Fuzzy likelihood ratio test for cooperative spectrum sensing in cognitive radio

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\textbf{Abstract}
Efficient and reliable spectrum sensing is an essential requirement in cognitive radio networks. One challenge faced in the spectrum sensing is the existence of the noise power uncertainty. This paper proposes a cooperative spectrum sensing scheme using fuzzy set theory to mitigate the noise power uncertainty. In this scheme, the noise power uncertainty in each Secondary User (SU) is modeled as a Fuzzy Hypothesis Test (FHT). We deploy the likelihood ratio test on the FHT to derive a fuzzy energy detector with a threshold that depends on the noise power uncertainty bound. The fusion center combines the received local hard decisions from the SUs and makes a final decision to detect the absence/presence of a Primary User (PU). We compare the performance of the proposed algorithm with some classical threshold-based energy detection schemes using receiver operating characteristic and detection probability versus the signal to noise ratio curves via Monte Carlo simulations. The proposed algorithm outperforms the cooperative spectrum sensing with a bi-thresholds energy detector and a simple energy detector.

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1. Introduction

Cognitive Radio (CR) technology has been introduced to alleviate the spectrum scarcity through opportunistically access of Secondary Users (SUs) to licensed spectrum bands. In a CR network, SUs must accurately sense the spectrum holes, use those for their transmissions and vacate the frequency band as soon as the Primary Users (PUs) start their transmissions\cite{1,2,3}. The main challenge in such systems is to avoid the harmful interference from SUs imposed on the PUs in their vicinity through spectrum sensing. The basic idea behind the spectrum sensing is to detect the weak signal of a PU. To alleviate some practical concerns of spectrum sensing such as multipath fading and hidden PUs, cooperative spectrum sensing has been proposed\cite{4}. In centralized cooperative spectrum sensing, each SU sends its local sensing results to a fusion center. The fusion center combines the received local information and makes a final decision to detect the absence/presence of the PU. In recent years, cooperative spectrum sensing schemes have been extensively studied in cognitive radio networks\cite{5,6}. In \cite{7}, the authors provide a comprehensive survey in the area of spectrum sensing, in particular in the context of cooperative approaches. To improve the detection probability, many signal detection algorithms are used in such cooperative spectrum sensing (e.g.\cite{6}). In most detection techniques, the noise power is supposed to be known a priori for threshold setting. However due to the noise power variations, the power of the noise is not known precisely which yields the noise uncertainty in the signal detection process. Due to the noise uncertainty and sensing time limitations, obtaining the accurate noise power is not
an easy task, thus the performance of such detection methods is susceptible to the noise power uncertainty. There are two types of the noise uncertainty; receiver device and environment noise uncertainties [8]. The receiver device noise uncertainty is caused by the time-varying thermal noise and the nonlinearity of the receiver components. Transmissions from other users, on the other hand, are the sources of the environment noise uncertainty. To achieve a desired performance in an energy detection-based system in the presence of the noise power uncertainty, the received Signal to Noise Ratio (SNR) must be more than a prespecified threshold level [9].

There exists some literature on the noise power uncertainty in cooperative spectrum sensing [10–12]. The study in [10] proposes a covariance and eigenvalue-based cooperative spectrum approaches in the presence of the noise uncertainty. However, the scheme in [10] has more complexity than common methods for cooperative spectrum sensing such as the energy detector. The authors in [11] propose an energy detector-based cooperative spectrum sensing method that uses three thresholds for local sensing. The methods proposed in [10,11] use soft decision combining mechanisms, however, the approaches occupy more bandwidth of the control channel for sending local sensing results. Some literature surpass this soft decision concern through utilizing a hard decision combining approach with a bi-thresholds energy detector in SU (e.g. [12]).

Throughout this paper, we address the hard decision-based cooperative spectrum sensing in the presence of the noise power uncertainty and utilize fuzzy set theory concepts [13] as a mathematical framework to model such uncertainty. In this scheme, the noise power uncertainty in each SU is modeled as a Fuzzy Hypothesis Test (FHT). We deploy the Likelihood Ratio Test (LRT) on the FHT [14] to derive a fuzzy energy detector with a threshold that depends on the noise power uncertainty. The local sensing results of SUs are sent to a fusion center and are combined to make a final decision to detect the absence/presence of the PU. We compare the performance of the proposed algorithm with some classical threshold-based energy detection schemes using the Receiver Operating Characteristic (ROC) and the detection probability versus the signal to noise ratio curves via Monte Carlo simulations. The proposed algorithm outperforms the cooperative spectrum sensing with a bi-thresholds energy detector in [12] and a simple energy detector.

