XPath: (P)DL on trees.

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ReasoningWeb2009

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Overview

1. Knowledge Representation on the Web

2. Logical research questions for XML

3. Getting familiar with XPath(s)

4. Zoom in
   i. Expressivity
   ii. Complexity

5. Conclusions
KR on the Web

**ABS2000** Edge labelled graphs queried by regular path expressions

**XML** Node labelled sibling ordered trees queried by XPath

**RDF** triples and non wellfounded sets

- ... but most web information is of course in the form of ...
KR on the Web

ABS2000 Edge labelled graphs queried by regular path expressions

XML Node labelled sibling ordered trees queried by XPath

RDF triples and non wellfounded sets

• ... but most web information is of course in the form of ... text
KR on the Web

**ABS2000** Edge labelled graphs queried by regular path expressions

**XML** Node labelled *sibling ordered* trees queried by XPath

**RDF** triples and non wellfounded sets

- ... but most web information is of course in the form of ... text sometimes generated from a *relational database*.

- This talk: XML.
Graphs and trees

- Edge labelled graphs can very directly encode ER diagrams.
- These can always be represented as trees
  - Sometimes as just trees
  - Cyclic information needs ID’s and IDREF’s.
Consequences of the choice of your representation

• query processing costs

• needed expressive power for your
  ★ query language
  ★ constraint language

• robustness for changes in the data-structures
Example: interviews

- Sigmod Record Distinguished DB Profiles

- Simple model:

  An interview consists of a list of questions each followed by a list of answers.
XPath: (P)DL on trees.

exemelify this
In practice

pdftohtml -xml |
saxon MakeInterviewTree.xsl >> interview.xml

Quiztime

1. How will the output of pdftohtml look as a tree?

2. What will be the easiest (and fastest) tree transformation?

3. Which of the 4 tree models?
In theory: TREE model

- **Query**: give me all QA pairs.
In theory: TREE model

- **Query:** give me all QA pairs.

- In “hybrid DL”:

  - for $q$ such that $q \models Q$, return
    
    
    $$ ($q, \{ a \mid a \models A \cap \exists.\text{parent } q \} )$$
In theory: TREE model

- In XPath 2.0:
  
  - for $q$ in //Q return ($q,$q/A)
**In theory: TREE model**

- In XPath 2.0:
  - for $q$ in //Q return ($q,$q/A)
**XPath and Description Logic**

- Specifying nodes from a different perspective

- In Description Logic you describe the node that you want as if you are standing on the wanted node.

- In XPath you describe how to get there, as if you are standing at the root.
Same query on the practical FLAT model

- **Query**: return all A-nodes answering a give Q node
- **Tree model**: simple ALC-formula using the tree-order
- **Flat tree model**: 

![Diagram showing a tree structure with nodes labeled Q, A, and interview.](image)
Same query on the practical FLAT model

- **Query**: return all A-nodes answering a given Q node

- **Tree model**: simple ALC-formula using the tree-order

- **Flat tree model**:
  - use the document-order or the sibling-order
  - all A nodes **after** the given Q, but **before** the next Q
  - 3 variables . . .
  - not modally expressible . . .
  - the wanted A-nodes must satisfy $A \land \text{since}($q, $\neg Q)$
Lesson Learned

• Choice of representation influences what query-language may be needed later-on.
Constraining the models: theory vs practice

- XML constraint languages are based on tree-automata

- languages use regular expressions over node-labels.

- these describe the children of a node read from left to right

Flat model
Constraining the models: theory vs practice

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- Languages use regular expressions over node-labels.
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**Flat model**

```
interview -> (Q,A+)+
```

**Tree model**
Constraining the models: theory vs practice

• XML constraint languages are based on tree-automata

• languages use regular expressions over node-labels.

• these describe the children of a node read from left to right

**Flat model**

```
interview -> (Q,A+)+
```

**Tree model**

```
interview -> Q+.  Q -> A+
```

**Data** Actual question and answer text is stored in attribute nodes.
Constraining the models: theory vs practice: robustness

- **Example:** Extend our constraints: every interview ends with a bye-bye question which receives no answer.

- In all models this is expressible as a FO sentence: thus a regular tree language.

*New Flat model*
Constraining the models: theory vs practice: robustness

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**Easy:** interview → (Q,A+)+,Q

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Constraining the models: theory vs practice: robustness

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*New Flat model*

**Easy**: interview -> (Q,A+)+,Q

*New Tree model*

**Hard**: Not expressible by a DTD. (Proof later)
Bad!

- Difficult to accept and understand non-expressibility by practitioners
- leads to underspecified documents
- leads to frustration and unsafe coupling
New Tree model

- We need types to express the last answerless question.

