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Simulation of Electromagnetic Wave Propagation in Microwave Directional Couplers and Rat-Race Hybrid Coupler Using Open Source Finite Element Software

Dr Ndagije Charles

Faculty of applied fundamental sciences, Institute of Higher Education of Ruhengeri(INES-Ruhengeri), Rwanda, P.O. *Box 155 Ruhengeri*

***Abstract*- Computer modeling and simulations of physical processes play an important role in teachinga-learning process of engineering courses and particularly for difficult courses such as electromagnetic and microwave engineering. In this paper we determined the s-parameters by analyzing electromagnetic wave propagation in passive microwave directional coupler and rat-race hybrid coupler using open source finite element software from the One Lab project: Gmsh and GetDP. The resulting fully parameterized finite element models will be then used in teaching and learning microwave communication course at the University of Rwanda and other national higher learning institutions where the course of microwave communication is taught for facilitating the students' understanding.**

***Keywords*- Simulation, Maxwell's equations, waveguide, direction coupler, rat-race hybrid coupler, s-matrix, finite element method, electromagnetic wave.**

I. INTRODUCTION

Many universities, especially those in the developing countries, suffer from the lack of (often expensive) laboratory equipments for conducting practical courses on important engineering topics, like electromagnetic and microwave engineering. One way to improve the effective teaching-learning process for these courses is to use simulation techniques.

From the experience acquired during the teaching of the course of microwave communication in third year electronics and communication systems at National University of Rwanda during the academic year 2011-2012 and 2012-2013, it was realized that modeling and simulation can help the students to better understanding engineering courses without using hardware equipment.

Electromagnetic wave propagation in microwave passive components is described by inhomogeneous partial differential equations (PDEs) that are often hard or impossible to solve analytically, which complicates the analysis of that process. In our previous work [12], we just simulated the wave propagation in microwave passive elements with a simple structure.

In this paper we propose to analyze the electromagnetic wave propagation in directional coupler and rat-race hybrid coupler which are microwave passive devices used in power division. Microwave directional coupler is a four-port circuit designed such a way that each pair of ports is always decoupled. The operational principle of the directional coupler and the rat-race hybrid coupler is based on the principle of electromagnetic wave interference. To satisfy the condition of constructive or destructive interference in a given port, it is required to know the operating wavelength in one port of the structure. However it is not easy to determine this wavelength which is the main parameter of a directional coupler or a rat-race hybrid coupler. The knowledge of that parameter will help to properly designing the structure which will fulfill the required characteristics and also it will help to know which modes are capable to propagate in the designed structure. The use of finite element method can facilitate the computation of that wavelength.

Another element which requires a special attention during the design of passive microwave power divider is to satisfy the condition of impedance matching for all ports. This problem is solved by changing the length of each port for a fixed wavelength, till we reach the situation where the structure is working properly. We assume that all ports are considered to have the same length.

Directional coupler and rat-race hybrid coupler are widely used in radar communication and in very high frequency systems where the power division and isolation is needed.

Modeling of such waveguides requires the solution of Maxwell's equations, which can be obtained numerically by using the finite element method [11], [12]. We propose to evaluate the time-dependent solution of Maxwell's equations in these structures. The main advantage of using the finite element method for analyzing the wave propagation in different structures is that it helps students to have a view of the geometric representation of a considered structure and also it makes easy to visualize the pattern of wave propagation in those structures.

The results of this work will be used in teaching and learning the course of microwave communication at University Rwanda and other higher learning institutions in Rwanda, in which this subject is taught. In fact, the developed parametric models can help students to easily understand that by changing the parameters of the cross section of the one port, the operating frequencies are also changing as well as the propagation characteristics of the corresponding modes for the whole structure.

All the models were prepared using the open source finite element tools from the OneLab project (http://www.OneLab.info), specifically the GetDP and Gmsh codes as developed by the team of Professor Christophe Geuzaine at the University of Liège in Belgium[1],[2]. Since the software tools are freely available on the internet, both students and lecturers can use them without restrictions.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

The dynamic simulation of electromagnetic wave in those structures is analyzed by solving Maxwell's equations with the application of boundary conditions. Let start our analysis by determining the operating frequency in one port of the directional coupler (see Figure 1 and 2) and the rat-race hybrid coupler (see figure 3 and 4).

