Successive Interference Cancellation in Multiple Data Rate DS/CDMA Systems*

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Abstract—In this paper we analyse a successive interference cancellation (IC) scheme for M-ary QAM modulation in direct sequences code division multiple access (DS/CDMA) systems. The performance under Rayleigh fading is analysed for single modulation systems and systems employing either of two different methods, mixed modulation or parallel channels, for handling multiple data rates. We show that the successive IC scheme for higher modulation together with mixed modulation or parallel channels give a considerable increase in performance and flexibility compared to single modulation systems employing a conventional detector.

I. Introduction

In the future we will demand mobile telephone systems to be able to handle services with other requirements than speech, like e.g. facsimile, Hi-Fi audio and computer data, which is not possible today. To achieve this we need to use a multiple-access method which is flexible and has the prospect of capacity increases and being able to handle variable data rates. Recently code division multiple access (CDMA) has been suggested to be a multiple-access method able to stand these requirements.

A direct sequence code division multiple access (DS/CDMA) system has several unique features. Some of them are spectrum sharing, rejection of multipath signal components or possibility to utilizing them for recombining [2] and frequency reuse factor of one in a cellular scenario [3]. These features are highly desirable, though a CDMA system employing a conventional matched filter detector is interference limited, which directly determines the system capacity. The presence of a number of users in the system introduces multiple-access interference (MAI), since the signature sequences used are not perfectly orthogonal, which leads to an irreducible error probability. In a mobile radio scenario the transmitters move in relation to the receiver and the energies of the received signals are to be neither equal nor constant. In this situation the conventional detector fails to demodulate weak users, even when the cross-correlation between the signals is relatively low. This is known as the near/far problem. One way to combat this problem is to use stringent power control [3]. Another approach is to use more sophisticated receivers which are near/far resistant. Recently a lot of attention has been directed to the area of multiuser detectors, which has the prospect of mitigating the near/far problem and by cancelling the MAI also increasing the total system capacity.

The optimum multiuser detector is very complex and that has initiated further research in the area of sub-optimal, lower complexity, detectors both using parallel detection [4]-[6] and serial or successive interference cancellation (IC) [7]-[9]. See also reference list in [1].

The motivation for our work is to evaluate an efficient detector for a multiuser and multirate DS/CDMA system. A method to handle multiple data rates is to let different users employ different modulation formats. A user could, depending on the need, choose between using e.g. BPSK, QPSK or 16-QAM modulation. Therefore, an IC scheme for M-ary QAM is analysed in this paper, based on the IC scheme for coherent BPSK modulation derived by Patel and Holtzman [8], [9]. Another approach to handle multiple data rates is to let each user transmit over one or several parallel channels according to requirements.

This paper will give an outline of the work, a deeper analysis will be found in [1]. We will consider the coherent case of demodulation and frequency-nonselective fading. The performance measure used is average bit error probability. We assume perfect knowledge of the phase, the time delay and the channel gain for each signal. Accordingly, we also assume perfect power ranking in the performance analysis.

II. System Model and Decoder Structure

We consider a system model for a system utilizing square lattice QAM, where the received signal, for a K user system, is

$$r(t) = \sum_{k=1}^{K} \alpha_k \sqrt{\frac{2E_0}{T}} d_k^I(t-\tau_k)\, c_k^I(t-\tau_k) \cos(\omega_c t + \phi_k) + \alpha_k \sqrt{\frac{2E_0}{T}} d_k^Q(t-\tau_k)\, c_k^Q(t-\tau_k) \sin(\omega_c t + \phi_k) + n(t) \quad (1)$$

This is the sum of all the transmitted signals embedded in AWGN. The information-bearing signal, $d_k^{I/Q}(t)$, for each users is an infinite sequence of $A_{k,l}^{I/Q}$ amplitude, rectangular pulses of duration T. $A_{k,l}^{I/Q}$ is the amplitude of the quadrature carriers for the k^{th} user's l^{th} symbol element, which together generate M equiprobable and independent symbols. $2E_0$ is the energy of the signal with lowest amplitude. Each user k