- Specialized DTD's = MSO = regular tree languages [Papakonstantinou, Vianu 00]

- NormalQ and EndQ are types of Q

- interview -> NormalQ+, EndQ

- NormalQ -> A+

- EndQ -> EMPTY
New Tree model

- We need types to express the last answerless question.

- Specialized DTD’s = MSO = regular tree languages  
  [Papakonstantinou, Vianu 00]

- NormalQ and EndQ are types of Q

- interview $\rightarrow$ NormalQ+,EndQ

- NormalQ $\rightarrow$ A+

- EndQ $\rightarrow$ EMPTY

- This is not expressible in XML Schema!
Relax

- But it is expressible in Relax NG.
- In exactly the way given.
- Relax NG is a Schema Language by Clark and Murata.
KR on the web: wrap up

- Most information on the web is in implicitly structured text.
- **Asking complex queries to the web** thus means to extract and make this structure explicit.
- This often leads to rather flat ("reading text-ordered") XML.
- KR languages are important to **describe, constrain and validate** the XML,
- because these **XML files are themselves often input** to other knowledge-extraction programs (tree-transformations, queries)
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XML-tasks

[Schwentick 04] distinguishes the following four:

- Validation
- Transformation
- Navigation
- Querying

Every task must be described in some (logical) language.
Usual research questions

Given some language $L$

- What tasks can I express in $L$? How well can I express them in $L$?
- Given an $L$ expression and data, what are the computation costs to perform the task?
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Given some language $L$

- What tasks can I express in $L$? How well can I express them in $L$?

- Given an $L$ expression and data, what are the computation costs to perform the task?

- Each task may involve more specialized questions: e.g.

  - **Typechecking**: given input conform $I_1 \in L_1$, given a transformation $T \in L_T$, will the output always be conform $I_2 \in L_2$?

- [Milo, Suciu, Vianu, 00] Decidable for DTD and Core XSLT.
This talk: focus on validation and navigation

Expressive power on trees

- relative to yardsticks as CQ, FO, MSO, tree automata
- semantic characterizations
- succinctness questions
- rewrite systems
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Expressive power on trees

- relative to yardsticks as CQ, FO, MSO, tree automata
- semantic characterizations
- succinctness questions
- rewrite systems

Complexity

- Model checking: given a tree $T$ and a formula $F$, does $T$ satisfy $F$?
- Static analysis: containment, equivalence, satisfiability of expressions.
Major techniques and strategies

• Similar research strategy as in DL: understand a language landscape by asking the same question for many different fragments.

• Where are the borders of decidability and tractability?

• Develop handy tools to show that something is not expressible in some fragment.

• Techniques include
  • Finite models
  • Tree automata, regular tree languages
  • tree decompositions
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XPath

- two sorted language, just as (P)DL
  - path sort binary relation between nodes
  - node sort set of nodes

- interpreted on a special class of models:
  - finite, sibling ordered, node-labelled unranked trees

- XPath, like DL, is not a language, more a “style”, a “family”
Operators on node sort are very familiar

- atomic tests
- test for being in the domain of a relation. (just like $\exists R. F$)
- closed under the booleans.
- (sometimes) $n \models R= S$ iff $\exists m. (n, m) \in R \cap S$. 
Operators on node sort are very familiar

- atomic tests
- test for being in the domain of a relation. (just like $\exists R.F$)
- closed under the boolean.
- (sometimes) $n \models R=S$ iff $\exists m. (n, m) \in R \cap S$
- term-definable from $w \models R_{\text{loop}}$ iff $(w, w) \in R$
- $R=S \equiv (R; S^{-1})_{\text{loop}}$
Primitive relations are tree relations

- down, up, left, right
- their transitive closures: descendant, ancestor, ...
- often syntactic sugar: following = ancestor*/right+/descendant*
- stay relation with a test:
Operators on path sort are also very familiar

- **Regular operators:** union, concatenation, Kleene closure

- **Boolean operators:** intersect and except

- **Variables and binders:** as in hybrid logic.
  - `for $x$ in PATH1 return PATH2`
  - **Meaning:** $\downarrow y.PATH1/\downarrow x. @y/PATH2$
Immediate relations to known formalisms

- node and path-formulas of PDL
- almost all operators can be found in some DL-language
- **Trees**: CTL, tree logics of [Blackburn, de Rijke, Meijer-Viol ’96]
- without Kleene *, all languages are inside FO.
Real life complications (1)

- Two syntaxes

- Unix path style:
  
  `/book//section[./paragraph[contains(.,'''XML''')]]`

- Official style:
  
  `/child::book/descendant::section[child::paragraph[contains(.,'''XML''')]]`

- Unix style only “up and down”. Official style: everything.
Real life complications (2)

XPath has many uses and interpretations.

