Interfaces with Other Disciplines

Borrowing cost reduction by interest rate swaps—an option pricing analysis

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Abstract

Interest rate swaps generally involve two firms with different credit ratings. A quality spread differential (QSD) is observed to exist at different maturities for firm debts with different credit ratings. The quality spread differential allows two firms with different credit ratings to decrease their borrowing costs through interest rate swaps by utilising their comparative advantage in borrowing in different markets. The credit ratings of firms are determined by credit risk factors such as leverage and volatility of earnings asset value. This paper investigates the effect of the leverage and the volatility on the behaviour of risk premia between firm debts with different credit ratings by using the contingent claim analysis. Our results show that the quality spread differential can be explained by the differences in leverage and volatility. Thus two firms with different leverage and volatility of earnings asset value will benefit from interest rate swaps. However, it is found that the duration within which the QSD exists is limited by the values of the leverage and the volatility of two firms. In conclusion, this paper shows that interest rate swaps can help the firms to lower borrowing costs without necessarily relying on the arbitrage argument asserted by existing literature.

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1. Introduction

Since the swap arrangement between the World Bank and IBM in 1981, swaps have become one of the most important tools for corporations to improve their financial performance. Financial managers use swaps to reduce borrowing costs, to increase asset returns or to hedge risk. Major market participants include commercial and investment banks, securities firms, savings and loan institutions, corporations, and government agencies. Swap markets have grown rapidly in the last two decades. At the end of 1982, the aggregate of swap contracts outstanding was US$5 billion. By the end of 2001, contracts outstanding exceeded US$58 trillion. The compound annual growth rate was about 60% during the past twenty years.

The justification for swap arrangements rests on the well-known economic principle of comparative advantage. Bicksler and Chen (1986) find that a
higher credit-rated firm has a comparative advantage in borrowing long-term, fixed-rate, while a lower credit-rated firm has a comparative advantage in borrowing short-term, floating-rate. Both firms can reduce their borrowing costs by performing a fixed/float-float interest rate swap.

A default risky firm needs to pay a risk premium on its debt over the default free government rate. The default risk premium depends on the credit rating of a firm. The lower the credit rating, the higher the default risk premium. Quality spread is the difference of the default risk premium for the same maturity that a lower credit-rated firm has to pay over that of a higher credit-rated firm. In both short-term and longer-term borrowing, a lower credit-rated firm has to pay a quality spread over that paid by a higher credit-rated firm. Thus, the latter firm has the absolute advantage in raising funds in both short-term and long-term markets. However, Bicksler and Chen (1986) observe that the quality spreads in the long-term fixed-rate market and in the short-term floating-rate market are not identical. The quality spread is narrower in the floating-rate market than the fixed-rate market. This implies that a lower credit-rated firm has a comparative advantage in borrowing floating-rate while a higher credit-rated firm has a comparative advantage in borrowing fixed-rate. This quality spread differential (QSD) presents a borrowing cost reduction opportunity in that both the higher and the lower credit-rated firms can borrow in the market where they have the comparative advantage and swap the borrowing with each other. A lower credit-rated firm borrows at floating-rate and then swaps its floating-rate loan for a fixed-rate loan borrowed by a higher credit-rated firm. Through this transaction, both firms lower their borrowing costs.

Many researchers argue that the QSD mainly arises from market inefficiencies and imperfections. Examples are Turnbull (1987), Wall and Pringle (1988); Litzenberger (1992) and Hull (2000). If that is the case, the borrowing cost savings to two firms in an interest rate swap would derive mainly from the arbitrage of market inefficiencies or imperfections. However, this kind of arbitrage opportunity should disappear when the financial markets become more efficient and perfect. In that case, interest rate swaps should have declined rather than have increased over the past twenty years. This claim notwithstanding, the interest rate swap market is still growing fast. In our opinion, the arguments made in the literature are based upon a misinterpretation of the comparative advantage argument. In the very simplest international trade theory comparative advantage is based on relatively different factor endowments. A country tends to specialise in the product which is relatively intensive in its relatively abundant factor of production. Prices end up the same in the trading countries but trade continues to take place. Similarly, two firms with different leverages and asset return volatilities will engage in a swap. The differences in leverage and asset volatility arise mainly from the differences in the nature of business undertaken by firms. These differences cannot be arbitraged away just as differences in the factor endowments of countries cannot be arbitraged away.

This paper seeks to characterise the source of comparative advantage as the default or credit risks resulting from differences in the firms’ leveraging and volatilities of asset returns. The analysis will be carried out in terms of contingent claims theory which assumes perfect market conditions. We find that two firms with different leverages or volatilities can decrease their costs of borrowing by dealing in an interest rate swap. However, the time to maturity of the interest rate swap cannot be at arbitrary length but is limited by the values of the leverages and asset return volatilities.

This paper is organised as follows. In Section 2, we apply the contingent claims model to analyse quality spread and the quality spread differential. We develop the pricing equations in subsequent sections and determine the effect of the variables, leverage, volatility and time to maturity, on the quality spread and the QSD. Finally we examine the conditions under which the quality spread differential exists.

