

Radiation via Tunneling from a de Sitter Cosmological Horizon

by

A.J.M. Medved

*Department of Physics and Theoretical Physics Institute
University of Alberta
Edmonton, Canada T6G-2J1
[e-mail: amedved@phys.ualberta.ca]*

ABSTRACT

Hawking radiation can usefully be viewed as a semi-classical tunneling process that originates at the black hole horizon. The same basic premise should apply to de Sitter background radiation, with the cosmological horizon of de Sitter space now playing the featured role. In fact, a recent work [hep-th/0204107] has gone a long way to verifying the validity of this de Sitter-tunneling picture. In the current paper, we extend these prior considerations to arbitrary-dimensional de Sitter space, as well as Schwarzschild-de Sitter spacetimes. We also provide commentary on the dS/CFT correspondence in this context.

1 Introduction

In light of recent astronomical observations, it has been suggested that our universe will asymptotically approach a de Sitter spacetime [1]. This realization has sparked a sense of urgency in resolving the quantum-gravitational mysteries of de Sitter space [2]. With prompting from the very successful anti-de Sitter/conformal field theory correspondence [3, 4, 5], much of this work has focused on finding a holographic description [6, 7] of de Sitter space. In particular, there has been much ado about establishing an analogous duality; that is, the so-called dS/CFT correspondence [8]. (Also see, for instance, [9]-[13].) Although there has been considerable success along this line, the proposed duality is still marred by various ambiguities. For example, a dual boundary theory that appears to be a non-unitary one [8], a conspicuous absence of measurable quantities (at least those with operational meaning to a de Sitter observer [2]), and logistic breakdowns [14] that can be attributed to the finite entropy of de Sitter space [15].

The above issues should probably be viewed as significant philosophical roadblocks as opposed to mere technical difficulties. Which is to say, their resolution will, in all likelihood, necessitate dramatic departures from our current ways of thinking. Hence, it may be an appropriate juncture to “take a step back” and re-enforce our understanding of de Sitter space at a semi-classical level. With this as our mind-set, let us now proceed to consider the central topic of the paper; namely, de Sitter radiation as a semi-classical tunneling process.

First, let us briefly review the concept of tunneling as it applies to a radiating black hole. According to this interpretation, Hawking radiation [16] can be attributed to the spontaneous creation of particles at a point just inside of the black hole horizon. One of the particles then tunnels out to the opposite side of the horizon, where it emerges with positive energy. Meanwhile, the negative-energy “partner” remains behind and effectively lowers the mass of the black hole.

The above point of view formed the foundation for a program of study that was initiated by Kraus and Wilczek (KW) [17]¹ and is central to the current work. The essence of the KW methodology is a dynamical treatment

¹For further developments and generalizations, see [18]-[25]. For another perspective on radiation via tunneling, see [26].

of black hole radiation. More to the point, KW considered the effects of a self-gravitating matter shell propagating outwards through a spherically symmetric black hole horizon. Two particularly significant points of this work are as follows. *(i)* The background geometry is allowed to fluctuate so that the formalism incorporates a black hole of varying mass. In this manner, the total energy of the spacetime is naturally conserved. Notably, energy conservation is often overlooked in other formal treatments of Hawking radiation [27]. *(ii)* Boundary conditions are imposed by foliating the spacetime with somewhat unconventional ‘‘Painleve coordinates’’ [28]. Significantly, these coordinates are regular at the horizon, as well as stationary but not static (i.e., time-reversal asymmetry is manifest). This gauge seems quite appropriate for describing the geometry of a slowly-evaporating black hole.

Let us now return the discussion to de Sitter space. As it is well known, there are many similarities between the thermodynamic properties of a de Sitter cosmological horizon and those of a black hole horizon [29]. Hence, it would seem natural to extend the tunneling picture and KW treatment to the background radiation associated with de Sitter space. Just such a study was recently carried out by Parikh [25] with considerable success. This author, however, focused on the interesting but unphysical case of 3-dimensional de Sitter space. The main purpose of the current paper is to generalize considerations to a de Sitter spacetime of arbitrary dimensionality. We will also provide some commentary on the dS/CFT correspondence in the context of this study.

The remainder of the paper is organized as follows. In Section 2, we consider a radiating cosmological horizon in an empty, $n+2$ -dimensional de Sitter spacetime and, with guidance from [17, 20, 25], calculate the semi-classical emission rate. The consistency of the derived expression is then verified for the case of $n = 3$. We accomplish this by extrapolating the emission spectrum and comparing the lowest order term with standard de Sitter thermodynamics. In Section 3, we further extend the formalism to a Schwarzschild-de Sitter spacetime. Here, we also comment on thermal stability and touch upon the subject of dS/CFT renormalization group flows [30]. In Section 4, we take a step towards the ethereal and reconsider the tunneling picture from an outside-of-the-horizon perspective. The dS/CFT correspondence provides the motivating factor for this portion of the analysis. Finally, Section 5 contains a brief summary.