The rest of the paper is organized as follows. Section 2 provides an overview of the system model, the FHT concept and the likelihood ratio test for FHT. In Section 3, we derive a detector using LRT for FHT at each SU and deploy this detector for the proposed cooperative spectrum sensing approach. Simulation results are presented in Section 4. Finally, Section 5 provides a brief summary and conclusions.

Notations: Throughout this paper, we use boldface lower case letter to denote vector. A Gaussian random variable with mean $m$ and variance $\sigma^2$ is represented by $\mathcal{N}(m,\sigma^2)$. Also, we use $\mathbb{E}()$ as the expectation operator, $\text{sup}(.)$ for representing the supremum, and $\mathbb{P}(.)$ for the probability of the given event. The signs “$\preceq$” and “$\succeq$” mean “almost smaller” and “almost greater”, respectively. Finally $Q(.)$ is the complementary cumulative distribution function, which calculates the tail probability of a zero mean and unit variance Gaussian variable, i.e. $Q(x) = \int_x^\infty (1/(\sqrt{2\pi})) \exp(-t^2/2) dt$.

2. System model and FHT formulation
2.1. System model

In this work, we consider a homogeneous cognitive radio network in the sense that all the SUs use the same protocol for local spectrum sensing. The network consists of $M$ secondary users indexed by $\{i = 1,\ldots,M\}$ that monitor the frequency band of the interest as shown in Fig. 1. The spectrum sensing problem for the $i$th SU can be represented by the binary hypothesis test as follows:

$$
\begin{align*}
H_0 : x_i[k] &= w_i[k], \\
H_1 : x_i[k] &= s_i[k] + w_i[k]
\end{align*}
$$

for $k = 0,1,\ldots,N-1$, where $N$ is the number of samples, $x_i[k]$ and $s_i[k]$ are the baseband representations of the $k$th samples of the received signal (at the $i$th SU’s receiver) and the PU signal, respectively, $w_i[k]$ denotes a complex additive white Gaussian noise with mean zero and the estimated variance $\sigma^2_{w_i}$. Throughout this paper, we assume that $s_i[k]$s are independent Gaussian random variables with zero mean and variance $\sigma^2_{s_i}$. This assumption is used in many literature related to spectrum sensing in CR networks (e.g. [15,16]). We also assume that $s_i[k]$ and $w_i[k]$ are independent. The performance of detecting $H_1$ against $H_0$ is defined as the probability of false alarm and the probability of detection. In CR networks, a larger detection probability indicates a less interference between the PU and SUs, and a smaller false alarm probability indicates a higher spectrum efficiency. Due to the uncertainty, the noise variance is not known precisely. To represent the noise power uncertainty, let the estimated noise variance $\hat{\sigma}^2_{o_i}$.

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![Fig. 1. Centralized cooperative spectrum sensing in a cognitive radio network.](image-url)
parameter that defines the noise power uncertainty bound [9]. In practice, the range of ρ is normally between 0.12 and 0.6 [16]. One heuristic approach to encounter the noise power uncertainty is to use the fuzzy hypothesis test. The following subsections present a brief introduction to the FHT and related likelihood ratio.

2.2. Fuzzy hypothesis test

The traditional detection problem is to decide between two crisp hypotheses \( H_1 \) known as the alternative hypothesis versus the null hypothesis \( H_0 \) [17]. Similar to (1), \( H_0 \) represents only the noise, while in \( H_1 \), the noise with signal is presented. In some situations, there are many practical problems that deal with uncertainty. During the past decades, there have been some attempts to model this uncertainty using fuzzy set theory concepts. The uncertainty consideration in the hypothesis test introduces an interesting problem called fuzzy hypothesis test (FHT). To introduce the FHT, suppose \( \theta \) is the mean parameter of a normal probability density function (pdf) and we would want to test the hypotheses related to the value of \( \theta \) based on the observations. In the crisp hypothesis test, we use the hypotheses expressions as \( H_0 : \theta = \theta_0 \) versus \( H_1 : \theta \neq \theta_0 \). Due to the uncertainty, a more realistic hypothesis can be written as

\[
\begin{aligned}
H_0 : \theta &\text{ close to } \theta_0, \\
H_1 : \theta &\text{ away from } \theta_0.
\end{aligned}
\]

Such hypothesis test is called fuzzy hypothesis test (or FHT) which has been extensively investigated in the literature [18–21].