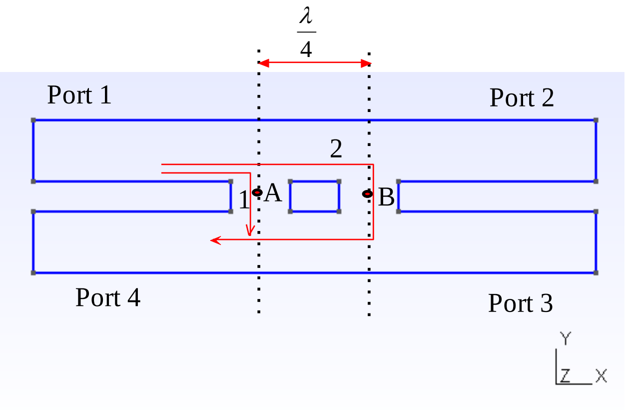

Figure 1: Geometric representation of a directional coupler in 2D.

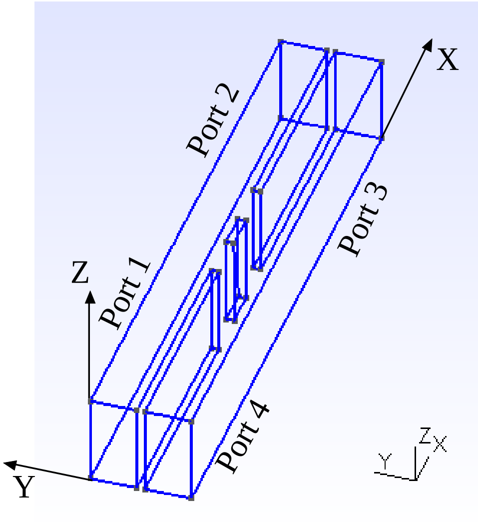

Figure 2: Geometric representation of a directional coupler in 3D.

The figure 1 and 2 indicate the geometrical structure of a microwave directional coupler, which is a four-port network consisting of two transmission lines that are electromagnetic coupled to each other. The first port is the input port, the second one is the output port, the third one is the coupled port and the fourth one is the isolated port. The operation principle of this structure is based on the interference of electromagnetic waves. On the figure 1; we have two waves such that the wave 2 will meet the wave 1 after traveling a distance equals to $\frac{\lambda}{2}$. When the path difference between two waves is an odd multiple of $\frac{\lambda}{2}$, the interference will be destructive.

The wave number one is described by the expression

$$E_1(x_1,t) = E_0 \sin(\omega t - kx_1)\,, \qquad (1)$$

Where $E_1(x_1,t)$ indicates the electrical field of the electromagnetic wave passing in the hole A, E_0 is the maximum amplitude, ω is the operating angular frequency, k is the wave number and x_1 is the distance traveled by the first wave.

The wave number 2 is also described by the following expression

$$E_2(x_2,t)=E_0\sin(\omega t-kx_2)\,, \qquad (2)$$

Where $E_2(x_2,t)$ indicates the electrical field of the electromagnetic wave passing in the hole B, x_2 is the distance traveled by the second wave. The total intensity in point A after that the wave number 2 passes through the point B and go back to the plane A, is given by

$$I=I_{\max}\cos\left[\frac{2\pi}{\lambda}(x_2-x_1)\right] \qquad (3)$$

Where $I_{\max}\propto 4E_0^2$ is the maximum intensity of electromagnetic wave and λ is the wavelength of the wave propagating in the structure. In order to have an isolated port it is necessary that

$$x_2-x_1=\left(m+\frac{1}{2}\right)\lambda,\; m=0,\pm1,\pm2,... \qquad (4)\,.$$

This expression shows the reason why the distance between the two holes must be equal to $\frac{\lambda}{4}$. In fact the wave number 2 has to travel a distance which is two times $\frac{\lambda}{4}$ while the wave number 1 still at the same position. If the impedance matching condition is respected for all ports, no signal will propagate in port number 4.

The relation (4) is the required condition for having a destrictive interference.

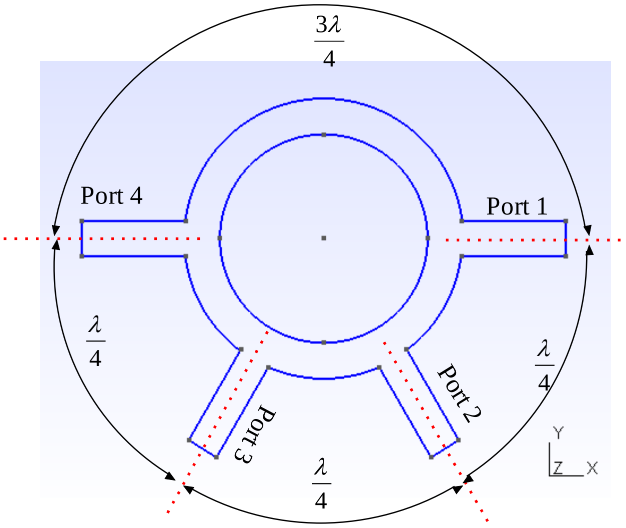

Figure 3: Geometric representation of a rat-race hybrid coupler in 2D.

