# Mapping of Ising models onto injection-locked laser systems

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An Ising model provides a prototype framework for studying various magnetic orders in frustrated spin lattice and random spin glasses [1] [2]. Hence, an Ising machine that can find a ground state of efficiently has been extensively searched in both classical and quantum domains [3]. Quantum annealing (or quantum adiabatic computation) is proposed to solve Ising models by utilizing quantum uncertainty, more specifically quantum mechanical tunneling across a potential energy landscape (PEL) [4][5]. Experimental realization of quantum annealing employed either a sample of real magnetic crystal [6] or molecular NMR technique [7]. However, in order to map a given mathematical problem onto such a quantum annealing Ising machine, non-local interaction $J_{ij}$ must be implemented artificially irrespective of actual distance between two sites. This is extremely hard to achieve in real crystal or molecules.

In this presentation, we propose a new Ising machine based on one master laser and $M$ mutually injection-locked slave lasers [8]. An Ising model is implemented by coherent feedback network using optical interference circuits instead of incoherent electrical measurement-feedback circuits. A spin degree of freedom $\sigma_{iz}$ at each site is represented by right or left circular polarization states of each slave laser. The ground state of an implemented Ising model which is represented with polarization configuration of $M$ slave lasers emerges spontaneously through the natural mode competition induced by cross-gain saturation among all candidate polarization configurations. The injection-locked laser system oscillates with a specific polarization configuration which minimizes the cost function (photon decay rate). In our proposed system $2^M$ different polarization configurations compete with each other through the cross gain saturation of a mutually injected laser system. In order to readout the computational result, we can access each partial wave existing in a specific slave laser. The Ising interaction term $J_{ij}$ can be implemented by mutual injection between the two slave lasers $i$ and $j$ via a horizontal linear polarizer, phase shifter (0 or $\pi$ □phase) and attenuator. The Zeeman term $\lambda_i$ can be implemented by injecting a horizontal linearly polarized master laser signal in addition to the vertical linearly polarized master laser output into the slave laser *i*.

We have confirmed numerically that the proposed Ising machine outputs correct answers for most cases even if the quantum noise source is neglected. For some other cases, the proposed system outputs wrong answers due to trapping in local minimum states. It is found that the Poissonian photon number noise can make the system to escape from the local minimum state and to find the global minimum state. A time delay to reach a steady state condition after the Ising and Zeeman energy terms are switched on can be considered as an upper bound of computational time. The computational time does not show a strong dependence on the problem size $M$ according to numerical simulations. It is on the order of a few *nsec* from $M$ = 2 to $M$ = 10. The relation of the computational power of the proposed Ising machine to the NP completeness will be a subject of future study.

**References**

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