2.3. Likelihood ratio test for FHT

This subsection is devoted to some definitions and preliminaries about the likelihood ratio test for FHT. Let \( X=(X_1, \ldots, X_n)^T \) be a random sample vector, with observed value \( x=(x_1, \ldots, x_n)^T \) where \( X_i \) has the pdf \( f(x_i; \theta) \) with unknown parameter \( \theta \in \Theta \), in which \( \Theta \) is the parameter space.

**Definition 1.** Any hypothesis of the form “\( H : \theta = H(\theta) \)” is called a fuzzy hypothesis, where “\( H : \theta = H(\theta) \)” implies that \( \theta \) is in a fuzzy set of \( \Theta \) with membership function \( H(\theta) \), i.e. a function from \( \Theta \) to [0, 1].

Note that the crisp hypothesis “\( H_0 : \theta = \theta_0 \)” is a fuzzy hypothesis with the membership function “\( H_0(\theta) = 1 \)” at \( \theta = \theta_0 \), and zero otherwise”. Therefore, in the fuzzy hypothesis test, we have

\[
\begin{aligned}
H_0 : \theta &\text{ is } H_0(\theta), \\
H_1 : \theta &\text{ is } H_1(\theta),
\end{aligned}
\]

where \( H_0(\theta) \) and \( H_1(\theta) \) are known. The aim is to accept or reject \( H_0 \) on the basis of \( x \). In other words, we would want to make a test function \( \phi(X) \) such that \( \phi(x) \) is the probability of rejecting \( H_0 \) if \( X=x \) is observed.

**Definition 2.** For the fuzzy hypothesis test in (3) and the test function \( \phi(X) \), the false alarm probability is defined as [22]

\[
P_{fa} = \sup_{\theta \in \Theta} H_0(\theta) E_\theta[\phi(X)],
\]

where

\[
E_\theta[\phi(X)] = \int \phi(x)f(x; \theta) \, dx
\]

and \( f(x; \theta) \) is the joint pdf of \( x \) with unknown parameter \( \theta \in \Theta \).

**Definition 3.** For the fuzzy hypothesis test problem in (3), a likelihood ratio test is a statistical test that rejects \( H_0 \) if and only if \( \lambda(x) < \kappa \), where \( \kappa \in [0,1] \) and \( \lambda(x) \) is the likelihood ratio statistic defined as [14]

\[
\lambda(x) = \frac{\sup_{\theta \in \Theta} H_0(\theta)f(x; \theta)}{\sup_{\theta \in \Theta} f(x; \theta)}.
\]

3. Cooperative spectrum sensing using fuzzy likelihood ratio test

3.1. Local spectrum sensing at SUs

In this section, we provide an efficient algorithm to encounter the noise power uncertainty using the FHT. To achieve this goal, we reconsider the conventional detection using the energy detector for local spectrum sensing at each SU. For such energy detector, the decision rule is given by

\[
U_i = \sum_{k=0}^{N-1} |x(k)|^2 \geq \eta_i
\]

where \( U_i \) and \( \eta_i \) are the test statistic and the detection threshold in the ith SU, respectively. According to the central limit theorem, \( U_i \) is approximated by the Gaussian random variable when \( N \) is large enough [23]. Therefore, the statistics of \( U_i \) is given by

\[
U_i \sim \begin{cases} 
\mathcal{N}(N\sigma_n^2, 2N\sigma_n^2) & \text{under } H_0, \\
\mathcal{N}(N(\sigma_n^2 + \sigma_k^2), 2N(\sigma_n^2 + \sigma_k^2 + 2\sigma_n^2\sigma_k^2)) & \text{under } H_1.
\end{cases}
\]

