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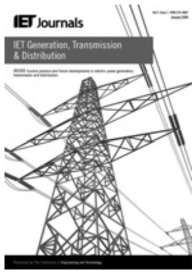

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On electrical power evaluation in *dq* coordinates under sinusoidal unbalanced conditions

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Abstract: This study develops a phasor-based analytical expression for the reactive power in *dqo* coordinates, which exhibits compatibility with IEEE Standard 1459-2010 under sinusoidal unbalanced conditions. The *dqo* coordinates are often employed in power systems, for instance in machine modelling, but the power expressions are provided in the IEEE Standard only in terms of the *abc* coordinates and the sequence components. In addition, the usual complex product $v_{dq}i_{dq}^*$ for power evaluation in the *dqo* coordinates does not exhibit reactive power consistency with the IEEE standard but for the balanced case. Such consistency becomes a central issue in the context of distributed generation systems since they are often subject to voltage/current imbalance, and the active power, the reactive power, and the power factor are important control variables and, as such, require calculation and measurement. This study provides a complete set of phasor-based analytical expressions (including the instantaneous, active, reactive, complex and apparent powers) in *dqo* coordinates which help stability and control developments of unbalanced distributed generation systems. An example of a distributed generation system with sinusoidal unbalanced voltage is given to illustrate the contribution.

1 Introduction

It is common practice to assume that loads, voltages and currents are balanced in power system stability analysis at the transmission system level, since that is very close to what happens in the actual bulk power transmission system [1]. Motivated by this fact, many simulation softwares assume a one-wire system model, especially when the focus is on the electromechanical transient phenomena.

Even though this assumption is reasonable at the transmission system level, in general this is not the case at the distribution system level. Among the factors that contribute to voltage imbalance are: the presence of single-phase loads; loads connected at several points along radial lines; larger voltage drops causing voltage imbalance along lines because of unbalanced loads connected to them. Nonetheless, correct power measurement and evaluation under such conditions is fundamental for the control and the operation of the distributed generation systems [2].

Under balanced conditions, three-phase systems can be reduced to a single-phase equivalent representation and, as such, instantaneous and rms values of electrical power can be computed by very simple expressions, with reasonably simple conceptual meaning. When the unbalanced conditions or non-sinusoidal signals come into place, there still are debates in the literature regarding the conceptual aspects, time against frequency-based characterisations, and, consequently, formulae for evaluating electrical power [3–11]. In this context, the IEEE Standard 1459-2010 [12] provides definitions for electrical power quantities under balanced, unbalanced, sinusoidal or non-sinusoidal conditions which can be taken to convey well-established concepts.

When dealing with the sinusoidal three-phase unbalanced conditions, instantaneous power values present oscillations with twice the voltage frequency, similar to what happens in single-phase circuits [1, 13, 14]. In distributed generation systems, among other situations, it might be important to quantify the amplitude of the power oscillation, besides its average value, so that other internal machine phenomena can be analysed, such as electromagnetic torque pulsation [13], for instance. In addition, in the context of electrical machinery modelling and analysis, often *dqo* Park coordinate models are employed [1, 15] and it is thus important to be able to correctly evaluate and interpret both the average values and the oscillating amplitudes of the active and the reactive power in the *dqo* coordinates under unbalanced conditions, which are, however, not provided in [12].

The traditional evaluation of the complex power as the phasor product $\tilde{V}\tilde{I}^*$ carries over to an equivalent complex product in the *dqo* coordinates when the voltages and the currents are balanced. Unfortunately, it does not carry over when the conditions are unbalanced, a fact that may easily mislead power evaluations in the *dqo* coordinates. The p–q

power theory introduced the instantaneous real and imaginary powers, defined in the dq coordinates [16–19], which coincide with the active and the reactive power under balanced conditions. However, under unbalanced conditions, the imaginary power has other interpretations and differs from the reactive power as defined in [12].

The goal in this paper is 2-fold: (i) one is to provide expressions for the reactive power evaluation in the dqo coordinates under the sinusoidal unbalanced conditions in a way that exhibits compatibility with the abc coordinates power definitions given in [12] and, by doing so, also to develop expressions for the complex and the apparent powers; (ii) the second is related to the instantaneous quantities (whose average values provide the active and the reactive powers), and focuses on the oscillations induced by the voltage/current imbalance, developing phasor-based expressions to determine the amplitude of these oscillations in the dqo coordinates. This paper contributes to the analysis of the distributed generation systems, which will often operate under unbalanced conditions, by providing a complete set of electrical power expressions in the dqo coordinates describing both the average value and the oscillating amplitude, in terms of the phasor quantities.

This paper is structured as follows: Section 2 presents the problem that is addressed in the paper. Then, Section 3 develops the expressions for electrical power evaluation in the dqo coordinates under the sinusoidal unbalanced conditions. A discussion is then presented in Section 4, relating the developed expressions with the others in the literature. In Section 5, the power expressions in the dq coordinates are applied for the analysis of a distributed generation example subject to the voltage/current imbalance. Finally, Section 6 provides concluding remarks.

