An optimal solution to resource allocation among soft QoS traffic in wireless network

Liansheng Tan1,*,†, Zhongxun Zhu2, Wei Zhang1 and Gong Chen1

1Department of Computer Science, Central China Normal University, Wuhan 430079, China
2College of Mathematics and Statistics, South Central University for Nationalities, Wuhan 430074, China

SUMMARY

Optimization theory and nonlinear programming method have successfully been applied into wire-lined networks (e.g., the Internet) in developing efficient resource allocation and congestion control schemes. The resource (e.g., bandwidth) allocation in a communication network has been modeled into an optimization problem: the objective is to maximize the source aggregate utility subject to the network resource constraint. However, for wireless networks, how to allocate the resource among the soft quality of service (QoS) traffic remains an important design challenge. Mathematically, the most difficult comes from the non-concave utility function of soft QoS traffic in the network utility maximization (NUM) problem. Previous result on this problem has only been able to find its sub-optimal solution. Facing this challenge, this paper establishes some key theorems to find the optimal solution and then present a complete algorithm called utility-based allocation for soft QoS to obtain the desired optimal solution. The proposed theorems and algorithm act as designing guidelines for resource allocation of soft QoS traffic in a wireless network, which take into account the total available resource of network, the users’ traffic characteristics, and the users’ channel qualities. By numerical examples, we illustrate the explicit solution procedures.

1. INTRODUCTION

The utility optimization model has been widely adopted in the research of end-to-end congestion control [1–6] by viewing variant congestion control protocols as distributed algorithms to solve some basic network utility maximization (NUM) problems. Consider a communication network (e.g., the Internet) with \( L \) links, each with a fixed capacity of \( c_l \) bps and \( S \) sources (i.e., end users or flows), each transmitting at a source rate of \( x_s \) bps. It is assumed that each source \( s \) emits only one flow, using a fixed set \( L(s) \) of links in its path, and has a utility function \( U_s(x_s) \). The utility function describes the degree of user satisfaction when allocated by a certain amount of bandwidth. Each link \( l \) is shared by a set \( S(l) \) of sources. NUM is shown in the following problem of maximizing the total utility of the network

\[
\sum_s U_s(x_s),
\]
over the source rates $x_s$, subject to linear flow constraints

$$\sum_{s \in L(l)} x_s \leq c_l$$

for all links $l$. The solution strategy of the preceding NUM problem has successfully been developed [4] into distributed primal-dual algorithms for congestion controlling of elastic traffic in a wire-lined network (e.g., the Internet).

In telecommunication networks, quality of service (QoS) refers to several related aspects that allow the transport of traffic with special requirements. QoS is the ability to provide different priority to different applications or to guarantee a certain level of performance to a data flow. For example, a required bit rate, delay, jitter, packet dropping probability, and/or bit error rate may be guaranteed. QoS guarantees are important if the network capacity is insufficient, especially for real-time streaming multimedia applications such as voice over IP, online games and IP-TV, and in networks where the capacity is a limited resource, for example, in cellular data communication.

In order to guarantee the QoS in future wireless networks, it is desirable to implement certain resource allocation mechanism to allocate the limited resource (e.g., bandwidth) among all the users fairly and efficiently. The paper of Ma et al. [7] is concerned with the bandwidth allocation problem for cooperative relay networks. The bandwidth allocation problem is therein formulated as a Nash bargaining problem, and then the bandwidth allocation algorithm is discussed using the sub-gradient method. For wireless mesh networks, a solution to address channel assignment termed as the extended level-based channel assignment scheme is presented in [8]. In [9], the theoretically achievable average channel capacity (in the Shannon sense) per user of a hybrid cellular direct sequence/fast frequency hopping code-division multiple-access system, operating in a Rayleigh fading environment, was examined. The paper of Oh and Woo [10] presents a performance analysis of dynamic channel allocation on the basis of the greedy approach for orthogonal frequency-division multiple-access downlink systems over Rayleigh fading channels. Relevant work also includes that of Tan et al. [11], where for mobile ad hoc networks, energy-efficient routing approach is presented to study the resource allocation issue.

It is established that the wireless communication system may allocate its resource to users according to some performance metrics such as throughput and fairness [12–14] or according to the type of traffic [15, 16]. Because of the difficulty of trading off throughput and fairness, the allocation problem is usually approached by focusing on users’ satisfaction [13,17], which is specific to traffic type. The degree of user satisfaction can be described by the utility function of the traffic type under consideration. Different shapes of utility functions lead to optimal resource allocations that satisfy the existing definitions of fairness [18], such as max–min fairness [19], proportional fairness [3], and $(\alpha-n)$ proportional fairness [20]. And utility functions can provide a new metric to define optimality of resource allocation efficiency.

