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Damping of primordial gravitational waves from generalized sources

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It has been shown that a cosmological background with an anisotropic stress tensor, appropriate for a free-streaming thermal neutrino background, can damp primordial gravitational waves after they enter the horizon, and can thus affect the cosmic microwave background B-mode polarization signature due to such tensor modes. Here we generalize this result and examine the sensitivity of this effect to nonzero neutrino masses, extra neutrino species, and also a possible relativistic background of axions from axion strings. In particular, additional neutrinos with cosmologically interesting neutrino masses at the $O(1)$ eV level will noticeably reduce damping compared to massless neutrinos for gravitational wave modes with $k\tau_0 \approx 100$–200, where $\tau_0 \approx 2/H_0$ and H_0 is the present Hubble parameter, while an axion background would produce a phase-dependent damping distinct from that produced by neutrinos.

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I. INTRODUCTION

A generic prediction of inflation in the early universe [1–3] is the production of gravitational waves (GW) with a nearly flat spectrum [4,5]. There are ongoing observational efforts to detect such a spectrum and recent, comprehensive reviews of past, present, and future experimental efforts can be found in Refs. [6,7]. The dominant signature in the near term involves the effect that long-wavelength gravitational waves can have on the cosmic microwave background (CMB) through the generation of B-mode polarization (e.g., [8–14]). The amplitude of the gravitational waves can be related to the energy scale at which inflation occurred, and the ratio of the power spectrum of gravitational waves to that of the scalar power spectrum, also known as the "tensor-to-scalar ratio," r, can give vital information into the nature of the inflaton—the field which drives inflation—via the Lyth bound [15,16]. Additionally, primordial gravitational wave spectra produced by various noninflationary mechanisms have been suggested [17–21]. Therefore any observation of a primordial gravitational wave spectrum would be an immensely powerful tool in the study of the very early universe.

A question that naturally arises is—are there any effects that can intervene and alter the nature of the GW spectrum from the time of its production until the time of observation? If the answer is yes, then one must account for such effects in order to accurately describe the primordial spectrum. As is well known, just such an intervening effect does arise due to the fact that an anisotropic stress from free-streaming particles can damp the amplitude of GWs from their primordial value. Weinberg showed in [22] that the damping effect of free-streaming neutrinos on the GW spectrum can be quite significant with up to a 35.6% loss in amplitude, and following this work, the issue has been the subject of some attention [23–33].

The original study of Weinberg, and much of the following work has been focused on the effects of three massless neutrinos. However, recent cosmological observations have shown hints of deviations from the standard cosmological value of three effective neutrino degrees of freedom [34–38]. Because of neutrino oscillation experiments, it is also known that neutrinos are not massless, as described in a recent global analysis of neutrino properties [39]. There are also some, albeit statistically insignificant presently, hints that the addition of extra neutrino species can improve fits to short baseline neutrino oscillation data [40]. Recently issues regarding light sterile neutrinos and cosmology have been addressed in [41–45].

Prompted by these results, we broaden the scope by calculating GW damping in more general scenarios, including the effect of neutrino masses, additional (beyond three) massive and massless neutrino species, and extra *bosonic* degrees of freedom. (Note that other recent work [33] also touches on some of these issues related to extra degrees of freedom and gravitational waves.)

This paper is organized as follows. In Sec. II we derive in some generality the formula for anisotropic stress. The results for particular cases are presented in Sec. III, and we then present our conclusions in Sec. IV.

II. CALCULATION

To first order, a perturbed Friedmann-Robertson-Walker metric with scale factor $a(t)$ can be written as

$$\begin{aligned} ds^2 = & -(1+2\psi)dt^2 + a(\partial F_i + G_i)dx^i dt \\ & + a^2(t)[\delta_{ij}(1-2\phi) + h_{ij} + \partial_i\partial_j B \\ & + \partial_j C_i + \partial_i C_j]dx^i dx^j. \end{aligned} \tag{1}$$

Gravitational waves arise as the transverse, traceless components of the metric fluctuations, which are characterized by h_{ij}. These modes satisfy the transverse and traceless conditions

$$\partial^i h_{ij} = 0; \qquad h_{ii} = 0. \tag{2}$$

The Fourier transformed k-space modes, h_k, satisfy the Einstein equation of the form

$$h_k'' + 2Hh_k' + k^2 h_k = 16\pi G_N a^2(\tau)\Pi_k, \tag{3}$$

where the prime denotes differentiation with respect to conformal time $d\tau = dt/a(t)$, and Π_k is the anisotropic stress.

To solve this equation, we first turn to the Boltzmann equation, which determines the evolution of the phase space density of the particles, $F(x, P)$, given as a function of the four-momentum P, which has components $P^\mu = dx^\mu/d\lambda$. One can then determine the anisotropic stress by perturbing the distribution function about the background as $F(x, P) = \bar{F}(P^0) + \delta F(x, P)$, and employing the Boltzmann equation $dF(x, P)/dt = 0$. For the scenario of three massless neutrinos, the details of this calculation have been nicely presented in Appendix D of [26], which explores the impact of collisionless damping. We will follow this treatment, but we will generalize to the case of massive particles with the number of degrees of freedom left as an input parameter. We will also examine the situation where a bosonic degree of freedom is incorporated, as well as the case of relativistic axions produced by axionic cosmic strings. These relativistic axions have a nonthermal spectrum.