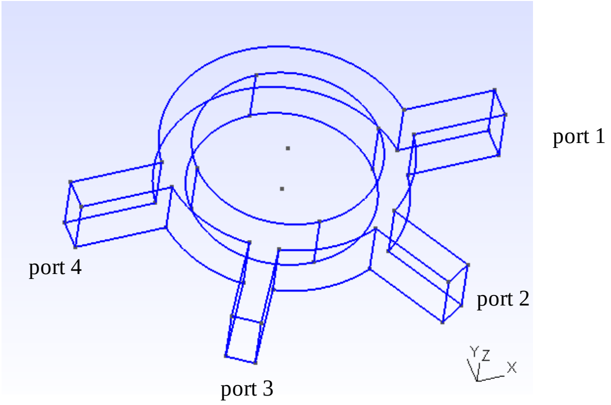

Figure 4: Geometric representation of a rat-race hybrid coupler in 3D.

For this structure, the wave propagating in port 1 from the source is split into 2 waves. The first wave travels towards port 2, while the second one travels towards port 4. If all ports are matched terminated, no signal will propagate in port 3, because the path difference between the distances traveled by the two waves is equal to $\frac{\lambda}{2}$ and according to the formula (4), the total intensity in port 3 will be equal to zero. To determine the wavelength λ we have to solve Maxwell's equations by using the finite element method. The domain of resolution is the cross section of one port.

Maxwell's equations in linear media can be read [11],[12],[14]:

$$\begin{cases} \vec{\nabla} \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} \\ \vec{\nabla} \times \vec{H} = \vec{j} + \varepsilon \dfrac{\partial \vec{E}}{\partial t} \\ \vec{\nabla} \bullet \vec{E} = \dfrac{\rho}{\varepsilon} \\ \vec{\nabla} \bullet \vec{B} = 0 \end{cases} \qquad (5)$$

Where $\vec{E}(V/m)$ is the electrical field intensity , $\rho(Coul/m^3)$ is the electrical charge density, ε is the permittivity of the medium, $\vec{B}(Web/m^2)$ is magnetic flux density, μ is the permeability of the medium, $\vec{H}(A/m)$ is the magnetic field intensity, $\vec{j}(A/m^2)$ is the conduction current density. If the considered medium is the air, the permittivity becomes ε_0, and the permeability becomes μ_0, where $\varepsilon_0 = 8.854.10^{-12} Farad/m$ and $\mu_0 = 4\pi.10^{-7} henry/m$.

By using the properties of vector analysis, the system (1) of first order partial differential equations (PDEs) can be rewritten as the following second order equation [12]:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} + \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad (6)$$

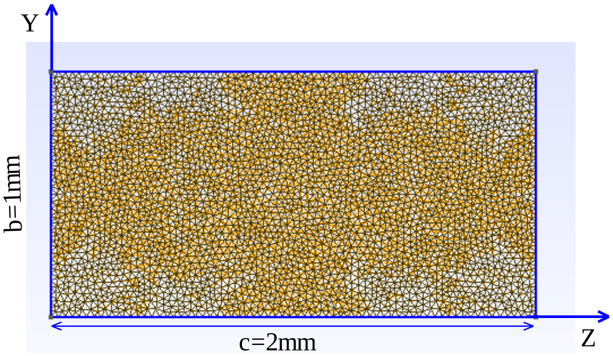

Figure 5: Discretized cross section of each port.

III. FINITE ELEMENT METHOD

The finite element method is an appropriate method to approximate solutions to the equation (6). Its application was demonstrated in details in our previous works [11], [12]. In this section, we just want to remind the main steps of this method. The first step is the weak formulation of the problem, the second one is the discrete formulation of the problem, and then the last step is the numerical solution of the problem [5],[6],[7],[8],[9],[10]. For numerical solution we have used the Gmsh and GetDP software [1].

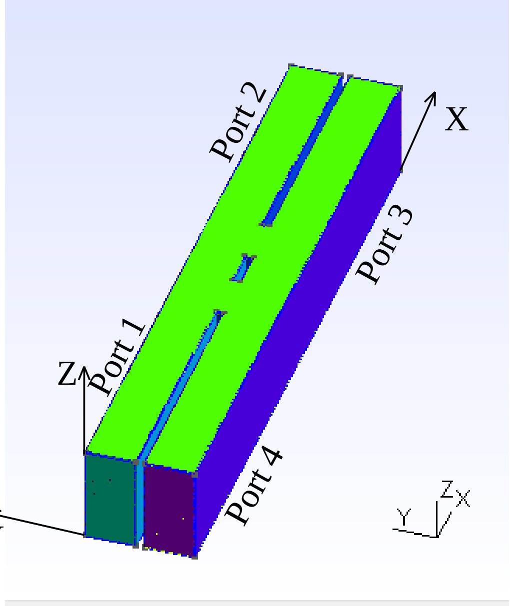

Figure 6: Discretized microwave directional coupler in 3D.