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has two signature sequences, $c_k^{I/Q}(t)$, which are used for spreading the signal in the in-phase (I) and the quadrature (Q) branch. Each sequence consists of a sequence of, antipodal, unit amplitude, rectangular pulses of duration T_c. The period of all the users' signature sequences is $N = T/T_c$, hence there is one period per data symbol. In the asynchronous, though symbol-synchronous, case the time delay, τ_k, and the phase, ϕ_k, are i.i.d. uniform random variables over $[0, T)$ and $[0, 2\pi)$, respectively. Both parameters are assumed to be known in the analysis. ω_c represents the common centre frequency, α_k represents the channel gain and $n(t)$ is the AWGN with two-sided power spectral density of $N_0/2$.

Fig. 1 shows the structure of the i^{th} user's receiver. The I-branch as well as the Q-branch is correlated with both the I and Q signature sequence of the i^{th} user to form four different $Z_{i,l}$ factors at each instant of *T*. These factors, which are the sufficient statistics, are then combined to obtain the decision variables, $S_{i,l}^I$ and $S_{i,l}^Q$, for the I and Q components of the i^{th} user. They consist of a constant, the symbol amplitude, and a noise component.

III. Successive Interfernces Cancellation Scheme with M-ary QAM

A receiver with IC is shown in Fig. 2. The received signal is correlated with each signature sequence and the correlator outputs are used both for deciding which user is the strongest and in the cancellation of that users signal. The strongest user, the one with the largest α_k, is decoded first and cancelled at baseband from the received signal. (The detector is a coherent detector and it is assumed to perform ideally.) Let us assume we have the means of deciding which user is the strongest and therefore also the one most likely to be decoded correctly. A suggestion of such a scheme is given in [9]. Without loss of generality we consider the decision of symbol element 0 and that user 1 has the strongest signal. Accordingly, $A_{1,0}^{I/Q}$ is decoded first at time $\tau_1 + T$ and cancelled at baseband from the composite signals. Subsequently all the users are decoded and cancelled in decreasing order of their powers. We use the previous defined decision variable, $S_{1,0}^{I/Q}$, to estimate user 1's

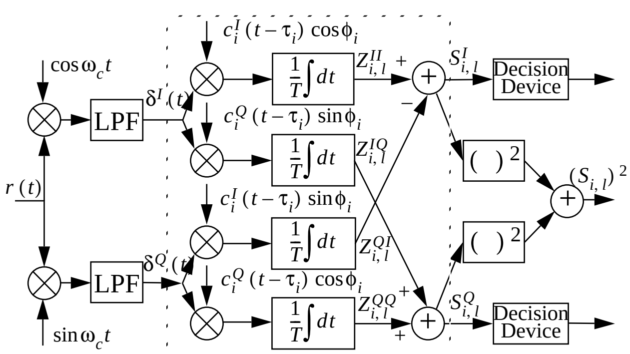

Fig. 1. M-ary QAM DS/CDMA receiver.

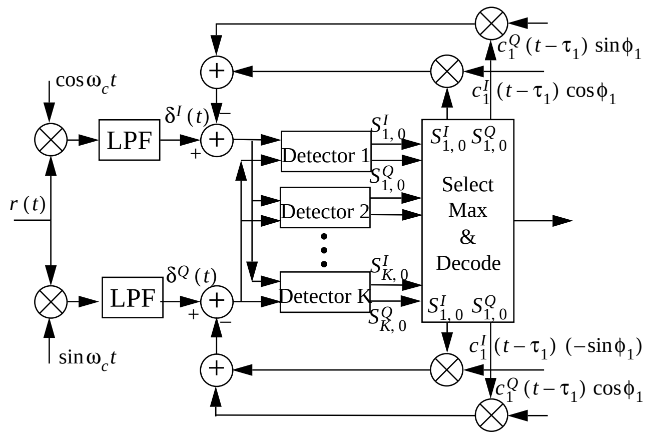

Fig. 2. M-ary QAM receiver with IC.