For particles with a thermal distribution, the background phase space density is given by

$$\bar{F}(P^0) = \frac{g}{e^{P^0/T_\nu} \pm 1}, \tag{4}$$

where the plus sign is for fermions while the minus sign is for bosons, and g gives the number of degrees of freedom. For the two cases of interest in this work, $g = g_\nu = 2$ for a single neutrino, and $g = g_B = 1$ for a real scalar. T_ν is the temperature of neutrinos that is related to the photon temperature at times after the neutrinos' decoupling as $T_\nu = (4/11)^{1/3}T_\gamma$. One begins with the relation

$$g_{\mu\nu}P^\mu P^\nu = -(P^0)^2 + g_{ij}P^i P^j = -m^2. \tag{5}$$

We write this as

$$\tilde{p}_0^2 = g_{ij}P^i P^j, \tag{6}$$

where we have defined a new variable through a shift

$$P^0 \equiv \sqrt{m^2 + \tilde{p}_0^2}. \tag{7}$$

A. Damping from neutrinos

We follow the derivation in [26] with this change (their $P_0 =$ our$\tilde{p}_0$). The full neutrino distribution function satisfies the relativistic collisionless Boltzmann equation,

$$\begin{aligned} \frac{dF(t, x^i, \gamma^i, \tilde{p}_0)}{dt} & = \frac{\partial F}{\partial t} + \frac{dx^i}{dt}\frac{\partial F}{\partial x^i} + \frac{d\tilde{p}_0}{dt}\frac{\partial F}{\partial \tilde{p}_0} + \frac{d\gamma^i}{dt}\frac{\partial F}{\partial \gamma^i} \\ & = 0. \end{aligned} \tag{8}$$

Using the variable $\tilde{p}$, one finds that to first order Eq. (8) becomes the Einstein-Vlasov equation,

$$\begin{aligned} \left(\frac{\partial F}{\partial t}\right)_{\text{first order}} & = \frac{\partial \delta F}{\partial t} + \frac{\gamma_i \tilde{p}_0}{a\sqrt{m^2 + \tilde{p}_0^2}}\frac{\partial \delta F}{\partial x^i} \\ & \quad - \frac{\dot{a}}{a}\frac{(m^2 + \tilde{p}_0^2)}{\tilde{p}_0}\frac{\partial \delta F}{\partial \tilde{p}_0} - \frac{1}{2}\frac{\partial \bar{F}}{\partial \tilde{p}_0}\tilde{p}_0\frac{\partial h_{ij}}{\partial t}\gamma^i\gamma^j \\ & = 0, \end{aligned} \tag{9}$$

and $\gamma_i = \gamma^i$ are directional cosines.

Defining $\mu \equiv \gamma^i k_i/k$ and using the mode decomposition of h_{ij} and δF,

$$h_{ij} = \sum_{\lambda=+,\times}\int \frac{d^3k}{(2\pi)^3} h_{\lambda,k}(t) Q_{ij}^\lambda(\vec{x}), \tag{10}$$

$$\delta F = \sum_{\lambda=+,\times}\int \frac{d^3k}{(2\pi)^3} f_{\lambda,k}(t, \tilde{p}_0, \mu)\gamma^i\gamma^j Q_{ij}^\lambda(\vec{x}), \tag{11}$$

where Q_{ij}^λ are symmetric, traceless and divergenceless tensors that satisfy $Q_{ij}^\lambda = Q_{ji}^\lambda$, $Q_{ij;a}^{\lambda\ ;a}(\vec{x}) + k^2 Q_{ij}^\lambda(\vec{x}) = 0$ and $Q_{ij}^{\lambda\ ;j} = 0$. The covariant derivative is with respect to the unperturbed spatial Friedmann-Robertson-Walker metric. Thus, the first order Einstein-Vlasov equation in terms of the decomposed modes becomes

$$\begin{aligned} & \frac{\partial f_k}{\partial t} + \frac{ik\mu}{a}\left(\frac{\tilde{p}_0}{\sqrt{m^2 + \tilde{p}_0^2}}\right)f_k - \frac{\dot{a}}{a}\left(\frac{m^2 + \tilde{p}_0^2}{\tilde{p}_0}\right)\frac{\partial f_k}{\partial \tilde{p}_0} \\ & \quad = \frac{1}{2}\tilde{p}_0\frac{\partial \bar{F}}{\partial \tilde{p}_0}\frac{\partial h_k}{\partial t}. \end{aligned} \tag{12}$$

Once again, following [26] we define new variables $q^\mu = aP^\mu$ and $q^0 = aP^0 \equiv q$ and conformal time, $d\tau = dt/a(t)$. Then Eq. (12) can be written as

$$\frac{\partial f_k}{\partial \tau} + \frac{ik\mu\tilde{p}_0}{\sqrt{m^2 + \tilde{p}_0^2}}f_k = \left(\frac{q^2 - a^2m^2}{q}\right)\frac{\partial \bar{F}}{\partial q}\frac{1}{2}\frac{\partial h_k}{\partial \tau}. \tag{13}$$

This equation determines the time evolution of the perturbation of distribution function δF, which in turn determines the anisotropic stress part of the perturbed energy-momentum tensor that goes into the right-hand side (RHS) of Eq. (3).

$$\delta \mathcal{T}_{ij} = a^2 \sum_{\lambda=+,\times} \int \frac{d^3k}{(2\pi)^3} \Pi_{\lambda,k} Q^{\lambda}_{ij}(\vec{x}) \tag{14}$$

and

$$\mathcal{T}_{ij} = \frac{1}{\sqrt{-g}} \int \frac{d^3q}{(2\pi)^3 q} q_i q_j F(q)$$
$$\Rightarrow \delta\mathcal{T}_{ij} = \frac{1}{\sqrt{-g}} \int \frac{d^3q}{(2\pi)^3 q} q_i q_j \delta F(q). \tag{15}$$