2. Borrowing costs reduction through interest rate swaps

The argument that interest rate swaps help firms to reduce borrowing costs is a hot debate
between market participants and finance academics. On one hand, market participants advocate that comparative advantage exists between firms borrowing in different credit markets and swaps help firms to exploit the comparative advantage. On the other hand, finance academics tend to argue that the source of comparative advantage mainly comes from market imperfections or inefficiencies. Accordingly, the comparative advantage will disappear when the financial markets become more perfect and efficient. This argument notwithstanding, the growth of interest rate swaps is still very fast nowadays. The motivation of this paper is to illustrate that comparative advantage between firms borrowing in different credit markets can exist with the option pricing model that assumes perfect and efficient market conditions. As such, we add one more piece of evidence supporting the comparative advantage argument for the development of interest rate swaps. In the following we illustrate how two firms can reduce borrowing costs through interest rate swaps.

Fig. 1 below shows a graph of the costs of borrowing faced by two firms, A and B, assuming that firm A is a higher credit-rated firm and firm B is a lower credit-rated firm.

\[ R_{A1} \text{ and } R_{B1} \text{ represent the costs of borrowing of firms A and B for different time to maturity, } t. \]

The quality spreads are represented by \((R_{B1} - R_{A1})\) and \((R_{B2} - R_{A2})\). \((R_{B1} - R_{A1})\) represents the quality spread of short-term borrowing whereas \((R_{B2} - R_{A2})\) represents that of long-term borrowing. Fig. 1 shows that firm A has the absolute advantage in borrowing because it can borrow at lower costs both short and long-term. However, it is important to note that as long as \((R_{B2} - R_{A2})\) is greater than \((R_{B1} - R_{A1})\), a QSD arises and a swap will be beneficial to both firms. Thus, the QSD is represented by

\[ \text{QSD} = (R_{B2} - R_{A2}) - (R_{B1} - R_{A1}). \quad (1) \]

If there is a QSD, then comparative advantage exists and both firms can lower their borrowing costs through interest rate swaps. The total gain in interest rate swaps will not be equal to the QSD due to transaction costs which are mainly the fees charged by the financial intermediaries. How the two firms divide the QSD between themselves depends on the relative credit worthiness and bargaining power of the two firms. For simplicity, the procedures of how the QSD is exploited through an interest rate swap are shown in Table 1 by as-

<table>
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<tr>
<td>Sharing of QSD through interest rate swap</td>
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<tr>
<th>Firm A</th>
<th>Firm B</th>
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<td>Borrow in long-term (fixed-rate) market at Swap, where swap payment to firm B at floating swap receipt from firm B at fixed Effective cost of borrowing</td>
<td>( R_{A2} ) ( R_{B1} ) ( R_{A1} ) ( (R_{A2} + \text{QSD}/2) ) ( (R_{A1} - \text{QSD}/2) )</td>
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<tr>
<td>Swap, where swap payment to firm A at fixed swap receipt from firm A at floating Effective cost of borrowing</td>
<td>( R_{A1} ) ( (R_{A2} + \text{QSD}/2) ) ( (R_{A1}) ) ( R_{B1} + R_{A2} + \text{QSD}/2 - R_{A1} )</td>
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suming that the QSD is shared equally between two firms at no transaction costs. Firm A has effectively converted its fixed-rate borrowing to floating-rate and it can save QSD/2 through swap rather than by borrowing in the floating-rate market directly itself. Through Eq. (1), it can be shown that

\[ R_{B1} + R_{A2} + \frac{QSD}{2} - R_{A1} = R_{B2} - \frac{QSD}{2}. \]

Thus, firm B has effectively converted its floating-rate borrowing to fixed-rate and it can also save QSD/2 through swap rather than borrowing in the fixed-rate market directly itself. Fig. 2 shows the direction of cash flow in the interest rate swap. If the quality spread differential is expressed in terms of the first derivative with respect to time, \( \partial(R_{Bt} - R_{At})/\partial t \), then the comparative advantage occurs when \( \partial(R_{Bt} - R_{At})/\partial t > 0. \)

3. The model

3.1. Model assumptions

In applying the contingent claims model, we make the usual perfect and efficient market assumptions as well as the following assumptions (A1, A2, A3 and A4) specifically relating to the payoffs of debts and the asset value generating process:

(A1): The firm has two classes of claims in its balance sheet structure, namely, (a) a single homogeneous class of debt (\( D \)), and (b) the shareholder equity (\( E \)).

(A2): The indenture of the debt issue contains the following provisions and restrictions. Firstly, the firm promises to pay a total of \( F \) dollars to the creditors on the maturity date \( T \). Secondly, in the event that this payment is not met, the creditors immediately take over the firm’s assets and the shareholders receive nothing. Lastly, the firm cannot issue any new senior (or its equivalent rank) claims on the firm nor can it pay cash dividends or conduct share repurchase prior to the maturity of the debt.

(A3): There is a risk free interest rate, \( r \), which is fixed and will remain constant.

(A4): The firm’s asset value is generated by Ito’s process,

\[ dV = \rho V dt + \sigma V dZ, \]

where \( dZ \) is the Wiener process

\[ dZ = \varepsilon \sqrt{dT}, \]

where \( \varepsilon \) is a standard normal deviate (i.e. \( E(\varepsilon) = 0 \) and \( Var(\varepsilon) = 1 \)), \( \rho \) is the net instantaneous return and \( \sigma \) is the volatility of the firm’s asset return.