The performance of the energy detector in the ith SU is defined as the probability of false alarm denoted by \( P_{fa}^{(i)} \) and the probability of detection denoted by \( P_{d}^{(i)} \) which are given by

\[
P_{fa}^{(i)} = \mathbb{P}[U_i > \eta_i | H_0] = Q\left(\frac{\eta_i - N\sigma_n^2}{\sqrt{2N}}\right)
\]

and

\[
P_{d}^{(i)} = \mathbb{P}[U_i > \eta_i | H_1] = Q\left(\frac{\eta_i - N(\sigma_n^2 + \sigma_k^2)}{\sqrt{2N(\sigma_n^2 + \sigma_k^2)}}\right).
\]

The threshold \( \eta_i \) is determined based on the false alarm probability as follows:

\[
\eta_i = \sigma_n^2 \sqrt{2N} Q^{-1}(P_{fa}^{(i)}) + N.
\]

The energy detector described in (7) is equivalent to the utilization of the variance of the received signal at the SU for detection of the PU signal. Under each hypothesis, the
variance of the received signal on the observation interval at the ith SU denoted by $\sigma_i^2$ can be written as

$$\sigma_i^2 = \begin{cases} 
\sigma_{0,i}^2 & \text{under } H_0, \\
\sigma_{1,i}^2 + \sigma_{2,i}^2 & \text{under } H_1.
\end{cases}$$

(12)

For decision making, $\sigma_i^2$ is compared with the threshold $\eta_i$, i.e.,

$$\begin{cases} 
H_0 : \sigma_i^2 \leq \eta_i, \\
H_1 : \sigma_i^2 > \eta_i.
\end{cases}$$

(13)

According to the fuzzy hypothesis test in (3) and using crisp membership functions, the hypothesis test in (13) can be written as

$$\begin{cases} 
H_1 : H_1(\sigma_i^2) = u(\sigma_i^2 - \eta_i), \\
H_0 : H_0(\sigma_i^2) = 1 - H_1(\sigma_i^2),
\end{cases}$$

(14)

where $u(\cdot)$ is the unit step function.

Now, we generalize the above conventional hypothesis testing to a fuzzy hypothesis testing problem. As mentioned before, due to the uncertainty, the noise variance is not known precisely. One heuristical approach to encounter the noise power uncertainty is to use the fuzzy hypothesis test that applies the membership functions of hypotheses. The corresponding fuzzy hypothesis test can be implemented as follows:

$$\begin{cases} 
H_0 : \sigma_i^2 \leq \eta_i, \\
H_1 : \sigma_i^2 \geq \eta_i,
\end{cases}$$

(15)

where $\eta_i = \sigma_{0,i}^2((2NQ)^{-1}(p_0^{(n_i)} + N)$. Through extending the crisp membership functions in (14), we propose the membership functions for the fuzzy hypothesis test as follows:

$$\begin{cases} 
H_1 : H_1(\sigma_i^2) = \frac{1}{1 + e^{-(1/\rho)\sigma_i^2 - \eta_i}}, \\
H_0 : H_0(\sigma_i^2) = 1 - H_1(\sigma_i^2),
\end{cases}$$

(16)

where $\eta_i$ is computed by (11). The proposed membership functions yield the desirable properties $H_1(\sigma_i^2) = 0$ for $\sigma_i^2 < \eta_i$ and $H_1(\sigma_i^2) = 1$ for $\sigma_i^2 > \eta_i$. In addition, in the ideal case where there is no noise power uncertainty or $\rho_i \to 0$, the membership functions have correct responses. More precisely, in the presence of the PU signal at the ith SU, we have $(\sigma_i^2 - \eta_i) > 0$ and $e^{-(1/\rho_i)\sigma_i^2 - \eta_i} \to 0$. Thus, from (16) we have $H_1(\sigma_i^2) = 1$ and $H_0(\sigma_i^2) = 0$. Also, in the absence of the PU signal, if $(\sigma_i^2 - \eta_i) < 0$, we have $e^{-(1/\rho_i)\sigma_i^2 - \eta_i} \to \infty$, then the membership function values will be $H_1(\sigma_i^2) = 0$ and $H_0(\sigma_i^2) = 1$. Fig. 2 demonstrates the fuzzy membership function of $H_0$ for the crisp and fuzzy hypothesis tests for different values of uncertainty factor $\rho$. It is seen that the uncertainty in the fuzzy hypothesis model causes a smooth transition from $H_0$ hypothesis to $H_1$ and vice versa. In addition, the fuzzy hypothesis model tends to the crisp hypothesis model when $\rho$ decreases.