Using Eqs. (11), (14), and (15), one finds that the anisotropic stress is

$$\Pi_{\lambda,k} Q^{\lambda}_{ij}(\vec{x}) = a^{-4} \int \frac{d^3q}{(2\pi)^3 q} q^2 \gamma^i\gamma^j\gamma^l\gamma^m f_{\lambda,k} Q^{\lambda}_{lm}(\vec{x}), \tag{16}$$

where $f_{\lambda,k} \equiv f_{\lambda,k}(\tau, q, \mu)$. On the other hand, Eq. (13), which is a first order differential equation, has the following solution:

$$f_k(\tau, q, m, \mu) = \frac{q^2}{2}\frac{\partial \bar{F}}{\partial q} \int_{\tau_{\rm dec}}^{\tau} d\tau' h'_k(\tau') \alpha(m, \tau', q)^2 e^{-i\mu\alpha^2 k(\tau-\tau')}, \tag{17}$$

where we have defined

$$\alpha(\tau, m, q)^2 = 1 - \frac{m^2 a(\tau)^2}{q^2} \tag{18}$$

and

$$\mu = \frac{\gamma^i k_i}{k} \Rightarrow \mu = \hat{\gamma}\cdot\hat{k} \tag{19}$$

and used the fact that $f_k(\tau_{\rm dec}, q, m, \mu) = 0$ because there is no anisotropic stress at neutrino decoupling since the neutrinos just start to free-stream at decoupling. The polarization index λ is suppressed on both sides of Eq. (17). Finally using the identity

$$\int d\Omega_q \gamma^i\gamma^j\gamma^l\gamma^m e^{-i\gamma_i k^i u} Q^{\lambda}_{lm} = \frac{1}{8}(\delta^{il}\delta^{jm} + \delta^{im}\delta^{jl}) \times \int d\Omega_q e^{-i\mu u};$$
$$d^3q = q^2 dq d\Omega_q, \tag{20}$$

one can write the anisotropic stress in momentum space (again the polarization index λ is suppressed)

$$\Pi_k = \frac{1}{8a(\tau)^4} \int d\tau' \frac{d^3q}{(2\pi)^3} (1-\mu^2)^2 e^{-i\mu b} h'_k(\tau') \frac{\partial \bar{F}(q)}{\partial q} q^2 \alpha^2, \tag{21}$$

where

$$b \equiv k(\tau - \tau')\alpha^2, \tag{22}$$

$$\alpha^2 \equiv 1 - \frac{m^2 a(\tau')^2}{q^2}, \tag{23}$$

$$\mu = \hat{\gamma}\cdot\hat{k} = \cos\theta_q, \tag{24}$$

where we have taken $\hat{k}^i$ to be in the z direction in q space. We also define

$$u \equiv k\tau, \tag{25}$$

$$s \equiv k\tau'. \tag{26}$$

We can perform the integrations over $d\Omega_q = d\phi_q d(\cos\theta_q)$ and find that anisotropic stress is then given by

$$\Pi_k = \frac{\pi}{2a(u)^4} \int_{u_{\rm dec}}^{u} ds dq \frac{dh_k(s)}{ds} \frac{\partial \bar{F}(q)}{\partial q} q^4 \alpha^2 \Big[\frac{\sin b}{b} + 2\alpha^2\Big(-\frac{\sin b}{b} - 2\frac{\cos b}{b^2} + 2\frac{\sin b}{b^3}\Big) + \alpha^4\Big(\frac{\sin b}{b} + 4\frac{\cos b}{b^2} - 12\frac{\sin b}{b^3} - 24\frac{\cos b}{b^4} + 24\frac{\sin b}{b^5}\Big)\Big]. \tag{27}$$

Furthermore we define

$$x \equiv \frac{q}{aT} = \frac{q}{a_0 T_0} = \frac{q}{T_0}, \tag{28}$$

where the second equality holds for our normalization that the present day scale factor is $a_0 = 1$. This allows us to write the distribution $\bar{F}(q)$ and the function α as

$$\bar{F}(x) = \frac{g_\nu}{e^x + 1}, \tag{29}$$

$$\alpha^2(m, x) = 1 - \frac{m^2 a^2}{T_0^2 x^2}. \tag{30}$$

With this we find

$$\Pi_k = \frac{1}{16\pi^2 a(u)^4}\int_{u_{\rm dec}}^{u} ds dx \frac{dh_k(s)}{ds}\frac{\partial \bar{F}(x)}{\partial x} x^4 T_0^4 \alpha^2(m,x)\Bigg[\frac{\sin((u-s)\alpha^2(m,x))}{(u-s)\alpha^2(m,x)} + 2\alpha^2(m,x)\bigg(-\frac{\sin((u-s)\alpha^2(m,x))}{(u-s)\alpha^2(m,x)} - 2\frac{\cos((u-s)\alpha^2(m,x))}{((u-s)\alpha^2(m,x))^2} + 2\frac{\sin((u-s)\alpha^2(m,x))}{((u-s)\alpha^2(m,x))^3}\bigg) + \alpha^4(m,x)\bigg(\frac{\sin((u-s)\alpha^2(m,x))}{(u-s)\alpha^2(m,x)} + 4\frac{\cos((u-s)\alpha^2(m,x))}{((u-s)\alpha^2(m,x))^2} - 12\frac{\sin((u-s)\alpha^2(m,x))}{((u-s)\alpha^2(m,x))^3} - 24\frac{\cos((u-s)\alpha^2(m,x))}{((u-s)\alpha^2(m,x))^4} + 24\frac{\sin((u-s)\alpha^2(m,x))}{((u-s)\alpha^2(m,x))^5}\bigg)\Bigg]. \tag{31}$$

The full gravitational wave equation is

$$\frac{d^2h_k(u)}{du^2} + 2\bigg(\frac{da(u)/du}{a(u)}\bigg)\frac{dh_k(u)}{du} + h_k(u) = \frac{16\pi G_N a^2(u)}{k^2}\Pi_k = \frac{6}{\rho(u)}\bigg(\frac{da(u)/du}{a(u)}\bigg)^2\Pi_k. \tag{32}$$

This gives an equation for the transverse-traceless tensor modes as a general function of the mass of the particle creating the anisotropic stress for a general phase space distribution $\bar{F}(x)$. It can be seen to reduce to the standard form for three massless neutrinos when $g_\nu = 6$ and $m = 0$ [26].