Utility-based allocation schemes in wireless networks are proposed in [21, 22]. Kuo and Liao [23] studied the issue of resource allocation for two types of traffic, that is, best effort and hard QoS traffic. For soft QoS traffic, which demands a preferred amount of bandwidth with some flexibility for normal operation, Kuo and Liao [17] proposed a utility-based resource allocation scheme. Unfortunately, the proposed algorithm is only sub-optimal, and the authors pointed out that it was difficult to give an approach to obtain the optimal solution. The present paper has to face this challenge and obtain the optimal solution for this problem. We establish some key theorems to find the optimal solution to this problem; on this basis, we then present a complete algorithm called utility-based allocation for soft QoS (USQ) to obtain the desired optimal solution. The proposed theorems and algorithm act as designing guidelines for resource allocation of soft QoS traffic in a wireless network, which take into account the total available resource of network, the users’ traffic characteristics, and the users’ channel qualities. By numerical examples, we illustrate the explicit solution procedures.

Our approach establishes that in a wireless network, the base station can optimally allocate the resource among the competing soft QoS traffic to maximize the total utility of all users. This allocation takes into account the users’ utility characteristic, the users’ channel conditions, and the total available resource of the system. The new developments such as the adaptive modulation and
coding schemes [24] in wireless networks enable the upper layers to access user channel conditions. Therefore, our algorithm can be implemented at the base station in a wireless network to perform the bandwidth allocation function by using these new technologies.

The rest of this paper is organized as follows. In Section 2, the theoretical analysis and the main results are presented. The USQ algorithm is proposed in Section 3. In Section 4, we then verify the performance of this algorithm via some numerical examples. Finally, Section 5 concludes this paper.

2. PROBLEM DESCRIPTION AND OPTIMAL SOLUTION OF UTILITY MAXIMIZATION

A utility function is defined as a curve mapping the amount of bandwidth received by the application to the performance as perceived by the end user. Utility function is monotonically non-decreasing; in other words, more bandwidth allocation should not lead to degraded application performance. The key advantage of utility function is that it can inherently reflect the QoS requirements of the end user. The exact expression of a utility function may depend on the type of traffic and can be derived from some metrics that reflect the user’s perception or the content quality.

2.1. Utility function of soft QoS traffic and the utility maximization problem

Soft QoS traffic refers to the applications that have flexible bandwidth requirements. In case of congestion, they can gracefully adjust their transmission rates to adapt to various network conditions. However, such applications have an intrinsic bandwidth requirement because the data generation rate is independent of the network congestion. Typical examples of soft QoS traffic are interactive multimedia services and video on demand. The utility function of soft QoS traffic is assumed to be a sigmoidal function (see, e.g., [17]) in NUM approach, which is a non-concave function with a shape needed to be determined by a couple of parameters.

Without loss of generality, we study the following sigmoid utility function $U(r)$:

$$U(r) = \begin{cases} q e^{p(r-rc)} & r < rc, \\ 1 - (1 - q) e^{-p(r-rc)} & r \geq rc. \end{cases}$$  \hspace{1cm} (2.1)

where $r$ is the bandwidth being allocated to the user, which may also refer to timeslots or radio frequency occupied by this user. The component $rc$ denotes the preferable amount of resource for the soft QoS traffic. The component $q$ is the channel quality parameter, which represents the ratio of actual amount of resource received by the user to the amount of resource allocated by the base station to the user, and it is in the range of $[0, 1]$. It is the utility value when $r = rc$. The parameter $p$ determines the slope of the utility function, and it characterizes the flexibility level to bandwidth requirement of the flow. When $p$ increases, the shape of the sigmoid utility function becomes closer to that of a unit-step function. Note that a unit-step function represents the utility function of hard QoS traffic. Hard QoS traffic refers to the applications with stringent bandwidth requirements. A call belonging to this type of traffic requires strict end-to-end performance guarantees and does not show any adaptive properties. For $q = 0.4$, $p = 0.12$, $rc = 40$, we depict a utility function in Figure 1.

The utility function $U(r)$ given by (2.1) is a non-decreasing function with respect to the amount of allocated resource $r$: the more the resource is allocated, the more the user is satisfied. To see this feature, we take a look at its marginal utility function $u(r)$, which is the derivative of the utility function $U(r)$ with respect to $r$. Namely

$$u(r) = U'(r) = \begin{cases} qpe^{p(r-rc)} & r < rc, \\ (1 - q)e^{-p(r-rc)} & r \geq rc. \end{cases}$$  \hspace{1cm} (2.2)

By taking $q = 0.4$, $p = 0.12$, $rc = 40$, for example, the preceding marginal utility function $u(r)$ is depicted in Figure 2. The utility function given by (2.1) is a non-concave function with respect to $r$. This is seen from the following derivative of the marginal utility function:

$$u'(r) = U''(r) = \begin{cases} q p^2 e^{p(r-rc)} & r < rc, \\ -p^2(1 - q)e^{-p(r-rc)} & r \geq rc. \end{cases}$$  \hspace{1cm} (2.3)
Again by taking \( q = 0.4, p = 0.12, rc = 40 \), the preceding function is depicted in Figure 3. Observing from (2.3) and Figure 3, one notes that there is a special point \( r = rc \). \( rc \) is referred to as the preferable amount of resource for the soft QoS traffic. It is the single inflexion point of the derivative of the marginal utility function satisfying \( U''(rc) = 0 \). We observe that if \( r < rc \),
$u'(r) > 0$; if $r \geq rc$, $u'(r) \leq 0$. It can be interpreted as follows: when the amount of bandwidth $r$ is given insufficiently, it is unsatisfactory for user for real-time application; as the bandwidth value $r$ approaches its desired value, the flow becomes gradually operational, and thus the marginal utility increases dramatically. Once the allocated amount of resource $r$ exceeds $rc$ too much, allocating more resource may not be helpful for operation, the user still runs around its normal operation, and therefore the marginal utility drops hence forth.