3.2. Model formulation

Before going into the details of model formulation, we have the following list of notations for the sake of clarity:
At time $T$, if the firm's asset value ($V_T$) is greater than the face value of the debt ($F$), then the debt value ($D_T$) of the firm at time $T$ is $F$; and the equity value of the firm is equal to $V_T - F$. On the other hand, if $V_T < F$, then $D_T = V_T$ and $E_T = 0$. Therefore, we have the following relations for $D_T$ and $E_T$.

$$D_T = F + \min(V_T - F, 0) = \min(V_T, F), \quad (4)$$

$$E_T = \max(V_T - F, 0). \quad (5)$$

From the accounting identity, "Assets = Liabilities + Equity", we have $V = D + E$ and $D_T = V_T - \max(V_T - F, 0)$.

Next, the firm's asset value generating process is given by (2). By Ito's lemma, the solution of (2) is given by

$$\ln \left( \frac{V_T}{V_0} \right) = \left( \rho - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T}. \quad (6)$$

By (5), $E_T = \max(V_T - F, 0)$, which is the expected value of the equity. It follows that the expected value of $E_T$ at time $T$ will be

$$E(E_T) = \int_F^\infty (V - F) \Phi'(V) dV, \quad (7)$$

where $\Phi'(\cdot)$ is the density function of the lognormal distribution.

Based on the possibility of constructing a perfectly hedged portfolio consisting of the firm's assets, the firm's equities and the riskless debt, we can apply the Cox and Ross (1976) risk neutrality assumption, setting $\rho = r$ (risk free rate), and discounting equation (7) by $r$ yields:

$$E_0 = V_0 \Phi(x_1) - F \exp(-rT) \Phi(x_2), \quad (8)$$

where

$$x_1 = \frac{\ln \left( \frac{V_T}{F} \right) + \left( \frac{r + \frac{\sigma^2}{2} }{\sigma} \right) T}{\frac{\sigma}{\sqrt{T}}} = \frac{\frac{r}{\sqrt{T}}}{\sqrt{T}}, \quad (9)$$

$$x_2 = x_1 - \frac{1}{\sqrt{T}} = x_1 - \sqrt{T}, \quad (10)$$

$$d = \frac{F \exp(-rT)}{V_0} \quad \text{and} \quad \tau = \frac{\sigma^2}{2}. \quad (11)$$

Now also by the accounting identity $V = D + E$ and (8),

$$D_0 = V_0 \Phi(-x_1) + F \exp(-rT) \Phi(x_2), \quad (12)$$

where

$$\Phi(-x_1) = 1 - \Phi(x_1).$$

Since $D_0 = F \exp(-rT)$, we obtain

$$F \exp(-rT) = V_0 \Phi(-x_1) + F \exp(-rT) \Phi(x_2). \quad (13)$$

Dividing both sides of (13) by $F \exp(-rT)$ and taking natural logarithms, we get the following equation by rearranging terms:

$$R - r = -\frac{1}{T} \ln \left[ \frac{V_0}{F \exp(-rT)} \Phi(-x_1) + \Phi(x_2) \right]$$

$$= -\frac{1}{T} \ln \left[ \frac{1}{d} \Phi(-x_1) + \Phi(x_2) \right]. \quad (14)$$

We note that $R - r$ is simply the default risk premium, which the risky firm has to pay over the risk-free rate.

Now let us apply this principle to the situation of two firms, say firm A and B. If we let $H = R - r$, then for firm A, the default risk premium function $H_A$ is

$$H_A = R_A - r$$

$$= -\frac{1}{T} \ln \left[ \frac{1}{d_A} \Phi(-x_{1A}) + \Phi(x_{2A}) \right], \quad (15)$$
and for firm B, $H_B$ is

$$H_B = R_B - r$$

$$= -\frac{1}{T} \ln \left[ \frac{1}{d_B} \Phi(-x_{1B}) + \Phi(x_{2B}) \right].$$  \hspace{1cm} (16)

Then the difference of the default risk premium function between two firms $\Delta H$ is given by

$$\Delta H = H_B - H_A$$

$$= \frac{1}{T} \left\{ \ln \left[ \frac{1}{d_A} \Phi(-x_{1A}) + \Phi(x_{2A}) \right] \right.$$  \hspace{1cm} (17)

$$\left. - \ln \left[ \frac{1}{d_B} \Phi(-x_{1B}) + \Phi(x_{2B}) \right] \right\},$$

and

$$\frac{\partial (\Delta H)}{\partial T} = \frac{\partial H_B}{\partial T} - \frac{\partial H_A}{\partial T}. \hspace{1cm} (18)$$

Hence, $\frac{\partial (\Delta H)}{\partial T}$ can be expressed explicitly as follows:

$$\frac{\partial (\Delta H)}{\partial T} = \frac{1}{T} \ln \left( \frac{P_B}{P_A} \right) + \frac{1}{T} \left[ \frac{1}{P_A} \left( \frac{r}{d_A} \Phi(-x_{1A}) \right) \right.$$

$$\left. - \frac{\sigma_A}{2\sqrt{2\pi}T} e^{-x_{1A}^2/2} \right]$$

$$- \frac{1}{T} \left[ \frac{1}{P_B} \left( \frac{r}{d_B} \Phi(-x_{1B}) \right) \right.$$

$$\left. - \frac{\sigma_B}{2\sqrt{2\pi}T} e^{-x_{1B}^2/2} \right],$$  \hspace{1cm} (19)

where $\phi(x) = \Phi(-x) = (1/\sqrt{2\pi})e^{-x^2/2}$ is the standard normal density function. It is difficult to show that $\frac{\partial (\Delta H)}{\partial T} > 0$ because $\frac{\partial (\Delta H)}{\partial T}$ depends on several variables. The exact relations are very complicated mathematically. However, we may examine $\frac{\partial (\Delta H)}{\partial T}$ in some special cases. The mathematical detail is given in Appendix A. In order to ameliorate the mathematical complications, it is necessary to express the QSD in a simpler way.