From the relation Eq. (31), one can then include additional degrees of freedom by simply using $g_\nu = 8$ (10) for four (five) massless neutrino species. We use the simplifying assumption that the neutrinos all have the same decoupling temperature. (To generalize to arbitrary decoupling temperatures we would change the lower integration limit of the anisotropic stress for that species.) For a mixed scenario where particles of different masses contribute, one can use Eq. (31) for the anisotropic stress, $\Pi_{k,i}$, generated by a single species of mass $m = m_i$ with $g_\nu = g_i$ degrees of freedom, and then add another anisotropic stress term of this form for any additional species of mass m_j with degrees of freedom g_j. In other words, the total anisotropic stress due to i particles is given by the sum $\Pi_{k,\rm tot} = \sum_i \Pi_{k,i}$.

To graphically display the effect of adding nonzero neutrino masses we adopt the simple analytic form for the scale factor $a(\tau)$ in a matter plus radiation universe given by

$$a(\tau) = \bigg(\frac{\tau}{\tau_0}\bigg)^2 + 2\bigg(\frac{\tau}{\tau_0}\bigg)\sqrt{a_{\rm eq}}; \qquad \tau_0 = \frac{2}{\sqrt{\Omega_M}H_0}. \tag{33}$$

Note that today, the relation between radiation and matter densities is given by

$$\Omega_r = a_{\rm eq}\Omega_M. \tag{34}$$

For the standard cosmological scenario with $N_{\rm eff} = 3$, i.e., three effective neutrino degrees of freedom, we have $a_{\rm eq} = 1/3600$, $\Omega_M = 0.3$ and $\Omega_r = \Omega_\gamma + \Omega_\nu$ since the free-streaming neutrinos are relativistic. Further it can be shown that [26]

$$\frac{\Omega_\nu}{\Omega_\nu + \Omega_\gamma} = 0.40523; \qquad g_\nu = 6. \tag{35}$$

When adding extra neutrino species, we use the above relation and keep Ω_M fixed but change the ratio in Eq. (35) accordingly to get a new redshift for matter-radiation equality [46]. So, for $N_{\rm eff} = 4$, $a_{\rm eq} = 1/3172$ and for $N_{\rm eff} = 5$, $a_{\rm eq} = 1/2834$.

One sees that Eq. (32) is an integro-differential equation since the source term on the right-hand side is the integral in Eq. (31). To put the equation into a suitable form for a numerical solution, we adopt the method in Appendix A of [47], which consists of rewriting the single, second order integro-differential equation as a system of coupled first-order Volterra type integro-differential equations. This can then be solved by standard methods of numerical integration [48]. There is a slight difference in our method of solution from that in [26,47] due to the form of the integral kernel. Namely, we do not have the simplifying option of integrating out the distribution function (which would give the energy density in the standard case of massless neutrinos) due to the additional x dependence of other factors in the integrand. We therefore were forced to generate numerical values for the integrand at each value of u and s, after which the procedure of [47] could be implemented.

B. Damping from axions

The fine-tuning of $\theta_{\rm QCD}$ can be avoided in models of particle physics that contain an extra $U(1)$ Peccei-Quinn (PQ) symmetry. A consequence of these models is a light pseudoscalar particle, the axion. In a cosmological setting, the axion is massless above the QCD temperature but gains a small mass below this temperature. Even though the axion is very light, with a typical mass $m_a = \mathcal{O}(10^{-3}\ \rm eV)$, it can be nonrelativistic because it is produced coherently throughout the cosmological horizon and has momenta given by $\sim t^{-1}$ at cosmic time t. This argument, however, ignores the topology that accompanies the breaking of the PQ symmetry, which is relevant if the PQ symmetry breaking scale occurs below the scale of inflation. In this case, the spontaneous breaking of the PQ symmetry leads to the production of axionic cosmic strings with energy density set by the PQ energy scale. As the strings oscillate, they radiate relativistic axions. At the QCD temperature scale, the strings get connected by axionic domain walls, and the whole network of strings and walls collapses, dissipating energy again into *relativistic* axions. Hence the axion density in the Universe contains two separate components: the nonrelativistic component due to coherent oscillations of the axion field, and the relativistic component due to the radiation from topological defects. The latter component can be significant and may even dominate the nonrelativistic component for large values of the Peccei-Quinn symmetry breaking scale. Relativistic axions can also have anisotropic stress, and hence they can couple to gravitational waves just as neutrinos do.