Notation: Lowercase letters, f, denote time dependent variables, either real or complex. Capital letters, F, denote rms values. Superscript $\tilde{F}$ denotes phasor variables, $\bar{F}$ denotes complex variables, which will be written as either $F\angle\theta$ or $F\mathrm{e}^{\mathrm{j}\theta}$. Bold letters, $\boldsymbol{f}$, denote vector variables. Subscripts a, b and c denote the phases of the three-phase system. Subscripts 1, 2 and 0 refer to the positive, the negative and the zero sequence components, respectively, and $a = \mathrm{e}^{\mathrm{j}2\pi/3}$. Subscripts d, q and o refer to the standard dqo coordinates components and the superscript ^{e} denotes the synchronously rotating reference frame. For the index set $\{x, y, z\}$, $\boldsymbol{f}_{xyz}$ denotes the vector $[f_x\, f_y\, f_z]^{\mathrm{T}}$. $\Re\{x\}$ is the real part of x and $\Im\{x\}$ is its imaginary part.

2 Problem statement

Consider a general three-phase and four wire system, here represented by the unbalanced sinusoidal voltage and current signals

$$\boldsymbol{v}_{abc} = \sqrt{2}\Re\{\tilde{\boldsymbol{V}}_{abc}\,\mathrm{e}^{\mathrm{j}\omega_s t}\} = \sqrt{2}\Re\{A\,\tilde{\boldsymbol{V}}_{120}\,\mathrm{e}^{\mathrm{j}\omega_s t}\} \quad (1)$$

$$\boldsymbol{i}_{abc} = \sqrt{2}\Re\{\tilde{\boldsymbol{I}}_{abc}\,\mathrm{e}^{\mathrm{j}\omega_s t}\} = \sqrt{2}\Re\{A\,\tilde{\boldsymbol{I}}_{120}\,\mathrm{e}^{\mathrm{j}\omega_s t}\} \quad (2)$$

where $\tilde{\boldsymbol{V}}_{abc}$, $\tilde{\boldsymbol{I}}_{abc}$, $\tilde{\boldsymbol{V}}_{120}$ and $\tilde{\boldsymbol{I}}_{120}$ are the three-phase phasor vectors and A is the standard Fortescue sequence component decomposition.

By applying Park's transformation

$$K(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (3)$$

voltages in the dqo coordinates can be written in the form [18, 20]

$$v_{dq} = \bar{V}^e_{1dq}\,\mathrm{e}^{-\mathrm{j}(\theta-\omega_s t)} + \bar{V}^{-e}_{2dq}\,\mathrm{e}^{-\mathrm{j}(\theta+\omega_s t)} \quad (4)$$

$$v^e_o = \Re\left\{\bar{V}_{0dq}\,\mathrm{e}^{\mathrm{j}\omega_s t}\right\} \quad (5)$$

where $v_{dq} = v_d + jv_q$ and the complex variables

$$\bar{V}^e_{1dq} = \sqrt{2}\,\tilde{V}_1, \quad \bar{V}^{-e}_{2dq} = \sqrt{2}\,\tilde{V}^*_2, \quad \bar{V}_{0dq} = \sqrt{2}\,\tilde{V}_0$$

are directly related to the positive, the negative and the zero sequence component phasors, respectively. In (4), $\bar{V}^e_{1dq}$ correspond to v_{dq} in the synchronously rotating reference frame $K(\omega_s t)$ when only the positive sequence is present. Similarly, $\bar{V}^{-e}_{2dq}$ correspond to v_{dq} in the reference frame $K(-\omega_s t)$ when only the negative sequence is present.

As is well known, under the balanced conditions, the product $(3/2)v_{dq}i^*_{dq}$ yields the complex power, and thus [1, 18]

$$\frac{3}{2}v_{dq}i^*_{dq} = \frac{3}{2}\bar{V}^e_{1dq}\,\bar{I}^{e*}_{1dq} = 3\,\tilde{V}_1\,\tilde{I}^*_1 = \bar{S} = P + jQ \quad (6)$$

In the context of the p–q power theory, for which a vast literature can be found, the real and the imaginary parts of $1.5v_{dq}i^*_{dq}$ are defined as the instantaneous real power $p_p(t)$ and the instantaneous imaginary power $q_p(t)$ [16–19], covering the unbalanced non-sinusoidal three-phase systems. Even though the p–q theory is mostly aimed at instantaneous power characterisations, average values, the real power P_p and the imaginary power Q_p, are also characterised, for instance in [18].

It is known that the real and the imaginary powers coincide with the active and the reactive powers only under sinusoidal balanced conditions. Relations between the instantaneous active power and the instantaneous real power are given in [18, 19], for the general case. Since the imaginary power differs from the reactive power in the general case, in what follows, we want to develop expressions for power evaluation in the dq coordinates which are consistent with the reactive, the complex and the apparent power defined in the IEEE Standard [12] for the sinusoidal unbalanced three-phase systems. Besides the average values, expressions are also derived to evaluate the oscillation amplitudes that are induced by the voltage/current imbalance.