4. Analysis of the QSD by using guarantee cost as substitute for the risk premium

Merton (1977) shows that the risk premium can be related to the cost of a guarantee on the borrowing of a firm. Suppose at the time of borrowing, the firm purchases a guarantee on the debt from a third party which can guarantee that the promised payment, $F$, will always be paid at the time of maturity. The debt of the firm then becomes default free and will yield the risk free rate of interest only. The firm must bear the cost of this guarantee which depends on the risk premium it must originally pay. The funds the firm will get at the time of borrowing equals $F \exp(-rT) - G_0$ where $G_0$ is the cost of the guarantee. It must be the same as $F \exp(-rT)$. Therefore,

$$G_0 = F \exp(-rT) - F \exp(-rT).$$  \hspace{1cm} (20)

The value of the guarantee can thus be used as a substitute for the risk premium. In the quality spread analysis, we are interested in the cross-sectional difference of guarantee costs between two firms at a point in time. The arrangement of the guarantee is that if at the debt maturity date, $V_T > F$, then the guarantee will simply expire. If $V_T < F$, the guarantor takes over the assets of the firm and pays the promised payment $F$ to the creditors. The deficiency is the loss to the guarantor. The guarantee will possess a pay-off structure as follows:

if $V_T > F : G_T = 0,$ \hspace{1cm} (21)

if $V_T < F : G_T = -(V_T - F).$ \hspace{1cm} (22)

Therefore,

$$G_T = -\min(V_T - F, 0) = \max(F - V_T, 0).$$  \hspace{1cm} (23)

The payoff structure of the loan guarantee is identical to that of a put option. By purchasing a loan guarantee, the firm has effectively purchased a put option on its assets which gives the firm the right to sell the assets for $F$ dollars on the maturity date of the debt. The guarantee is a put option on the firm’s assets with exercise price $F$. Following the same arguments in the valuation of equity (a call option of the firm’s assets), we can derive a formula for the value of the guarantee, and it can be written as

$$G_0 = F \exp(-rT) \Phi(-X_2) - V_T \Phi(-X_1).$$  \hspace{1cm} (24)

Since we are interested in the cross-sectional difference of guarantee costs amongst firms at a point in time, it is useful to work with the ratio $g \equiv G_0/F \exp(-rT)$ rather than the absolute cost level $G$. Thus, $g$ is the cost of the guarantee per
dollar of the guarantee loan payment discounted at riskfree rate and is always less than or equal to 1. For \( g = G_0/F \exp(-rT) \), Eq. (24) can be rewritten as

\[
g = \Phi(-X_2) - \frac{1}{d} \Phi(-X_1).
\]  

Eq. (25) shows that \( g \) is a function of the two variables \( d \) and \( \tau \). The effect of the change in either variable on \( g \) can be seen by taking the first derivative.

\[
\frac{\partial g}{\partial d} = \frac{\phi(X_1)}{d} > 0,
\]  

\[
\frac{\partial g}{\partial \tau} = \frac{\phi(X_1)}{2d\sqrt{\tau}} > 0,
\]

where \( \phi(X_1) \) denotes the first derivative. The change in the guarantee cost is thus an increasing function of both variables \( d \) and \( \tau \). We will next consider how \( d \) and \( \tau \) affect the credit ratings and the quality spreads in the following paragraphs.

Standard and Poor’s (1992) defines a credit rating as, “an assessment of an issuer’s ability to pay interest and repay capital in a timely manner.” Thus, credit rating is a measure of default risk of the debt issuer. In practice, the credit-rating agencies will give a high credit rating to a firm with low default risk and a low credit rating to a firm with high default risk. The risk premium required by investors will be higher for high default risk than for low default risk. Ederington and Yawitz (1987) find that there is persistent existence of such a quality spread between lower and higher credit-rated firms. Financial measurements such as coverage, profitability and leverage are found to be important determinants of ratings. In the case of leverage, Ederington and Yawitz (1987) empirically document an inverse relationship between ratings and leverage. The higher the leverage, the lower the credit rating and vice versa. Although the volatility measurement is seldom used in the rating process, we can easily see the inverse relationship between ratings and the volatility of a firm’s earnings asset value. A high volatility of earnings asset value implies a high volatility of profitability and coverage. As such, the default risk will also be higher with higher volatility in these measurements. The credit rating will naturally be lower as a result. The difficulty in using the volatility of the firm’s earnings asset value is that it is not observable empirically. However, Christie (1982) shows that the volatility of the firm’s asset value can be inferred from the volatility of the market value of equity which is easily observable. We can conclude that the higher the leverage or the volatility, the lower the credit rating and vice versa. From the results of Eqs. (26) and (27), we can see that a lower credit-rated firm has to pay a higher guarantee cost for its borrowing which is equivalent to the quality spread it has to pay over that of a higher credit-rated firm. As a result, the option pricing technique indicates that the existence of quality spread between two different credit-rated firms at a point in time can be explained by the different leverages and volatilities of earnings asset values between the two firms.