The spectral energy density of relativistic axions has been debated and there has been some disagreement between Davis [49] and Hagmann *et al.* [50]. We will be using Davis's spectral distribution, which has recently been verified by field theoretic lattice simulations in [51,52],

$$\rho_a(t) = \frac{4\pi f_a^2}{t^2}\int_{\Omega^{2/3}/\tilde{t}}^{\Omega^{5/6}/\sqrt{\tilde{t}t^*}}\left(\frac{q^2+(t/\tilde{t})m_a^2}{q^2+\tilde{m}^2}\right)^{1/2} \times\left[\ln\left(\frac{\Omega^{5/3}}{\tilde{t}q^2\delta}\right)-\frac{1}{2}\right]\frac{dq}{q}, \tag{36}$$

where, as previously defined, q is the comoving momentum, $f_a \le 2\times 10^{10}$ GeV is the PQ symmetry breaking scale, $\delta = 1/f_a$, $\Omega = 2\pi$, $\tilde{m} \equiv 1/\tilde{t} = 10^{-9}$–$10^{-8}$ eV and t^* is the time at which the axions decoupled. The mass of the axion m_a and the decoupling temperature T_d^* of axions is related to the scale f_a through [53]

$$m_a = 6\times 10^{-6}\ \mathrm{eV}\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right) = \frac{6\times 10^{15}\ \mathrm{eV}^2}{f_a}, \tag{37}$$

$$T_d^* = 5\times 10^{11}\ \mathrm{GeV}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^2. \tag{38}$$

Since $T \propto 1/a(t)$, we can find t^* using T_d^* and the scale factor $a(t)$ in (33). We will consider damping from relativistic axions with the spectrum (36) for three different f_a values 10^8, 10^9 and 10^{10} GeV. This lies within the range 10^7 GeV $< f_a < 2\times 10^{10}$ GeV where the lower bound on f_a comes from astrophysical constraints [54] and the upper bound comes from the requirement that the energy density in relativistic axions remain below critical energy density to avoid overclosing the Universe.

To calculate the anisotropic stress Π_k, we need the unperturbed phase space distribution function $\bar{F}(q)$ of these relativistic axions, which we can read off from (36)

$$\bar{F}^{\mathrm{axion}}(q) = \frac{f_a^2 a(t)^4}{t^2 q^3 (q^2 + a(t)^2 m_a^2)^{1/2}}\left(\frac{q^2+(t/\tilde{t})m_a^2}{q^2+\tilde{m}^2}\right)^{1/2} \times\left[\ln\left(\frac{\Omega^{5/3}}{\tilde{t}q^2\delta}\right)-\frac{1}{2}\right]. \tag{39}$$

However, there are few differences from the neutrino case of (31). Since axions have a nonthermal spectrum, we do not do the substitution of (28) and retain the expression for α^2 in (18). Thus, the expression for Π_k for axions becomes

$$\Pi_k^{\mathrm{axion}} = \Pi_k(\bar{F} \to \bar{F}^{\mathrm{axion}}), \tag{40}$$

where Π_k is defined in Eq. (31) and $\alpha(m, q, \tau)^2$ is the same as in Eq. (18)

III. RESULTS AND DISCUSSION

For massless neutrinos, the effects of damping are determined solely by the neutrino energy density contribution, (which falls as a^{-4}) once one enters the matter-dominated era, and will thus be most significant for $k \gtrsim k_{\mathrm{eq}} = a_{\mathrm{eq}}H_{\mathrm{eq}} \approx 170/\tau_0$.

The effect of nonzero neutrino masses will be to add an extra k dependence to damping, as free streaming, and hence damping, will be reduced when the temperature is of order of the mass. The k modes that come inside the horizon while neutrinos are relativistic, and contribute significantly to the overall energy density, will be damped more. On the other hand, those modes that come inside the horizon at later times, either when neutrino masses become significant or during matter domination, when the neutrino energy density fraction may have been reduced considerably due to redshifting, will be damped less. This heuristic behavior is validated by our detailed calculations, which quantitatively explore this effect. For demonstrations purposes here, we display the damping as a function of neutrino mass for three different values of $k\tau_0 = 100, 200, 1000$.

In Fig. 1, we plot the damping from 3 massless and a 1 eV neutrino and compare it to the case of 4 massless neutrinos. In doing so, we have adjusted a_{eq} to $N_{\mathrm{eff}} = 4$ for both cases, which is a good approximation since neutrino masses of $O(1)$ eV are largely relativistic at matter-radiation equality. Note that a cosmological scenario with a 1 eV neutrino and $N_{\mathrm{eff}} = 4$ is consistent with the current Planck data [55].

For $k\tau_0 = 100$, the ratio between the minima of the 3 massless plus 1 eV case and the homogeneous (*undamped*) case is 0.94, and for the 4 massless vs homogeneous case is 0.92, a difference of order 2%. For $k\tau_0 = 200$, the difference in damping is of order 1.5%. And finally, for $k\tau_0 = 1000$ the difference in damping is now only 0.7%.

Similarly the effect of additional massless degrees of freedom is to increase the damping of gravitational waves. As seen in Fig. 2 this effect varies slightly with conformal time, τ. We can compare the effect of extra species, for