3 Power expressions in the *dqo* coordinates

This section develops analytical power expressions in the dqo coordinates exhibiting compatibility with [12] under the sinusoidal unbalanced conditions. Let us start with a review of the active power expressions. The most basic power quantity is the instantaneous power, which is given by the authors [12, 17, 18, 21]

$$p(t) = \boldsymbol{v}_{abc}^{\mathrm{T}}\boldsymbol{i}_{abc} = \frac{3}{2}(v_d i_d + v_q i_q + 2v_o i_o) = \frac{3}{2}\left(\Re\{v_{dq}i_{dq}^*\} + 2v_o i_o\right) \tag{7}$$

By using (1) and (2), the instantaneous power can be written as

$$p(t) = 2\Re\left\{\tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\ \mathrm{e}^{\mathrm{j}\omega_s t}\right\}\Re\left\{\tilde{\boldsymbol{I}}_{abc}^{\mathrm{T}}\ \mathrm{e}^{\mathrm{j}\omega_s t}\right\} = P + P_{osc}\cos(2\omega_s t + \varphi) \tag{8}$$

where

$$P = \Re\{\tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{abc}^*\} = 3\quad \Re\{\tilde{\boldsymbol{V}}_{120}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{120}^*\} \tag{9}$$

$$P_{osc} = |\tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{abc}| = 3|\tilde{\boldsymbol{V}}_1\tilde{\boldsymbol{I}}_2 + \tilde{\boldsymbol{V}}_2\tilde{\boldsymbol{I}}_1 + \tilde{\boldsymbol{V}}_0\tilde{\boldsymbol{I}}_0| \tag{10}$$

Thus, the instantaneous power presents an average value, the active power P, and an oscillatory part with twice the voltage frequency and amplitude P_{osc}. Clearly, for the balanced systems, the active power equals the instantaneous power. In terms of the *dqo* variables, by employing (4) and (5) one obtains

$$P = \frac{3}{2}\Re\left\{\tilde{V}_{1dq}^{e}\,\tilde{I}_{1dq}^{e*} + \tilde{V}_{2dq}^{-e*}\,\tilde{I}_{2dq}^{-e} + \tilde{V}_{0dq}\,\tilde{I}_{0dq}^*\right\} = \frac{3}{2}\Re\{\tilde{\boldsymbol{V}}_{120dq}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{120dq}^*\} \tag{11}$$

$$P_{\mathrm{osc}} = \frac{3}{2}\left|\tilde{V}_{1dq}^{e}\,\tilde{I}_{2dq}^{-e*} + \tilde{V}_{2dq}^{-e*}\,\tilde{I}_{1dq}^{e} + \tilde{V}_{0dq}\,\tilde{I}_{0dq}\right| \tag{12}$$

where

$$\tilde{\boldsymbol{V}}_{120dq}^{\mathrm{T}} = [\,\tilde{V}_{1dq}^{e}\quad \tilde{V}_{2dq}^{-e}\quad \tilde{V}_{0dq}\,]$$

and similarly for $\tilde{\boldsymbol{I}}_{120dq}$.

3.1 Reactive power

The reactive power for unbalanced sinusoidal signals is defined [12] as the average value of the instantaneous quantity

$$q(t) = \omega_s\left(\boldsymbol{i}_{abc}^{\mathrm{T}}(t)\int \boldsymbol{v}_{abc}(\tau)\,\mathrm{d}\tau\right) \tag{13}$$

For balanced systems the reactive power equals this instantaneous quantity, that is, $Q = q(t)$, where $q(t)$ has a constant value. For unbalanced systems, the instantaneous quantity $q(t)$ presents an average value and oscillations with twice the voltage frequency, analogously to the instantaneous power (7), and can be obtained in the following form, by using (1) and (2)

$$q(t) = 2\Re\left\{\tilde{\boldsymbol{I}}_{abc}^{\mathrm{T}}\ \mathrm{e}^{\mathrm{j}\omega_s t}\right\}\quad \Re\left\{-j\tilde{\boldsymbol{V}}_{abc}\ \mathrm{e}^{\mathrm{j}\omega_s t}\right\} = Q + Q_{\mathrm{osc}}\sin(2\omega_s t + \varphi) \tag{14}$$

where

$$Q = \Im\{\tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{abc}^*\} = 3\quad \Im\left\{\tilde{\boldsymbol{V}}_{120}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{120}^*\right\} \tag{15}$$

$$Q_{\mathrm{osc}} = |\tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{abc}| = 3|\tilde{\boldsymbol{V}}_1\tilde{\boldsymbol{I}}_2 + \tilde{\boldsymbol{V}}_2\tilde{\boldsymbol{I}}_1 + \tilde{\boldsymbol{V}}_0\tilde{\boldsymbol{I}}_0| \tag{16}$$

To obtain an expression in the *dqo* coordinates, let us replace the integral in (13), whose effect is to introduce a 90° phase lag in the voltages, by a time delay of a quarter of a period introduced in the sinusoidal voltage signals, that is, $q(t) = \boldsymbol{i}_{abc}^{\mathrm{T}}(t)\boldsymbol{v}_{abc}(t_{90})$ where $t_{90} = t - \pi/2\omega_s$ is the time delay corresponding to a 90° phase lag. Writing $\boldsymbol{v}_{abc}(t_{90})$ in terms of the sequence components yields

$$\boldsymbol{v}_{abc}(t_{90}) = M\ \boldsymbol{v}_{1abc}(t) - M\ \boldsymbol{v}_{2abc}(t) + \boldsymbol{v}_{0abc}(t_{90})$$

where

$$M = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Now, taking $q(t)$ into the *dqo* coordinates, leads to

