MODELING OF PASSENGER WAITING TIME IN INTERMODAL STATION WITH CONSTRAINED CAPACITY ON INTERCITY TRANSIT

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Abstract: This paper aims to develop a mathematical model to analyze the passenger waiting time in an intermodal station in which intercity transit system served with feeder buses. Passengers are categorized into two groups, named scheduled and random, respectively. The analytic model is developed for quantifying the relationships of passenger waiting time to reliability of feeder bus services and capacity of intercity transit. The numerical results identify that mean waiting time of intercity transit passengers is greater than half of the intercity transit headway, with the condition that feeder service is provided with poor reliability; and on the contrary, mean waiting time will be less than half of the intercity transit headway. Moreover, to avoid long waiting time, headway deviation of feeder buses should be less than 3.5 minutes. In terms of intercity transit capacity, it is suggested that at least 80% of total arrival passengers should be satisfied.

Keywords: intermodal, transfer, waiting time, passenger behaviors, headway.

1. INTRODUCTION

When analyzing mean waiting time of an intensive urban transit service, passenger arrivals are assumed to follow uniform distribution; and thus consequently obtained the outcomes of “mean waiting time is half of the headway”. However, for the intercity transit passengers, they tend to arrive at the station close to the departure time, because of the regular schedule and relatively long headway. Thus, the waiting behaviors are different from those of urban bus transit passengers.

Moreover, intercity transit passengers will go to intermodal transit station by various feeder systems, e.g. mass rapid transit, light rail, bus, taxi, auto, bicycle and even walk. Reliability of these feeder services should have great influences on the passengers’ behavior of arranging their arrival time to the intermodal transit station. If the urban bus service is considered as a major feeder system for the intercity transit, passenger waiting time and relevant behaviors for
the intercity transit system should be influenced by the service reliability of feeder buses. And of course, the capacity constraints of intercity transit will also make some impacts on the passenger waiting time.

For the urban bus systems, if considering the influence of service reliability, the mean random waiting time $w_r$ can be characterized as Equation (1), where $h$ represents the mean headway and $\sigma$ represents the deviation of bus headways (Welding, 1957; Osuna & Newell, 1972).

$$w_r = \frac{h}{2} \left[ 1 + \left( \frac{\sigma}{h} \right)^2 \right]$$

Equation (1) shows mean waiting time will decrease with the increase of bus service reliability (i.e., decrease of $\sigma$). Turnquist (1981) then tried to expand the study of Welding. He divided the passengers into two categories: random and non-random, according to their behaviors of taking buses. Waiting time of the random passengers is calculated based on Equation (1) and a log-normal distribution of arrivals is assumed to calculate the waiting time of non-random passengers. However, Turnquist did not make further efforts on passenger behaviors.

Another study finds that due to traffic congestion, reliability of service usually decreases when feeder buses go into the CBD area or intercity transit station area (Orloff & Ma, 1975). They assume that the feeder bus headway follows normal distribution, and let the deviation of headway represents the reliability index of service, then service reliability of feeder system will not only be affected by the route environment but also by the congestion of intercity transit station area.

In 1995, Knoppers & Muller show the waiting time of intercity transit passengers will be changed according to the arrival time of feeder buses and departure time of intercity transit. Expected mean transfer waiting time $E(w)$ can be obtained by the integral of passengers’ actual waiting time $w(p)$ and their corresponding probability, as shown in Equation (2). It improves the accurate of waiting time calculation. However, capacity constraints and passenger behaviors are not considered in their research.

$$E(w) = \int_{-\infty}^{\infty} w(p) \cdot Pr(p) dp$$

Distinguished and improved from the former studies, this paper aims to develop a mathematical model for analyzing the integrated influence of “feeder service reliability”,

“passenger behaviors”, and “capacity constraint of intercity transit” on passenger waiting time for intercity transit systems (e.g., intercity bus, conventional rail, and high speed rail), and identifying the potential benefits of intercity transit users by increase of feeder service reliability. Hereinafter, the paper first discusses assumptions of model, then analyzes the influences of different feeder and intercity transit service scenarios, and finally makes some useful suggestions to system operation according to the numerical case.

