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ACTIVE CONTROL OF FLUTTER IN TURBOMACHINERY USING OFF BLADE ACTUATORS AND SENSORS. PART I: MODELING FOR CONTROL

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Abstract:
We describe a linear control-oriented model for fan or compressor blades flutter in gas turbine engines. We model the dynamics of blade rows in turbo-machinery as similar to those of a flexible disk. Aeromechanical modes form travelling waves as seen by the rotor. Since we chose the Spatial Fourier Coefficients to represent the state, the state space involves complex variables, which makes the model non-standard. Modeling for control involves translation from rotating to stationary frame and including actuation and sensor signal models.

Keywords: Flutter, turbomachinery, modeling

1. INTRODUCTION

Blade failures due to flow induced vibrations are a long standing, endemic problem for the turbomachinery industry. Flutter and resonant stress fundamentally constrain the design and operation of gas turbine engines. Ensuring aeromechanical operability often requires compromises in turbomachine efficiency, performance and cost and can result in development delays and increased maintenance costs.

In this paper we describe a control-oriented model for fan and compressor blade flutter in gas turbine engines. We model the dynamics of blade rows in turbo-machinery as similar to those of a flexible disk. Aeromechanical modes form travelling waves as seen by the rotor. This means that when viewed from the rotating frame the peak of deflection appears to travel around the disk. The deflection of the disk at a given point on the fixed frame along the circumference of the blade-row can be decomposed into sinusoids of frequencies separated by integer multiples of the rotor frequency. At any fixed point in time, the deflection of the disk can also be decomposed into sine-waves function of the angular position around the rotor. Therefore each aeromechanical mode has a characteristic shape and a characteristic frequency. Each of

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these modes can lose stability as operating conditions change. The objective of a flutter control is to enhance the region of stable operation by adding damping to the aeromechanical modes.

The derivation of the flutter control algorithm is described in (Banaszuk *et al.*, 2002) and the demonstration of the flutter control in a transsonic fan rig operating at 9000 RPM is described in (Banaszuk *et al.*, 2003).

We will use the following notation. N will denote the number of blades, $n = \dots, -2, -1, 0, 1, 2, \dots$ will denote the index of a flutter mode. $-\zeta_n$ and ω_{ns} will denote the real and imaginary part of the n-th flutter mode pole (in the rotating frame). ξ_n will denote the damping coefficient of the n-th flutter mode, $\xi_n := \frac{\zeta_n}{\sqrt{\zeta_n^2+\omega_{ns}^2}}$. δ_n will denote the logarithmic decrement of the n-th flutter mode, $\delta_n := 2\pi\frac{\zeta_n}{\omega_{ns}} = 2\pi\frac{\xi_n}{\sqrt{1-\xi_n^2}}$. θ_r will denote angle in the rotating frame, θ_s will denote the angle in the stationary frame. $\alpha_{nr}(t,\theta_r)$ and $\alpha_{ns}(t,\theta_s)$ will denote blade deflection angle at time t at angle θ_r in the rotating frame and stationary frame, respectively. ω_r will denote the circular rotor frequency. θ_{s0} will denote the angle between the fixed reference points on the rotor and the stator at time $t = 0$. $\widetilde{(\cdot)}_n$ will denote the n-th spacial Fourier coefficient and $\hat{(\cdot)}$ will denote the temporal Fourier transform.

2. FLUTTER MODELS

For an integer n (positive, zero, or negative) we model the n-th flutter mode, or n-th nodal diameter flutter mode, as a travelling wave in which all blades are oscillating harmonically with a constant phase angle $\theta_n := \frac{2\pi n}{N}$ relative to each other (Forsching, 1984).