$$q(t) = \frac{3}{2}\left[(v_{1q} - v_{2q})i_d - (v_{1d} - v_{2d})i_q + 2v_o(t_{90})i_o\right] = \frac{3}{2}\left[\Im\{(v_{1dq} - v_{2dq})i_{dq}^*\} + 2v_o(t_{90})i_o\right] \tag{17}$$

which allows us to obtain the following, by using (4) and (5), along with similar expressions for the currents

$$Q = \frac{3}{2}\Im\left\{\tilde{V}_{1dq}^{e}\,\tilde{I}_{1dq}^{e*} + \tilde{V}_{2dq}^{-e*}\,\tilde{I}_{2dq}^{-e} + \tilde{V}_{0dq}\,\tilde{I}_{0dq}^*\right\} \tag{18}$$

$$Q_{\mathrm{osc}} = \frac{3}{2}\left|\tilde{V}_{1dq}^{e}\,\tilde{I}_{2dq}^{-e*} + \tilde{V}_{2dq}^{-e*}\,\tilde{I}_{1dq}^{e} + \tilde{V}_{0dq}\,\tilde{I}_{0dq}\right| \tag{19}$$

Equation (17) is consistent with (13), whose average value gives the reactive power (18), which is compliant with the reactive power definition of [12]. Thus, (17)–(19) provide expressions for the reactive power in the *dqo* coordinates, in terms of its instantaneous and average value, which exhibit compatibility with the IEEE Standard definitions [12].

The reactive power expression in the *dqo* coordinates (18) is seen to have a special form, which cannot be reduced to a vector product as in the active power case (11), because of the conjugated form of the second term in (18). As a result, the natural *dqo* version of $Q = \Im\{\tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{abc}^*\}$, that is, $(3/2)\Im\{\tilde{\boldsymbol{V}}_{120dq}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{120dq}^*\}$, does not yield the reactive power, and neither does $(3/2)\Im\{v_{dq}i_{dq}^*\}$, which may easily mislead the reactive power evaluation in the *dq* coordinates (see [13] for instance).

3.2 Complex power

Equations (9) and (15) are actually the real and the imaginary parts of the same quantity, which is the complex power $\bar{S} = P + jQ$

$$\bar{S} = P + jQ = \tilde{\boldsymbol{V}}_{abc}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{abc}^* = 3\tilde{\boldsymbol{V}}_{120}^{\mathrm{T}}\tilde{\boldsymbol{I}}_{120}^* \tag{20}$$

In the *dqo* coordinates, the complex power takes a special form, which can be obtained directly from (11) and (18) and is given by

$$\bar{S} = \frac{3}{2}\left(\bar{V}_{1dq}^{e}\,\bar{I}_{1dq}^{e*} + \bar{V}_{2dq}^{-e*}\,\bar{I}_{2dq}^{-e} + \bar{V}_{0dq}\,\bar{I}_{0dq}^*\right) \tag{21}$$

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The special form is due to the reactive power expression (18), which implies that $\bar{S} \neq (3/2)\,\tilde{\boldsymbol{V}}^{\mathrm{T}}_{120dq}\,\tilde{\boldsymbol{I}}^{*}_{120dq}$ even though $P = \Re\{(3/2)\,\tilde{\boldsymbol{V}}^{\mathrm{T}}_{120dq}\,\tilde{\boldsymbol{I}}^{*}_{120dq}\}$. Equation (21) can also be seen as the average value of the complex instantaneous quantity $p(t) + jq(t)$, denoted here by $s(t)$, which, from (8) and (14), is given by

$$s(t) = p(t) + jq(t) = \bar{S} + S_{\mathrm{osc}}\mathrm{e}^{\mathrm{j}(2\omega_s t + \varphi)} \tag{22}$$

where S_{osc} is numerically equal to P_{osc} and Q_{osc}.

3.3 Apparent power

Analytical expressions for the apparent power are developed here motivated by their importance in the power factor evaluation, which is one of the most commonly controlled variables in distributed generation systems.

Different definitions for the apparent power are found in the literature such as, for instance, the following ones (which are given in [4, 11]), known as the arithmetic, geometric and Buchholz apparent powers, respectively

$$S_A = V_a I_a + V_b I_b + V_c I_c \tag{23}$$

$$S_G = \sqrt{P^2 + Q^2} \tag{24}$$

$$S_B = \sqrt{V_a^2 + V_b^2 + V_c^2}\sqrt{I_a^2 + I_b^2 + I_c^2} \tag{25}$$

The reader is referred to [4, 11], and the references therein, for further discussions on the apparent power definitions, which are out of the scope of this paper. In [12], S_B is given a slightly different expression, referred to as the effective apparent power.