2 De Sitter Tunneling

Let us begin by considering an $n+2$ -dimensional de Sitter spacetime (with $n \geq 1$). There are many different coordinate systems that can be used to provide a local description of de Sitter space [10], including the following explicitly static coordinates:

$$ds^2 = - \left(1 - \frac{r^2}{l^2}\right) dt^2 + \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\Omega_n^2. \quad (1)$$

Here, l is the curvature radius of de Sitter space (i.e., $\Lambda = n(n+1)/2l^2$ is the positive cosmological constant), $d\Omega_n^2$ represents an n -dimensional spherical hypersurface of unit radius, and the non-angular coordinates range according to $0 \leq r \leq l$ and $-\infty \leq t \leq +\infty$. Keep in mind that the boundary at $r = l$ describes a cosmological horizon for an observer located at $r = 0$.

The above coordinates fail, of course, to cover the entire de Sitter manifold. Eq.(1) does, however, precisely cover the so-called ‘‘southern causal diamond’’ [10], which is the region of spacetime that is fully accessible to an observer at the south pole ($r = 0$). A particularly attractive feature of this coordinate gauge is the existence of a timelike Killing vector (∂_t), thus leading to a sensible notion of time evolution for a south-pole observer. Note that such a timelike Killing vector is notoriously absent in *any* global description of de Sitter space.

As discussed in Section 1, it is most convenient, in the tunneling picture, to use stationary coordinates that are manifestly asymmetric under time reversal. In the case of a Schwarzschild black hole, the following Painleve coordinates [28] have been utilized for just this purpose [17]:

$$ds^2 = - \left(1 - \frac{2M}{r}\right) d\tau^2 - 2\sqrt{\frac{2M}{r}} d\tau dr + dr^2 + r^2 d\Omega_2^2. \quad (2)$$

Along with the above-mentioned properties, this system has the distinguishing features of horizon regularity and flat constant-time surfaces. Further note that such coordinates comply with the perspective of a free-falling observer, who is expected to experience nothing out of the ordinary upon passing through the horizon.

To obtain an analogous coordinate system for de Sitter space or ‘‘Painleve-de Sitter’’ coordinates [25], we first employ the following transformation:

$$t = \tau + f(r), \quad (3)$$

so that the static metric (1) takes on the form:

$$\begin{aligned}
ds^2 = & - \left(1 - \frac{r^2}{l^2}\right) d\tau^2 - 2\frac{df}{dr} \left(1 - \frac{r^2}{l^2}\right) d\tau dr \\
& + \left[\left(1 - \frac{r^2}{l^2}\right)^{-1} - \left(1 - \frac{r^2}{l^2}\right) \left(\frac{df}{dr}\right)^2 \right] dr^2 + r^2 d\Omega_n^2. \quad (4)
\end{aligned}$$

Next, we enforce that constant- τ slices reduce to $n+1$ -dimensional flat space (i.e., $dr^2 + r^2 d\Omega_n^2$), so that:

$$\frac{df}{dr} = \pm \frac{r}{l} \left(1 - \frac{r^2}{l^2}\right)^{-1}. \quad (5)$$

Hence, we can rewrite the metric (4) as:

$$ds^2 = - \left(1 - \frac{r^2}{l^2}\right) d\tau^2 - 2\frac{r}{l} d\tau dr + dr^2 + r^2 d\Omega_n^2, \quad (6)$$

up to an arbitrary choice of sign in the off-diagonal term.

Clearly, these “new” coordinates exhibit all of the priorly discussed features of Painleve coordinates; including horizon ($r = l$) regularity and time-reversal asymmetry. The feature of horizon regularity has special significance in de Sitter space, as the revised coordinates are no longer restricted to the southern causal diamond. In fact, Eq.(6) covers the entire causal future (of an observer at $r = 0$), which translates to precisely one half of the complete de Sitter manifold. Meanwhile, one can describe the remaining half (or the causal past) by “flipping” the sign in front of the off-diagonal term. In this sense, the Painleve-de Sitter coordinate system is closely related to de Sitter planar coordinates [10]; a point which has been elaborated on in [25]. Of further interest, ∂_τ is a Killing vector throughout the Painleve-de Sitter system, although this vector changes character (timelike \rightarrow spacelike) upon passing through the horizon.

For later usage, let us evaluate the radial, null geodesics described by Eq.(6). Under these conditions ($d\Omega_n^2 = ds^2 = 0$), we can re-express this line element as follows:

$$(\dot{r})^2 - 2\frac{r}{l}\dot{r} - \left(1 - \frac{r^2}{l^2}\right) = 0, \quad (7)$$

where a dot denotes differentiation with respect to τ . Solving the quadratic, we then have:

$$\dot{r} = \frac{r}{l} \pm 1, \quad (8)$$

where the $+/-$ sign can be identified with outgoing/incoming radial motion.