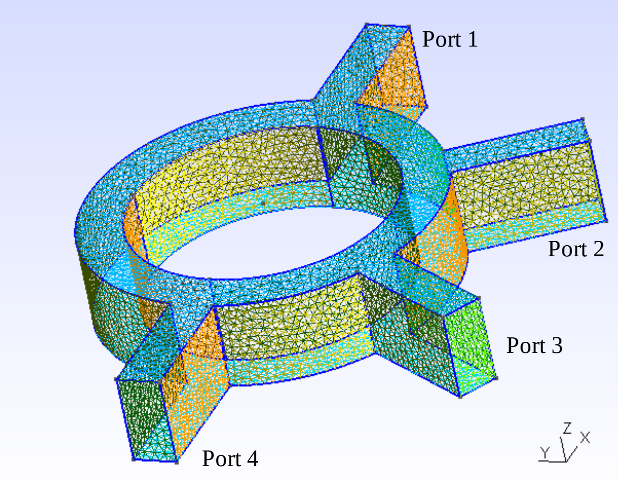

Figure 7: Discretized rat-race hybrid coupler in 3D.

The discretized structures have been obtained by using the Gmsh software.

IV. NUMERICAL SOLUTION

The operating frequency has been determined by solving the following discrete equation [11]

$$\sum_{j=1}^{N} C_j \left[\int_{\Omega} Curl\vec{W}^{j} . Curl\vec{W}^{k} d\Omega + \int_{\Omega} \varepsilon_0 \mu_0 \frac{\partial^2 \vec{W}^{j}}{\partial t^2} \vec{W}^{k} d\Omega \right] = 0 \qquad (7)$$

Where $\vec{W}^j$ represent the shape functions, $\vec{W}^k$ represent the test (basis) functions and C_j represent the unknown values to be determined. Once the values C_j are computed, the electromagnetic field can be calculated as follows [11]

$\sum_{j=1}^{N} C_j \vec{W}_j$, where N is the total number of edges in the mesh.

TABLE I
CRITICAL ANGULAR FREQUECIES WHICH ARE ABLE TO PROPAGATE IN THE ANALYSED DIRECTION COUPLER

Type of mode	Critical angular frequency (rad/s)	Wavelength(m)
TM_{11}	1.05×10^{12}	1.794×10^{-3}
TM_{21}	1.33×10^{12}	1.417×10^{-3}
TM_{31}	1.7×10^{12}	1.108×10^{-3}
TM_{12}	1.95×10^{12}	9.662×10^{-4}
TM_{32}	2.36×10^{12}	7.983×10^{-4}
TM_{51}	2.54×10^{12}	7.417×10^{-4}
TM_{42}	2.67×10^{12}	7.056×10^{-4}
TM_{13}	2.87×10^{12}	6.564×10^{-4}

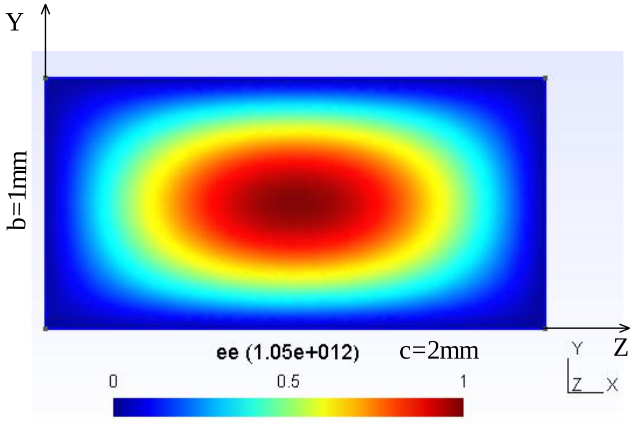

Figure 8: Intensity of magnetic field in the cross section of a rectangular microwave guide for TM_{11} mode.