baseband signal for symbol element 0 and cancel it from the composite signal. We assume that all the symbol elements prior to the zeroth have been cancelled for all users, and the resulting baseband signal, $\delta_1^{I/Q}(t)$, will consist of all K users' baseband signals from symbol element 0 and onwards, except user 1's zeroth symbol element. The cancellations have however not been performed perfectly. The signature sequences are not completely orthogonal and the correlation outputs contain Gaussian noise and therefore some additional noise is included in $\delta_1^{I/Q}(t)$. In a situation like the one shown in Fig. 3, the strongest user, user 1, has a time delay τ_1 which is shorter than any other user's time delay as e.g. user 2's. In this case it is not enough to cancel only symbol element 0 from the composite signal to reduce the interference noise since user 2's zeroth symbol element is partly correlated with user 1's symbol element 1. To overcome this we decode and cancel both user 1's zeroth and first symbol element before we continue the IC scheme for the second strongest user. Consequently, without loss of generality, we may assume in the analysis that $\tau_1 > \tau_2 > \ldots > \tau_K$.

Proceeding in the same manner we cancel symbol element 0 for the second strongest user, i.e. user 2, and all the other users in decreasing order of their powers. The decision varia-

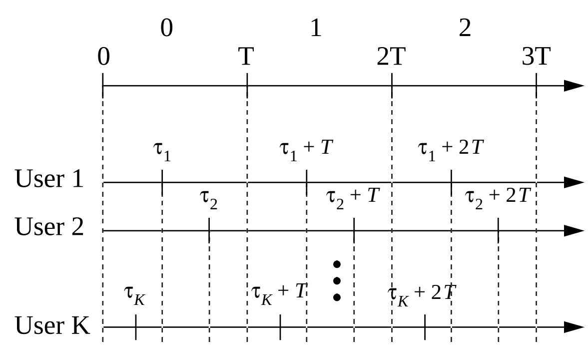

Fig. 3. Asynchronous system.

ble, $S_{h,0}^{I/Q}$, before the h^{th} cancellation will be as follows

$$S_{h,0}^{I/Q} = \sqrt{\frac{E_0}{2T}}\alpha_k A_{h,0}^{I/Q} + N_h^{I/Q} \quad (2)$$

where $N_h^{I/Q}$ is the total noise component which consists of noise caused by the remaining *K-h* interfering users, noise caused by imperfect cancellations of the *h-1* stronger users and AWGN [1].

IV. Performance Analysis

In the case of fading, we consider a system where the *K* users are received through independent, frequency-nonselective slowly fading channels. When the symbol period, *T*, is less than the coherence time, the multiplicative process, caused by fading, may be regarded as constant during at least a few signal intervals. This can be considered the condition for the term slowly fading [2]. The channel has to fade sufficiently slow for making it possible to estimate the phase shift, ϕ_k, from the received signal of the k^{th} user adequately. This is a requirement for coherent detection [2]. Operating an IC scheme for M-ary QAM also demands that the fading is slow, since the channel gain, α_k, has to be estimated for correct decoding of the symbols and the ranking of the users. No instantaneous power control is applied, only average power control which takes care of shadowing and distance attenuation.

In this paper we use the same method, for analysing the single-path Rayleigh fading channel, as was used in [9] for BPSK modulation. We assume that α_k is estimated perfectly and we start with calculating the variance of the I and Q channel decision variables conditioned on α_k, i.e.