With the de Sitter cosmological horizon (at $r = l$) in mind, let us now focus on a semi-classical treatment of the associated radiation, as advocated in (for instance) [17, 20, 25]. First of all, we adopt the picture of a pair of particles spontaneously created just outside of the horizon. The positive-energy particle tunnels through the horizon to emerge as an inward-moving, self-gravitating energy shell; whereas the negative-energy particle remains behind and effectively lowers the energy of the background spacetime. Because of the infinite blue-shift near the horizon, the emerging energy shell can be treated as a point particle; meaning that a WKB-type of approximation may be appropriately employed. For sake of simplicity, we will further invoke an “s-wave” approximation; in particular, we assume a massless shell and symmetry with respect to the angular coordinates.

Given this semi-classical, WKB framework, it has been shown that the logarithm of the emission rate (Γ) can be expressed in terms of the imaginary part of the “total” (particle plus gravitational) action, \mathcal{I} [17]. More specifically:

$$\Gamma \approx e^{-2Im\mathcal{I}}. \quad (9)$$

Alternatively, one can re-express this relation in the following spectral form:

$$\frac{\omega}{T(\omega)} \approx 2Im\mathcal{I}, \quad (10)$$

where $\omega > 0$ is the particle energy and $T(\omega)$ can be identified with the effective temperature.

For a positive-energy s-wave, the imaginary part of the action has been found to have a conveniently simple form [17]:

$$Im\mathcal{I} = Im \int d\tau \dot{r} p_r = Im \int_{r_i}^{r_f} \int_0^{p_r} dp'_r dr, \quad (11)$$

where p_r is the canonical momentum (conjugate to r). Also, r_i and r_f indicate (roughly) the point of particle creation and the classical turning point of motion.

To proceed with an explicit calculation, it is useful to apply Hamilton's equation:

$$\dot{r} = \frac{dH}{dp_r} = \frac{d(E - \omega)}{dp_r} = -\frac{d\omega}{dp_r}. \quad (12)$$

Here, E represents the total conserved energy of the system, whereas $E - \omega$ can be regarded as the (varying) gravitational energy stored in the background spacetime. Let us re-emphasize that, by keeping E fixed, energy conservation will be enforced in a natural way.

Substituting Eq.(12) into Eq.(11), we find:

$$Im\mathcal{I} = -Im \int_{r_i}^{r_f} \int_0^\omega \frac{d\omega' dr}{\dot{r}}. \quad (13)$$

Before evaluating the above integral, we must necessarily obtain an expression for \dot{r} as a function of ω' . The form of this expression depends on the answer to the following question: what effective metric does the energy shell see as it propagates through the background spacetime? Considering that the de Sitter background loses some of its energy to the propagating shell, we propose that the effective metric in question is that of a Schwarzschild-de Sitter geometry. The reasoning is somewhat subtle and based on the observation that the total energy of a Schwarzschild-de Sitter spacetime is always more negative than that of empty de Sitter space. That is to say, the energy of a Schwarzschild-de Sitter spacetime is known to decrease with increasing black hole mass [31]. Although counter-intuitive, this inverted correspondence can be attributed to a negative binding energy between a positive-mass object and a de Sitter gravitational field [32].

With the above discussion in mind, let us now consider Schwarzschild-de Sitter static coordinates:

$$ds^2 = - \left(1 - \frac{r^2}{l^2} - \frac{M\epsilon_n}{r^{n-1}} \right) dt^2 + \left(1 - \frac{r^2}{l^2} - \frac{M\epsilon_n}{r^{n-1}} \right)^{-1} dr^2 + r^2 d\Omega_n^2, \quad (14)$$

where $\epsilon_n \equiv 16\pi G_{n+2}/n\mathcal{V}_n$ and M is the conserved mass [31]. (Also, G_{n+2} is Newton's constant and \mathcal{V}_n is the volume of the spherical hypersurface described by $d\Omega_n^2$.) It is a straightforward process to generalize the prior Painleve-de Sitter formalism for this black hole spacetime. In particular, Eq.(6) and Eq.(8) should respectively be modified as follows:

$$ds^2 = - \left(1 - \frac{r^2}{l^2} - \frac{M\epsilon_n}{r^{n-1}} \right) d\tau^2 - 2\sqrt{\frac{r^2}{l^2} + \frac{M\epsilon_n}{r^{n-1}}} d\tau dr + dr^2 + r^2 d\Omega_n^2, \quad (15)$$

$$\dot{r} = \sqrt{\frac{r^2}{l^2} + \frac{M\epsilon_n}{r^{n-1}}} \pm 1. \quad (16)$$

