream cook, mcc and emcc zones at a sampling period of 5 min so as to enable control of the Kappa profile. The faster sampling period of 5 min will ensure effective control. The digester model (see Appendix A) described by 874 ODEs may be summarized in the state space form as follows:

\[
\begin{align*}
\dot{x} &= f(x, u, d) \\
y &= g(x, d)
\end{align*}
\]

where \( x \), \( u \) and \( d \) represents states, manipulated inputs and various disturbances, respectively, while the available measurements are denoted by \( y \). Below, we briefly describe the components of these vectors.

**Measured outputs (\( y \)):** We consider availability of 17 measurements for the estimation and control problem of the continuous digester. Table 1 summarizes these measurements along with their sampling rates. Among these the emcc Kappa number is the slow measurement available every 20 min. As discussed by Perala and Kirby [24], several commercial Kappa analyzers are available that provide online measurement of the amount of lignin in the pulp. Among the fast sampled measurements, the effective alkali (EA) of the upper and lower extracts may be obtained using conductivity measurements. Measurements of dissolved solids (DS) and dissolved lignin (DL) have been reported by Paulonis and Krishnagopalan [2,3] and Andersson and Wilson [5]. Andersson and Wilson [5] also report spectroscopic methods for 14 measurements of the black liquor. The EKF provides a consistent framework to use these measurements for estimation and control.

**Controlled outputs (\( y^c \)):** The main aim of this work is to explore control of the Kappa number profile characterized by the cook, mcc and emcc Kappa numbers of which the cook and mcc Kappa numbers are inferred using the MR-EKF. The EA concentrations at the upper and lower extraction screens determine the black liquor quality sent to the recovery section and represent the other two controlled outputs considered in the present study.

**Manipulated inputs (\( u \)):** Seven manipulated inputs were used in this work; these are white liquor flowrate added to the impregnation vessel along with the pre-steamed wood chips, cook, mcc, and emcc temperatures and cook, mcc and emcc trim flowrates (see Fig. 1 for location of the manipulated variables).

**Unmeasured disturbances (\( d \)):** We do not assume any structural differences between the simulation plant and its model. However, to simulate plant uncertainty, large errors in parameters and initial states are considered. Also, uncertainty is introduced in the plant by addition of white noise to measurements as well as the states. The details of these uncertainties are as follows:

- values of all 874 initial states of model were higher in magnitude than the plant states by a factor of 1.05,
- magnitude of the heat of reaction heat of reaction \( (\Delta H_R) \) in Eq. (A.14) in the model was higher relative to the plant by a factor of 10\%, (plant: \(-581 \text{ kJ/kg}\); model: \(-639.1 \text{ kJ/kg}\))
- overall heat transfer coefficient \( (U \) in Eqs. (A.14) and (A.17)) in the model was lower relative to the plant by 10\%, (plant: \( 827 \text{ kJ/min K m}^3 \); model: \( 744.3 \text{ kJ/min K m}^3 \))
- white noise was added to both, the states and measurements of plant. The standard deviations used were 0.2\% of the current value of the state and measurement in SI units.
- \( +2\% \) perturbation in activation energies \( (E_{1,i} \text{ and } E_{2,i} \text{ in Eq. (A.8) in Appendix A}) \) and a randomly assigned perturbation in pre-exponential constants of the kinetic equations of all five solid components \( (A_{1,i} \text{ and } A_{2,i} \text{ in Eq. (A.8) in Appendix A}) \),
- 5\% decrease in the high reactive lignin composition of raw chips entering the impregnation vessel \( (\rho_{s,1,b}) \).

The first four items listed above were included in all simulation studies reported in this paper. Use of 0.2\% of nominal value as the standard deviation of noise injected in all seven measurements and all 874 states ensured a significant presence of stochastic disturbances, which also capture the effect of feedstock variability and test the estimator and controller performance in a highly uncertain environment. Further, the deterministic uncertainties listed above assess the steady-state performance of the estimator and controller designs. For example, the above deviation in the heat of reaction alone results in a \(-3.4 \) and \(-2.2 \) change in the mcc and emcc Kappa numbers, respectively. Preliminary results showed that these uncertainties result in steady-state offsets in estimation. We therefore considered the use of input disturbance model to account for sources of uncertainty.