Let us consider a wireless network that consists of a base station and a set of soft QoS traffic (users) denoted by $\Gamma$. All the soft QoS traffic are competing for the total resource (e.g., bandwidth) valued at $C$ at the base station in communications. The notations of all the variables are given in Table I. We depict this communication system in Figure 4.

Assume that each flow belonging to $\Gamma$ has a sigmoid utility function $U(\bullet)$ given by (2.1). Let $r_i$ denote the amount of resource allocated to flow $i$, $i \in \Gamma$, and $q_i$ the channel quality of flow $i$, where $0 \leq q_i \leq 1$. The bigger the value of $q_i$, the better the channel quality. Let $p_i$ be the parameter determining the flexibility level to its preferred bandwidth value for the soft QoS traffic $i$. It is reasonable to assume that (i) the base station has the information of each user’s parameters such as $q_i$, $p_i$, and (ii) users under different channel conditions can transmit data at different data rates.

Given the channel quality $q_i$ of flow $i$, if it is allocated by the bandwidth value $r_i$ at the base station, the amount of bandwidth it receives for operation is actually $r_i q_i$. Therefore, the actual utility function of flow $i$ can be expressed as $U_i(\bullet) = U(r_i q_i)$, where $U(\bullet)$ is the utility function of the soft QoS traffic under consideration (formulated by (2.1)) and $U_i(\bullet)$ is the utility function for the

<table>
<thead>
<tr>
<th>Table I. Nomenclature.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
</tr>
<tr>
<td>$C$</td>
</tr>
<tr>
<td>$r_i$</td>
</tr>
<tr>
<td>$q_i$</td>
</tr>
<tr>
<td>$U(\bullet)$</td>
</tr>
<tr>
<td>$rc$</td>
</tr>
<tr>
<td>$U_i(\bullet)$</td>
</tr>
<tr>
<td>$rc_i$</td>
</tr>
<tr>
<td>$u_i(\bullet)$</td>
</tr>
<tr>
<td>$u'_i(\bullet)$</td>
</tr>
</tbody>
</table>

Figure 4. The network utility optimization for bandwidth allocation in a wireless network.
specific traffic described by $U(\bullet)$ but taking into account the channel quality parameter $q_i$ and others. One sees that the actual beneficial utility value the user $i$ 'received' is less than the 'allocated' utility value for a bandwidth allocation, that is, $U_i(r_i,q_i) \leq U_i(r_i)$. For flow $i$, the marginal utility function of $U_i(\bullet)$, denoted by $u_i(\bullet)$, is defined by $u_i(r_i) = \frac{dU_i(r_i,q_i)}{dr_i} = q_i \times u(q_i \times r_i)$. Considering that the channel condition of each flow may not be identical, the preferable amount of resource for flow $i$ is denoted by $r_{c_i}$. For each flow $i$, $i \in \Gamma$, $U_i(r_{c_i}) = U(r_{c_i}) = U_c$, and thus $r_{c_i} = \frac{U_c}{q_i}$.

Now we are ready to describe the targeted problem as follows (refer to Figure 4):

\[ P1 : \max \sum_{i \in \Gamma} U_i(r_i) = \sum_{i \in \Gamma} U(r_i,q_i) \]

\[ \text{s.t.: } \sum_{i \in \Gamma} r_i \leq C, r_i \geq 0, i \in \Gamma, \]

where

\[ U(r_i,q_i) = \begin{cases} q_i e^{P_i(r_i,q_i,-r_i)}, & r_i q_i < r_{c_i}, \\ 1 - (1-q_i) e^{-P_i(r_i,q_i,-r_i)}, & r_i q_i \geq r_{c_i}. \end{cases} \] (2.4)

2.2. The optimal solution to the utility maximization problem

Definition 2.1
The solution $\mathcal{R} = \{r_i, i \in \Gamma\}$ to the preceding problem $P1$ is optimal if, for any other allocation solution $\mathcal{R}' = \{r'_j, j \in \Gamma\}$, we have $U(\mathcal{R}) \geq U(\mathcal{R}')$, where

\[ U(\mathcal{R}) = \sum_{i \in \Gamma} U_i(r_i), U(\mathcal{R}') = \sum_{i \in \Gamma} U_i(r'_i). \]

According to Lemma 2.2 in [17], the optimal solution of $P1$ is at most one flow's derivation of the marginal utility function, which is positive. To obtain the optimal solution, we only need to consider the following two classes:

(i) $\forall i \in \Gamma$, $u_i'(r_i) < 0$ if $r_i > 0$;
(ii) $\forall i \in \Gamma$, there exists unique flow $i$ whose $u_i'(r_i) > 0$ if $r_i > 0$.

For the first case, Kuo and Liao [17] proposed a sub-optimal algorithm. In the meantime, they pointed out that it was difficult to give an approach to obtain the optimal solution of the whole problem.