On the basis of our prior discussion, it follows directly that the positive-energy shell sees the effective metric of Eq.(15), although with M replaced by ω' . The same substitution in Eq.(16) yields the desired expression for \dot{r} as a function of ω' . (Note that we must choose the negative sign in Eq.(16), as the shell is propagating from larger to smaller r .) Thus, we can now rewrite Eq.(13) in the following explicit manner:

$$Im\mathcal{I} = -Im \int_{r_i}^{r_f} \int_0^\omega \frac{d\omega' dr}{\sqrt{\frac{r^2}{l^2} + \frac{\epsilon_n(\omega' - i\delta)}{r^{n-1}}} - 1}. \quad (17)$$

Here, we have also added a small imaginary part to the effective energy (i.e., $\omega' \rightarrow \omega' - i\delta$ with $\delta \ll 1$), so that the above integral can be evaluated via contour techniques. Let us further point out that $\delta > 0$ is to be implied, as this choice ensures that the positive-frequency solution ($\sim e^{-i\omega\tau}$) decays exponentially in time.

To explicitly evaluate this integral, let us temporarily treat r as a constant and make the following change of variables: $x = \sqrt{\frac{r^2}{l^2} + \frac{\epsilon_n\omega'}{r^{n-1}}}$. This leads to:

$$Im\mathcal{I} = -\frac{2}{\epsilon_n} Im \int_{r_i}^{r_f} r^{n-1} dr \int_{x(0)}^{x(\omega)} \frac{x dx}{x - 1 - i\bar{\delta}(r)}, \quad (18)$$

where $\bar{\delta}(r) = l\epsilon_n\delta/2r^n \approx 0^+$. Given that x monotonically increases with ω' and $\bar{\delta} > 0$, it is appropriate to integrate in a counter-clockwise direction in the upper half of the complex- x plane. Following this prescription, we obtain:

$$Im\mathcal{I} = -\frac{2\pi}{\epsilon_n} Im i \int_{r_i}^{r_f} r^{n-1} dr. \quad (19)$$

The integration over r can now be trivially performed to give:

$$Im\mathcal{I} = \frac{2\pi}{n\epsilon_n} [r_i^n - r_f^n]. \quad (20)$$

Note that, by construction, $r_i > r_f$, and so the sign of $Im\mathcal{I}$ comes out positive as required; cf. Eqs.(9,10). Generating the correct sign in de Sitter thermodynamics is not as trivial as one may think. Indeed, naive application

of the first law of thermodynamics to a cosmological horizon can often lead to an erroneous negative sign [10, 32].

The above formula is the key quantitative result of this paper. We can substantiate its validity by considering a specific value of n . It is readily shown that the case of $n = 1$ (i.e., 3-dimensional de Sitter space) is in agreement with the analogous expression found in [25]. Another convenient choice is $n = 3$ (i.e., 5-dimensional de Sitter space), as the Schwarzschild-de Sitter horizon can then be solved for via a quadratic relation.

With our attention on the $n = 3$ case, it follows (cf. Eq.(14)) that the Schwarzschild-de Sitter cosmological horizon is described by the largest root of:

$$\frac{r_H^4}{l^2} - r_H^2 + M\epsilon_3 = 0. \quad (21)$$

That is:

$$r_H^2(M) = \frac{l^2}{2} \left[1 + \sqrt{1 - \frac{4M\epsilon_3}{l^2}} \right]. \quad (22)$$

Recalling our prior definitions of r_i and r_f , we have $r_i^2 = r_H^2(0) = l^2$ and $r_f^2 = r_H^2(\omega)$. Hence, Eq.(20) can be re-expressed as:

$$Im\mathcal{I} = \frac{2\pi l^3}{3\epsilon_3} \left[1 - \frac{1}{2^{3/2}} \left(1 + \sqrt{1 - \frac{4\omega\epsilon_3}{l^2}} \right)^{3/2} \right]. \quad (23)$$

When the particle energy is small (i.e., $\omega \ll l^2/\epsilon_3$), the above expression can be expanded to yield:

$$Im\mathcal{I} = \pi l\omega + \mathcal{O}(\omega^2). \quad (24)$$

Incorporating the above expansion into Eq.(10), we are able to deduce the temperature of radiation:

$$T(\omega) \approx \frac{1}{2\pi l} + \mathcal{O}(\omega). \quad (25)$$

Reassuringly, the leading-order, energy-independent term is the well-known background temperature of empty de Sitter space [29]. Meanwhile, the energy-dependent corrections, which can easily be computed to any desired order in ω , are indicative of a “greybody” factor in the emission spectrum;

that is, a deviation from pure thermality. That such deviations occur for Hawking-like radiation is well known [16], but this point is rarely stressed in the relevant literature.