### 3.1. Input disturbance model

Modeling errors, either due to parametric or structural mismatch, as well as unmeasured process disturbances can lead to steady-state offsets in the state estimates unless

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Sampling rate (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kappa number at the end of emcc zone (1)</td>
<td>20</td>
</tr>
<tr>
<td>DS of free liquor at upper and lower extract (2)</td>
<td>5</td>
</tr>
<tr>
<td>DL of free liquor at upper and lower extract (2)</td>
<td>5</td>
</tr>
<tr>
<td>Chip temp at the end of IZ, cook, mcc, emcc zone (4)</td>
<td>5</td>
</tr>
<tr>
<td>Free liquor temp at the end of IZ, cook, mcc, emcc zone (4)</td>
<td>5</td>
</tr>
<tr>
<td>EA concentration at upper and lower extract (2)</td>
<td>5</td>
</tr>
<tr>
<td>HS concentration at upper and lower extract (2)</td>
<td>5</td>
</tr>
</tbody>
</table>
precautions are taken in the estimator design. Disturbance models have been effectively used to account for the plant-model mismatch, by augmenting the model with disturbance states. Assuming zero order hold and a sampling time of $T_s$, a discrete form of the model represented by Eqs. (1) and (2) is given by,

$$
\begin{align*}
x_k &= f_T(x_{k-1}, u_{k-1}, d_{k-1}) \\
y_k &= g(x_k, d_k)
\end{align*}
$$

where $k$ is the index for the discrete time sequence and $f_T(x_{k-1}, u_{k-1}, d_{k-1})$ is the state vector obtained by integrating the model (Eq. (1)) for one sampling time with an initial condition $x_{k-1}$ and constant inputs $u_{k-1}$ and $d_{k-1}$. The disturbance vector $d_k$ is considered to be a realization of a linear stochastic process and may be modeled as follows:

$$
\begin{align*}
x_k^w &= A^w x_{k-1}^w + B^w w_{k-1} \\
d_k &= C^w x_k^w
\end{align*}
$$

For linear systems, it has been shown, using observability considerations, that only as many extra states as the number of measurements available may be augmented [25,26]. The process model may be augmented with the disturbance model as follows:

$$
\begin{align*}
\begin{bmatrix} x_k \\ x_k^w \\ y_k \\ y_k^w 
\end{bmatrix} &= \begin{bmatrix} f_T(x_{k-1}, u_{k-1}, d_{k-1}) \\ 0 \\ A^w x_{k-1}^w \\ B^w 
\end{bmatrix} w_{k-1} \\
y_k &= g(x_k, d_k) + v_k \\
y_k^w &= Hs_k + H^d d_k
\end{align*}
$$

where $w_{k-1}$ and $v_k$ are white noise signals with covariances $Q$ and $R$, respectively. This augmented model will form the basis for all the further discussions. The augmented model as shown in Eq. (7) does not specify how the disturbance states interact with the process states. Prasad et al. [27] provide guidelines for design of the disturbance model using a local observability analysis on the augmented system, a priori information about the location of the disturbance and choice of measurements. In the current implementation, the rank of the local observability matrix of the process model linearized at nominal operating conditions is 384. This rank is based on the measurement set discussed previously. Further, we do not assume any a priori knowledge about the plant-model mismatch and simply attribute all differences between the predicted output and the measurement to a load disturbance in the manipulated inputs as follows:

$$
u_{k+1} = u_k + d_k$$

Assumption of load disturbance is appropriate in many cases. However, due to lack of observability, the augmented states will aid in removal of offsets only in the subset of states of the original model (Eq. (1)) that are observable in the augmented system. The remainder states may continue to exhibit steady-state offsets. Thus, in the context of the continuous digester, elimination of the steady-state offsets in the estimates of all states, given the lack of observability of the augmented system, is not feasible. However, for a properly designed disturbance model, the observability of states necessary in inferring the Kappa number values may be enhanced. This aspect of estimator design needs to be further studied. We assume that the unknown disturbance $d_k$ is likely to change in the random walk fashion, $d_k = d_{k-1} + w_{k-1}$ through an appropriate choice of the disturbance model matrices $\{A^w, B^w, C^w\}$.

3.2. Multi-rate extended Kalman filter based nonlinear model predictive control

Model predictive control of nonlinear dynamic systems has emerged as a major tool for optimal operation and control. The NMPC techniques range from extending linear MPC design by successive linearization of the nonlinear model to computationally intensive techniques such as discretization of the nonlinear model followed by nonlinear programming. Although the nonlinear model of the digester is available, an NMPC implementation is a formidable task due to requirement of an online solution of nonlinear program. To retain the simplicity of quadratic program, Garcia [28] suggested that the future predictions needed in MPC may be obtained by adding the nonlinear unforced response of the system with a forced response based on a linear model. Such an approach seems to be particularly well suited for large-scale systems such as the continuous digester. A key feature in the design of NMPC deals with modeling of unmeasured disturbances. For nonlinear systems, the use of EKF and its variants for state estimation have been widely reported [29]. State estimation techniques that represent improvements over the EKF are areas of active research [30,31]. We briefly discuss the implementation of NMPC using a MR-EKF.