For local spectrum sensing at each SU, we construct the fuzzy likelihood ratio statistic according to (6) that is given by

$$\lambda(\mathbf{x}_i) = \frac{\sup_{\sigma_i^2} G(\sigma_i^2)}{\sup_{\sigma_i^2} f(\mathbf{x}_i; \sigma_i^2)}.$$
As mentioned before, the false alarm probability for the fuzzy hypothesis testing is given by
\[
P_{fa}^0 = \sup_{\sigma_0^2} H_0(\sigma_0^2) \mathbb{E}_{\sigma_0^2}[P(X_i)] = \sup_{\sigma_0^2} \left\{ \frac{e^{-(1/\rho_0)\sigma_0^2 - \eta_0}}{1 + e^{-(1/\rho_0)\sigma_0^2 - \eta_0}} \mathbb{E}_{\sigma_0^2}[P(X_i)] \right\},
\]
(25)
where
\[
\mathbb{E}_{\sigma_0^2}[P(X_i)] = \mathbb{P}(U_i \geq \eta_{H_{1f_i}}) = \int_{\eta_{H_{1f_i}}}^{\infty} f_{U_i}(u_i) \, du_i.
\]
(26)
Since \(U_i\) is the squares summation of \(N\) independent and identically distributed Gaussian random variables with zero-mean and variance of \(\sigma_i^2\), \(U_i\) is approximated by a Gaussian random variable with distribution \(N(\sigma_i^2, 2\sigma_i^2)\) [23]. Therefore, we have
\[
\mathbb{E}_{\sigma_i^2}[P(X_i)] = \frac{\eta_{H_{1f_i}} - \sigma_i^2}{\sqrt{2N\sigma_i^2}}.
\]
(27)
Thus, the false alarm probability at the \(i\)th local SU is
\[
P_{fa}^0 = \sup_{\sigma_i^2} \left\{ \frac{e^{-(1/\rho_i)\sigma_i^2 - \eta_i}}{1 + e^{-(1/\rho_i)\sigma_i^2 - \eta_i}} \mathbb{E}_{\sigma_i^2}[P(X_i)] \right\}.
\]
(28)
In each secondary user, this equation can be solved by an iterative optimization algorithm.

In the proposed method, \(\sigma_i^2\) and \(\rho_i\) are determined according to the noise conditions. Based on a desired false alarm probability \(P_{fa}^0\) and the nominal noise power \(\sigma_i^2\), the amount of \(\eta_i\), is computed from (11). Having \(\rho_i, \eta_i,\) and \(P_{fa}^0, \eta_{H_{1f}}\), can be obtained from the complicated Eq. (28). This equation can be solved by an iterative optimization algorithm. For example, Algorithm 1 suggests an iterative method for solving Eq. (28). In this algorithm, the smaller values for \(d\rho\) and \(d\eta\) give the higher accuracy, but the running time mutually increases. In our simulations, we have selected \(d\rho = 0.0015\) and \(d\eta = 0.15\). Although calculation of the threshold values in this method is complicated, the threshold values can be calculated offline, tabulated and used by the detector at each SU. Table 1 gives some tabulated thresholds for several values of \(P_{fa}^0\) and \(\rho_i\) for the fuzzy method and the energy detector. It is observed that unlike the energy detector, the threshold values for the fuzzy method change with noise uncertainty bound. So far, we perform local spectrum sensing at each SU. In the next section, we use cooperative spectrum sensing for the performance improvement.

**Algorithm 1.** Threshold determination for the fuzzy method.

1: Given \(\sigma_0^2, \rho_i,\) and \(P_{fa}^0\)
2: Obtain \(\eta_i\), from \(\eta_i = \sigma_i^2 \sqrt{2\eta Q^{-1}} (P_{fa}^0) + N\)
3: Initialize \(k = 0\) and \(\eta_{H_{1f}} = \eta_i\)
4: \(k = k + 1\)
5: Compute \(P_{fa}^{(k)} = \sup_{\sigma_i^2} \left\{ \frac{e^{-(1/\rho_i)\sigma_i^2 - \eta_i}}{1 + e^{-(1/\rho_i)\sigma_i^2 - \eta_i}} \mathbb{E}_{\sigma_i^2}[P(X_i)] \right\}.
6: if \(P_{fa}^{(k)} - P_{fa}^{(k-1)} > d\rho\) then \(\eta_{H_{1f}}^{(k)} = \eta_{H_{1f}}^{(k-1)} - d\rho\), Go to step 4
7: else if \(P_{fa}^{(k)} - P_{fa}^{(k-1)} < d\rho\) then \(\eta_{H_{1f}}^{(k)} = \eta_{H_{1f}}^{(k-1)} + d\rho\), Go to step 4
8: else if \(P_{fa}^{(k)} - P_{fa}^{(k-1)} < d\rho\) then \(\eta_{H_{1f}}^{(k)} = \eta_{H_{1f}}^{(k-1)}\) and Exit
9: end if