As a further check on our formalism (this time for any n), we can consider the change in entropy during the process of emission. The first law of thermodynamics indicates that:

$$\Delta S = -\frac{\omega}{T} \approx -2Im\mathcal{I} = \frac{4\pi}{n\epsilon_n} [r_f^n - r_i^n], \quad (26)$$

where we have also applied Eqs.(10,20). We can compare this outcome with that predicted by the Bekenstein-Hawking “area” law [33, 34], which tells us:

$$\Delta S = S_f - S_i = \frac{\mathcal{V}_n}{4G_{n+2}} [r_f^n - r_i^n]. \quad (27)$$

Since $\epsilon_n = 16\pi G_{n+2}/n\mathcal{V}_n$, these two independent formulations of ΔS are, indeed, in perfect agreement.

3 Schwarzschild-de Sitter Tunneling

In the discussion to follow, we will consider the implications (on the tunneling picture) when the initial state is described by a Schwarzschild-de Sitter spacetime. Let us, once again, consider the incoming radiation from the cosmological horizon and, for the moment, ignore the outgoing radiation from the black hole horizon. It is readily observed that the key result of last section, Eq.(20), remains valid; although r_i and r_f must be appropriately re-defined. Recalling the inverse correspondence between black hole mass and background energy, we expect a black hole of initial mass M to have a final mass of $M + \omega$ (where, as before, ω is the energy of the emitted particle). It follows (cf. Eq.(14)) that the radii in question correspond to the largest roots of:

$$\frac{r_i^2}{l^2} - 1 + \frac{\epsilon_n M}{r_i^{n-1}} = 0, \quad (28)$$

$$\frac{r_f^2}{l^2} - 1 + \frac{\epsilon_n (M + \omega)}{r_f^{n-1}} = 0. \quad (29)$$

As in the preceding section, let us turn to the case of $n = 3$ as a check on our formalism. In this 5-dimensional case, the above equations can be explicitly solved to yield:

$$r_i^2 = \frac{l^2}{2} \left[1 + \sqrt{1 - \frac{4\epsilon_3 M}{l^2}} \right], \quad (30)$$

$$r_f^2 = \frac{l^2}{2} \left[1 + \sqrt{1 - \frac{4\epsilon_3(M + \omega)}{l^2}} \right]. \quad (31)$$

Substituting these expressions into Eq.(20) and expanding, we find:

$$Im\mathcal{I} = \frac{\pi l^2 r_i}{2r_i^2 - l^2} \omega + \mathcal{O}(\omega^2). \quad (32)$$

From the above result and Eq.(10), the corresponding temperature is found to be:

$$T \approx \frac{2r_i^2 - l^2}{2\pi r_i l^2} + \mathcal{O}(\omega). \quad (33)$$

It is not difficult to verify that the leading-order term agrees with the usual Hawking definition [16] (translated to a cosmological horizon [29]); that is:

$$T = \frac{1}{4\pi} \left| \frac{d}{dr} \left[1 - \frac{r^2}{l^2} - \frac{\epsilon_3 M}{r^2} \right] \right|_{r=r_i}. \quad (34)$$

Furthermore, the change in entropy during emission can again be shown to agree with that predicted by the Bekenstein-Hawking area law. (See the end of Section 2 for details.)

There is an intriguing observation that follows from the emission rate, $\Gamma \approx e^{-2Im\mathcal{I}}$, being a measurable and, hence, real quantity. Again focusing on the case of $n = 3$ (although the discussion throughout this section is quite general²), we can see from Eqs.(20,30,31) that the condition:

$$M + \omega \leq \frac{l^2}{4\epsilon_3} = \frac{3\pi l^2}{32G_5} \quad (35)$$

²The generality of this discussion does not, however, necessarily apply to the $n = 1$ case. This is because the 3-dimensional Schwarzschild-de Sitter solution describes a conical deficit angle rather than a black hole [10]. For related discussion that highlights this 3-dimensional scenario, see [25].

must always be enforced. It is of interest that this upper bound corresponds precisely with the mass of the (5-dimensional) Nariai black hole [35]. Significantly, the Nariai solution describes the coincidence of the black hole and cosmological horizons (the black hole horizon is located by changing the explicit $+$ in Eq.(30) to a $-$); meaning that this solution represents the most massive black hole in an asymptotically de Sitter spacetime. Hence, the tunneling formalism provides a natural mechanism for censoring against larger values of mass. Similar observations have been made with regard to charged (Reissner-Nordstrom) black holes, where the tunneling formalism has been shown to censor against naked singularities [18, 20].