2. MODELING PASSENGER WAITING TIME

In this study, reliability of feeder bus services, passenger behaviors and service characteristics of intercity transit are taken into consideration to develop the intercity passenger waiting time model. Concept of system operation is shown as Figure 1. Some basic assumptions of this model are listed as follows:

- Operation of feeder and main services (i.e. intercity transit) is considered as a continuous flow.
- Feeder buses have a good scale of fleet.
- There is intensive passenger demand in service area.
- Intercity transit system is always operated promptly.
- The in-station walking and processing time are neglected, e.g., the waiting time represents total transfer time.
- Passengers are categorized into two groups:
  - Scheduled passengers: passengers who have decided to ride specific transit or even have the seats reserved already.
  - Random passengers: the others not belong to scheduled passengers.

![System Topology](image)

Figure 1. System Topology

According to the characteristics of these two types of passengers, the probability distribution functions of passenger arrivals and mean waiting time model of intercity transit passengers will be derived step by step. Detailed definition and reasonable values of variables and parameters deployed in the model are listed as in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
<th>Reasonable Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Access behavior parameter of random passengers</td>
<td>1</td>
</tr>
<tr>
<td>$A$</td>
<td>Total arrival passengers by feeder buses during $t = 0 \sim H$</td>
<td>—</td>
</tr>
<tr>
<td>$c$</td>
<td>Crush capacity of feeder buses (passengers/veh), including “standee” and “seated”</td>
<td>55</td>
</tr>
<tr>
<td>$C$</td>
<td>Seat capacity of intercity transit (seats/$H$)</td>
<td>70</td>
</tr>
<tr>
<td>$h$</td>
<td>Headway of feeder buses</td>
<td>10 min</td>
</tr>
<tr>
<td>$H$</td>
<td>Headway of intercity transit</td>
<td>30 min</td>
</tr>
<tr>
<td>$H_p$</td>
<td>Planned arrival time in intermodal transit station</td>
<td>—</td>
</tr>
<tr>
<td>$I$</td>
<td>Reliability factor of feeder bus information system</td>
<td>0.8</td>
</tr>
<tr>
<td>$k$</td>
<td>Percentage of scheduled passengers</td>
<td>80%</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Missed percentage of additional waiting cycle $r$, due to the capacity constraint of intercity transit</td>
<td>—</td>
</tr>
<tr>
<td>$n$</td>
<td>Arrival number of feeder buses during the intercity transit headway</td>
<td>—</td>
</tr>
<tr>
<td>$N(t)$</td>
<td>Aggregated arrival probability density function of intercity transit passengers</td>
<td>—</td>
</tr>
<tr>
<td>$N_s(t)$</td>
<td>Arrival probability density function of scheduled passengers</td>
<td>—</td>
</tr>
<tr>
<td>$N_r(t)$</td>
<td>Arrival probability density function of random passengers</td>
<td>—</td>
</tr>
<tr>
<td>$p$</td>
<td>Passengers’ planned punctuality deviation to the intercity transit station</td>
<td>5 min</td>
</tr>
<tr>
<td>$s$</td>
<td>Additional waiting cycles for intercity transit</td>
<td>1</td>
</tr>
<tr>
<td>$W$</td>
<td>Mean waiting time of intercity transit passengers</td>
<td>—</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Access behavior parameter of scheduled passengers</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Access behavior parameter of random passengers</td>
<td>2</td>
</tr>
<tr>
<td>$\beta(t)$</td>
<td>Delayed-access parameter of random passengers</td>
<td>—</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>Scheduled passengers’ reduced-probability factor for riding earlier feeder buses</td>
<td>—</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>Mean arrival time of feeder buses</td>
<td>—</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>Headway deviation of feeder buses - index of service reliability</td>
<td>5 min</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Average load factor of feeder buses</td>
<td>0.9</td>
</tr>
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</table>
2.1 Distribution of Passenger Arrivals