**Step 1: Initiation.** At the sampling instant, $k = 0$, start with the initial values of inputs $u_0$, states $x_0$ and sampling time $T_s = 5$ min. Choose the initial state error covariance matrix ($P_0$), and the state and measurement noise covariances ($Q$ and $R$). Select control parameters such as the prediction horizon ($p$), control horizon ($q$), and the weighting matrices for the projected error ($W_u$) and the manipulated inputs ($W_d$) needed during minimization of the MPC objective function (see Eqs. (14)–(18) and Appendix B).

**Step 2: Model update.** Set $k = k + 1$, and linearize the digester model (Eq. (3)) to get the linear state space structure identified by matrices $A_k$, $B_k$, $C_k$ and $D_k$. We used the `linmod` function in MATLAB to linearize the system. Select the appropriate set of available measurements, the measurement noise covariance matrix ($R$), and the state to output matrix $C$ for the multi-rate state estimator. Note that while 17 measurements are available every 20 min (major sampling period), only 16 measurements are available every 5 min (minor sampling
Step 5: Control move computation.

The prediction of the controlled variables from their respective reference trajectories over the prediction horizon. The objective function: \( \text{min} \left\| W_s (Y_{k+1/k} - R_{k+1/k}) \right\|_H + \left\| W_e \Delta U_k \right\|_H^2 \)

subject to the constraints:

\( \Delta u_{k+q} = \cdots = \Delta u_{k+p-1} = 0 \)

\( u_{k+l}^{\text{low}} \leq u_{k+l} \leq u_{k+l}^{\text{high}}, \quad 0 \leq l \leq q - 1 \)

\( \Delta u_{k+l}^{\text{min}} \leq \Delta u_{k+l} \leq \Delta u_{k+l}^{\text{max}}, \quad 0 \leq l \leq q - 1 \)

\( y_{k+l}^{\text{low}} \leq y_{k+l} \leq y_{k+l}^{\text{high}}, \quad 1 \leq l \leq p \)

where \( R_{k+1/k} = [r_{k+1/k}^T \cdots r_{k+p/k}^T]^T \) is the reference trajectory over the prediction horizon. The objective function captures the deviation of the controlled variables from their respective reference trajectories over the prediction horizon. As the prediction equation and the constraints are linear in the decision variables \( \Delta U_k \), the above optimization constitutes a quadratic program (solved using \textit{quadprog} in MATLAB).

Step 6: Implementation. Consistent with the receding horizon strategy, the first element of the calculated control moves \( \Delta U_k \) is implemented. If control has to be continued, then repeat step 2.

3.3. Application of NMPC to continuous pulp digester

The objective of the controller is to monitor the quality of the pulp by controlling the three Kappa numbers (cook, macc, and emcc) as well as the upper and lower extract alkali concentrations in presence of mismatch in the initial state estimates, parameters and unmeasured input disturbances. Pulping in a continuous digester is a slow process with long residence times in the range of 8–12 h. Hence, the local linearization based NMPC technique with a 5 min sampling interval is expected to adequately capture the nonlinear attributes and yield expected state estimates and control performance. Given the large transport lags, size of the system, and scarcity of Kappa measurements, our control problem represents a severe test of the various attributes of the NMPC based on the multi-rate EKF. Among the various challenges we encountered, arriving at a single tuning of the estimator so as to provide consistent estimates for the various parametric mismatches as well as unmeasured disturbances was the most difficult.

3.4. EKF tuning parameters

The overall performance of the EKF is sensitive to the various design parameters like initial state error covariance \( (P_0) \), state noise covariance \( (Q) \), and measurement noise covariance \( (R) \). It was observed that a non-judicious choices of these parameters yielded infeasible estimates (such as negative Kappa number values). In particular, a very large value of \( P_0 \) or \( Q \), implying low confidence in the model, resulted in large corrections to the predicted state estimates, parameters and control performance. On the other hand, a small value of \( P_0 \) resulted in slower convergence of the estimates to the plant states. Similar issues existed with choice of the measurement noise covariance, \( R \). A small value of \( R \) resulted in state estimates, which ignored the a priori knowledge in the fundamental model of the digester. It was also observed that a set of tuning parameters that worked well for a particular instance of a parametric mismatch might result in infeasible or poor estimates when a different mismatch is introduced. In the current work, the covariance matrices were tuned by trial and error. The values of \( P_0 \) and \( Q \) have been assumed as 100\( P_{874} \) and 0.016\( I_{874} \), respectively. Depending on the
sampling instant, the value of $R$ is either 0.14$I_{16}$ (minor sampling instant) or 0.14$I_{17}$ (major sampling instant). These values have been used for all simulation studies for state estimation reported in the paper.