3.2. Cooperative spectrum sensing

Cooperative spectrum sensing is proposed to mitigate the impact of multipath fading, shadowing and hidden PU problem. In addition, it improves the detection performance at the low SNR and decreases the sensing time. In this paper, we adopt centralized CR networks in which each SU sends its sensing results to a fusion center via a dedicated control channel. The fusion center combines the sensing results and makes a decision on the presence or absence of the PU signal. In order to minimize the communication overhead, we use a hard decision combining approach. In this method, the fusion center combines the binary local decisions sent from the SUs through prespecified rules such as \(K\) out of \(M\). In this rule, the fusion center decides the PU signal is present if at least \(K\) out of \(M\) SUs decide \(H_1\). In this paper, we use “OR” and “AND” rules which correspond to the case of \(K=1\) and \(K=M\), respectively. Based on the OR rule, the fusion center decides the presence of the PU signal meaning that there is no spectrum hole, if at least one SU decides \(H_1\). In addition, based on the AND rule, the fusion center decides the presence of the PU signal, if all SUs decide \(H_1\). It is observed that the OR rule behaves very conservatively for accessing the licensed band. Using the OR rule, the probability of interference between SUs with PU would be minimum. The false alarm and the detection probabilities based on the OR and AND rules in the cooperative spectrum sensing are given by [24]

\[
Q_{f-OR} = 1 - \prod_{i=1}^{M} (1 - P_{fa}^{(i)}),
\]
(29)
\[
Q_{d-OR} = 1 - \prod_{i=1}^{M} (1 - P_{fa}^{(i)}),
\]
(30)
\[
Q_{f-AND} = \prod_{i=1}^{M} P_{fa}^{(i)},
\]
(31)
\[
Q_{d-AND} = \prod_{i=1}^{M} P_{fa}^{(i)},
\]
(32)
where \(P_{fa}^{(i)}\) and \(P_{fa}^{(i)}\) are the false alarm and the detection probabilities at the \(i\)th SU, respectively, and \(Q_f\) and \(Q_d\) denote the false alarm and the detection probabilities in the fusion center, respectively.

**Table 1**

<table>
<thead>
<tr>
<th>(P_{fa} = 0.15)</th>
<th>(P_{fa} = 0.3)</th>
<th>(P_{fa} = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED threshold</td>
<td>220.72</td>
<td>210.48</td>
</tr>
<tr>
<td>Fuzzy method threshold</td>
<td>210.57</td>
<td>199.0</td>
</tr>
<tr>
<td>ED threshold</td>
<td>220.72</td>
<td>210.48</td>
</tr>
<tr>
<td>Fuzzy method threshold</td>
<td>209.58</td>
<td>194.97</td>
</tr>
</tbody>
</table>
4. Simulation results

In this section, we evaluate the performance of the proposed algorithm using the Receiver Operating Characteristic (ROC) and the detection probability versus SNR curves by Monte Carlo simulations. In our simulation, we assume that the number of SUs ($M$) is equal to 5 and the number of samples ($N$) is equal to 200. In addition, $\rho_i$ and $\text{SNR}_i$ indicate the noise power uncertainty bound and the SNR at the $i$th SU, respectively. Fig. 3 shows the ROC curves of a single SU for the proposed fuzzy method compared to the energy detector for $\text{SNR} = -5$ dB and different values of the noise power uncertainty. It is observed that the proposed fuzzy method outperforms the energy detector in all cases. In addition, an increase in the noise power uncertainty results in a decrease in the performance of both methods.