Evaluation of S_G is obvious from P and Q. Regarding S_B, recall that [11]

$$V_a^2 + V_b^2 + V_c^2 = \frac{1}{T}\int_0^{\mathrm{T}} \boldsymbol{v}^{\mathrm{T}}_{abc}\boldsymbol{v}_{abc}\,\mathrm{d}t = \tilde{\boldsymbol{V}}^{\mathrm{T}}_{abc}\,\tilde{\boldsymbol{V}}^{*}_{abc}$$

Furthermore, since the Park transformation preserves the inner product, that is,

$$\frac{1}{T}\int_0^{\mathrm{T}} \boldsymbol{v}^{\mathrm{T}}_{abc}\boldsymbol{v}_{abc}\,\mathrm{d}t = \frac{1}{T}\int_0^{\mathrm{T}} \boldsymbol{v}^{\mathrm{T}}_{dqo}R\boldsymbol{v}_{dqo}\,\mathrm{d}t \tag{26}$$

where $R = (3/2)\mathrm{diag}\{1,\ 1,\ 2\}$, three-phase rms voltage and current values can easily be found in the dqo coordinates and

$$S_B = \frac{3}{2}\sqrt{V_d^2 + V_q^2 + 2V_o^2}\sqrt{I_d^2 + I_q^2 + 2I_o^2} \tag{27}$$

$$= \frac{3}{2}\sqrt{|\,\bar{V}^{e}_{1dq}\,|^2 + |\,\bar{V}^{-e}_{2dq}\,|^2 + |\,\bar{V}_{0dq}\,|^2} \times \sqrt{|\,\bar{I}^{e}_{1dq}\,|^2 + |\,\bar{I}^{-e}_{2dq}\,|^2 + |\,\bar{I}_{0dq}\,|^2} \tag{28}$$

$$= \frac{3}{2}\sqrt{\bar{\boldsymbol{V}}^{\mathrm{T}}_{120dq}\,\bar{\boldsymbol{V}}^{*}_{120dq}\,\bar{\boldsymbol{I}}^{\mathrm{T}}_{120dq}\,\bar{\boldsymbol{I}}^{*}_{120dq}} \tag{29}$$

where V_d, V_q and V_o are the rms values of the d, q and o axes voltages, respectively, and similar definitions hold for the currents. Equation (28) comes from the facts that

$$\boldsymbol{v}^{\mathrm{T}}_{dqo}R\boldsymbol{v}_{dqo} = \frac{3}{2}\left(v_{dq}v^{*}_{dq} + 2v_o^2\right)$$

and

$$\frac{1}{T}\int_0^{\mathrm{T}}\left(v_{dq}v^{*}_{dq} + 2v_o^2\right)\mathrm{d}t = \bar{\boldsymbol{V}}^{\mathrm{T}}_{120dq}\,\bar{\boldsymbol{V}}^{*}_{120dq}$$

both of which hold analogously for the currents. Thus, the three-phase rms values in the dqo coordinates can be evaluated as the moduli of the complex variables $\bar{V}^{e}_{1dq}$, $\bar{V}^{-e}_{2dq}$ and $\bar{V}_{0dq}$, similar to what can be done in the abc coordinates with the modulus of the phasor variables. However, this relation cannot be decomposed per axis as it can be done per phase in the abc coordinates, that is, even though $V_i = \sqrt{\tilde{V}_i\,\tilde{V}^{*}_i}$ for $i = a,\ b,\ c$, in the dqo coordinates one has

$$V^{e}_{d} = \sqrt{\Re\{\,\bar{V}^{e}_{1dq}\,\}^2 + \frac{1}{2}\bar{V}^{-e}_{2dq}\,\bar{V}^{-e*}_{2dq}}$$

$$V^{e}_{q} = \sqrt{\Im\{\,\bar{V}^{e}_{1dq}\,\}^2 + \frac{1}{2}\bar{V}^{-e}_{2dq}\,\bar{V}^{-e*}_{2dq}}$$

$$V^{e}_{0} = \frac{1}{\sqrt{2}}\sqrt{\bar{V}_{0dq}\,\bar{V}^{*}_{0dq}}$$

In [5], a time-varying apparent power is defined as

$$s_B(t) = \sqrt{\boldsymbol{v}^{\mathrm{T}}_{abc}\boldsymbol{v}_{abc}\ \boldsymbol{i}^{\mathrm{T}}_{abc}\boldsymbol{i}_{abc}}$$

By using (26) and following a development similar to the one that was made for (28), this definition can easily be taken to the dqo coordinates. It can be written as

$$s_B(t) = \sqrt{\boldsymbol{v}^{\mathrm{T}}_{dqo}R\boldsymbol{v}_{dqo}\ \boldsymbol{i}^{\mathrm{T}}_{dqo}R\boldsymbol{i}_{dqo}}$$

which can be further developed to result in

$$s_B(t) = \frac{3}{2}\sqrt{\bar{\boldsymbol{V}}^{\mathrm{T}}_{120dq}\,\bar{\boldsymbol{V}}^{*}_{120dq} + V_{\mathrm{osc}}\cos(2\omega_s t + \gamma_V)} \times \sqrt{\bar{\boldsymbol{I}}^{\mathrm{T}}_{120dq}\,\bar{\boldsymbol{I}}^{*}_{120dq} + I_{\mathrm{osc}}\cos(2\omega_s t + \gamma_I)} \tag{30}$$

where

$$V_{\mathrm{osc}} = |\,\bar{V}^{e}_{1dq}\,\bar{V}^{-e*}_{2dq} + \bar{V}^{-e*}_{2dq}\,\bar{V}^{e}_{1dq} + \bar{V}_{0dq}\,\bar{V}_{0dq}\,|$$

An analogous development also holds for I_{osc}. Note that, in general, $S_B^2 \neq (1/T)\int_0^{\mathrm{T}} s_B^2(t)\,\mathrm{d}t$, but the equality holds for the balanced conditions.