3.5. Model predictive controller tuning parameters

Control moves are calculated by solving the control problem outlined in Eqs. (14)–(18). One must choose several tuning parameters namely, the prediction horizon ($p$), the control horizon ($q$), controlled output weighing factor ($W_o$) and the manipulated input weighing factor ($W_u$), in order to implement the control algorithm. Elements of $W_o$ are selected on the basis of their relative importance of a particular controlled variable with respects to the others. The elements of $W_u$ must also account for the different units as well as magnitudes of the controlled variables. In our control problem the most important controlled variable is emcc Kappa as it defines the quality of the pulp at the exit of the digester, hence maximum weight is given to the emcc Kappa number. Selection of the $W_u$ is based on two factors: (1) the importance of the input move suppression term in Eq. (14) with respect to the setpoint tracking error term, and (2) the relative importance or availability of a given manipulated input resource with respect to other manipulated inputs. An increase in the prediction horizon $p$ leads to robust control at the cost of more computation time necessary for integration. In the current application, we chose a prediction horizon greater than the settling time of the process (of the order of 10–12 h). If the control horizon $q$ is chosen to be small, it typically leads to a less aggressive controller. At the same time, a large value of the control horizon increases the size of the optimal control problem that needs to be solved at every instant. These tuning parameters were determined after extensive simulations and are fixed as follows: $p = 160$, $q = 1$, $W_o = \text{diag}(0.8, 5.7, 35.3, 21.6, 40.7)$, $W_u = \text{diag}(12245, 3.125, 3.125, 3.125, 3.125, 3.653 \times 10^5, 95345, 3.847 \times 10^5)$. The various input/output constraints in Eqs. (13)–(16) are determined by safety and actuation limits and have been summarized in Table 2.

4. Results and discussion

The above multi-rate EKF based NMPC was extensively tested in MATLAB to evaluate its efficacy. The efficacy of the estimator is determined by its ability to provide quick and offset free state estimates. In the present work, the states corresponding to the five solid components are key in providing accurate inference of Kappa numbers. Thus, the ability of the estimator in providing reasonable estimates of the Kappa numbers consistently in presence of a variety of parametric mismatches and disturbances served as the performance criteria during estimator tuning. With respect to the NMPC design, the performance was judged by the ability of the controller to efficiently reject disturbances and track set point changes with emphasis on control of the emcc Kappa number. Simulation studies show that the speed of convergence of the estimator critically depends on the availability of Kappa number measurement. However, as discussed previously, Kappa number measurements are scarce. Absence of emcc Kappa number in the set of available measurements results in poor convergence to plant states, particularly for those states that represent wood concentration. To highlight the performance of the MR-EKF and NMPC designs, we considered the following case studies:

Case study 1: Estimator design: Benefits of disturbance modeling in state estimation.

Case study 2: Estimator design: Effect of uncertainty in delignification kinetic parameters in state estimation.

Case study 3: Controller design: Servo control.

Case study 4: Controller design: Disturbance in high reactive lignin concentration in the raw wood chips.

4.1. Case study 1: Benefits of disturbance modeling in state estimation

In this case study, we compared the performance of the state estimator for the following cases:

- Open-loop model prediction (without measurement feedback)
- EKF with 17 measurements
- EKF with 17 measurements and using the input disturbance model

In all the above cases the following conditions were identically applied, namely the plant was at a steady state at time $t = 0$ and the first four disturbances described in Section 3 were implemented. The estimation results are shown in Fig. 4. Fig. 4(a) and (b) depict the inferential measurement of cook and mce Kappa numbers for which no measurements exist. It is observed that the estimator using the input disturbance model (dashed line) provides offset free estimation although the time needed for the inferred cook Kappa number to converge to the plant Kappa number is in excess of 10 h. Predictions with the open-loop model (dash-dotted) and the EKF predictions without input disturbance model (dotted) exhibit steady-state offset. In Fig. 4(c), we observe that the EKF without the disturbance model and open-loop model exhibit large offsets despite the fact that emcc Kappa number measure-

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max free liquor flow rate change</td>
<td>0.35 m$^3$/min</td>
</tr>
<tr>
<td>Max cook temperature change</td>
<td>4 K</td>
</tr>
<tr>
<td>Max mce temperature change</td>
<td>4 K</td>
</tr>
<tr>
<td>Max emce temperature change</td>
<td>4 K</td>
</tr>
<tr>
<td>Max change in cook trim</td>
<td>0.03 m$^3$/min</td>
</tr>
<tr>
<td>Max change in mce trim</td>
<td>0.05 m$^3$/min</td>
</tr>
<tr>
<td>Max change in emce trim</td>
<td>0.03 m$^3$/min</td>
</tr>
</tbody>
</table>
Kappa number was available. The open-loop model also shows a large settling time of 700 min, which is larger than the Kappa number estimates using EKF. Between the two EKF designs, the estimator with the input disturbance model provides offset-free inference of the emcc Kappa number with a settling time of 400 min (80 samples). The spikes observed in estimation of the emcc Kappa number reflect the fact that a measurement of emcc Kappa number becomes available every 20 min. The apparent increase in the emcc Kappa number measurement (solid line) from an initial value of 28 at 0 min to 30 at 1000 min is due to the presence of drifts introduced by noise in the plant states. Upon comparing among the estimated cook, mcc, and emcc Kappa numbers using

Fig. 4. Case study 1: Kappa profile at the end of (a) cook zone (b) mcc zone (c) emcc zone. Solid line (real plant), dashed line (EKF with input disturbance model), dotted line (EKF without disturbance model) and dash-dot line (open-loop model prediction). Measurements of cook and mcc Kappa numbers are shown only for comparison.