Let θ_r denote the angle measured relative to a fixed point on the rotor in the direction of the rotation. Assume that we have continuum of blades and there is no external forcing. We postulate that the n-th nodal component of the blade deflection at angle θ_r at time t is given by the formula

$$\alpha_{nr}(t,\theta_r) = A_n e^{-\zeta_n t}\cos(\omega_{nr}t - n\theta_r + \phi_{nr}) \quad (1)$$

where $-\zeta_n$ and ω_{nr} are, respectively, the real and imaginary part of the n-th flutter mode pole. Note that ω_{nr} is also the (pseudo) frequency of the n-th flutter mode in the rotating frame, and A_n and ϕ_{nr} are the initial magnitude and phase angle of the n-th flutter mode. The damping of the n-th flutter mode is usually described by one of two coefficients: the *damping coefficient* $\xi_n := \frac{\zeta_n}{\sqrt{\zeta_n^2+\omega_{ns}^2}}$ or the *logarithmic decrement* $\delta_n := 2\pi\frac{\zeta_n}{\omega_{ns}} = 2\pi\frac{\xi_n}{\sqrt{1-\xi_n^2}}$. Note that:

(1) The m-th blade is moving according to equation (1) with the corresponding angle $\theta_r = \frac{2\pi m}{N} + \theta_1$, where θ_1 is the position of the first blade relative to the fixed reference point on the rotor, $m = 1, 2, \dots, N$.
(2) For a fixed time t and $n \neq 0$ the blade deflection $\alpha_{nr}(t,\theta_r)$ considered as a function of the angle θ_r has a sinusoidal shape with $|n|$ nodes. For $n = 0$ and a fixed time t the deflection is the same for each blade.
(3) For $\zeta_n = 0$ and $n \neq 0$ the blade deflection $\alpha_{nr}(t,\theta_r)$ is a wave with a fixed sinusoidal shape travelling around the annulus. The speed and the direction of rotation can be obtained by considering movement in time of the angle corresponding to one of the peaks of the wave. For instance, the first peak is obtained by solving the equation $\omega_{nr}t - n\theta_r + \phi_{nr} = \frac{\pi}{2}$ for θ_r. We have

$$\theta_r = \frac{1}{n}(\omega_{nr}t + \phi_{nr} - \frac{\pi}{2}). \quad (2)$$

Therefore, the speed of the wave is $\frac{\omega_{nr}}{|n|}$ and the direction is positive (the same as the direction of rotation of the rotor) for $n > 0$ and negative (the opposite to the rotor's rotation direction) for $n < 0$. We call the flutter modes travelling in the same direction as the rotor the *forward* travelling modes and the ones travelling in the direction opposite to the rotor's direction the *backward* travelling modes.
(4) For a fixed angle θ_r, the blade deflection $\alpha_{nr}(t,\theta_r)$ considered as a function of time represents a response of a damped oscillator, i.e., a second order system with poles $-\zeta_n + i\omega_{nr}$ and $-\zeta_n - i\omega_{nr}$.
Note that each particular blade oscillates with frequency ω_{nr}, which is n times bigger than the frequency of the corresponding travelling wave.

Now we express the motion of a blade due to a particular flutter mode as measured at an arbitrary angle on the stator.

Let ω_r denote the circular rotor frequency. Fix a reference point on the stator. The angles in the stationary frame will be measured relative to this point with positive direction corresponding to the rotor's rotation direction. Let θ_{s0} denote the angle at which the reference point on the stator is seen from the reference point on the rotor at time $t = 0$. Then, for an arbitrary time t, a fixed angle θ_s on the stator is related to the corresponding point on the rotor θ_r (measured in the rotating frame) by the formula $\theta_r = \theta_s + \theta_{s0} - \omega_r t$. Therefore, the deflection of the blade passing a fixed angle θ_s on the stator at time t is given by the formula

$$\begin{aligned}&\alpha_{ns}(t,\theta_s) = \alpha_{nr}(t, \theta_s + \theta_{s0} - \omega_r t) = \\ &A_n e^{-\zeta_n t}\cos((\omega_{nr} + n\omega_r)t - n\theta_s + \phi_{ns})\end{aligned} \quad (3)$$

where $\phi_{ns} := \phi_{nr} - n\theta_{s0}$ is the initial phase of the n-th mode in the stationary frame.

Note that for $\zeta_n = 0$ and $n \neq 0$ the blade deflection $\alpha_{ns}(t, \theta_s)$ in the stationary frame is a wave with a fixed sinusoidal shape travelling around the rotor. In particular, a single blade vibration frequency in the stationary frame is $\omega_{ns} := \omega_{nr} + n\omega_r$.