5. Conditions under which QSDs occurs

On the basis of the analysis of the previous section, the problem has become one of examining the change of the difference in guarantee costs between two firms against the change in the time to maturity of the loan. Let \( \Delta g \) be the difference of the guarantee costs between two firms, A and B, i.e. \( \Delta g = g_B - g_A \); if \( \Delta g \) is not identical for different times to maturity, \( T \), then a quality spread differential exists. Two firms will gain through the sharing of the QSD by an interest rate swap.

Quality spread differential occurs when

\[
\frac{\partial (\Delta g)}{\partial T} = \frac{\partial (g_B - g_A)}{\partial T} > 0.
\]  

From (24), the first derivative of \( g \) with respect to \( T \) will be

\[
\frac{\partial g}{\partial T} = \frac{\sigma}{2d\sqrt{T}} \phi(X_1).
\]  

Then

\[
\frac{\partial (\Delta g)}{\partial T} = \frac{\sigma_B}{2d_B\sqrt{T}} \phi(X_{1B}) - \frac{\sigma_A}{2d_A\sqrt{T}} \phi(X_{1A}).
\]

Eq. (30) shows that the change of the difference in guarantee costs with respect to time to maturity is
also a function of the variables \( d, \sigma \) and \( T \). We are assuming that firm A is the higher credit-rated firm and firm B is the lower credit-rated one. From our previous arguments on the relation between credit rating and leverage and volatility, it follows that either the leverage or the volatility of firm B will be higher than that of firm A, i.e. \( d_B > d_A \) or \( \sigma_B > \sigma_A \) or both. It is worthwhile to identify the conditions under which \( \partial \Delta g / \partial T > 0 \) to see for which values of \( d \) and \( \sigma^2 \), the QSD occurs. The results are as follows:

(a) \( \sigma^2 \) of two firms are identical, \( d_A < d_B \) and \( d \leq 1 \)
\[
\frac{-\ln d_A + \ln d_B}{\sigma^2} > T.
\]

(b) \( d_A = d_B = d \) and \( d < 1 \), \( \sigma_A^2 < \sigma_B^2 \)
\[
\frac{\ln(\sigma_B)}{\sigma_A} > \left( \frac{\sigma_B^2 - \sigma_A^2}{\ln d + \frac{\sigma_A \sigma_B T}{d}} \right) \left( \ln d - \frac{\sigma_A \sigma_B T}{2d} \right).
\]

Mathematical derivations are shown in Appendix B. Table 2 shows the maximum value of \( T \) for which \( \partial \Delta g / \partial T > 0 \) and for some representative values of \( d \) and \( \sigma^2 \). Figs. 3 and 4 show \( \partial \Delta g / \partial T \) versus \( T \) for various values of \( d \) and \( \sigma^2 \). Several points require comment.

First, we ignore the case where \( d > 1 \). A firm with \( d > 1 \) is technically insolvent and may go bankrupt at any time. The default risk of this firm is extremely high and it may not get any credit-rating at all. If there is any credit rating at all for this firm, it will be classified into speculative grade which is below BBB according to Standard and Poors’ ratings. Swap markets are generally very credit risk averse and firms that can deal in these markets are mainly confined to those of very good credit-rating, i.e. investment grade of BBB or above. In fact, it is now common for swap contracts to include a rating trigger which is designed to shorten or terminate the swap transaction in the event of a counterparty rating downgrade in order to reduce the potential for credit losses in advance of counterparty bankruptcy. We therefore focus our analysis for \( d \leq 1 \). Second, we find that the QSD does not increase indefinitely but only for some limiting values of \( T \), the time to maturity of debt. The traditional argument that the quality spread between two firms of different credit ratings will become greater as time to maturity, \( T \), becomes longer is valid only up to a certain value of \( T \). Merton (1974) uses the option pricing technique to analyze and depict the graphs of corporate risk premia against the time to maturity of debt which is later refined by Lee (1981) and Pitts and Selby (1983). Their results show that for \( d < 1 \), the term structure of risk premia is concave for medium leveraged firms and upward sloping for low leveraged firms. The risk premia for a higher leveraged firm is always greater than that for a lower leveraged firm but the difference will become smaller and smaller as time to maturity increases. We perform a similar analysis by using the guarantee costs as substitutes for risk premia. Fig. 5 shows the guarantee costs versus time to maturity.

<table>
<thead>
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<th>( \sigma^2 )</th>
<th>( d_A )</th>
<th>( d_B )</th>
<th>( T )</th>
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<td>( \sigma_B^2 )</td>
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Note: If \( d \) and \( \sigma^2 \) are year measurements, then \( T \) will be expressed in terms of number of years.
of debt for firms of different leverages; while Fig. 6 shows those for firms of different volatilities. All guarantee costs are upward sloping and the gap between them first increases, then decreases and finally approaches zero as the time to maturity increases. The changes in the gap between the guarantee costs explain why the QSD is concave as depicted in Figs. 3 and 4.

Thus the QSD increases up to a certain value of $T$ and then starts decreasing. This applies not only to firms with different leverages (as shown in Fig. 3) but also to firms with different volatilities (as shown in Fig. 4) as long as the leverage is lower than 1. Our analysis on the behaviour of quality spread supports the analysis of risk structure of corporate debt by Merton (1974), Lee (1981) and Pitts and Selby (1983). In addition, we add more information on the behaviour of quality spread between firms with different volatilities. Third, the value of $T$ for $\partial \Delta g / \partial T > 0$ depends on the values of $d$ and $\sigma^2$. $T$ will be longer for lower values of $d$ or $\sigma^2$ (as shown in Table 2). A traditional fixed/ floating interest rate swap, where a higher credit-rated firm borrows fixed and a lower credit-rated firm borrows floating, will occur only with maturity within the limit of $T$. This implies that short-term interest rate swaps will occur more often and that long-term interest rate swaps will be done mainly by firms with low leverages and volatilities.