4 Discussion

The power expressions developed in the previous section included the oscillating part of the instantaneous power $p(t)$ and the other instantaneous quantities, whose amplitude was given in terms of the phasor-based formulae. These formulae are not given in [12]. Since the active power is

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closely related to the electromagnetic torque in electrical machines, the oscillating amplitude of the instantaneous power indicates the presence of torque pulsation in the generators and/or motor loads in the distributed generation systems analysis.

The instantaneous power is given by a vector inner product and as such is preserved by the Park transformation. Thus, the presence of the voltage/current imbalance does not introduce any problem besides the need to consider the o axis component. In fact, in the absence of the o axis component, (7) reduces to the same expression as in (6) and the instantaneous power is obtained from the complex product $(3/2)\Re\{v_{dq}i^*_{dq}\}$, under balanced or unbalanced conditions. The developed expression is equivalent to others existing in the literature, for instance, (7) is equivalent to the total instantaneous real power of [19] for the unbalanced three-phase systems, either 3-wired or 4-wired, with the sinusoidal signals.

In the case of the instantaneous quantity $q(t)$, besides the vector inner product, there is also a 90° phase shift. The Park transformation maps the phase shift differently for the positive and the negative sequence components, which is the main reason for the reactive power expression to have the special form (18). As a consequence, the complex power expression also requires the special form (21).

The instantaneous quantity $q(t)$ is only introduced in such a way that it exposes the property that its average value is the reactive power defined in [12]. We do not claim that $q(t)$ has the meaning of the instantaneous reactive power. Several different notions of the instantaneous reactive power can be found in the literature [5, 16, 19, 22] but they do not exhibit such a property under unbalanced conditions. For instance, (17) is not equivalent to the instantaneous reactive power of [19], even though they coincide for the balanced systems. Similarly, (17) is not equivalent to the instantaneous reactive (imaginary) power defined in [22].

An important special case is when the o axis component is not present. Even in this case, the instantaneous quantity $q(t)$, whose average value is the reactive power according to the [12] definition, differs from the imaginary power $q_p(t)$ [18] as it can be easily seen by

$$\begin{aligned} q(t) &= \frac{3}{2}\Big[(v_{1q} - v_{2q})i_d - (v_{1d} - v_{2d})i_q\Big] \\ &\neq \frac{3}{2}(v_q i_d - v_d i_q) = \frac{3}{2}\Im\{v_{dq}i^*_{dq}\} = q_p(t) \end{aligned} \tag{31}$$

All these notions and the expressions coincide under the balanced conditions, hence one might be tempted to employ the usual complex product $(3/2)v_{dq}i^*_{dq}$ to yield the complex power in the general case (see [13], for instance). In fact, such a product yields $p_p(t) + jq_p(t)$, the instantaneous real and imaginary powers, which reduces to the complex power in the balanced case. However, in general, even in the absence of the o axis component, one has

$$\begin{aligned} s(t) &= p(t) + jq(t) \\ &= \frac{3}{2}(v_d i_d + v_q i_q) + j\frac{3}{2}\Big[(v_{1q} - v_{2q})i_d - (v_{1d} - v_{2d})i_q\Big] \\ &\neq \frac{3}{2}(v_d i_d + v_q i_q) + j\frac{3}{2}(v_q i_d - v_d i_q) = \frac{3}{2}v_{dq}i^*_{dq} \end{aligned} \tag{32}$$

Noted that the expressions hold for any reference frame, hence the issue is not related to the choice of the reference frame. It is, indeed, because of the existence of the voltage or the current imbalance. Moreover, the issue is not related to the fact that $s(t)$, $p(t)$ and $q(t)$ are instantaneous quantities.

In fact, consider

$$\begin{aligned} \frac{3}{2}v_{dq}i^*_{dq} &= \frac{3}{2}\left(\bar{V}^e_{1dq}\,\mathrm{e}^{-\mathrm{j}(\theta-\omega_s t)} + \bar{V}^{-e}_{2dq}\,\mathrm{e}^{-\mathrm{j}(\theta+\omega_s t)}\right) \\ &\quad\times\left(\bar{I}^e_{1dq}\,\mathrm{e}^{-\mathrm{j}(\theta-\omega_s t)} + \bar{I}^{-e}_{2dq}\,\mathrm{e}^{-\mathrm{j}(\theta+\omega_s t)}\right)^* \\ &= \frac{3}{2}\left(\bar{V}^e_{1dq}\,\bar{I}^{e*}_{1dq} + \bar{V}^{-e}_{2dq}\,\bar{I}^{-e*}_{2dq}\right) \\ &\quad+\frac{3}{2}\left(\bar{V}^e_{1dq}\,\bar{I}^{-e*}_{2dq}\,\mathrm{e}^{\mathrm{j}2\omega_s t} + \bar{V}^{-e}_{2dq}\,\bar{I}^{e*}_{1dq}\,\mathrm{e}^{-\mathrm{j}2\omega_s t}\right) \end{aligned} \tag{33}$$

Clearly, $(3/2)\Re\{v_{dq}i^*_{dq}\}$ recovers (8), but $(3/2)\Im\{v_{dq}i^*_{dq}\}$ does not recover (14), not even in its average value Q. As a result, even in the absence of the 0 sequence component, in the *dqo* coordinates, the complex power has to be evaluated as given in (21), not as the average value of (33), which would be a natural extension of the balanced case.