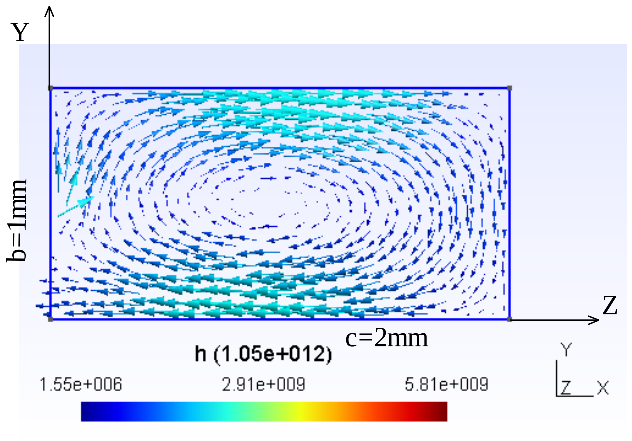

Figure 9: Pattern of magnetic field in the cross section of rectangular microwave guide for TM_{11} mode.

A. Electromagnetic wave propagation in rat-race hybrid coupler

To describe the electromagnetic wave propagation in the considered structures, we have solved the following equation [12]

$$\sum_{j=1}^{N} C_j \left[\int_{\Omega} Curl\vec{W}^{j}.Curl\vec{W}^{k} d\Omega + \int_{\Omega} \varepsilon_0 \mu_0 \frac{\partial^2 \vec{W}^{j}}{\partial t^2} \vec{W}^{k} d\Omega \right] = -\int_{\Omega} \mu_0 \frac{\partial \vec{j}}{\partial t} \vec{W}^{k} d\Omega \quad (8)$$

Where $\vec{j}$ is the source current and Ω is the domain of resolution . Equation (8) has been obtained from the weak and discrete formulation of the problem.

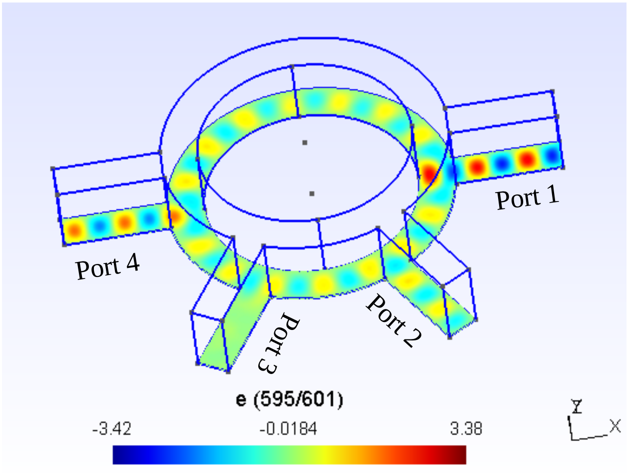

Figure 10: Propagation of TM_{11}-mode in microwave hybrid coupler when port 1 is the input port.

The signal entering in port 1 is split equally into two signals, one propagating in clockwise and the other in counterclockwise.

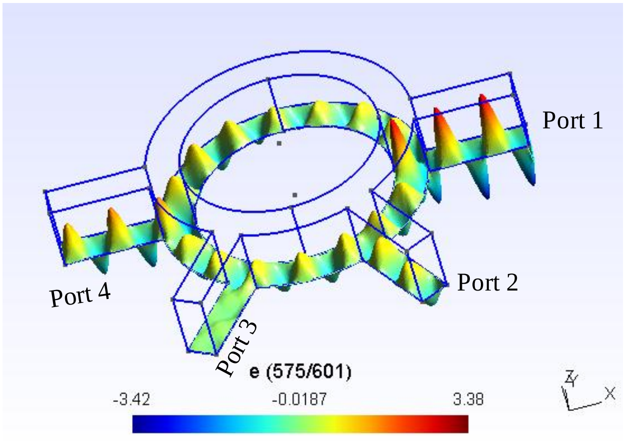

Figure 11: Raised form of electromagnetic propagation in rat-race hybrid coupler when port 3 is the input port.

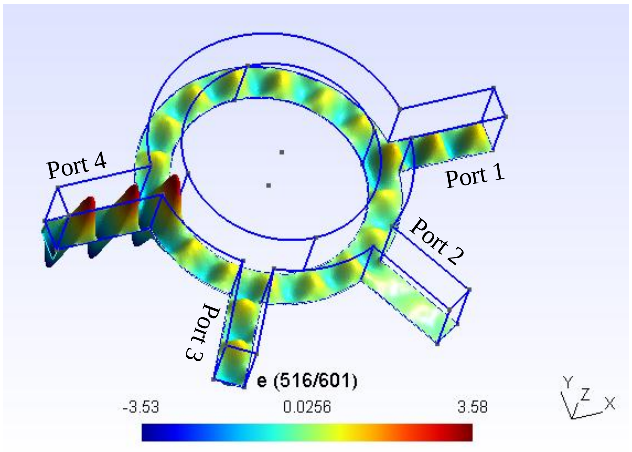

Figure 12: Raised form of electromagnetic wave propagation in rat-race hybrid coupler when port 4 is the input port.