Fig. 5. Case study 2: Kappa profile at the end of (a) cook zone, (b) mcc zone and (c) emcc zone. Solid line (real plant), dashed line (EKF with input disturbance model). Measurements of cook and mcc Kappa numbers are shown only for comparison.
the disturbance model based EKF (dashed lines in Fig. 4(a)–(c)), it is observed that the estimation of the cook Kappa number is the slowest. Since the MR-EKF with the input disturbance model shows the most promising performance, all further simulations are based on the input disturbance model.

4.2. Case study 2: Effect of uncertainty in delignification kinetic parameters in state estimation

In Kraft pulping, delignification occurs primarily by the OH\(^-\) ion contributed by alkali in the white liquor; while the HS\(^-\) ion from the sulfide in white liquor prevents re-precipitation of lignin. The delignification kinetics is described by complex equations based on summation of Arrhenius type equations for both, the OH\(^-\) and HS\(^-\) ions (see Eqs. (A.7) and (A.8) in Appendix A). Accurate information of the various kinetic parameters (pre-exponential constants \(A_{1,i}\) and \(A_{2,i}\), activation energies \(E_{1,i}\) and \(E_{2,i}\), and stoichiometry) is not always available. In this case study, we explore the performance of the MR-EKF in presence of uncertainty in (a) pre-exponential constant and (b) activation energy.

4.2.1. Uncertainty in the pre-exponential constant

Here, the 10 pre-exponential constants corresponding to the five solid components were perturbed simultaneously in the controller model. The percentage deviations in pre-exponential constants \(A_{1,i}\) for the five solid components are 2.3, 0.3, 0.8, 1.9 and 1, and for \(A_{2,i}\), are 5, 4.1, 3, 4.9 and 1.9, respectively (see Eq. (A.8)). The resulting estimation performance is shown in Fig. 5. It is observed that the EKF provides offset free estimation of emcc Kappa number (Fig. 5(c)) although the rate of convergence is slow. The small steady-state offsets in the estimations of cook and mcc Kappa numbers (Fig. 5(a) and (b)) can be attributed to the lack of observability of the model. Inclusion of cook and mcc Kappa number measurements eliminates these offsets (results not shown). The input disturbance model used in the current implementation ascribes the source of estimation error due to parametric mismatch to the manipulated inputs and accordingly updates the state estimates. Simulations performed in absence of the disturbance model resulted in very poor estimation (results not shown).

4.2.2. Uncertainty in the activation energy

Fig. 6 represents the estimation results for an approximately +2% perturbation in the activation energies of the 10 delignification kinetic rates (reaction of the five solid components with HS and OH ions). Fig. 6(a) and (b) shows that the estimation of the cook and mcc Kappa number is unacceptable. Fig. 6(c) shows the comparison between the

| Table 3 |
| Reference trajectory for NMPC |
| Controlled variable | Initial setpoint | New setpoint |
| Kappa number at the end of emcc zone | 28.3 | 22.8 |
| Kappa number at the end of mcc zone | 56.8 | 48.9 |
| Kappa number at the end of cook zone | 126.75 | 119.75 |
| Upper extract EA concentration (kg/m\(^3\)) | 17.07 | 18.59 |
| Lower extract EA concentration (kg/m\(^3\)) | 13.56 | 14.47 |
estimated Kappa number (dashed) and the plant measurements (solid). The estimated emcc Kappa number is close to the plant emcc Kappa number measurement for initial 200 min, after which the estimation quality degrades significantly. On availability of the measurement of emcc Kappa number at the major sampling rate, the estimated Kappa number moves closer to the true emcc Kappa number. However, during the minor sampling instant, lack of emcc Kappa number measurement severely compromises the estimator performance. The spikes demonstrate the trade-

Fig. 7. Case study 3: Servo control: Kappa profile at the end of (a) emcc zone, (b) mcc zone, (c) cook zone, (d) upper extract EA concentration and (e) lower extract EA concentration. Solid line (real plant), dashed line (EKF with input disturbance model), dash-dot line (reference trajectory) and dotted line (desired band of upper and lower control limits).
off between the model and the measurement. These results show that the estimation performance is highly sensitive to the activation energy and knowledge of accurate values is critical from an estimation viewpoint.