5 Unbalanced operation in the distributed generation systems

The active and the reactive power considerations developed so far are of central relevance in analysis of the distributed generation problems, where voltage and current imbalance are almost ubiquitous and the *dqo* coordinates representations are commonly employed for the machine models. This is illustrated in the present section by considering the analysis of a simple distributed generation system model, representing the main unbalanced operation phenomena involving typical distributed generators.

The considered model is shown in Fig. 1, and consists of a squirrel-cage induction generator connected to the electrical grid, represented here as an ideal voltage source (infinite bus) with a series impedance (connection line). A fixed capacitor bank is connected to the generator terminals to compensate for its reactive power absorption from the grid. The voltage imbalance at the generator terminals is the result of a single-phase load present in the system.

The system parameters are as follows, given in per unit at the base $V^b = 690$ V (line-to-line) and $P^b = 2$ MW (three-phase). The infinite bus voltage is balanced at 1 pu. The line parameters are: $R_l = 0.0928$ pu and $X_l = 0.2321$ pu, balanced. The shunt capacitor bank is connected in grounded-Y, balanced, with rated reactive power $Q_c = -0.2$ pu. The single-phase load is represented as a constant impedance

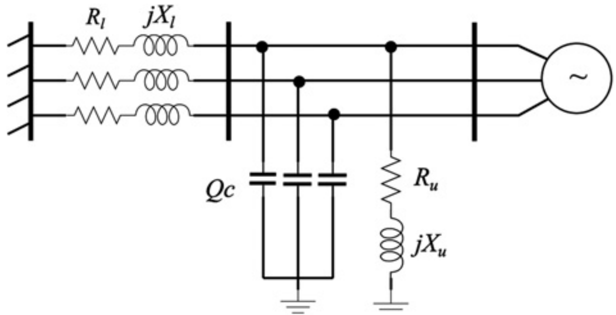

Fig. 1 *Simplified distributed generation system*

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load with $R_u = 1$ pu and $X_u = 1$ pu, which represents a 0.471 MVA load with a 0.71 inductive power factor. The generator parameters are: 2 MW, 690 V, 50 Hz, $R_s = 0.0049$ pu, $X_{ls} = 0.0924$ pu, $X_{lr} = 0.0996$ pu, $R_r = 0.0055$ pu, $X_m = 3.9528$ pu and $H = 3.5$ s, without friction.

A mechanical torque of $T_m = 0.99$ pu is applied to the generator under the given unbalanced voltage condition. The generator convention is adopted, that is, the active power is positive when delivered by the generator and the positive current direction is from the generator to the grid.

Since the generator is connected in Y, there is no 0 sequence current component. Furthermore, since the stator voltages are measured with respect to the internal neutral point of the Y connection, the 0 axis component of the stator voltage is zero. With this, the steady-state condition of the distributed generation system is evaluated by solving the steady-state model and obtaining the generator voltages and currents in the *dqo* coordinates

$$\begin{aligned} \bar{V}^{e}_{1dq} &= 0.9508\angle 14.34 = 0.9211 + j0.2355 \\ \bar{V}^{-e}_{2dq} &= 0.0232\angle 133.59 = -0.0160 + j0.0168 \\ \bar{I}^{e}_{1dq} &= 1.1377\angle 38.93 = 0.8851 + j0.7149 \\ \bar{I}^{-e}_{2dq} &= 0.1223\angle 41.33 = 0.0919 + j0.0808 \end{aligned} \tag{34}$$

By employing (11), (12) and (18), the active and the reactive powers, and also the oscillation amplitude of the instantaneous power, are obtained as

$$P = \Re\{ \bar{V}^{e}_{1dqs} \bar{I}^{e*}_{1dqs} + \bar{V}^{-e*}_{2dqs} \bar{I}^{-e}_{2dqs} \} = 0.9835 \tag{35}$$

$$Q = \Im\{ \bar{V}^{e}_{1dqs} \bar{I}^{e*}_{1dqs} + \bar{V}^{-e*}_{2dqs} \bar{I}^{-e}_{2dqs} \} = -0.4529 \tag{36}$$

$$P_{\text{osc}} = | \bar{V}^{e}_{1dqs} \bar{I}^{-e*}_{2dqs} + \bar{V}^{-e*}_{2dqs} \bar{I}^{e}_{1dqs} | = 0.1287 \tag{37}$$

Also, by using (28), $S_B = 1.0883$, and note that $S_B \neq \sqrt{P^2 + Q^2} = 1.0828$, the difference because of the unbalanced power [11]. In the light of (33), by using (4), the mean value of $(3/2)v^{e}_{dqs}i^{e*}_{dqs}$ results in $0.9835 - j0.4472$ which does not yield $P + jQ = 0.9835 - j0.4529$, as expected.