1. Path formula denotes **binary relation**
   
   when used for navigation within other languages

2. Path formula denotes **set of nodes**
   - when used as a stand-alone query language
   - Meaning of PATH is **range** of PATH.
   - Natural with /PATH (all nodes reachable by PATH from the root)

3. Path formula denotes a **set of trees**
   - XPath used as a constraint language
   - “all trees having a PATH from the root”
Example

- **Task** Express the tree-like interview model in XPath.

- For **N** a node-formula (“modal formula”), **N** holds everywhere iff the root starts path

  \[. \text{[not //*[not N]]}.\]

\[Q \rightarrow \mathcal{A}^+\]
Example

- **Task** Express the tree-like interview model in XPath.

- For $N$ a node-formula ("modal formula"), $N$ holds everywhere iff the root starts path

  $$ \text{.}[\text{not } //*[\text{not } N]]. $$

- $Q \rightarrow A^+$  

  Q and (not child::A or child::*[not A])
Example

- **Task** Express the tree-like interview model in XPath.

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last $Q$ without $A$
Example

• Task Express the tree-like interview model in XPath.

• For $N$ a node-formula ("modal formula"), $N$ holds everywhere iff the root starts path

  $.[\text{not } //*[\text{not } N]].$

$Q \rightarrow A+$  

Q and (not child::A or child::*[not A])

last Q without A  

Q and not right::Q and child::A
Real life benefits

• Firefox and IE support XPath.

• Fast free XPath evaluators (Saxon, Libxslt)

• Good editors for XPath available
  ✴ syntax highlighting
  ✴ help with debugging
  ✴ evaluation on XML docs
XPath practice

We define two information needs in terms of XPath.

1. a descendant with lots of specific ancestors along the way

2. question-answer pairs
Practice 1

Q return all q descendants of current node
Practice 1

Q return all q descendants of current node

A descendant::q or .//q
Practice 1

**Q** return all \( q \) descendants of current node

**A** `descendant::q` or `./q`

**Q** return all \( q \) descendants reachable through \( p_1, \ldots, p_n \) nodes
Practice 1

Q return all q descendants of current node

A descendant::q or ./q

Q return all q descendants reachable through $p_1, \ldots, p_n$ nodes

A1 $./p_1//q$ intersect $\ldots$ intersect $./p_n//q$
Practice 1

Q return all q descendants of current node

A descendant::q or .//q

Q return all q descendants reachable through p₁, . . . , pₙ nodes

A₁ .//p₁//q intersect ... intersect .//pₙ//q

A₂ big union for all permutations ρ of 1, . . . , n of

.//p_{ρ(1)}///p_{ρ(2)}/// . . . //p_{ρ(n)}///q
Practice 2: question-answers pairs

• Flat (QA+)+ models

• Find an XPath expression $x/\ldots$ which returns
  ★ when $x$ is bound to a Q node
  ★ all following A nodes until the next Q.

\[ x \]

\[ \ldots QAA \quad Q \quad AAAAQAQAAAAA \ldots \]
Kleene style
Kleene style

\[ \text{\$x/(right::A)^+.} \]

- \((.)^+\) is the transitive closure operator.
- But \((.)^+\) is not available (and not expressible) in W3C XPath dialects (because that is just FO).
Tarski style
Tarski style

\[ \text{XPath: (P)DL on trees.} \]

\[ \frac{x}{\text{following-sibling::A except following-sibling::Q/following-sibling::A}} \]

- Expressible in XPath with Booleans on path expressions [Hidders, 2003]
Frege (or first-order) style
Frege (or first-order) style

$x$/following-sibling::A[ not preceding-sibling::Q/preceding-sibling::Q[. is $x]]$

- Uses variables bound to nodes
- Test $\text{is } x$ is the hybrid logic variable test.
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Expressivity questions on trees

Rabin’s theorem sets a clear upper bound:

$$\text{MSO} = \text{tree automata} = \text{regular tree languages} = \text{decidable}.$$ 

Questions we will survey:

- expressivity relative to yardsticks
- succinctness
- semantic characterizations

Signature of the languages:

equality, unary predicates for nodes, child, descendant, right, right+
Four XPath dialects

Four flavours of XPath strictly below MSO [ten Cate, M. 2007 survey]

**Core XPath** $\approx$ PDL without *

**XPath 2.0 no vars** $\approx$ Boolean modal logic $\approx$ Core XPath plus booleans on paths

**XPath 2.0** $\approx$ hybrid Boolean modal logic

**Regular XPath** PDL with the four one-step tree relations.
Characterization of Core XPath

- On unary trees (= the line), this is Prior’s temporal logic with $F$ and $P$.

- Kamp’s theorem ’68 not enough to capture $\text{FO}(x)$ on the line.