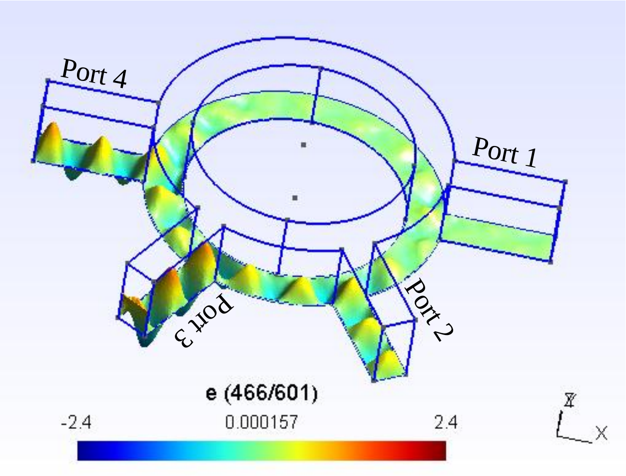

Figure 13: Raised form of electromagnetic wave propagation in rat-race hybrid coupler when port 3 is the input port.

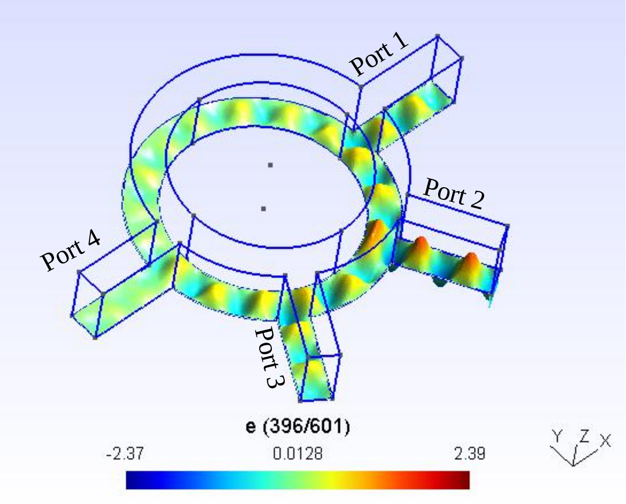

Figure 14: Raised form of electromagnetic wave propagation in rat-race hybrid coupler when port 2 is the input port.

The following table summarizes the results indicated on the above indicated figures.

TABLE II
DIFFERENT POSSIBILITIES FOR EXCITATION OF THE RAT-RACE HYBRID COUPLER

Input port	Output ports		Isolated port
Port 1	Port 2	Port 4	Port 3
Port 2	Port 1	Port 3	Port 4
Port 3	Port 2	Port 4	Port 1
Port 4	Port 1	Port 3	Port 3

This table shows that the rate- race hybrid coupler is a reciprocal network and it leads to the following equality $S_{31}=S_{24}=S_{13}=S_{42}=0$, where for S-parameter subscripts "*ij*", *j* is the input port and "*i*" is the output port. Considering that all ports are impedance matching terminated, we get the following S-matrix

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

If the network is lossless [14], we get the following s-matrix,

$$\begin{bmatrix} 0 & jq & 0 & p \\ jq & 0 & p & 0 \\ 0 & p & 0 & jq \\ p & 0 & jq & 0 \end{bmatrix}$$

Where $S_{14}=S_{23}=p$ and $S_{12}=S_{34}=jq$[14].

B. *Electromagnetic wave propagation in directional coupler*

The electromagnetic wave propagation in the directional coupler has been described by solving equation (8).

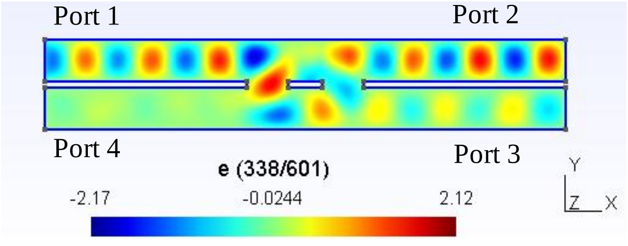

Figure 15: Propagation of electromagnetic wave in directional coupler when the input is the port 1 in case of 2D representation.