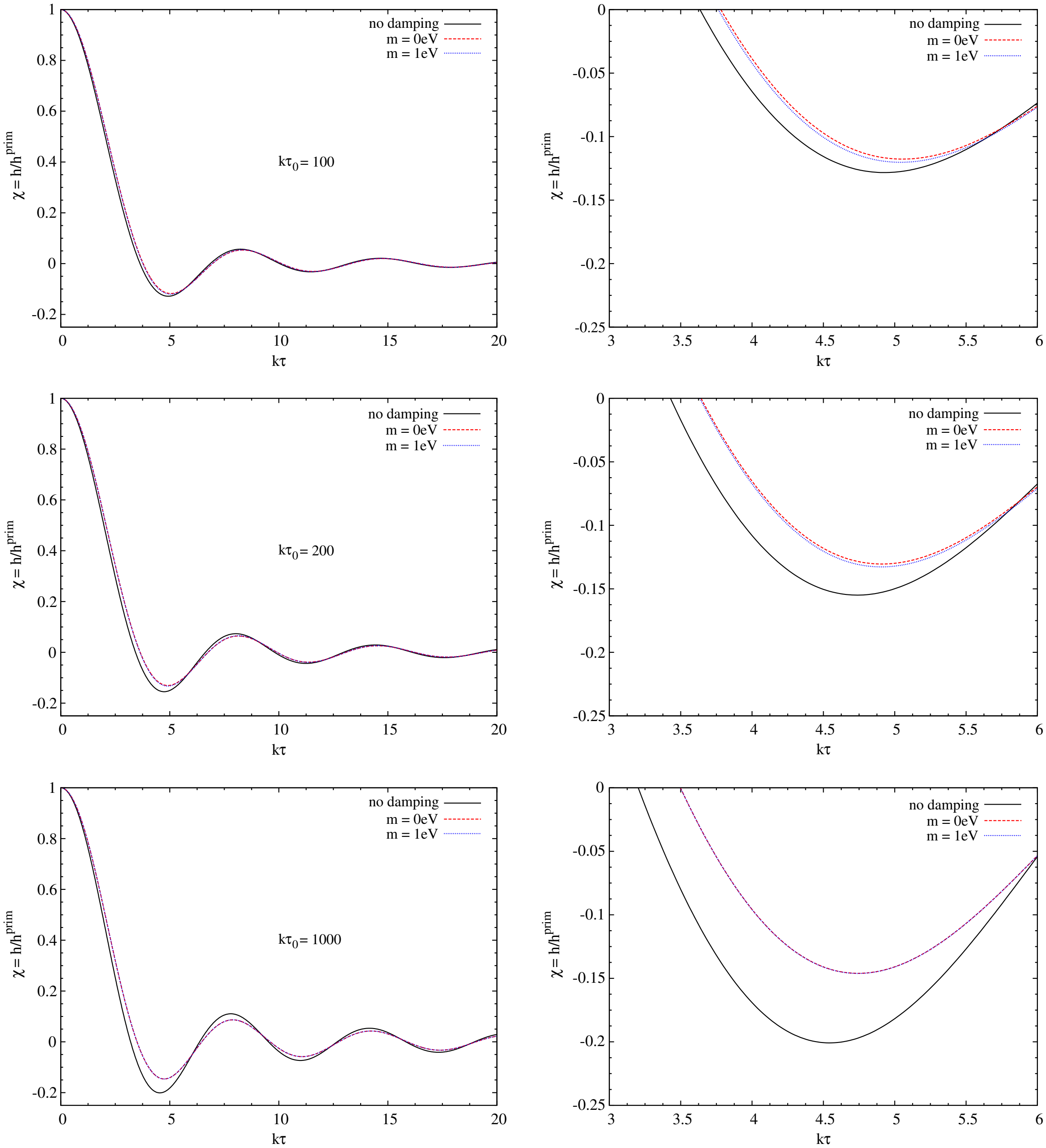

FIG. 1 (color online). The k dependence of the damping of an extra massive neutrino is demonstrated. The plots show that the damping is reduced for gravitational wave modes that enter the horizon as neutrinos are beginning to become nonrelativistic. The damping is also less at later times when the neutrino energy fraction has been reduced due to redshifting. The region in each plot on the left around the first minimum is zoomed in the adjacent plot on the right. For the $k\tau_0 = 1000$, the red and blue lines are overlapping, implying a negligible effect of mass.

example, at the first minimum. For the undamped case, this minimum occurs at $u = 4.54$ independent of $N_{\rm eff}$. However, including GW damping through free streaming, this minimum shifts to $u = 4.72, 4.74, 4.76$ ($N_{\rm eff} = 3, 4, 5$, respectively). With respect to the homogeneous case, the mode amplitude at the minimum is 76.5%, 73.1% and 70.5% as large as for $N_{\rm eff} = 3$, 4, 5, respectively. Thus, tensor modes are damped more, with increasing $N_{\rm eff}$, as expected.

Since the identity of the source of any possible extra degrees of freedom is currently unknown, one may want to expand the realm of possibilities to include bosonic degrees of freedom. As expected on the basis of the number of degrees of freedom, and hence $N_{\rm eff}$, the damping due to a single boson is less by about 19% than that of a single, massless neutrino species. Two bosonic degrees of freedom are virtually indistinguishable from a single neutrino.

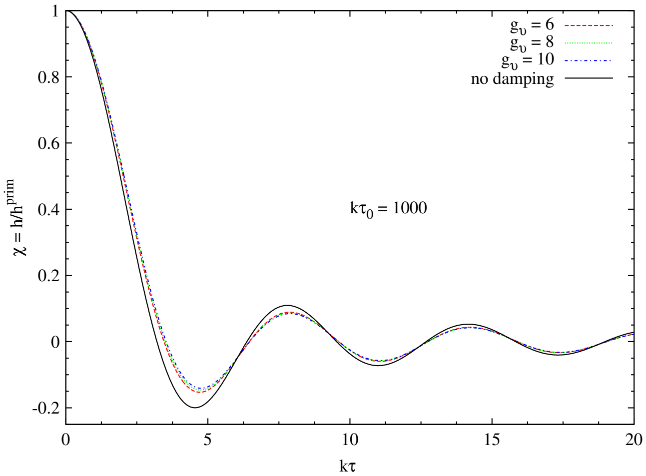

FIG. 2 (color online). The damping effect of extra massless neutrino species is shown. Each neutrino species has 2 degrees of freedom; thus $g_\nu = 6$, 8, and 10 correspond to 3, 4, and 5 neutrinos.

In Fig. 3, we examine the damping of gravitational waves caused by relativistic axions. We compare the results for axions with $f_a = 10^9$ GeV versus 3 massless neutrinos since relativistic axions have 26% and 3 massless neutrinos constitute 10% of critical density at the last scattering. With respect to the no-damping case, the mode amplitudes at the minima are damped by 76.5% for 3 neutrinos versus 77% for axions. However, note that the minimum for neutrinos is at $u = 4.72$, but for axions it is at $u = 4.42$. And as can be seen this phase shift persists throughout the time evolution. Thus, although axions damp the amplitudes by the same amount as neutrinos for these parameters, their phase shift is an important distinguishing feature.