In 2001, the Bank for International Settlements
surveys of over-the-counter derivative transactions showed that interest rate swaps with maturity of less than one year persistently shared the largest component of the interest rate swap market, followed by swaps with maturity between one and five years, and swaps with maturity over five years always remained the smallest component of the interest rate swap market. Thus, empirical observations agree with our analysis here.

6. Conclusion

The principle of comparative advantage in trade theory asserts that international trade is always beneficial whenever, in the absence of trade, there is a difference in the opportunity costs of production between countries. An interest rate swap transaction is analogous to international trade in that both parties can benefit from a swap whenever there is a difference in the quality spreads between short-term and long-term borrowing. The existence of the quality spread differential between two firms forms one of the sources of economic benefits in that both firms can decrease their costs of borrowing through the sharing of the QSD. In this paper, we have identified the differences in leverages and volatilities of earnings asset values as sources of the QSD. The difference in the balance sheet structure between two firms
forms the basis of comparative advantage. Our results show that the behaviour of the difference in risk premia of debts between firms with different leverages or volatilities along different maturities gives rise to the QSD. Given that the leverage and the volatility of asset value are important factors in credit ratings, the results of this paper explain why interest rate swaps are usually transacted between two different firms with different credit ratings. The fact that interest rate swaps generally involve two firms with different credit ratings has a further implication on the pricing and default risk of interest rate swaps. The probability of default is usually higher for a lower credit-rated firm than a higher credit-rated firm. In dealing with a lower credit-rated firm in interest rate swaps, the higher credit-rated firm will take on more risk than the lower credit-rated firm. It will then be interesting to see how the higher credit-rated firm determines the price of the interest rate swap in order to compensate the higher risk it takes. It will be especially important for banks that act as swap dealers since they are usually the higher credit-rated partner. The default risk of interest rate swaps also has an impact on the regulatory requirements for banks that have swaps as their assets. Further research in the pricing and the default risk of interest rate swaps with focus on individual firm’s balance sheet structure which relates to the firm’s credit risk will shed additional light on the sources of comparative advantage underlying swaps.

Acknowledgements

We wish to thank two anonymous referees for their helpful comments and suggestions.
Appendix A

From Eq. (14), we can write

$$\frac{\partial H}{\partial T} = \frac{1}{T^2} \ln P$$

$$- \frac{1}{TP} \left[ \frac{r}{d} \Phi(-x_1) - \frac{\sigma_A}{2\sqrt{2\pi}T} e^{-\left(x_1^2/2\right)} \right],$$

(A.1)

where

$$P = \frac{D_0}{F} \exp(-rT) = \frac{D_0 \exp(rT)}{F}$$

$$= \Phi(x_2) + \frac{1}{d} \Phi(-x_1).$$

Eq. (A.1) is either one or the other of the components of Eq. (17) by dropping the subscript A or B.

Let $S = V/F$ and $f(S, T) = Se^{rT} \Phi(-x_1) + \Phi(x_2)$. Then $H(S, T) = -(1/\tau) \ln f(S, T)$ and

$$x_1 = \frac{\ln(S) + \left(r + \frac{\sigma_1^2}{\tau}\right)}{\sigma/\sqrt{\tau}}$$

and

$$x_2 = \frac{\ln(S) + \left(r - \frac{\sigma_1^2}{\tau}\right)}{\sigma/\sqrt{\tau}}.$$
in order to discuss the sign of $\partial H(S, T)/\partial T$, we must consider the ratio $S = V/F$. $S$ is the ratio of the firm’s initial asset value $V_0$ and the face valued $F$ of the debt.

Since

$$rT - \frac{x_1^2}{2} = -\frac{x_2^2}{2} - \ln S,$$

we have

$$\frac{\partial f(S, T)}{\partial T} = rxe^{rT} \Phi(-x_1) + \frac{1}{2\sqrt{2\pi}T^{3/2}}$$

$$\times \left[ e^{-(x_2^2/2)} \left( \left( r - \frac{\sigma^2}{2} \right) T - \ln S \right) \right.$$

$$- e^{-(x_2^2/2)} \left( \left( r + \frac{\sigma^2}{2} \right) T - \ln S \right) \left. \right]$$

$$= rxe^{rT} \Phi(-x_1) + \frac{1}{2\sqrt{2\pi}T^{3/2}} e^{-(x_2^2/2)}$$

$$\times \left[ \left( r - \frac{\sigma^2}{2} \right) T - \ln S \right.$$

$$- e^{lnS} \left( \left( r + \frac{\sigma^2}{2} \right) T - \ln S \right) \left. \right]$$

$$= rxe^{rT} \Phi(-x_1) - \frac{\sigma}{2\sqrt{2\pi}T} e^{-(x_2^2/2)}. \quad (A.3)$$

In particular, if $r = \sigma^2/2$, we have

$$\frac{\partial f(S, T)}{\partial T} = rxe^{rT} \Phi(-x_1) - \frac{\sigma}{2\sqrt{2\pi}T} \exp \left\{ \frac{-(\ln S)^2}{2\sigma^2T} \right\}. \quad (A.4)$$