$$\eta_h^{I/Q} = \mathrm{Var}\left[N_h^{I/Q}\middle|\alpha_h\right] \quad (3)$$

We assume that the random variables in $N_h^{I/Q}$ are independent and it can be shown that they have zero mean. Consequently we can use the Gaussian approximation and model $N_h^{I/Q}$ as an independent Gaussian random variable with zero mean and variance $\eta_h^{I/Q}$. When using the Gaussian approximation it is easy to obtain the probability of error from the theory of single transmission of QAM signals over an AWGN channel [2]. The symbol error rate with ideal coherent detection can be expressed as

$$\hat{P}e_h = 1 - (1 - \hat{P}e_h^I)(1 - \hat{P}e_h^Q) \quad (4)$$

where $\hat{P}e_h^{I/Q}$ is the error probability in the I or Q channel. We get the distributions of the ordered amplitudes by using order statistics [10]. Then the unconditional error probability for each stage of cancellation is obtained from the conditional probability of error as follows

$$\hat{P}e_h^{I/Q} = \int_0^\infty Pe_h^{I/Q} f_{\alpha_h}(x)\,dx \quad (5)$$

where $f_{\alpha_h}(x)$ is the distribution of the h^{th} strongest user's amplitude and $Pe_h^{I/Q}$ is that user's conditional probability of error for the transmission over the I channel, respectively the Q channel, given by [2]

$$Pe_h^{I/Q} = 2\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)Q\left(\rho_h^{I/Q}\right) \quad (6)$$

where the Q-function is the complementary Gaussian error function. $\rho_h^{I/Q}$ in (6) is a signal-to-noise ratio for the I or Q channel defined as

$$\rho_h^{I/Q} = \frac{\sqrt{\frac{E_0}{2T}}\alpha_h}{\sqrt{E_{\alpha_k}\left[\eta_h^{I/Q}\right]}} \quad (7)$$

where $E_{\alpha_k}\left[\eta_h^{I/Q}\right]$ is the expected value of the conditional variance with respect to α_k [1]. Transmission under Rayleigh fading for systems with IC employing different modulation formats, though the same total transmitted bit rate, and the single BPSK and 16-QAM user bounds are shown in Fig. 4. There are 60 BPSK, 30 QPSK respectively 15 16-QAM users, with the code length 127, in the systems. The performance of the systems is depicted as average bit error rate (BER) as a function of bit energy per noise spectral density (E_b/N_0). The amplitudes of the users determines the order of cancellation, thus the order will change continuously with fading. This is the reason for using average BER as a measure of performance, since we are not interested in the best or worse performance of a user at one instant of time. The result shows that the IC scheme gives a considerable decrease in BER and even

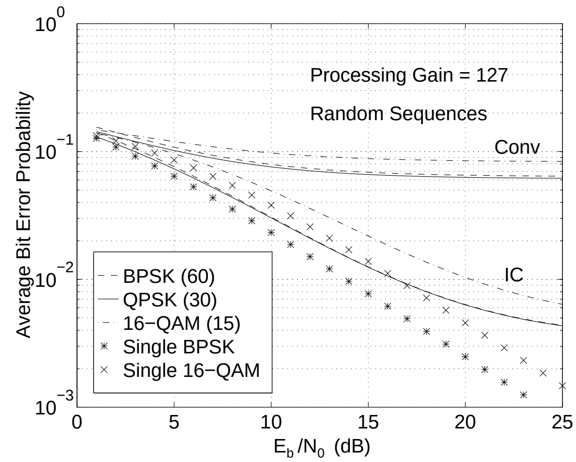

Fig. 4. Performance of single modulation systems with and without IC under Rayleigh fading.

with a large number of users in the systems the performance is close to the single user bounds.