The current paper attacks this problem. Our procedure works according to the following mechanism: first, to obtain the optimal solution, which satisfies (ii), then to compare it with the optimal solution which satisfies (i). As a result, the solution achieving larger utility is the optimal one.

Let $\mathcal{R}_j^1, \mathcal{R}_j^2$ denote the optimal solutions, which allocate a resource to a total of $j$ flows and satisfy class (i) and class (ii), respectively. Note that $\mathcal{R}_j^1, \mathcal{R}_j^2$ are n-dimensional vectors in the form of $(r_1, r_2, \cdots, r_n)$, where $r_{j+1} = \cdots = r_n = 0$.

For simplicity, we assume that all flows in the queue are sorted in the decreasing order of their channel qualities.

Lemma 2.1
For $\mathcal{R}_j^2$, we have $u_j'(r_j) > 0$. 

Copyright © 2013 John Wiley & Sons, Ltd.  
DOI: 10.1002/dac
Let a set of bandwidth allocation \( E \) be the inverse function of \( \theta \), hence \( u \) is increasing. Let \( \theta_{ij} = \theta_i - \theta_j = q_j r_j \), so
\[
 r_i \geq \frac{q_j r_j}{q_i} > \frac{r_c}{q_i} = r_{ci},
\]
hence \( u'(r_i) < 0 \) is a contradiction. \( \square \)

By Lemma 2.1, we find that in \( S_j, j = 1, 2, \ldots, n \), inequality \( r_i > r_{ci} \) holds for each allocated flows \( i, i = 1, 2, \ldots, j - 1 \) and \( r_j < r_{cj} \).

Let \(
\hat{u}_i(x_i) = u_i(x_i + r_{ci}), \quad i = 1, 2, \ldots, j - 1,
\)
then \( \hat{u}_i(x_i) \) is a decreasing function with respect to \( x_i \), and \( r_i = x_i + r_{ci} \) for \( i = 1, 2, \ldots, j - 1 \) and \( u_j(r_j) \) is increasing. Let \( \hat{u}_i^{-1}(\bullet) \) be the inverse function of \( \hat{u}_i(\bullet), i = 1, 2, \ldots, j - 1 \) and \( u_j^{-1}(\bullet) \) be the inverse function of \( u_j(\bullet) \).

To find the optimal solution of \( P1 \), we first establish the sufficiency and necessity condition of optimal solution. On the basis of the optimization theory, we have the following result.

**Theorem 2.2**
Let a set of bandwidth allocation \( \vec{r} = (r_1, r_2, \ldots, r_n) \) and
\[
L(r, \lambda, \lambda_1, \cdots, \lambda_n) = \sum_{i=1}^{n} (-U_i(r_i)) - \lambda \left( C - \sum_{i=1}^{n} r_i \right) - \sum_{i=1}^{n} \lambda_i r_i.
\]
The bandwidth allocation \( \vec{r} \) is an optimal solution to \( P1 \) if and only if there exists Lagrange multiplier \( (\lambda, \lambda_1, \cdots, \lambda_n) \geq 0 \) such that
\[
u_i(r_i) = \lambda > 0, \text{ for } r_i > 0,
\]
\[
C = \sum_{i=1}^{n} r_i.
\]

**Proof**
\( P1 \) is equivalent to the following form
\[
\min_{i \in \Gamma} \sum_{i=1}^{n} (-U_i(r_i)) \quad h(r) = C - \sum_{i=1}^{n} r_i \geq 0,
\]
\[
r_i \geq 0, i \in \Gamma.
\]
Let \( I = \{i | r_i = 0\} \), because at least one \( r_{io} > 0 \), so gradients
\[
\nabla_{\vec{r}} h(\vec{r}) = (-1, \cdots, -1)^T,
\]
\[
\nabla_{\vec{r}} r_i = (0, \cdots, 0, 1, 0, \cdots, 0)^T, i \in I.
\]
(where the $i$th component is 1, and the others are 0) are linear independent. If $\hat{r}$ is an optimal solution of $P1$, by the Karush–Kuhn–Tucker condition [25], we have

$$-u_i(r_i) + \lambda - \lambda_i = 0,$$

$$\lambda h(r) = 0, \lambda_i r_i = 0,$$

$$\lambda, \lambda_i \geq 0, \ i = 1, \cdots, n.$$  

If $r_i > 0$, then $\lambda_i = 0$; hence $u_i(r_i) = \lambda$. As $U_i(\cdot), i \in \Gamma$ are sigmoid utility functions, hence $u_i(\cdot) > 0$; and $\lambda_i \geq 0$, so $\lambda > 0$, hence $h(\hat{r}) = 0$, that is, $C = \sum_{i \in \Gamma} r_i$.

As $\hat{u}_i(x_i)(i = 1, \cdots, j)$ is decreasing for the class (i), $\hat{u}_i(x_i)(i = 1, \cdots, j - 1)$ is decreasing, and $u_j(r_j)$ is increasing for class (ii), then the solution of $\lambda = u_i(r_i)$ is always unique. Then if $\hat{r}$ satisfies (2.1), it must be optimal. Hence we obtain our desirable result. □

Now we can consider the solution of $P1$.