In the rest of the simulations, we use the centralized cooperative spectrum sensing, in which a fusion center receives the local decisions from SUs and combines them using “OR” and “AND” rules. We compare the proposed fuzzy method with the simple energy detector and the method proposed in [12], in which the SUs use a bi-thresholds detector for local spectrum sensing. The decision equation of the bi-thresholds detector in the $i$th SU is given as

$$\phi_{bi}(x_i) = \begin{cases} 0, & U_i \leq \eta_i \, \text{AND} \, \eta_l < U_i < \eta_h, \\ D_i, & \eta_l < U_i < \eta_h, \\ 1, & \eta_h \leq U_i, \end{cases}$$

where $D_i$ indicates that the decision falls into a confused region and the two thresholds are computed based on the low and high bounds of the noise power uncertainty as follows [12]:

$$\eta_l = (1 - \rho)\sigma_i^2(\sqrt{2}NQ^{-1}(P_{fa}) + N),$$

$$\eta_h = (1 + \rho)\sigma_i^2(\sqrt{2}NQ^{-1}(P_{fa}) + N).$$

Fig. 4 shows the ROC curves for cooperative spectrum sensing that compares the performance of the proposed fuzzy method with one and bi-thresholds energy detector methods for $\rho = 0.26$ and $\text{SNR} = -10$ dB for each SU. It is seen that the performance of the proposed fuzzy method with “OR” and “AND” rules is better than that of the other detectors. In the proposed fuzzy method, the introduced membership functions take different values, depending on the noise power uncertainty and these values are utilized for the threshold determination. While, in the simple energy detector the threshold is set regardless of the noise power uncertainty and the bi-thresholds energy detector uses only two thresholds that depend on the noise power uncertainty bound. In Fig. 5, we simulate a more practical example related to the case of Fig. 4. Here, we assume that $\rho = 0.26$ and $\text{SNR} = -18$ dB that are reasonable values for the IEEE 802.22 standard. It is observed that the proposed fuzzy method with the “OR” and after that “AND” rules have the best performance, and the other detectors, become random detectors. In [25], it
is shown that for many practical cases, the "OR" rule has the better performance than that of other rules.

In Fig. 6, we consider the case that the uncertainty factor for each SU is different. For this simulation, we have set SNR = −15 dB, \(\rho_1 = 0.6, \rho_2 = 0.26, \rho_3 = 0.6, \rho_4 = 0.12\) and \(\rho_5 = 0.12\). It is seen from Fig. 6 that the proposed fuzzy method along with the "OR" rule outperforms the other detectors. A more practical case is when both uncertainty factor and SNR are different in all SUs. For this case, Fig. 7 depicts the ROC curves of the aforementioned detectors in which we have set SNR1 = −10 dB, \(\rho_1 = 0.12, \rho_2 = 0.26, \rho_3 = 0.12\) and \(\rho_4 = 0.12\) and \(\rho_5 = 0.26\). It is observed that the proposed fuzzy method with the "OR" rule outperforms the other detectors. The difference between the starting points of the ROC curves is because of computational limitations for small values of false alarm probability, and also the difference between the values of \(Q_f - OR\) and \(Q_f - AND\) when they are computed by (29) and (31), respectively.

Another common way to evaluate the performance of the algorithms is to use the curve of probability of detection versus SNR. For this purpose, we assume that all SUs are in the same conditions from the uncertainty factor and false alarm probability perspectives. Fig. 8 shows these curves for the aforementioned detectors when the probability of false alarm at the fusion center denoted by \(Q_{fa - FC}\) equals to 0.1. In this figure, it is assumed that the uncertainty factor for all users is \(\rho = 0.6\). As observed in this figure, the proposed fuzzy method with the "OR" rule again outperforms the other detectors.

5. Conclusion

In this paper, a cooperative spectrum sensing algorithm based on the fuzzy hypothesis test was proposed to confront the noise power uncertainty. We derived the detector using the likelihood ratio test for the fuzzy hypothesis test at each SU. It was shown that the obtained detector is an energy detector except that its threshold depends on the noise power uncertainty. A fusion center receives the local hard decisions from the SUs and fuses them using the "AND" and "OR" rules for final decision. Simulation results revealed that the proposed algorithm outperforms the cooperative spectrum sensing by a simple energy detector and a bi-thresholds energy detector and is more suitable for spectrum sensing applications.

References