V. Performance Analysis of Mixed Modulation Systems

One way to handle multirate systems is to let each user choose a modulation format in correspondence with required transmitted data rate [11]. We have analysed the performance of a system where the users employ different forms of modulation, e.g., a combination of K_1 BPSK, K_2 QPSK and K_3 16-QAM users. To make a comparison between different forms of modulation we let the average SNR/bit be equal for all users. That is, the transmitted bit energy, E_b, is equal and independent of the employed modulation format. We rewrite the energy, E_0, as a function of E_b. We then use this expression in (7) where we redefine $\eta_h^{I/Q}$ as the variance of a sum of noise components, including noise caused by interference from both BPSK, QPSK as well as 16-QAM users and Gaussian noise [1]. When calculating the average BER for the whole system we weight together each users' BER using the rates as weights. A mixed system with 20 BPSK, 10 QPSK and 5 16-QAM users, with code length 127, is shown in Fig. 5, together with the same mixed system employing a conventional detector. In each mixture the order of the users is random, independent of modulation format. The result is an average over 100 mixtures. The average over a finite number of mixtures may affect the result, but because of the time consuming calculations the number had to be limited. The average BER for a BPSK, QPSK respectively 16-QAM user in the mixture is also shown in the figure. Mixed modulation systems offers more flexibility with the cost of a slight increase in average BER compared to pure asynchronous QPSK systems. The 16-QAM users in the mixed system, who are most sensitive to noise, will have the highest BER.

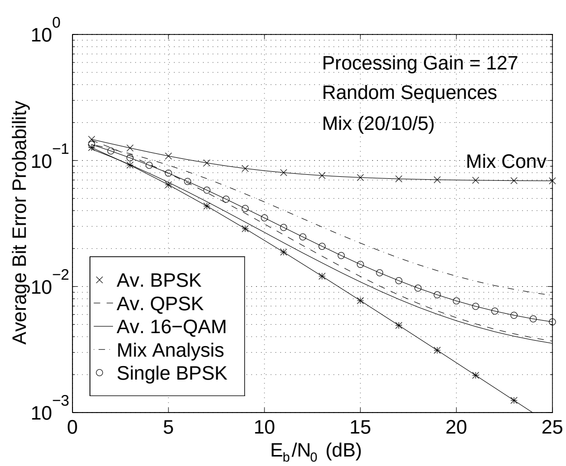

Fig. 5. Performance of mixed modulation system with and without IC under Rayleigh fading.

VI. Unequal Powers of the Users within the System

The disadvantage of using mixed modulation as a method of handling multiple data rates is that the 16-QAM users have a higher average BER than the BPSK and QPSK users. A possible way to reduce the BER of the 16-QAM users is to increase their transmitted power. The increase in power, however, should not be too large since this causes additional interference to the BPSK and QPSK users. Though, since we know that the 16-QAM users are the ones most sensitive to interference [1] a small increase in power may endorse them with minor effect on the other users.

To evaluate the performance of a mixed system with unequal powers we use two different Rayleigh distributions for the 16-QAM users respectively the BPSK and QPSK users. Thus, we have a system with K_3 16-QAM users distributed with one power level and K_1 BPSK and K_2 QPSK users distributed with another. In this case the theory of order statistics can not directly be used. We have a distinct number of users from two distributions and therefore the distribution for each ordered user has to be calculated separately. Fig. 6 shows the average BER for a mixed system with 20 BPSK, 10 QPSK and 5 16-QAM users with the code length 127. The power of the 16-QAM users is increased in steps corresponding to 1 dB E_b/N_0 compared to the power of the other users, which is kept constant. The E_b/N_0 for the other users, BPSK and QPSK, is 18 dB. The performance is evaluated from an average of 200 mixtures. The mixtures were formed as follows: $K_1 + K_2$ amplitudes were picked from the Rayleigh distribution with constant variance and K_3 from the other. Then the order of the different users was determined from the order of the amplitudes. The graph shows that the average BER of the 16-QAM users may be decreased by increasing the power of these users without severe degradation of the performance of

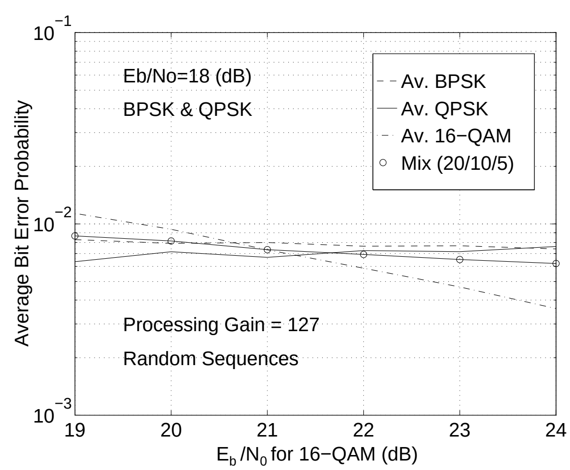

Fig. 6. Performance of mixed modulation system under fading with users employing different power levels.