- [Etessami, Vardi and Wilke ’97]: expressive power is exactly $\text{FO}_2(x)$, with an exponential succinctness gap.
  - “any two nodes that agree on $p_1, \ldots, p_n$ also agree on $p_0$”
  - linear constraint in $\text{FO}_2$, exponential in $\text{TL}$.

- Generalizes to sibling-ordered trees and Core XPath.
Core XPath plus booleans on paths

- Kamp’s thm on unary trees: $\text{FO}(x) = \text{FO}_3(x)$.

- [M. 2005]: Generalizes to XML-trees and paths: $\text{FO}(x,y) = \text{FO}_3(x,y)$

- Tarski’s thm: $\text{FO}_3(x,y) = \text{Tarski relation algebras}$.

- on trees: Tarski relation algebras = Core XPath plus booleans on paths

- Core XPath plus booleans on paths = $\text{FO}(x,y)$ on XML trees.
Regular XPath

- Captures \( FO(x, y) \) (because it captures “since and until”).

- [ten Cate 06] With additional loop it captures \( FO^*(x, y) \).

  \[
  T, x \models R^{\text{loop}} \iff T, (x, x) \models R.
  \]

- [ten Cate, Segoufin 08] With additional subtree relativization it captures FO extended with monadic TC.

  \[
  T, x \models W\phi \iff T_x, x \models \phi.
  \]

- [ten Cate, Segoufin 08] Both are strictly less expressive than MSO.
## Summary

<table>
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<tr>
<th>XPath dialect</th>
<th>Core XPath 1.0</th>
<th>Variable-free Core XPath 2.0</th>
<th>Core XPath 2.0</th>
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<tbody>
<tr>
<td>FO-dialect</td>
<td>$\exists FO_{tree}^{\text{mon}}$</td>
<td>$FO_{tree}$</td>
<td>$FO_{tree}$</td>
</tr>
<tr>
<td>(exponential succinctness gap)</td>
<td>(at least exponential succinctness gap)</td>
<td>(no succinctness gap: linear translations)</td>
<td>(non-elementary succinctness gap)</td>
</tr>
</tbody>
</table>
Semantic characterizations

• class of trees $C$ is definable in $L$ iff $C$ is closed under . . .

• Useful for inexpressivity results.

• Real-life languages (W3C standards) often have practical constraints with unexpected theoretical effects

• DTD’s: must be deterministic

\[(a+b)^*a(a+b)\] is not expressible by a DTD \[\text{[Brüggemann-Klein Wood 98]}\]

• XML schema’s must be single-typed specialized DTD’s \[\text{[Murata, Lee, Mani ’01]}\]
Characterization of single type SDTD

- [Martens, Neven, Schwentick 05] For $T$ a regular tree language, $T$ is definable by a single type SDTD iff $T$ is closed under ancestor-guarded subtree exchange.

**Ancestor-Guarded Subtree Exchange**

$T$ a regular tree language
(QA+) + Q is not definable on hierarchical models

- Interviews ending in a Q without an A.
- We could not find a DTD specifying this in the hierarchical model.
- Now we can prove it:
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## Complexity questions: evaluation

- **Model checking.** Validation, querying

  **Input** Tree, node(s), formula. **Output** Boolean

- **PSPACE complete for FO. PTIME for fixed variable FO.**

<table>
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<th>Fragment</th>
<th>Evaluation complexity</th>
<th>Reference</th>
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<td>Core XPath</td>
<td>PTime (linear)</td>
<td>[Gottlob, Koch, Pichler]</td>
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<td>Core XPath 2 no vars</td>
<td>PTime (quadratic)</td>
<td>(from FO)</td>
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<td>Core XPath 2</td>
<td>Pspace</td>
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<td>Regular XPath</td>
<td>PTime (linear)</td>
<td>(from PDL)</td>
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<td>Regular XPath+</td>
<td>PTime (linear)</td>
<td>[Gottlob Koch 04]</td>
</tr>
<tr>
<td>TMNF tests (MSO)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Complexity: Static analysis

- Satisfiability, equivalence, . . .

- Decidable for MSO. Non-elementary hard already for FO on unary trees [Rabin; Meyer]

- Complexity overview [ten Cate, Lutz, 2007]
  - Satisfiability.
  - Lower bound is EXPTIME, already for Core XPath
  - Small language extensions may yield large leaps in complexity
XPath: (P)DL on trees.
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XML language research and (P)DL: close relations

- both rooted in KR
- trees as fundamental models
- strong emphasis on working systems
- huge tables with acronyms and complexity classes ;-)

XPath: (P)DL on trees.
strong Description Logic–XML interplay

• KR aspects

• Data integration and mediation [Halevy, Rome school] (certain answers are hard to compute)

• Design, maintenance, reuse, integration of ontologies is daily headache for XML/web-engineers

• DL’s research on modularity of TBoxes [Manchester school] seems useful.
Thank you
Thank you
Thank you