4.3. Case study 3: servo control

In this case study, we evaluate the performance of the NMPC for a step change in the setpoints of the five controlled variables documented in Table 3. The mismatches present in the parameters and initial conditions are same as in case study 1. Use of the EKF designed for case studies 1 and 2 resulted in aggressive initial control action with high variability due to excessive variability in the innovations. We therefore used a CUSUM filter [33] to reduce reaction to the stochastic variability in measurements. We also retuned the EKF for use with control as follows. $P_0$ was selected as a diagonal matrix with the elements representing five times of a 4% squared perturbation of the initial model states. Similarly, $Q$ was selected as 4% squared perturbation of the initial model states. This ensured a proper scaling to account for the states of different magnitudes. To account for the differing magnitudes of the measurements, the noise covariance matrix $R$ was also modified. The tuning parameters of the controller provided in Section 3.3 were selected by trial and error for setpoint implementation with emphasis on tight control of emcc Kappa number. The plant was assumed to be at steady state at time $t = 0$ at which time the step change in the setpoint is implemented. The new and old setpoint values are shown in Table 3. The estimator was operational 1000 min prior to activating the controller. This was necessary in view of the slow convergence observed in Figs. 4 and 5. We use a performance band of ±1, ±2 and ±4 Kappa for the emcc, mcc and cook Kappa numbers. In Fig. 7(a) we observe that the controller steers the emcc Kappa number within the performance band in approximately 410 min. After 410 minutes, the controller maintains the mean value of the emcc Kappa number at 23.1, which is in the vicinity of desired value of 22.8 in presence of large stochastic disturbances in both, the plant states and measurements.

The NMPC performance for the estimated mcc Kappa number is also observed to be satisfactory, as the mcc Kappa number is comfortably in the desired band of ±2 Kappa number as shown in Fig. 7(b). The settling time for mcc Kappa number (300 min) is smaller than the settling time for emcc Kappa number (410 min). Moreover, given the distributed parameter nature of the process, changes in certain manipulated inputs effect only downstream outputs. For example, both the mcc and emcc trims affect the emcc Kappa number but not the cook Kappa number. Fig. 7(d) and (e) show the control of the upper extract and lower extract EA concentrations, respectively by the NMPC strategy. Both the variables settle in the vicinity of their new setpoints. It is noted that the measurements do violate the control limits. This variability is partly due to the noise injected in the plant states and measurements as well as the tuning of the controller. The manipulated inputs are shown in Fig. 8. As the new setpoints imply lower values of all three Kappa numbers (see Table 3), the white liquor and temperatures should increase so as to displace more lignin from the wood fibers. The manipulated inputs exhibit such behavior.

4.4. Case study 4: unmeasured disturbance in high reactive lignin concentration

Here, the digester is operating at the new setpoint condition as implemented in Case study 3. At $t = 100$, the high reactive lignin concentration of raw wood chips is increased from 31.02 to 32.6 kg/m$^3$. Fig. 9(a) shows that the emcc Kappa number was not significantly affected by the change in solid chip concentration. In Fig. 9(b) and
(c), it can be observed the mcc and cook Kappa numbers are oscillatory. The real cook Kappa number fails to settle in the desired band of ±4 Kappa number. However, recall that the cook and the mcc Kappa numbers are not measured but inferred by the EKF. Fig. 9(d) and (e) show that the upper and lower extract EA concentrations are also oscillatory. Fig. 10 documents the trajectory of the manipulated inputs as determined by the NMPC controller.
5. Conclusions

The continuous pulp digester represents a large-scale distributed parameter system. Doyle and Kayihan [13] demonstrated that a tightly managed Kappa number profile provides a superior strategy than control of the endpoint Kappa number alone. The main objective of the current work is exploring the ideas of profile control of the continuous digester and the study of its potential as an alternative control strategy. Control of an inferential profile rather than the endpoint Kappa number throws up new challenges that must be addressed. The main issues are (1) quality of the inferred Kappa number profile including estimator tuning; and (2) computational efficiency for feasibility of real-time implementation. To this end, a MR-EKF based nonlinear inferential model predictive controller has been developed for the continuous pulp digester. The recursive nature of EKF makes it computationally attractive. However, arriving at a single set of estimator tuning parameters that consistently provides reasonable performance for a variety of plant-model mismatches is difficult. Superior performance was obtained when an input disturbance model was used to account for the various parametric uncertainty, unmeasured disturbances and errors in initial state. A study of the effect of uncertainty in various kinetic parameters demonstrated that accurate information regarding activation energy is necessary. However the estimator is robust for changes in other kinetic parameters such as the pre exponential constants.