We can understand this effect on physical grounds. The axion phase space distribution, Eq. (39), has an explicit time dependence that is not present in the thermal neutrino distribution function. As a result the integral over time of the anisotropic stress, which produces the damping, is modulated compared to the neutrino case, and hence modulates the resulting k-dependent damping of gravitational waves.

This phase difference will have an observational impact on the damping of CMB B-modes. Recall that it is $\dot{\chi}$ that enters into the Boltzmann equation for the temperature perturbations [56]. Following [22], we expect all tensor multipole coefficients to depend on $\chi(u)$ only through a factor of $|\chi'(u_{\rm LSS})|^2$, where $u_{\rm LSS} = (1 + z_{\rm EQ})/(1 + z_{\rm LSS})$ is the value of u at the last scattering surface (LSS). We take $z_{\rm LSS} = 1089$ and convert it into $u_{\rm LSS}$ using Eq. (33) and $\Omega_M = 0.3$. Moreover, we expect the dominant contribution to multipole l in the CMB will come from the wave number, $k \approx a_{\rm LSS} l/d_{\rm LSS}$ [22] where $a_{\rm LSS}$ is the scale factor at the surface of last scattering and $d_{\rm LSS}$ is the angular diameter distance of the surface of the last scattering. Using numerical values of $a_{\rm LSS}$ and $d_{\rm LSS}$ we get

$$l = 0.878 u_{\rm LSS}. \tag{41}$$

In Fig. 4 we show the ratio of χ'^2 for damped to undamped gravitational waves for axions and different numbers of massless neutrinos. We have extracted this ratio at the surface of last scattering for several different low l values. We expect the graph to look similar at higher l values, but the computations at high l become prohibitively expensive. Both neutrinos and axions produce an oscillatory pattern in the damping, but there is a phase shift between them. It is important to note that at certain l, "damped" gravitational waves can actually produce a larger signal than undamped waves by a factor of 2 or more. This surprising effect is due to that fact that for some $k\tau$ values there is actually a relative amplification caused by anisotropic stress, as can be seen from Figs. 1 and 2,

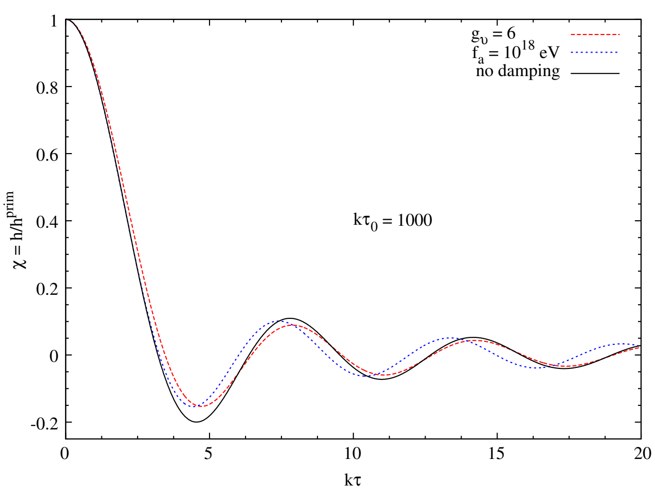

FIG. 3 (color online). The effect of axions produced by axionic strings.

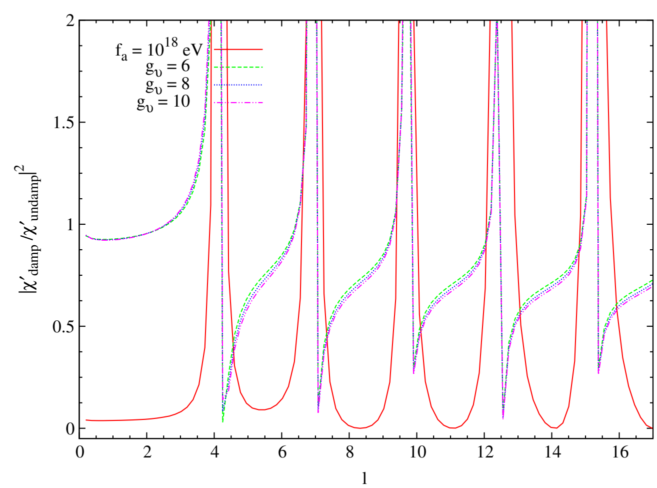

FIG. 4 (color online). The square of the ratio of the time derivative of damped modes to undamped modes, which is useful for calculating the B-mode correlation function C_l^{BB}.

where the mode amplitude does not decrease as rapidly as in the undamped case.

IV. CONCLUSIONS

The observation of a primordial gravitational wave spectrum would provide a direct window on physics of the very early universe. As has been stressed, to extract as much cosmological information as possible from such a signal, one must be mindful of any phenomena that may alter the primordial signal. One example of such a process is the damping of gravitational waves by free-streaming particles such as neutrinos.

In this work we have generalized the formalism for deriving the effects of damping of gravitational waves due to anisotropic stress caused by free streaming by deriving a general formula for the anisotropic stress as a function of mass and number of degrees of freedom, which should be useful for calculating the cosmological signature of possible additional nonstandard model relativistic species.

We find that for additional neutrino masses of current cosmological interest, the effects of nonzero mass on damping in comparison to the massless case is most pronounced for $k\tau_0 \approx 100$–200. For longer wavelength modes that enter the horizon later, the damping is suppressed for all cases because the neutrino energy density is less significant. In addition we have explored the possible impact of a relativistic axion background, as might be present due to radiation from axion strings. While the overall damping produced by such a background could perhaps be comparable to that due to three standard model neutrinos, we find that their nonthermal phase space distribution will produce a possibly measurable phase shift in the damping signature.