the BPSK and QPSK users. Hence, by increasing the power with an amount corresponding to about 2 dB E_b/N_0 the average BER of the 16-QAM users is just slightly higher than for the BPSK and QPSK users. Letting the 16-QAM users have a slightly higher BER is a fair price for a higher data rate.

VII. Performance Analysis of Parallel Channel Systems

We have previously discussed the possibility to handle multirate systems by the means of employing M-ary QAM modulation. Another alternative is to use parallel channels for transmission of information from each user [11]. We simply let a user send simultaneously over as many channels as required for a specific data rate. The disadvantage of such a scheme is that the number of channels will be increased, though, on the other hand, if codes with good cross-correlation properties are used the interference from the pure synchronous signals is considerably less than from the asynchronous signals. A comparison between a system with 15 QPSK users employing two parallel channels each and asynchronous systems with 15 16-QAM respectively 30 QPSK users under Rayleigh fading is shown in Fig. 7. In this case Gold codes, with the length 127, have been used. (Note that this gives a slightly better performance, see Fig. 4.) The parallel channel system is shown only in the case of IC and the average BER curve is very close to the one for the asynchronous QPSK system. (The two curves are almost not distinguishable.) This may be explained by the fact that the gain in performance obtained from good cross-correlation properties between synchronous signal is lost because of more interference. The next strongest user causes now twice as large interference since this user transmits over two channels. The result shows that it is better for a user to employ QPSK modulation and transmit over two channels than to employ 16-QAM modulation. (This is without unequal power levels.)

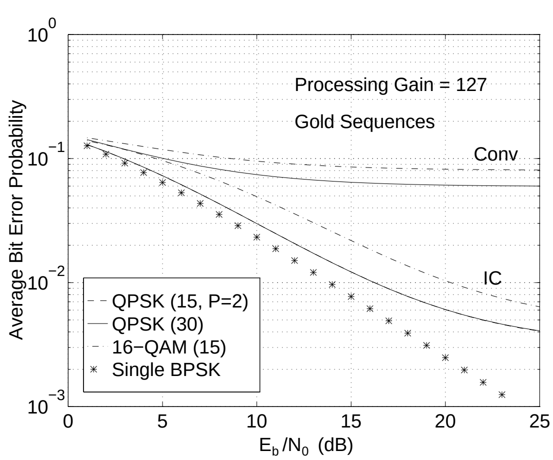

Fig. 7. Comparison between a parallel channel QPSK system and asynchronous 16-QAM and QPSK systems.

VIII. Summary and Conclusions

We demonstrate the use of M-level rectangular QAM modulation together with successive IC in single modulation systems and compare the average BER between the systems. We analyse two different methods, mixed modulation and parallel channels, for handling multiple data rates.

The conclusion is that the successive IC scheme has relatively low complexity even in the case of M-ary QAM modulation and yields considerable increase in performance. Mixed modulation systems offers more flexibility, though the different users in the system will have unequal average BER. We show, however, that if we let the noise sensitive 16-QAM users increase their power, compared to the BPSK and the QPSK users in the system, the average BER for the 16-QAM users may be decreased without considerably affecting the other users. The parallel channel system performs, in average, better or equally well as the mixed modulation system but the drawback with parallel channels is that sooner or later we run out of signature sequences. In some cases it is then better to add a 16-QAM user instead of a QPSK user with two parallel channels. Accordingly, a system which combine both mixed modulation and parallel channels will of course be the most flexible kind of variable data rate system.

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