Large time delays and the stochastic variability of feedstock, necessitate use of a model based control strategy. Doyle and Kayihan [13] demonstrated that model predictive control provides a more uniformly cooked pulp than a PI controller strategy in presence of stochastic variability in feedstock. We therefore used an inferential nonlinear model predictive control approach. It was shown that controller was able to track the set point change in emcc Kappa number well in presence of parametric perturbations and large stochastic disturbances in the plant states and measurements. The NMPC controller was also able to reject an unmeasured step disturbance in high reactive lignin concentration of entering wood chips. Developments in sensor technology resulting in availability of new measurements [5] can be exploited by systematically integrating these measurements in the estimation and control solution. Future work will address the issue of controlling the entire profile instead of the representative Kappa number at the end of the cook, mce and emcc zones.

Acknowledgements

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Appendix A. Summary of digester model equations

The impregnation and digester vessels were divided into 46 CSTRs in series and mass and energy balances were performed over each CSTR. Below are the equations that describe mass and energy conservation over each CSTR

Compaction

\[
(1 - \eta) = \frac{\text{volume of chip}}{\text{volume of CSTR}} \left[ \frac{V_c}{V} \right] \]

Fig. 10. Case study 4: Chip lignin disturbance rejection: manipulated inputs for disturbance in high reactive lignin concentration.
Porosity

\[ e = \frac{\text{volume of entrapped liquor}}{\text{volume of chip}} \frac{V_e}{V_c} \]  \hspace{1cm} (A.2)

Concentration of solid

\[ \rho_{s,i} = \frac{\text{mass of } i\text{th solid component}}{\text{volume of chip}} \]  \hspace{1cm} (A.3)

Concentration of entrapped liquor

\[ \rho_{e,i} = \frac{\text{mass of } i\text{th component of entrapped liquor}}{\text{volume of entrapped liquor in chip}} \]  \hspace{1cm} (A.4)

Concentration of free liquor

\[ \rho_i = \frac{\text{mass of } i\text{th component of free liquor}}{\text{volume of free liquor}} \]  \hspace{1cm} (A.5)

Solid phase component mass balance

\[ \frac{d}{dt}[\rho_{s,i}V_e] = \dot{V}_{e,0}\rho_{s,i,0} - \dot{V}_e\rho_{s,i} + \dot{R}_{es}; \quad i = 1, 2, \ldots, 5 \]  \hspace{1cm} (A.6)

where the reaction rate, \( \dot{R}_{es} \), term may be calculated as

\[ \dot{R}_{es} = -e_t \left[ k_{1,i}\rho_{e,1} + k_{2,i}\rho_{s,2}^{0.5}\rho_{s,3}^{0.5} \right] \left( \rho_{e,i}V_e - \rho_{s,i}V_e \right) \]  \hspace{1cm} (A.7)

The rate constants are based on the Arrhenius law,

\[ k_{1,i} = A_{1,i} \exp \left( -\frac{E_{1,i}}{RT_e} \right) \quad \text{and} \quad k_{2,i} = A_{2,i} \exp \left( -\frac{E_{2,i}}{RT_e} \right) \]  \hspace{1cm} (A.8)

\( E_{1,i} \) and \( E_{2,i} \) represent the activation energies and \( A_{1,i} \) and \( A_{2,i} \) the pre-exponential frequency factors. The free parameter \( e_t \) represents a tuning parameter to adjust the overall delignification kinetics.

The porosity may be calculated as follows:

\[ e = 1 - \sum_{i=1}^{5} \rho_{s,i} \]  \hspace{1cm} (A.9)

where \( \rho_s \) is the density of wood substance.

Entrapped liquor phase component mass balance

\[ \frac{d}{dt}[\rho_{e,i}V_e] = \dot{V}_{e,0}\rho_{e,i,0} - \dot{V}_e\rho_{e,i} + \dot{D}V_e(\rho_{e,i} - \rho_{s,i}) + \dot{R}_{e,i}; \quad i = 1, 2, \ldots, 6 \]  \hspace{1cm} (A.10)

where the diffusivity \( D \) necessary to describe transport of chemicals from the free liquor phase to the entrapped liquor phase is given as

\[ D = 6.1321 \sqrt{T_e} e^{4.870/1.98T_e} \left[ \frac{1}{\text{min}} \right] \]  \hspace{1cm} (A.11)

The reaction rate in the entrapped liquor phase \( \dot{R}_{e,i} \) is related to the reaction rates in the solid phase \( \dot{R}_{es} \) through the stoichiometric coefficients \( b_{i,j} \) as follows:

\[ \dot{R}_{e,i} = \sum_{j=1}^{s} b_{i,j}\dot{R}_{es,j}; \quad i = 1, 2, \ldots, 6 \]  \hspace{1cm} (A.12)

Free liquor phase component mass balance

\[ \frac{d}{dt}[\rho_{f,i}V_f] = \dot{V}_{f,0}\rho_{f,i,0} - \dot{V}_f\rho_{f,i} + \dot{D}V_f(\rho_{f,i} - \rho_{e,i}) \pm \dot{V}_{ext}\rho_{f,i,ext}, \quad i = 1, 2, \ldots, 6 \]  \hspace{1cm} (A.13)

where \( \dot{V}_{ext} \) represents flowrate of liquor entering into the CSTR externally (such as trim liquor flows).