If a nonzero tensor B-mode contribution is observed in future CMB experiments, one might hope to use these results to help constrain new physics beyond the standard model.

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[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] L. P. Grishchuk, Sov. Phys. JETP **40**, 409 (1975).
[5] A. A. Starobinsky, JETP Lett. **30**, 682 (1979).
[6] C. Pryke, arXiv:1209.2768.
[7] L. M. Krauss, S. Dodelson, and S. Meyer, Science **328**, 989 (2010).
[8] A. Kosowsky, Ann. Phys. (N.Y.) **246**, 49 (1996).
[9] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. D **55**, 7368 (1997).
[10] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997).
[11] W. Hu and M. J. White, Phys. Rev. D **56**, 596 (1997).
[12] W. Hu and M. J. White, New Astron. **2**, 323 (1997).
[13] P. Cabella and M. Kamionkowski, arXiv:astro-ph/0403392.
[14] D. Baumann *et al.* (CMBPol Study Team Collaboration), AIP Conf. Proc. **1141**, 10 (2009).
[15] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
[16] S. Choudhury and A. Mazumdar, arXiv:1306.4496.
[17] L. M. Krauss, Phys. Lett. B **284**, 229 (1992).
[18] K. Jones-Smith, L. M. Krauss, and H. Mathur, Phys. Rev. Lett. **100**, 131302 (2008).
[19] E. Fenu, D. G. Figueroa, R. Durrer, and J. Garcia-Bellido, J. Cosmol. Astropart. Phys. 10 (2009) 005.
[20] L. M. Krauss, K. Jones-Smith, H. Mathur, and J. B. Dent, Phys. Rev. D **82**, 044001 (2010).
[21] R. H. Brandenberger, arXiv:1104.3581.
[22] S. Weinberg, Phys. Rev. D **69**, 023503 (2004).
[23] S. Bashinsky, arXiv:astro-ph/0505502.
[24] D. A. Dicus and W. W. Repko, Phys. Rev. D **72**, 088302 (2005).
[25] L. A. Boyle and P. J. Steinhardt, Phys. Rev. D **77**, 063504 (2008).
[26] Y. Watanabe and E. Komatsu, Phys. Rev. D **73**, 123515 (2006).
[27] H. X. Miao and Y. Zhang, Phys. Rev. D **75**, 104009 (2007).
[28] A. Mangilli, N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. D **78**, 083517 (2008).
[29] W. Zhao, Y. Zhang, and T. Xia, Phys. Lett. B **677**, 235 (2009).
[30] R. Benini, M. Lattanzi, and G. Montani, Gen. Relativ. Gravit. **43**, 945 (2011).
[31] G. Shchedrin, arXiv:1204.1384.
[32] B. A. Stefanek and W. W. Repko, arXiv:1207.7285.
[33] R. Jinno, T. Moroi, and K. Nakayama, Phys. Rev. D **86**, 123502 (2012).
[34] Y. I. Izotov and T. X. Thuan, Astrophys. J. **710**, L67 (2010).
[35] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[36] J. Dunkley *et al.*, Astrophys. J. **739**, 52 (2011).
[37] R. Keisler *et al.*, Astrophys. J. **743**, 28 (2011).
[38] M. Archidiacono, E. Calabrese, and A. Melchiorri, Phys. Rev. D **84**, 123008 (2011).

[39] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A.M. Rotunno, Phys. Rev. D **86**, 013012 (2012).
[40] J.M. Conrad, C.M. Ignarra, G. Karagiorgi, M.H. Shaevitz, and J. Spitz, Adv. High Energy Phys. **2013**, 163897 (2013).
[41] C.M. Ho and R.J. Scherrer, Phys. Rev. D **87**, 065016 (2013).
[42] T.D. Jacques, L.M. Krauss, and C. Lunardini, Phys. Rev. D **87**, 083515 (2013).
[43] A. Mirizzi, G. Mangano, E. Borriello, C. Giunti, G. Miele, and O. Pisanti, arXiv:1303.5368.
[44] M. Archidiacono, E. Giusarma, S. Hannestad, and O. Mena, arXiv:1307.0637 [Adv. High Energy Phys. (to be published)].
[45] S. Joudaki, K.N. Abazajian, and M. Kaplinghat, Phys. Rev. D **87**, 065003 (2013).
[46] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).
[47] J.R. Pritchard and M. Kamionkowski, Ann. Phys. (Amsterdam) **318**, 2 (2005).
[48] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery, *Numerical Recipes in FORTRAN: The Art of Scientific Computing* (Cambridge University Press, New York, 1992).
[49] R.L. Davis, Phys. Lett. B **180**, 225 (1986).
[50] C. Hagmann, S. Chang, and P. Sikivie, Phys. Rev. D **63**, 125018 (2001).
[51] M. Yamaguchi, M. Kawasaki, and J. Yokoyama, Phys. Rev. Lett. **82**, 4578 (1999).
[52] T. Hiramatsu, M. Kawasaki, T. Sekiguchi, M. Yamaguchi, and J. Yokoyama, Phys. Rev. D **83**, 123531 (2011).
[53] P. Sikivie, Lect. Notes Phys. **741**, 19 (2008).
[54] G.G. Raffelt and D.S.P. Dearborn, Phys. Rev. D **36**, 2211 (1987).
[55] Planck Collaboration, arXiv:1303.5076.
[56] S. Weinberg, *Cosmology* (Oxford University Press, Inc., New York, 2008).