**Case 1** If $r_j > rc_j$, by Theorem 2.2, we have

$$\hat{u}_1(x_1) = \cdots = \hat{u}_j(x_j) = \lambda > 0.$$  

Let $rr_j = C - \sum_{i=1}^{j} rc_i$, then

$$rr_j = \sum_{i=1}^{j} \hat{u}_i^{-1}(\lambda).$$  

(2.6)

Denote its solution by $\lambda = k_0$. Then

$$r_i = rc_i + \hat{u}_i^{-1}(k_0), \ \ i = 1, \cdots, j;$$

$$r_i = 0, \ \ i = j + 1, \cdots, n.$$

(2.7)

**Case 2** If $r_j < rc_j$, set $rr_{j-1} = C - \sum_{i=1}^{j-1} rc_i$, then

$$rr_{j-1} = \sum_{i=1}^{j-1} \hat{u}_i^{-1}(\lambda) + u_j^{-1}(\lambda).$$  

(2.8)

Denote its solution by $\lambda = k_1$. Then

$$r_i = rc_i + \hat{u}_i^{-1}(k_1), \ \ i = 1, \cdots, j - 1,$$

$$r_j = u_j^{-1}(k_1),$$

$$r_i = 0, \ \ i = j + 1, \cdots, n.$$  

(2.9)

Note that $U_i(r_i), i = 1, 2, \cdots, n$ all are increasing, so $P1$ always has an optimal solution. Hence at least one of (2.6) and (2.8) has a solution. In (2.8), we have $\hat{u}_i(x_i)$ is a decreasing function with respect to $x_i$ for $i = 1, \cdots, j - 1$, and by adding them up as $\sum_{i=1}^{j-1} \hat{u}_i$ (as shown in Figure 5), at the same coordinate system, we can obtain the plots of $u_j$ and $\sum_{i=1}^{j-1} \hat{u}_i$. Figure 5 illustrates the procedures of how to construct Equations (2.6) and (2.8), which work as follows: Figure 5(a) is the plots of sigmoid function $U_i(\cdot) (i = 1, 2, \cdots, j - 1)$, and the plots of their derivations are shown in Figure 5(b). Along $rc_i$, we can divide them into two parts: in each part, they are all monotonous, parallel moving the last part to exactly intersect y-axis, which is shown in Figure 5(d), then adding up all of them obtain the right part of Figure 5(e). Combining this and the plot of the left plot of derivation of $U_j(\cdot)$, we have the fifth sub-figure. At the same system, we show them into Figure 5(f).

If $rr_{j-1} = a + c$, that is, the resource is allocated among the first $j$ users, and $r_j < rc_j$, then by (2.8), we have solution (2.9).

If $rr_{j-1} = b$, that is, the resource is allocated only among the first $j - 1$ users and $r_i \geq rc_i$, $i = 1, \cdots, j - 1$, then by (2.6), we have solution (2.7).

On the basis of the preceding discussions, we have the following main result:
Figure 5. The procedure of constructing equations (2.6) and (2.8).

**Theorem 2.3**

$\mathcal{R}^j_k$ is the one and the only one optimal solution, which satisfies class (ii), and its component must satisfy (2.9).
Proof

As

\[
\sum_{i=1}^{n} r_i = \sum_{i=1}^{j-1} (rc_i + \hat{u}_i^{-1}(k_1)) + u_j^{-1}(k_1)
\]

\[
= \sum_{i=1}^{j-1} rc_i + \sum_{i=1}^{j-1} (\hat{u}_i^{-1}(k_1)) + u_j^{-1}(k_1) = C.
\]

For \( i = 1, 2, \ldots, j - 1 \), we have

\[
u_i(r_i) = u_i \left( rc_i + \hat{u}_i^{-1}(k_1) \right) = \hat{u}_i \left( \hat{u}_i^{-1}(k_1) \right) = k_1;
\]

and

\[
u_j(r_j) = u_j \left( u_j^{-1}(k_1) \right) = k_1.
\]

For \( i = 1, \ldots, n \), if \( r_i > 0 \), set \( \lambda = k_1, \lambda_i = 0 \);

if \( r_i = 0 \), set \( \lambda_i = \lambda - u_i(0) = k_1 - u_i(0) > 0 \).

That is, there exists \( (\lambda, \lambda_1, \ldots, \lambda_n) \geq 0 \) such that

\[-u_i(r_i) + \lambda - \lambda_i = 0,\]

\[\lambda_i r_i = 0, \quad i = 1, \ldots, n,\]

\[C = \sum_{i=1}^{n} r_i.
\]

By Theorem 2.2, (2.9) is an optimal solution in which a total of \( j \) flows are allocated by resource. By the proof Theorem 2.2, we know that the solution of \( \lambda = u_i(r_i) \) is unique. Hence we obtain the desired result.

The preceding theorem is our new finding that enables us to find the desired optimal solution by using both conditions (2.7) and (2.9). In contrary, the paper of Kuo and Liao [17] is only able to find a sub-optimal solution using condition (2.7) alone. On the basis of Theorem 2.3, the explicit procedures on how to obtain the desired optimal solution are detailed in the next section.