Wood chip energy balance

\[ \frac{d}{dt}[C_p\rho_s + C_p\rho_{e,i}]V_cT_c = \dot{V}_{c,0}(C_p\rho_{e,0} + C_p\rho_{e,0})T_c + \dot{D}V_cT_c + \Delta H_R \sum_{i=1}^{5} R_{es}V_c \]  \hspace{1cm} (A.14)

where the heat capacity of the solid components, \( C_{ps} \), is assumed to be constant. On the other hand, to account for the dissolved solids in the entrapped and free liquors, their respective heat capacities are calculated using the following mixing rule:

\[ C_{pe} = C_{pe}\rho_{es} + C_{pe}\rho_{el} \]  \hspace{1cm} (A.15)

\[ C_{pf} = C_{pf}\rho_{f0} + C_{pf}\rho_{f1} \]

Here, \( \rho_{es}, \rho_{el}, \rho_{f0}, \) and \( \rho_{f1} \) represent the mass fractions of the dissolved solid and liquor components in the entrapped and free liquor phases, respectively. \( D_E \) represents the net energy gained by the free liquor phase due to the diffusion of the liquor chemicals from the entrapped lipid phase. Thus,

\[ D_E = \sum_{i=1}^{4} (\rho_{f,i} - \rho_{e,i})C_{pf}T_f + \sum_{i=5}^{6} (\rho_{f,i} - \rho_{e,i})C_{pf}T_f \]  \hspace{1cm} (A.16)

\[ T_f = T_f \quad \text{if} \quad \rho_{f,i} > \rho_{e,i} \]

\[ = T_e \quad \text{if} \quad \rho_{e,i} > \rho_{f,i} \]

Free liquor phase energy balance

\[ \frac{d}{dt}[C_p\rho_fV_fT_f] = \dot{V}_{f,0}C_p\rho_{f,0}T_{f,0} - \dot{V}_fC_p\rho_{f}T_f + \dot{D}V_f(C_p\rho_{f} - C_p\rho_{e}) \pm \dot{V}_{ext}C_p\rho_{f,ext}T_{ext} \]  \hspace{1cm} (A.17)

The last term represents the enthalpy brought in or taken out of CSTR due to an external flow. In all of the above equations, the subscript ‘0’ represents the entering stream. Thus, in the impregnation vessel and cook zone of the digester vessel the incoming free liquor stream enters from the top of the CSTR, whereas the incoming free liquor stream in the mec and emc zones enters from the bottom of the CSTR.

The above equations represent 19 ODEs per CSTR.
Appendix B. Model equations for EKF based NMPC

The state error covariance is propagated in step 2 described in Section 3.2 using the equation:

\[ P_{k/k-1} = A_k P_{k-1/k-1} A_k^T + Q \]  \hspace{2cm} (B.1)

The Kalman gain \( G_k \) may be calculated as follows:

\[ G_k = P_{k/k-1} C_k^T (C_k P_{k/k-1} C_k^T + R)^{-1} \]  \hspace{2cm} (B.2)

where the dimensions of the measurement matrix \( C \) and the noise covariance matrix \( R \) depend on the number of measurements available at sample \( k \). Finally, the state error covariance is updated at step 3 in Section 3.2,

\[ P_k = (1 - G_k C_k) P_{k/k-1} \]  \hspace{2cm} (B.3)

The matrices necessary for computing future model predictions in Eq. (12),

\[ Y_0 = \begin{bmatrix} H f_{T_1}(x_{k/k-1}, u_{k-1}, C^{w} x_k^w) \\ H f_{T_2}(x_{k/k-1}, u_{k-1}, C^{w} x_k^w) \\ \vdots \\ H f_{P,T_i}(x_{k/k-1}, u_{k-1}, C^{w} x_k^w) \end{bmatrix} \]

\[ S^w_k = \begin{bmatrix} H B_k^u & 0 & \cdots & \cdots & 0 \\ H (A_k B_k^u + B_k^p) & H B_k^p & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \sum_{i=0}^{p-1} H A_k^i B_k^u & \cdots & \cdots & \cdots & H B_k^p \end{bmatrix} \]  \hspace{2cm} (B.4)

where \( f_{p,T_i}(x_{k/k-1}, u_{k-1}, C^{w} x_k^w) \) represents the terminal state values resulting from integrating the nonlinear differential equation \( \dot{x} = f(x, u, d) \) for \( p \) sampling time intervals with initial condition \( x_{k/k} \), constant input \( u = u_{k-1} \) and piecewise constant input \( d \) taking the value \( C^{w}(A_n)x^w_k \) during the time interval \([k+i, k+i+1] \).

References