Scheduled Passengers

Suppose that $H$ represents headway of intercity transit and $h$ represents headway of feeder buses, and it is assumed the previous transit departed in $t = 0$. Then the analytic period of time will lie in $t = 0 \sim H$. Additionally, suppose that passengers will arrive at the station in $p$ minutes before the departure time of transit to make sure that they will not miss the transit. The operation characteristics of intermodal station, reliability of transit and feeder services will all affect $p$. For which, it comes the corresponding analytic time $H_p$, as shown in Equation (3).

$$H_p = H - p \quad 0 < H_p \leq H$$ (3)

In Equation (4), $n$ represents the arrival number of feeder buses within $t = 0 \sim H$. $n_1$ represents the arrival number of feeder buses within $H_p$, as shown in Equation (5); and $n_2$ represents the arrival number of feeder buses within time period $H_p \sim H$, which can be obtained by Equation (6).

$$n = \frac{H}{h} + 1$$ (4)

$$n_1 = \frac{H_p}{h} + 1$$ (5)

$$n_2 = n - n_1$$ (6)

Suppose that the arrivals of feeder buses follow normal distribution, and Equation (7) shows the arrival probability density function of the $q^{th}$ bus. Where $\mu_q$ means the expected arrival time of the $q^{th}$ feeder bus, as shown more detailed in Equation (8) and (9), and $\sigma_q$ is the headway deviation of $q^{th}$ feeder bus.

$$f_q = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_q^2}} e^{-\frac{(u-\mu_q)^2}{2 \sigma_q^2}}$$ (7)

$$\mu_i = H_p - (i-1)h \quad i=1\sim n_1$$ (8)
\[ \mu_i = H_p + j \cdot h \quad j = 1 - n_2 \]  

(9)

With the distribution of feeder bus arrivals, the passenger arrivals can be obtained via it. For the scheduled passengers, it is assumed that they can obtain some information from the feeder bus information system. However, reliability of bus information system is still not good enough, thus a parameter for reliability of feeder bus information system is set to be between 0.0 \( \sim \) 1.0. Meanwhile, because of the unreliability of feeder services, some of the scheduled passengers will take the earlier feeder buses to arrive at the intermodal station on their expected time and to minimize the probability of missing the anticipated transit. Since it is a planned behavior, the arrival number of these passengers will increase when it close to the departure time of intercity transit, and vise versa. In order to characterize this passenger behavior, let the arrival probability density function of feeder buses multiplied by a reduced-probability factor \( \gamma_i \), as shown in Equation (10) and (11).

\[ \gamma_i = 1 \quad i = 1 \]

\[ = \{I[H_p + (i - 1)h]\}^{-\alpha_i} \quad i = 2 \sim n_1 \]  

(10)

\[ \gamma_j = 0 \quad j = 1 \sim n_2 \]  

(11)

In Equation (10), \( \alpha_i \) represents the passenger behavior factor corresponding to feeder bus services. When the reliability of feeder services is poor, the passengers will tend to take the earlier buses and then \( \alpha_i \) declines. When \( \alpha_i \) approaches zero, the arrival distribution of scheduled passenger follows normal distribution. Based on the former study, \( \alpha_i \) is set to be 2 (Chang & Hsu, 2001).