3. THE ALGORITHM USQ TO OBTAIN THE OPTIMAL SOLUTION

Suppose that there are \( n \) flows sorted in decreasing order of their channel qualities in the system, and all flows have the same utility function: sigmoid utility function \( U(\cdot) \). To find the optimal solution, the following resource allocation algorithm, termed by USQ, is designed.

**USQ Algorithm**

1. Initialize \( r_i = 0, i = 1, 2, \ldots, n, rr_0 = C, \) and \( U(\mathcal{R}_{\text{sigmoid}}) = 0 \).
2. Sort all flows \( i \) in descending order of \( q_i \), and store them in the queue.
3. For \( j = 1 \) to \( n \)
   i. Solve the equation (2.6), (2.8).
   **Case 1:** If (2.6) has a solution but (2.8) does not have solution. Denote this solution by \( u = k_0 \). Then \( r_i = rc_i + \hat{u}_i^{-1}(k_0) \), for \( i = 1, \ldots, j \), and \( r_i = 0 \), for \( i = j + 1, \ldots, n \). Let \( \mathcal{R}_j = (r_1, \ldots, r_n) \).
Case 2: If (2.8) has a solution but (2.6) does not have solution. Denote this solution by \( u = \kappa_1 \). Then \( r_i = rc_i + \hat{u}_i^{-1}(k_1) \), for \( i = 1, \ldots, j - 1 \), \( r_j = u_i(k_1)^{-1} \) and \( r_i = 0 \) for \( i = j + 1, \ldots, n \). Let \( \mathcal{R}_j = (r_1, \ldots, r_n) \).

Case 3: If both (2.6) and (2.8) all have solution. Calculate \( U(\mathcal{R}_1), U(\mathcal{R}_2) \), let
\[
\mathcal{R}_j = \text{argmax} \left\{ U(\mathcal{R}_1), U(\mathcal{R}_2) \right\}.
\]
ii. Calculate \( U(R_j) \), if \( U(R_j) > U(R_{\text{sigmoid}}) \), \( R_{\text{sigmoid}} = R_j \), \( U(R_{\text{sigmoid}}) = U(R_j) \).

4. Return \( R_{\text{sigmoid}} \) and \( U(R_{\text{sigmoid}}) \).

Note that the algorithm given in [17] only solves Equation (2.6) repeatedly, then the obtained solution is certainly sub-optimal.

4. NUMERICAL EXAMPLES

In this section, we give two examples to demonstrate the explicit techniques of USQ and the solution procedures.

Example 1
We assume there are 10 soft QoS traffic in a wireless network, in which sigmoid utility functions are given [18] by
\[
U_i(r_i) = \frac{1}{1 + e^{-a_i r_i + b_i}}, \quad i = 1, 2, \ldots, 10,
\]
where
\[
(a_1, a_2, \ldots, a_{10}) = (1, 2, 1, 3, 2, 4, 1, 5, 3, 6),
\]
\[
(b_1, b_2, \ldots, b_{10}) = (5, 10, 3, 9, 8, 16, 16, 30, 6, 12).
\]

It should be noted that the preceding sigmoid utility function can be transformed into the form of (2.1). The transformation details are however omitted here. For the total capacity at the base station \( C = 58 \), we run the proposed algorithm USQ. The optimization problem at hand is therefore described into
\[
\max \sum_{i=1}^{10} \frac{1}{1 + e^{-a_i r_i + b_i}} \quad \text{s.t.: } \sum_{i=1}^{10} r_i \leq 58, r_i \geq 0.
\]

Mathematical manipulations arrive that
\[
\frac{dU_i(r_i)}{dr_i} = u_i(r_i) = \frac{a_i e^{-a_i r_i + b_i}}{(1 + e^{-a_i r_i + b_i})^2},
\]
\[
\frac{d^2 U_i(r_i)}{dr_i^2} = u_i'(r_i) = \frac{a_i^2 e^{-a_i r_i + b_i} (1 + e^{-a_i r_i + b_i}) (e^{-a_i r_i + b_i} - 1)}{(1 + e^{-a_i r_i + b_i})^4}.
\]
From
\[
\frac{d^2 U_i(r_i)}{dr_i^2} = 0,
\]
we obtain that
\[
rc_i = \frac{b_i}{a_i}.
\]
Furthermore,
\[
\hat{u}_i(r_j) = u_i(r_j + r_j) = \frac{a_i e^{-a_i r_j}}{(1 + e^{-a_i r_j})^2},
\]
\[
\hat{u}_i^{-1}(r_j) = \frac{1}{a_i} \ln \left( \frac{a_i - 2r_j + \sqrt{a_i^2 - 4a_i r_j}}{2r_j} \right),
\]
\[
u_i^{-1}(r_j) = \frac{1}{a_i} \left[ \ln \left( \frac{a_i - 2r_j - \sqrt{a_i^2 - 4a_i r_j}}{2r_j} + b_i \right) \right].
\]

When \( C = 58 \), we run Algorithm USQ. For the \( j \) \((j = 1, 2, \ldots, 10)\) allocated flows, we solve the equations (2.6) and (2.8), then we can obtain the corresponding allocated resource to every flow, we further compute the corresponding utility of the system, the allocated resource corresponding to larger utility is the optimal solution for the \( j \) allocated flows. This procedure has been displayed by Table II. Figure 6 plots the procedure and optimal solution to this utility optimization problem. Hence the resulted optimal solution of the system is

\( (5.8277, 5.9948, 3.8277, 3.8308, 4.9948, 4.7059, 16.8277, 6.6143, 2.8308, 2.5450) \).