The overall picture for Schwarzschild-de Sitter space is, however, much more complicated than we have alluded to above. This is because radiation is both propagating inwards from the cosmological horizon and outwards from the black hole horizon. A formal, complete analysis must consider both of these effects, and there would undoubtedly be scattering taking place between the black hole and cosmological contributions. Even without delving into calculational specifics, we can still comment on the stability of the total system. Once again turning to our $n = 3$ chestnut, let us take note of the following (lowest-order) expressions for the temperature (associated with the cosmological and black hole horizon, respectively):

$$T_{CH} = \frac{\sqrt{1 - \frac{4\epsilon_3 M}{l^2}}}{\sqrt{2\pi l} \left[1 + \sqrt{1 - \frac{4\epsilon_3 M}{l^2}} \right]}, \quad (36)$$

$$T_{BH} = \frac{\sqrt{1 - \frac{4\epsilon_3 M}{l^2}}}{\sqrt{2\pi l} \left[1 - \sqrt{1 - \frac{4\epsilon_3 M}{l^2}} \right]}. \quad (37)$$

Here, we have applied Eqs.(34 and 30) and again note that the black hole horizon can be found by reversing the explicit $+$ sign in Eq.(30).

With an inspection of the above, it becomes evident that $T_{CH} \leq T_{BH}$; with saturation occurring only at the Nariai value of mass ($M = l^2/4\epsilon_3$), in which case both temperatures are vanishing. With this observation, we are able to deduce that the net flow of radiation will always be (up to insignificant quantum fluctuations) towards the cosmological horizon. That is to say, the system will inevitably evolve towards empty de Sitter space. This phenomena is supported by the second law of thermodynamics, since the

total entropy of a Schwarzschild-de Sitter spacetime (or virtually any “well-behaved”³ asymptotically de Sitter spacetime) is known to be bounded from above by the entropy of empty de Sitter space [15].

The above viewpoint can also be substantiated by way of holographic (or dS/CFT duality) considerations. In particular, let us take note of Strominger’s realization [30] (also see [31, 37]) that time evolution in an asymptotically de Sitter spacetime is dual to an *inverted* renormalization group flow.⁴ On this basis, it follows that degrees of freedom will be integrated into the system with forward evolution in time. Moreover, the maximal entropic state (i.e., empty de Sitter space) will naturally correspond with a stable, ultraviolet fixed point for the flow.

On the other hand, because of the vanishing temperature associated with the Nariai solution, one might expect the system to stabilize precisely when the horizons coincide. Such stability would indeed be feasible at a strictly classical level; however, once quantum (or semi-classical) effects are accounted for, it becomes evident that the Nariai solution is unstable under the smallest of perturbations (see [38] and references within). In renormalization group language, this Nariai solution can be identified with an infrared fixed point that is unstable [37].

4 “The Dark Side of the Moon”

In this section, we will investigate the following question: how would the semi-classical tunneling picture be perceived by a hypothetical observer who is trapped outside of the cosmological horizon? Such a query may appear to be of little relevance, given that a “standard” de Sitter observer is causally restricted to the interior of his/her horizon. However, here we will argue that this question merits consideration on the basis of dS/CFT holography.

The dS/CFT duality, as we currently understand it, incorporates the entire spacetime into its framework and not just the causal diamond. Indeed, the dually related conformal field theory has been conjectured to “live” on the

³In this context, well-behaved implies no naked singularities and matter that satisfies the standard energy conditions [36].

⁴We remind the reader that the renormalization group is normally regarded as flowing from the ultraviolet (relatively large number of degrees of freedom) to the infrared (relatively small number of degrees of freedom).

spacelike asymptotic boundaries [8]; these being future (\mathcal{I}^+) and past (\mathcal{I}^-) infinity. Significantly, both of these boundaries lie outside of an observer’s causal diamond; in fact, an observer can only access precisely one point at either infinity. Moreover, the only measurable (gauge-invariant) quantities in de Sitter space would appear to be the elements of an S -like matrix [2] that can be expressed in terms of correlation functions of the dual boundary theory [12, 13]. To make operational sense of such “meta-observables” [2] clearly requires a “special” observer with a global view of the entire spacetime. To put it another way, if a quantum theory of de Sitter gravity is to be realized, we may yet have to adapt our intuitive ideas of what constitutes a physical observable.

With the above discussion in mind, let us return to the quantum-tunneling description of de Sitter radiation, as elaborated on in Section 2. From the perspective of someone (or something?) outside of the horizon, a negative-energy shell is tunneling outwards. Meanwhile, the positive-energy partner remains behind (i.e., in the vicinity of the horizon) and effectively raises the energy of the background spacetime. Hence, the effective metric, as seen by this negative-energy shell, must be one in which the background energy increases with increasing $|\omega'|$ (i.e., the magnitude of the shell energy, which increases from 0 to $|\omega|$). We can obtain just such an effective geometry by replacing M with $-|\omega'|$ in the Schwarzschild-de Sitter metric of Eq.(14). That is:

$$ds^2 = - \left(1 - \frac{r^2}{l^2} + \frac{\epsilon_n |\omega'|}{r^{n-1}} \right) dt^2 + \left(1 - \frac{r^2}{l^2} + \frac{\epsilon_n |\omega'|}{r^{n-1}} \right)^{-1} dr^2 + r^2 d\Omega_n^2. \quad (38)$$