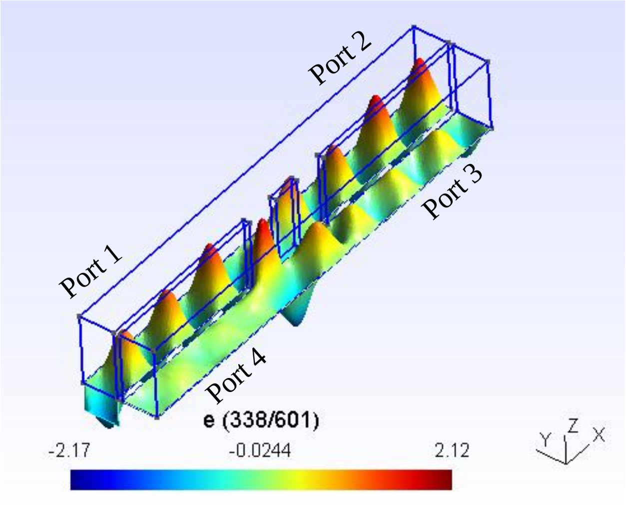

Figure 16: Propagation of electromagnetic wave in directional coupler when the input is the port 1 in case of 3D representation.

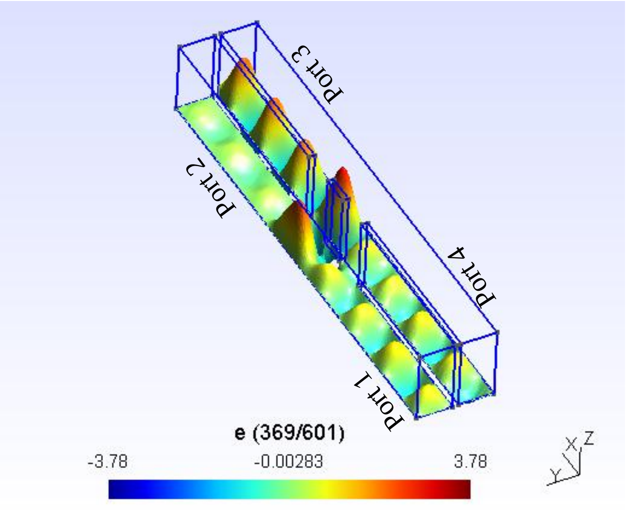

Figure 17: Propagation of electromagnetic wave in directional coupler when the input is the port 3.

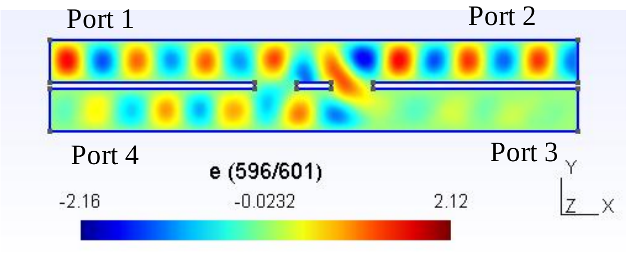

Figure 18: Propagation of electromagnetic wave in directional coupler when the input port is the port 3 for a 2D representation.

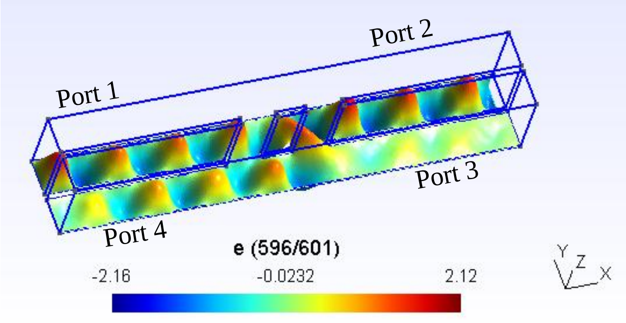

Figure 18: Propagation of electromagnetic wave in directional coupler when the input is the port 2.

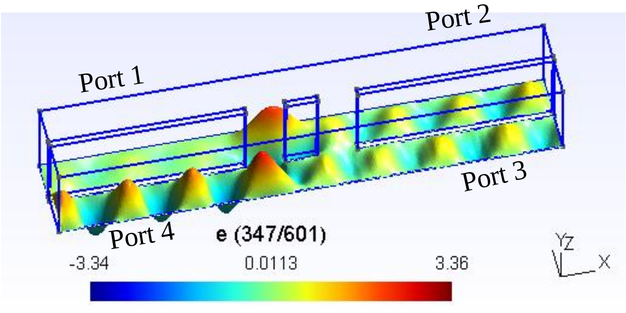

Figure 19: Propagation of electromagnetic wave in directional coupler when the input is the port 4.