**Example 2**

In a wireless network, we assume that the total resource is \( C \). The considered soft QoS traffic has the following sigmoid utility function

\[
U(r) = \begin{cases} 
q e^{0.2(r-9)} & r < 9, \\
1 - (1-q)e^{-0.2(r-9)} & r \geq 9.
\end{cases}
\]

<table>
<thead>
<tr>
<th>Solution of Equation (2.6)</th>
<th>Solution of Equation (2.8)</th>
<th>Optimal solution</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j = 1 )</td>
<td>( 9.6027 \times 10^{-24} )</td>
<td>No solution</td>
<td>( (58, 0, 0, 0, 0, 0, 0, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 2 )</td>
<td>( 1.5956 \times 10^{-14} )</td>
<td>No solution</td>
<td>( (36.7690, 21.2310, 0, 0, 0, 0, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 3 )</td>
<td>( 1.7495 \times 10^{-8} )</td>
<td>No solution</td>
<td>( (22.8614, 14.2773, 20.8614, 0, 0, 0, 0, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 4 )</td>
<td>( 4.6933 \times 10^{-7} )</td>
<td>( 3.7795 \times 10^{-9} )</td>
<td>( (19.5720, 12.6326, 17.5720, 8.2235, 0, 0, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 5 )</td>
<td>( 1.5383 \times 10^{-5} )</td>
<td>( 9.8938 \times 10^{-8} )</td>
<td>( (16.0822, 10.8877, 14.0822, 7.0603, 9.8877, 0, 0, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 6 )</td>
<td>( 1.1209 \times 10^{-4} )</td>
<td>( 2.0484 \times 10^{-5} )</td>
<td>( (14.0960, 9.8946, 12.0960, 6.3982, 8.8946, 6.6206, 0, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 7 )</td>
<td>( 2.5651 \times 10^{-2} )</td>
<td>( 1.6195 \times 10^{-3} )</td>
<td>( (8.6098, 7.1651, 6.6908, 4.5815, 6.1651, 5.2591, 19.6098, 0, 0, 0) )</td>
</tr>
<tr>
<td>( j = 8 )</td>
<td>( 9.7807 \times 10^{-2} )</td>
<td>( 7.2471 \times 10^{-2} )</td>
<td>( (7.0920, 6.4560, 5.0920, 4.1182, 5.4560, 4.9151, 18.0920, 6.7788, 0, 0) )</td>
</tr>
<tr>
<td>( j = 9 )</td>
<td>( 1.5649 \times 10^{-1} )</td>
<td>( 1.1463 \times 10^{-1} )</td>
<td>( (6.4229, 6.1845, 4.4229, 3.9466, 5.1845, 4.7894, 17.4229, 6.6797, 2.9466, 0) )</td>
</tr>
<tr>
<td>( j = 10 )</td>
<td>( 2.1163 \times 10^{-1} )</td>
<td>( 1.8775 \times 10^{-1} )</td>
<td>( (5.8277, 5.9948, 3.8277, 3.8308, 4.9948, 4.7059, 16.8277, 6.6143, 2.8308, 2.5450) )</td>
</tr>
</tbody>
</table>
Assume that there are five users in the system with its sigmoid utility function being $U_i(r_i)$ ($i = 1, 2, 3, 4, 5$). Set the channel quality of each user as followings $q_1 = 0.9, q_2 = 0.6, q_3 = 0.5, q_4 = 0.3, q_5 = 0.1$. Let $p_i = 0.2, i = 1, 2, 3, 4, 5$. By the analysis in Section 2, we know that $U(r)$ and $U_i(r_i)$ have the following relations: $U_i(r_i) = U(q_ir_i)$, where $q_i$ is the channel quality of flow $i$. So

$$U_i(r_i) = \begin{cases} q_i e^{0.2(q_ir_i - 9)} & q_ir_i < 9, \\ 1 - (1 - q_i)e^{-0.2(q_ir_i - 9)} & q_ir_i \geq 9, \end{cases}$$

for $i = 1, 2, 3, 4, 5$ and $r_{c1} = 10, r_{c2} = 15, r_{c3} = 18, r_{c4} = 30, r_{c5} = 90$. Therefore, the optimization problem can be described as follows:

$$\max \sum_{i=1}^{5} U_i(r_i),$$

s.t.: $\sum_{i=1}^{5} r_i \leq C, r_i \geq 0.$

When $C = 200$, we run Algorithm USQ. For the $j$ ($j = 1, 2, \ldots, 5$) allocated flows, we solve Equations (2.6) and (2.8), then we can obtain the corresponding allocated resource to every flow, and we then compute the corresponding utility of the system; the allocated resource corresponding to larger utility is the optimal solution for the $j$ allocated flows. This procedure has been displayed in Table III. Hence the resulted optimal solution of the system is

$$(22.9436, 42.5890, 51.5150, 82.9524, 0).$$

Figure 7 plots the procedure and optimal solution to this utility optimization problem.