The above metric can readily be identified with that of the so-called “topological” de Sitter spacetime⁵ [39, 40]. (Also see [41] for a recent discussion and references.) From a dS/CFT perspective, the topological de Sitter solution has the desirable property of an (apparently) unitary boundary [40, 42]. (Conversely, the conventional Schwarzschild-de Sitter solution would appear to have a non-unitary dual [8, 40].) On the other hand, topological de Sitter spacetimes have the detrimental feature of a naked singularity, as there is no longer a black hole horizon (although the cosmological horizon remains intact). The need to universally censor against such a singularity can,

⁵In its most general form, the topological de Sitter solution can allow for a hyperbolic, flat, or (as depicted above) a spherical horizon geometry. To obtain the hyperbolic (flat) topological solution, one can replace 1 with -1 (0) in the lapse function of Eq.(38).

however, be debated. That is to say, an observer outside of the cosmological horizon would be causally disconnected from the singularity and need not be aware of its existence.⁶

To obtain an “outside-of-the-horizon” emission rate, we can essentially repeat the calculations of Section 2, except using Eq.(38) for the effective metric and a few trivial modifications.⁷ Keeping a very careful track of the signs, we find the imaginary part of the action to be as follows:

$$Im\mathcal{I} = \frac{2\pi}{n\epsilon_n} [r_f^n - r_i^n]. \quad (39)$$

That is, the negative of the prior result (20). However, this sign reversal is a most welcome outcome, as now we have that $r_f > r_i$. (This must be the case by construction. It can also be verified with an explicit calculation of the horizon position as a function of particle energy. For instance, for $n = 3$, one finds that $r_i^2 = l^2$ and $r_f^2 = \frac{l^2}{2} \left[1 + \sqrt{1 + \frac{4\epsilon_3|\omega|}{l^2}} \right]$.) The positivity of Eq.(39) tells us that the effective temperature is strictly non-negative, even outside of the horizon, as is necessary for a sensible interpretation of the tunneling phenomena.

What (if anything) have we learned from this section? At the risk of straying from physics to philosophy, we propose the following pair of conjectural points:

(i) The topological de Sitter geometry should not be regarded as a substitute for its Schwarzschild-de Sitter counterpart but, rather, as a complementary description. The choice one should make depends on the side of the horizon under consideration.

(ii) The topological de Sitter solution is a necessary ingredient if one is to take a global view of de Sitter space. Let us re-emphasize that such a view is implicitly advocated by the dS/CFT correspondence.

⁶Although the topological de Sitter solution has recently been the subject of further criticism (based on string-theoretical considerations) [41], this analysis specifically applied to a hyperbolic horizon geometry and is not of issue in the current discussion.

⁷Specifically, the radial motion is now outgoing so that $\dot{r} = \sqrt{\frac{r^2}{l^2} - \frac{\epsilon_n|\omega'|}{r^{n-1}}} + 1$, and $dH \sim +d|\omega'|$ since the background energy increases with increasing $|\omega'|$.

5 Conclusion

In the preceding paper, we have considered de Sitter radiation as a semi-classical tunneling process. Adapting the methodology of Kraus, Wilczek [17] and others (including a recent, related work by Parikh [25]), we were able to calculate the rate of particle emission from a cosmological horizon. We then verified that this calculation agreed, up to higher-order corrections, with the known thermodynamic properties of de Sitter space, as well as Schwarzschild-de Sitter space. Meanwhile, these frequency-dependent corrections indicate that the emission spectrum of Hawking-like radiation deviates from perfect thermality; a well-known but often forgotten result [16].

Along the way, we have also touched base with certain aspects of the dS/CFT holographic correspondence [8]. It is quite possible that there are deep connections between semi-classical thermodynamics and de Sitter holography that await to be uncovered. We hope to report progress along these lines at a future date.

6 Acknowledgments

The author would like to thank V.P. Frolov for helpful conversations.

References

- [1] See, for instance, N. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, *Science* **284**, 1481 (1999) [astro-ph/9812133].
- [2] E. Witten, “Quantum Gravity in De Sitter Space”, hep-th/0106109 (2001).
- [3] J.M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998) [hep-th/9711200].
- [4] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Phys. Lett.* **B428**, 105 (1998) [hep-th/9802109].