Let the arrival distribution of individual feeder bus multiplied by the corresponding \( \gamma_i \), and take the summation, it can be obtained as Equation (12), which represents the aggregate arrival function of scheduled passengers.

\[ g(t) = \sum_{i=1}^{n_1} \gamma_i \cdot f(t; H_p - (i - 1)h, \sigma_i^2) + \sum_{j=1}^{n_2} \gamma_j \cdot f(t; H_p + j \cdot h, \sigma_j^2) \]  

(12)

To be noticed, few passengers may miss the planned intercity transit and have to wait for next transit because of the severely delay of feeder buses. However, since the service of entire system is continuous, this special case will also happen in previous intercity transit service interval, thus these cases for outliers can be eventually neglected.
Furthermore, let \( g(t) \) divided by the area below its function curve, it then turns out to be a probability density function of scheduled passengers, as shown in Equation (13).

\[
N_e(t) = \frac{g(t)}{\int_{-\infty}^{\infty} g(t)dt} \approx \frac{g(t)}{\int_{0}^{\infty} g(t)dt} \tag{13}
\]

**Random Passengers**

For the random passengers, since arrive by feeder buses, too; thus Equation (14) represents the distribution of random arrivals.

\[
f(t) = \sum_{i=1}^{n} f(t; H - (i - 1)h, \sigma_i^2) + \sum_{j=1}^{n} f(t; H_p + jh, \sigma_j^2) \tag{14}
\]

However, due to the relatively long headway of intercity transit, if the passengers arrive at the transit station and find it is still early for departure of transit services, passengers may leave the station for other activities and then come back in a specific time interval before the next departure. In order to characterize this behavior, a parameter named as delayed-access parameter \( \beta(t) \) is deployed, as shown in Equation (15). For the value of \( \alpha_2 \), as also based on the former study, is set to be 2 (Chang & Hsu, 2001).

\[
\beta(t) = \left[ \frac{t}{H_p} + a(H_p - t) \right]^{-\alpha_2} \quad t = [0, H_p] \\
= 1 \quad t = [H_p, H] \tag{15}
\]

Similarly, according to Equation (14) and (15), arrival function and probability density function of random passengers can be obtained, as shown in Equation (16) and (17), respectively.

\[
h(t) = \beta(t) \cdot f(t) \quad \tag{16}
\]

\[
N_b(t) = \frac{h(t)}{\int_{-\infty}^{\infty} h(t)dt} \approx \frac{h(t)}{\int_{0}^{\infty} h(t)dt} \tag{17}
\]

With both of the probability density functions of scheduled and random passengers, let \( k \) represents the proportion of scheduled passengers and “\( 1 - k \)” represents the proportion of random ones. Combining Equation (13) and (17), the probability density function of total
intercity transit passenger can be obtained as Equation (18).

\[ N(t) = k \cdot N_g(t) + (1 - k) \cdot N_h(t) \]  \hspace{1cm} (18)

2.2 Capacity Constraint on Intercity Transit

In Equation (19), \( A \) represents the total passenger arrivals during time period \( 0 \sim H \). And in Equation (20), due to the capacity constraints of intercity transit, \( m_r \) represents the missed percentage for additional waiting cycle \( r \).

\[ A = n \cdot c \cdot \rho \sum_{q=1}^{n} \int_{0}^{H} f_q(t)dt \]  \hspace{1cm} (19)

\[ m_r = \frac{A-r \cdot C}{A} \quad \text{for} \quad A \geq r \cdot C \quad r=1 \sim s \]

\[ = 0 \quad \text{for} \quad A < r \cdot C \]  \hspace{1cm} (20)

2.3 Mean Waiting Time

Since the probability and behavior equations have developed, mean waiting time of intercity transit passengers can be obtained by Equation (21).

\[ W = \int_{0}^{H} N(t) \cdot (1 - m_r)(H - t)dt + \sum_{i=1}^{z} \int_{0}^{H} N(t) \cdot m_r[(r+1)H - t]dt \]  \hspace{1cm} (21)