Table III
Different Possibilities For Excitation Of Directional Coupler

Input port	Output port	Isolated port	Coupled port
Port 1	Port 2	Port 4	Port 3
Port 2	Port 1	Port 3	Port 4
Port 3	Port 4	Port 2	Port 1
Port 4	Port 3	Port 1	Port 2

From this table we find that, for ideal directional coupler port 1 and port 4($S_{14} = S_{41} = 0$) are decoupled; also port 3 and port 2 are decoupled ($S_{23} = S_{32} = 0$). Considering that all four ports are matched terminated, the directional coupler represented in figure 2, will be characterized by the following S-matrix.

$$[S]=\begin{bmatrix}0 & S_{12} & S_{13} & 0\\ S_{12} & 0 & 0 & S_{24}\\ S_{13} & 0 & 0 & S_{34}\\ 0 & S_{24} & S_{34} & 0\end{bmatrix}$$

The table (III) indicate that the system is reciprocal and if all ports are impedance matched we get the following S-matrix [14]

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

If the network is lossless, the following conditions will be satisfied [14]

$\sum_{i=1}^{N} S_{ik}S_{ik}^{*}=1$ and $\sum_{i=1}^{N} S_{ik}S_{il}^{*}=0$. Using these conditions we get the following s-matrix [14]

$$[S]=\begin{bmatrix} 0 & q & jp & 0 \\ q & 0 & 0 & jp \\ jp & 0 & 0 & q \\ 0 & jp & q & 0 \end{bmatrix}$$

where $S_{13}=S_{24}=jp$ and $S_{12}=S_{34}=q$.

V. CONCLUSION

In this paper, the electromagnetic wave propagation in directional coupler and rat-race hybrid coupler has been described and simulated by using the finite element software.

The obtained results show that a directional coupler and a rat-race hybrid coupler are reciprocal networks and it was demonstrated that using simulation techniques with Gmsh and GetDP software, it easy to determine the S-matrix characterizing a microwave network. The results of this work also show that the simulation techniques can facilitate not only a better understanding but also a better preparation of teaching and learning process of an engineering course especially when the equipments are not available. One of the particular advantages of finite element method in this works is its capacity of computing the operating wavelength, which facilitate the design of the analyzed structures. Also in this work it was approved that the impedance matching condition in directional coupler and rat-race hybrid coupler can be reached by changing the size of the four ports.

This work is the continuity of our previous study which was focused only on the simulation of electromagnetic wave propagation in metallic waveguide, bend waveguide and tee-junctions [12].

REFERENCES

[1] C. Geuzaine and J.F. Remacle, Gmsh. A three- dimensional finite element meshes generation with built-in pre-and post-processing facilities. International for Numerical Method in Engineering, Volume 79, Issue 11, pages 1909-1331, 2009. http://www.geuz.org/gmsh.

[2] Patrick Dular, Christophe Geuzaine, A general environment for the treatment of Discrete Problems, University of Liege, 1997-2011. http://www.geuz.org/gmsh.

[3] Granino A. KORN, Advanced dynamic –System simulation, Willey & Sons, Inc., Arizona, 2007.

[4] Robert E. Collin, Foundations for Microwave Engineering, second edition, John Willey &Sons, Inc., New York, 2001

[5] G.R. Liu, S. S. Quek, The finite element method: A Practical course, Elsevier Science Ltd, Singapore, 2003.

[6] J.N. Reddy, An introduction to the finite element method, second edition, McGraw-Hill, In., Texas, 1993.

[7] Jean-Philippe Grivet, Méthodes numériques appliquées pour le scientifique et l'ingénieur, EDP Sciences, Paris 2009.

[8] Benoît Meys, Modélisation des champs électromagnétiques aux hyperfréquences par la méthode des éléments finis. Application au problème du chauffage diélectrique, Thèse présentée à la Faculté des sciences Appliquées, Université de Liège, 1999.

[9] David V. Hutton, Fundamental of finite element analysis, McGraw-Hill, New York, 2004.

[10] C. J. Reddy, Manohar D. Deshpande, C.R. Cockrell, and Fred B. Beck; Finite Element Method for Eigenvalue Problems in Electromagnetics, NASA Technical paper 3485, Virginia 23681-0001, 1994.

[11] C. Ndagije and C. Geuzaine, Simulation of Electromagnetic Field Distribution in Metallic Waveguides Using Open Source Finite Element Software, International Journal of Emerging Technology &Advanced Engineering, Volume3,Issue2, Page 91-98, February, 2013.

[12] C. Ndagijje and C. Geuzaine, Simulation of Dynamic Propagation of Electromagnetic wave in Microwave Passive Components Using Open Source Finite Element Software, International Journal of Emerging Technology &Advanced Engineering, Volume3,Issue8, Page 44-50, August, 2013.

[13] Matthew N. O. Sakidu, Numerical Techniques in Electromagnetics, Second edition, CRC Press LLC, Florida, 2001.

[14] David Pozar, Microwave Engineering, Third Edition, John Wiley and Sons, Inc; 2005.