It is interesting to note that, as displayed by Example 1, the optimal solution outputted from our algorithm is in accordance with the sub-optimal solution outputted by the algorithm in [17]; whereas as displayed in Example 2, the optimal solution outputted from our algorithm has deviations from the sub-optimal solution outputted by the algorithm in [17]. From these, we know that the optimal solution may obtain in either class (i) or (ii). Therefore, our algorithm inherits an advantage over that in [17] that one can always achieve the optimal solution by using our algorithm.
Table III. The results of running algorithm USQ when $C = 200$.

<table>
<thead>
<tr>
<th>$j$</th>
<th>Solution of Equation (2.6)</th>
<th>Solution of Equation (2.8)</th>
<th>Optimal solution</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.5258 \times 10^{-17}$</td>
<td>No solution</td>
<td>(200, 0, 0, 0, 0)</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>$0.1093 \times 10^{-6}$</td>
<td>$2.6424 \times 10^{28}$</td>
<td>(76.7306, 123.2694, 0, 0, 0)</td>
<td>2.0000</td>
</tr>
<tr>
<td>3</td>
<td>$0.5437 \times 10^{-4}$</td>
<td>$0.3119 \times 10^{-19}$</td>
<td>(42.2349, 71.5259, 86.2393, 0, 0)</td>
<td>2.9987</td>
</tr>
<tr>
<td>4</td>
<td>$0.1752 \times 10^{-2}$</td>
<td>$0.5299 \times 10^{-8}$</td>
<td>(22.9436, 42.5890, 51.5150, 82.9524, 0)</td>
<td>3.9290</td>
</tr>
<tr>
<td>5</td>
<td>$0.1713 \times 10^{-1}$</td>
<td>$0.2569 \times 10^{-6}$</td>
<td>(10.2754, 23.5866, 28.7122, 44.9477, 92.4782)</td>
<td>3.4488</td>
</tr>
</tbody>
</table>

Figure 7. Utility optimized bandwidth allocation in Example 2: procedures and optimal solution.

5. CONCLUSION

Because of the non-concavity of utility function of soft QoS traffic, it is a challenge to allocate the limited resource among the soft QoS traffic in wireless networks. Kuo and Liao [17] gave an algorithm to find its sub-optimal solution. In order to find the optimal solution, in this paper, we first establish some key theorems; on this basis, we then present a complete algorithm called USQ to obtain the desired optimal solution. The proposed theorems and algorithm act as designing guidelines for resource allocation of soft QoS traffic in a wireless network, which take into account the total available resource of network, the users’ traffic characteristics, and the users’ channel qualities. By numerical examples, we illustrate the explicit solution procedures. Future research would find it interesting to study the scenario where best effort, hard QoS, and soft QoS traffic coexist in a wireless communication system. A unified framework for resource allocation for this scenario hopefully will be developed by following the optimization approach proposed in this paper.

ACKNOWLEDGEMENTS

This research is supported by the National Natural Science Foundation of China (no. 61070197) and the Special Fund for Basic Scientific Research of Central Colleges, South Central University for Nationalities.
REFERENCES

AUTHORS’ BIOGRAPHIES

**Liansheng Tan** is now a Professor at the Department of Computer Science in Central China Normal University. Dr. Tan received his PhD degree from Loughborough University in the UK in 1999. He was a research fellow in Research School of Information Sciences and Engineering, The Australian National University, Australia, from 2006 till 2009. He was a postdoctoral research fellow in the School of Information Technology and Engineering at University of Ottawa, Canada, in 2001. He also held a number of visiting research positions at Loughborough University, University of Tsukuba, City University of Hong Kong, and University of Melbourne. Dr. Liansheng Tan is currently an Editor of *International Journal of Computer Networks and Communications*. He was an Associate Editor of *Dynamics of Continuous, Discrete & Impulsive Systems Series B: Applications & Algorithms* (2006–2008) and an Editor of *International Journal of Communication Systems* (2003–2008). He has published more than 100 referred papers widely in international journals and conferences. His research interests include modeling, congestion control analysis and performance evaluation of computer communication networks, resource allocation, and management of wireless and wireline networks, and routing and transmission control protocols.

**Zhongxun Zhu** is an Assistant Professor at the Faculty of Mathematics and Statistics, South-Central University for Nationalities, China, since 2010. Dr. Zhu received his PhD degree from Central China Normal University in China in 2011. His current research interests are in the area of wireless network, Potts model and its application, and graph theory.

**Wei Zhang** received his BSc, MSc, and PhD degrees from Central China Normal University, Wuhan, China, in 2002, 2005, and 2008, respectively. He is now a lecturer at the Department of Computer Science in Central China Normal University. His current research interests include network resource allocation and congestion control.

**Gong Chen** received his PhD degree from Central China Normal University in 2012. His current research interests lie in the areas of quality of service in wireless ad hoc networks and communication networks.