To further verify the agreement of the developed power expression in the *dqo* coordinates with the IEEE Standard, the distributed generation system was modelled and simulated with Matlab/Simulink™ software. The induction generator was represented by the standard fifth-order *dq* coordinates model in the synchronous reference frame [15]. The dynamics of the line, the load and the capacitor bank are also considered in the simulation, which is conducted in the time domain.

Fig. 2 shows the simulated generator terminal voltages (measured with respect to the ground) and the currents, in the *abc* coordinates. The voltage and the current imbalance can be observed from the slightly different amplitudes of the sine waves in Fig. 2. The voltage amplitude of the *a* phase is smaller than phases *b* and *c*, resulting from the voltage drop across the *a* phase line impedance caused by the single-phase load. It can be observed that the currents delivered by the generator are unbalanced with a larger unbalance level than the voltage.

The instantaneous power $p(t)$ and the quantity $q(t)$ were also obtained from the simulation and are shown in Fig. 3. We remark that these instantaneous quantities were determined from the voltages and the currents measurements directly in the *abc* coordinates. Compared with (35)–(37) one can realise that the average values of the instantaneous quantities shown in Fig. 3 agree with P and Q, respectively, whereas the oscillation amplitude agrees with P_{osc}.

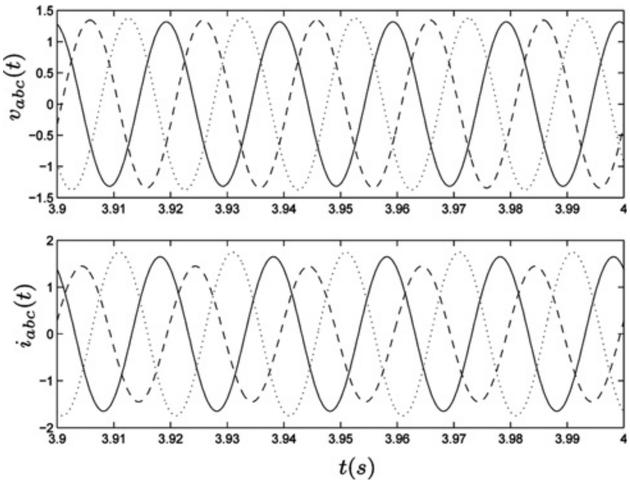

Fig. 2 *Generator terminal abc voltages and currents: phase a – solid; phase b – dashed; and phase c – dotted*

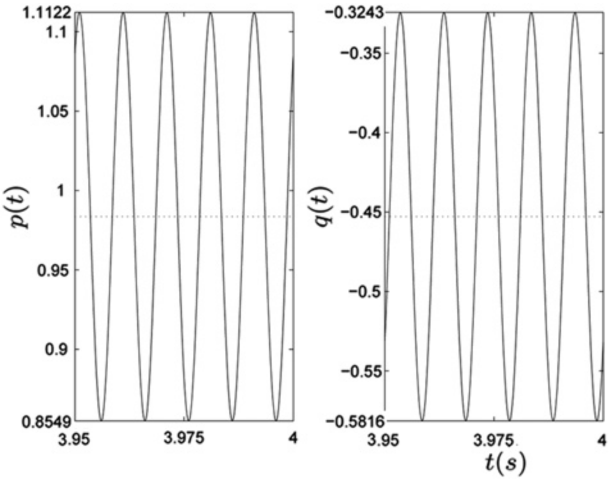

Fig. 3 *Instantaneous power p(t) and quantity q(t). Average values are shown in dotted lines*

6 Concluding remarks

This paper has dealt with the electrical power properties and their corresponding calculation, evaluation and/or measurement in the *dqo* coordinates under unbalanced sinusoidal voltage/current conditions. This is particularly useful for the distributed generation systems, in which the voltage/current imbalance cannot, in general, be neglected (especially in the stability studies) and, moreover, the *dqo* coordinates are often involved in machine modelling and analysis.

This paper developed an analytical expression for the reactive power evaluation in the *dqo* coordinates which exhibits consistency with IEEE Standard 1459-2010 under sinusoidal unbalanced conditions. The usual expressions, as the instantaneous imaginary power, obtained as the imaginary part of the product $1.5v_{dq}i^{*}_{qd}$, do not yield the reactive power if the system is not operating under the three-phase balanced conditions. The positive and the

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negative sequence components of either the voltage or the current contribute with opposite signs to the reactive power, a fact that has been accounted for in the developed analytical expression. It was then possible to provide a complete set of phasor-based expressions encompassing the instantaneous, the active, the reactive, the complex and the apparent powers, in the *dqo* coordinates, all of them exhibiting compatibility with IEEE Standard 1459-2010 under unbalanced voltage/current conditions.

Although there are other power definitions in the *dq* coordinates in the literature, the expressions and the interpretations in this paper were developed to be consistent with the *abc* coordinates statements of the IEEE Standard 1459-2010. This compliance is important for practical reasons, since the power transducers are built based on this type of standard. Furthermore, for control purposes, the correct measurement of the quantities such as the reactive power and the power factor is fundamental for a correct operation of the distributed generator controllers.

In this sense, the evaluation of the current practice for the power factor control and of the proposed practices for the reactive power control (for the voltage control purposes) in distributed generation systems and microgrids, and the application of the expressions developed in this paper for these two control strategies is among the next steps of this research, which can result in a more proper control of these important variables for the operation of these types of systems.

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