From the definition of $f(S, T)$, we have the following:

(a) for $S \geq 1$, $\lim_{r \to 0} f(S, T) = 1$,

(b) for $S < 1$, $\lim_{r \to 0} f(S, T) = S$,

(c) since

$$\lim_{r \to -\infty} e^{rT} \Phi(-x_1) = \lim_{r \to -\infty} \frac{\Phi(-x_1)}{e^{rT}}$$

$$= \lim_{r \to -\infty} \frac{\left( r + \frac{\sigma^2}{2} \right) T - \ln S}{2\sqrt{2\pi}T^{3/2}}$$

$$\times \exp \left\{ \frac{rT - x_1^2}{2} \right\}$$

$$= S \lim_{r \to -\infty} \frac{\left( r + \frac{\sigma^2}{2} \right) T - \ln S}{2\sqrt{2\pi}T^{3/2}} e^{x_2^2/2}$$

$$= 0, \quad (A.5)$$

we deduce, for any $S > 0$,

$$\lim_{r \to -\infty} f(S, T) = \lim_{r \to -\infty} \Phi(x_2)$$

$$= \lim_{r \to -\infty} \Phi \left( \frac{\ln S + (r - \frac{\sigma^2}{2}) T}{\sigma \sqrt{T}} \right)$$

$$= \begin{cases} 0, & r < \frac{\sigma^2}{2}, \\ \frac{1}{2}, & r = \frac{\sigma^2}{2}, \\ 1 & r > \frac{\sigma^2}{2}. \end{cases} \quad (A.6)$$

We can also obtain that, for any $T > 0$,

$$\lim_{S \to -\infty} f(S, T) = \lim_{S \to -\infty} \Phi \left( \frac{\ln S + (r - \frac{\sigma^2}{2}) T}{\sigma \sqrt{T}} \right) = 1. \quad (A.7)$$

Further, we discuss the monotonic behaviour of $f(S, T)$ with respect to $T$ for any fixed $S > 0$. For $T > \ln S/(r + \sigma^2/2)$,

$$\frac{d}{dT} \left( e^{rT} \Phi(-x_1) \right)$$

$$= re^{rT} \Phi(-x_1) - e^{rT} \left( \frac{r + \frac{\sigma^2}{2}}{2\sqrt{2\pi}T^{3/2}} e^{-(x_2^2/2)} \right)$$

$$= e^{rT} \left( r \Phi(-x_1) - \frac{r + \frac{\sigma^2}{2}}{2\sqrt{2\pi}T^{3/2}} e^{-(x_2^2/2)} \right)$$

$$\leq \frac{1}{\sqrt{2\pi}} e^{-(x_2^2/2)} \left[ r \left( \frac{r + \frac{\sigma^2}{2}}{2} \right) T - \ln S \right]$$

$$= S \frac{e^{-(x_2^2/2)} - \left( \frac{r + \frac{\sigma^2}{2}}{2} \right) T^2 + (\ln S)^2}{2\sigma T^{3/2} \left( \frac{r + \frac{\sigma^2}{2}}{2} \right) T + \ln S},$$

which implies that, for $T > \ln S/(r + \sigma^2/2)$, then $d/dT(e^{rT} \Phi(-x_1)) > 0$. This means that $e^{rT} \Phi(-x_1)$ is a strictly decreasing function with respect to $T$ for $T > \ln S/(r + \sigma^2/2)$ and for $S > 0$.

Since $\lim_{r \to -\infty} e^{rT} \Phi(-x_1) = 0$, we have $\lim_{T \to -\infty} T \Phi(-x_1) = 0$, and by (A.3).
\[
\lim_{T \to \infty} T e^T \Phi(-x_1) = \lim_{T \to \infty} \frac{T \Phi(-x_1)}{e^{-T}} \\
= -\lim_{T \to \infty} \Phi(-x_1) + \lim_{T \to \infty} \frac{\left(r + \frac{x_1^2}{2}\right) T - \ln \frac{T}{\sqrt{2\pi}}}{2\sqrt{2\pi} r \sigma^{3/2}} \\
\times \exp \left\{ \frac{\left(r - \frac{x_1^2}{2}\right)^2 T^2 + 2 \left(r + \frac{x_1^2}{2}\right) T \ln S + (\ln S)^2}{2\sigma^2 T} \right\} \\
= 0
\]

(A.8)

\[
\lim_{T \to \infty} \sqrt{T} e^{-\frac{x_1^2}{2}} = \lim_{T \to \infty} \sqrt{T} \Phi \left( \frac{\ln S - \left(r - \frac{x_1^2}{2}\right) T}{\sigma \sqrt{T}} \right) \\
= \begin{cases} 
0 & \text{if } r \neq \frac{x_1^2}{2} \\
\infty & \text{if } r = \frac{x_1^2}{2}.
\end{cases}
\]

(A.9)

Eqs. (A.8) and (A.9) imply that, for any fixed \( S > 0 \),

\[
\lim_{T \to \infty} T \frac{\partial f(S, T)}{\partial T} = rs \lim_{T \to \infty} T e^T \Phi(-x_1) \\
- \frac{\sigma}{2\sqrt{2\pi}} \lim_{T \to \infty} \sqrt{T} e^{-\left(\frac{x_1^2}{2}\right)} \\
= \begin{cases} 
0 & \text{if } r \neq \frac{x_1^2}{2} \\
\infty & \text{if } r = \frac{x_1^2}{2}.
\end{cases}
\]