- [5] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998) [hep-th/9802150]; *ibid.*, 505 (1998) [hep-th/9803131].
- [6] G. 't Hooft, “Dimensional Reduction in Quantum Gravity”, gr-qc/9310026 (1993).
- [7] L. Susskind, *J. Math. Phys.* **36**, 6377 (1995) [hep-th/9409089].
- [8] A. Strominger, *JHEP* **0110**, 034 (2001) [hep-th/0106113].
- [9] D. Klemm, *Nucl. Phys.* **B625**, 295 (2002) [hep-th/0106247].
- [10] M. Spradlin, A. Strominger and A. Volovich, “Les Houches Lectures on De Sitter Space”, hep-th/0110007 (2001).
- [11] B. McInnes, *Nucl. Phys.* **B627**, 311 (2002) [hep-th/0110062].
- [12] R. Bousso, A. Maloney and A. Strominger, *Phys. Rev.* **D65**, 104039 (2002) [hep-th/0112218].
- [13] M. Spradlin and A. Volovich, *Phys. Rev.* **D65**, 104037 (2002) [hep-th/0112223].
- [14] L. Dyson, J. Lindesay and L. Susskind, “Is There Really a de Sitter/CFT Duality?”, hep-th/0202163 (2002); L. Susskind, “Twenty Years of Debate with Stephen”, hep-th/0204027 (2002).
- [15] R. Bousso, *JHEP* **0011**, 038 (2000) [hep-th/0010252]; *JHEP* **0104**, 035 (2001) [hep-th/0012052].
- [16] S.W. Hawking, *Comm. Math. Phys.* **43**, 199 (1975).
- [17] P. Kraus and F. Wilczek, “A Simple Stationary Line Element for the Schwarzschild Geometry, and some Applications”, gr-qc/9406042 (1994); *Nucl. Phys.* **B433**, 403 (1995) [gr-qc/9408003].
- [18] P. Kraus and F. Wilczek, *Nucl. Phys.* **B437**, 231 (1995) [hep-th/9411219].
- [19] E. Keski-Vakkuri and P. Kraus, *Phys. Rev.* **D54**, 7407 (1996) [hep-th/9604151]; *Nucl. Phys.* **B491**, 249 (1997) [hep-th/9610045].

- [20] M.K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000) [hep-th/9907001].
- [21] S. Hemming and E. Keski-Vakkuri, Phys. Rev. **D64**, 044006 (2001) [gr-qc/0005115].
- [22] Y. Kwon, Nuovo. Cim. **B115**, 469 (2000).
- [23] E.C. Vagenas, Phys. Lett. **B503**, 399 (2001) [hep-th/0012134]; Mod. Phys. Lett. **A17**, 609 (2002) [hep-th/0108147]; Phys. Lett. **B533**, 302 (2002) [hep-th/0109108].
- [24] A.J.M. Medved, Class. Quant. Grav. **19**, 589 (2002) [hep-th/0110289].
- [25] M.K. Parikh, “New Coordinates for de Sitter Space and de Sitter Radiation”, hep-th/0204107 (2002).
- [26] S. Shankaranarayanan, K. Srinivasan and T. Padmanabhan, Mod. Phys. Lett. **A16**, 571 (2001) [gr-qc/0007022]; Class. Quant. Grav. **19**, 2671 (2002) [gr-qc/0010042].
- [27] See, for instance, N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, (Cambridge University Press, Cambridge) (1982).
- [28] P. Painleve, C.R. Acad. Sci. (Paris) **173**, 677 (1921).
- [29] G.W. Gibbons and S.W. Hawking, Phys. Rev. **D15**, 2738 (1977).
- [30] A. Strominger, JHEP **0111**, 049 (2001) [hep-th/0110087].
- [31] V. Balasubramanian, J. de Boer and D. Minic, Phys. Rev. **D65**, 123508 (2002) [hep-th/0110108].
- [32] Y.S. Myung, Mod. Phys. Lett. **A16**, 2353 [hep-th/0110123].
- [33] J.D. Bekenstein, Lett. Nuovo. Cim. **4**, 737 (1972); Phys. Rev. **D7**, 2333 (1973); Phys. Rev. **D9**, 3292 (1974).
- [34] J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. **31**, 161 (1973).
- [35] H. Nariai, Sci. Rep. Tohoku Univ. **34**, 160 (1950); *ibid* **35**, 62 (1951).

- [36] See, for instance, R.M. Wald, *General Relativity* (University of Chicago Press, 1984).
- [37] E. Halyo, JHEP **0203**, 009 (2002) [hep-th/0112093].
- [38] R. Bousso “Adventures in de Sitter Space”, hep-th/0205177 (2002).
- [39] R.-G. Cai, Y.S. Myung and Y.-Z. Zhang, Phys. Rev. **D65**, 084019 (2002) [hep-th/0110234].
- [40] R.-G. Cai, Phys. Lett. **B525**, 331 (2002) [hep-th/0111093].
- [41] B. McInnes, “dS/CFT, Censorship, Instability of Hyperbolic Horizons, and Spacelike Branes”, hep-th/0205103 (2002).
- [42] A.J.M. Medved, Class. Quant. Grav. **19**, 2883 (2002) [hep-th/0112226].