3. NUMERICAL STUDY

After developing the waiting time model, reasonable values (see Table 1) may be applied to it and to obtain the arrival distribution of scheduled and random passengers, as shown in Figure 2. For scheduled passengers, it exhibits that when \( t < 10 \), arrivals nearly zero, when \( t = 10 \sim 15 \), arrivals gently increase, and when \( t = 15 \sim 20 \), arrivals boom rapidly. For the random ones, arrivals increase significantly in the period of \( t = 10 \sim 25 \) and decline after \( t > 20 \). Because it is assumed that if random passengers reach the intermodal transit station too early before the transit departures, they will not wait and temporarily leave the station until some special time before the transit departures, thus the arrival rates of random passengers are nearly zero before \( t = 20 \). When \( t > 18 \) (12 minutes before departure of intercity transit), random arrivals emerges, and mainly intensive in 10 minutes before the
departure of intercity transit. Finally, for the distribution of total passenger arrivals, it is shown as Figure 3. For aggregated passengers, arrivals reach high when 15 ~ 5 minutes before the departure of intercity transit, and then decline.

Figure 2. Arrival of Passengers with Distinct Behaviors

Figure 3. Arrival of Total Passengers
4. SENSITIVITY ANALYSIS

Based on the waiting time model, it is found that the proportion of scheduled passengers, reliability of feeder buses, and capacity of intercity transit all play significant roles on mean waiting time. Therefore, a sensitive analysis for each of the important parameters is made in this section.

4.1 Capacity of Intercity Transit

In Figure 4, it is found that capacity of intercity transit plays an important role on mean waiting time. However, when the capacity is greater than the passenger arrivals “A”, the spared capacity becomes a waste, since it cannot improve mean waiting time any more. For this case, at least \( C/A = 0.63 \sim 0.8 \) is suggested to be supplied.

![Figure 4. Passenger Waiting Time vs. Capacity of Intercity Transit](image)

4.2 Headway Deviation of Feeder Buses

Suppose that the headway deviation “\( \sigma \)” stands for reliability index of the feeder buses, relation of mean waiting time to \( \sigma \) is shown as Figure 5. It exhibits that mean waiting time increases with the headway deviation. Mean waiting time is very sensitive to the change of reliability of feeder buses when \( \sigma \) is greater than 5 minutes, and become less sensitive when it is less than 5 minutes. For this case, at most \( \sigma = 3.5 \sim 4.6 \) minutes are recommended, when \( \sigma > 4.6 \) minutes, mean waiting time increase dramatically.
4.3 Percentage of Scheduled Passengers

Figure 6 shows the relation of mean waiting time to proportion of scheduled passengers. When \( k \) increase, the longer waiting time it will be. Thus, it is found that for the scheduled passengers, they will suffer more than that of the random ones, if the feeder service is not reliable. For the Figure 7, it exhibits the contrast situation, i.e., without capacity constraint on intercity transit. It is found that, comparing to Figure 6, \( k \) makes no significant influences on waiting time. This verifies again, for any negative factors in entire service, the scheduled passengers always damaged more.
5. CONCLUSIONS AND RECOMMENDATIONS

The conventional assumption on waiting time, i.e., waiting time is half of the headway or mean waiting time is some fixed multiplier of headway, is not completely to precisely represent the passenger waiting time in an intermodal transit station. This study exhibits that the waiting time can be modeled by considering the reliability of feeder service, passenger behaviors, and system characters of intercity transit.

According to the numerical results, it is found that passengers tend to arrive at 5~15 minutes before the departure of intercity transit. For the sensitivity analysis on feeder reliability, it identifies that mean waiting time of intercity transit passengers is greater than half of the transit headway if the feeder service is provided with poor reliability; and for the contrast, it is less than half of the transit headway as long as the feeder service is provided with good reliability. Moreover, it should be emphasized that, for any negative factors in feeder or intercity services, the scheduled passengers always affected more on average waiting time. For the capacity of intercity transit, it is suggested to satisfy at least 80% of total arrival passengers, if 100% is not available. In general, the intercity transit would really improve its service quality only if the feeder service also provides good reliability.

In this study, waiting time of random passengers is only calculated based on their re-arrival time, so that their waiting time is less than that of scheduled passengers. Moreover, peak and off-peak factors are not taken into considerations in this paper, they should be discussed in further study.
REFERENCES