(A.10)

By (A.6) and (A.10), we deduce that, for any fixed \( \delta > 0 \),

\[
\lim_{T \to \infty} \left[ f(S, T) \ln f(S, T) - T \frac{\partial f(S, T)}{\partial T} \right] \\
= \begin{cases} 
0 & \text{if } r \neq \frac{x_1^2}{2} \\
\infty & \text{if } r = \frac{x_1^2}{2}.
\end{cases}
\]

Also Eqs. (A.3), (A.4) and (A.6) yield that for any fixed \( T > 0 \),

\[
\lim_{S \to -\infty} \left[ f(S, T) \ln f(S, T) - T \frac{\partial f(S, T)}{\partial T} \right] = 0.
\]

Let \( F(S, T) = f(S, T) \ln f(S, T) - T \frac{\partial f(S, T)}{\partial T} \). It is easy to prove that \( \lim_{S \to -\infty} F(S, T) = 0 \).

To see that whether \( \partial(H) / \partial T \geq 0 \), it suffices to show that \( \partial H_a(S, T) / \partial T \) and \( \partial H_b(S, T) / \partial T \) are of different signs within a certain time period. However, it is difficult to determine under what conditions that \( \partial H(S, T) / \partial T > 0 \) or \( < 0 \) explicitly. Therefore we must resort to another method of measuring the default risk premium in order to obtain results for swapping of interest rates so that both firms have mutual benefit.

**Appendix B**

\[
\frac{\partial |\Delta|}{\partial T} = \frac{\sigma_B}{2d_B \sqrt{T}} \phi(X_{1b}) - \frac{\sigma_A}{2d_A \sqrt{T}} \phi(X_{1A})
\]

(B.1)

\( \phi(X_1) \) denotes the first derivative of the cumulative normal distribution \( \Phi(X_1) \) with respect to \( X_1 \), where by definition,

\[
\phi(x_1) = \frac{1}{\sqrt{2\pi}} e^{-\left(x_1^2 / 2\right)}.
\]

Therefore,

\[
\phi(X_{1b}) = e^{(X_{1b} - x_{1b}^2) / 2},
\]

where

\[
X_1 \equiv \frac{\ln(d) - \frac{\sigma^2 T}{2\sigma}}{\sigma \sqrt{T}}.
\]

Since the terms \( \sigma, d, T \) and \( \phi(X_{1A}) \) are always positive, the sign of \( \partial |\Delta| / \partial T \) is determined by the terms within the bracket in equation (B.1). That is,

if \( \frac{\partial \phi(X_{1b})}{\partial \phi(X_{1A})} > 1 \), then \( \partial |\Delta| / \partial T > 0 \)

or vice versa.

If we let \( \pi = \frac{\partial \phi(X_{1b})}{\partial \phi(X_{1A})} > 1 \), then if \( \pi > 1 \) or \( \ln(\pi) > 0 \), then \( \partial |\Delta| / \partial T > 0 \).
By simple algebra,
\[
\ln(\pi) = \frac{1}{2\sigma^2} \{[\ln(d_A) + \ln(d_B)] + \sigma^2 T \}
\times [\ln(d_A) - \ln(d_B)].
\]

For \( d_A < d_B \) and \( d \leq 1 \), then \( \ln(\pi) > 0 \) for the values of \( T \):
\[
- \frac{\ln(d_A) + \ln(d_B)}{\sigma^2} > T.
\]

(b) For identical \( d \), \( d \leq 1 \) and \( \sigma_A < \sigma_B \).

By factorising,
\[
\frac{\partial |\Delta g|}{\partial T} = \frac{\sigma_A}{2d\sqrt{T}} \phi(X_{1A}) \left[ \frac{\sigma_B \phi(X_{1B})}{\sigma_A \phi(X_{1A})} - 1 \right].
\]

(B.2)

Since the terms \( \sigma_A \), \( d \), \( T \) and \( \phi(X_{1A}) \) are always positive, the sign of \( \partial |\Delta g|/\partial T \) is determined by the terms within the bracket in equation (B.2). That is,

if \( \frac{\sigma_B \phi(X_{1B})}{\sigma_A \phi(X_{1A})} > 1 \), then \( \frac{\partial |\Delta g|}{\partial T} > 0 \)

or vice versa.

If we let \( \pi = (\sigma_B/\sigma_A) \phi(X_{1B})/\phi(X_{1A}) \), then if \( \pi > 1 \) or \( \ln(\pi) > 0 \), then \( \partial |\Delta g|/\partial T > 0 \).

It follows that
\[
\ln(\pi) = \ln \left( \frac{\sigma_B}{\sigma_A} \right) + \left( \frac{\sigma_B^2 - \sigma_A^2}{\sigma_B} \right)(\ln(d) + \frac{\sigma_B \sigma_T}{2})(\ln(d) - \frac{\sigma_A \sigma_T}{2}).
\]

For \( d \leq 1 \) and \( \sigma_A < \sigma_B \), then \( \ln(\pi) > 0 \) for the values of \( T \):
\[
\left| \ln \left( \frac{\sigma_B}{\sigma_A} \right) \right| > \left| \frac{(\sigma_B^2 - \sigma_A^2)(\ln(d) + \frac{\sigma_B \sigma_T}{2})(\ln(d) - \frac{\sigma_A \sigma_T}{2})}{2\sigma_A^2 \sigma_B^2 T} \right|.
\]

References