The velocity of the rotation of the wave can be obtained in a similar manner as in the rotating frame case. In particular, the velocity of the wave corresponding n-th flutter mode in the stationary frame is $\omega_r + \frac{\omega_{nr}}{n}$. Let us recall that the latter is the velocity at which a fixed point on the graph of the blade deflection as a function of angle (say, a peak) is travelling around the annulus at the stationary frame. This velocity should not be confused with an individual blade velocity due to n-th flutter modes, i.e., ω_{ns}, which is n times bigger.

In the sequel we are going to use the stationary frame only. Therefore, we will often skip the subscript s and use θ to denote the angles measured in the stationary frame.

Since at a fixed time the flutter modes and the corresponding forcing functions have a fixed sinusoidal shape, they can be represented via their *spatial Fourier coefficients* (SFCs). One complex Fourier coefficient can be used to describe a single sinusoidal travelling wave. A general travelling wave with n-th nodal spatial shape and a temporal frequency ω_0 has the form $f_n(t, \theta) := F_n(t)\cos(\omega_0 t - n\theta + \phi) = F_n(t)\cos(\omega_0 t + \phi)\cos(n\theta) + F_n(t)\sin(\omega_0 t + \phi)\sin(n\theta)$. The corresponding SFC is, for $n \neq 0$, equal to $\widetilde{f}_n(t) := \frac{1}{\pi}\int_0^{2\pi} f_n(t, \theta)e^{jn\theta}d\theta$. One has $\widetilde{f}_n(t) = \frac{1}{\pi}\int_0^{2\pi} F_n(t)\frac{1}{2}(e^{j(\omega_0 t - n\theta + \phi)} + e^{-j(\omega_0 t - n\theta + \phi)})e^{jn\theta}d\theta = \frac{1}{2\pi}F_n(t)\int_0^{2\pi}(e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t - 2n\theta + \phi)})d\theta = F_n(t)(\cos(\omega_0 t + \phi) + j\sin(\omega_0 t + \phi))$. For $n = 0$, one has $\widetilde{f}_0(t) := \frac{1}{2\pi}\int_0^{2\pi} f_0(t, \theta)d\theta$. Thus, $\widetilde{f}_0(t) = \frac{1}{2\pi}\int_0^{2\pi} F_0(t)\cos(\omega_0 t + \phi)d\theta = F_0(t)\cos(\omega_0 t + \phi) = f_0(t, \theta)$, for all θ.

To reconstruct a wave from its SFC one can use the inverse spatial Fourier tranform

$$f_n(t, \theta) = Re(\widetilde{f}_n(t)^* e^{jn\theta}) = Re(\widetilde{f}_n(t)e^{-jn\theta}), \quad (4)$$

where $(\cdot)^*$ stands for the complex conjugation.

Observe that for $n \neq 0$:

(1) The magnitude and phase of the complex number representing the spatial Fourier coefficient of the wave $f_n(t, \theta)$ are the same as magnitude and phase of the wave.

(2) The real and imaginary part of the spatial Fourier coefficient of the wave $f_n(t, \theta)$ are the *Fourier series coefficients* of $f_n(t, \theta)$, i.e., the coefficients of $f_n(t, \theta)$ represented as a linear combination of $\cos(n\theta)$ and $\sin(n\theta)$, respectively.

Assume that the magnitude and phase of the wave $f_n(t, \theta)$ are constant in time with $F_n(t) := F_n$, for some $n \neq 0$. Then $f_n(t, \theta)$, and hence $\widetilde{f}_n(t)$, is a periodic function of t and one can define the *temporal Fourier transform* of the spatial Fourier coefficient of the wave $f_n(t, \theta)$ $\hat{\widetilde{f}}_n(j\omega) := \int_{-\infty}^{\infty}\widetilde{f}_n(t)e^{-j\omega t}dt := \int_{-\infty}^{\infty} F_n(t)e^{j(\omega_0 t + \phi)}e^{-j\omega t}dt := F_n(t)e^{j\phi}\delta(\omega - \omega_0)$, where $\delta(\cdot)$ stands for the delta operator. Thus, the travelling waves with the temporal frequency ω_0 can be recognized in the (temporal) frequency domain as “spikes” at one single frequency ω_0. Spikes at positive frequencies represent the forward travelling waves, whereas the spikes at negative frequencies represent the backward travelling waves.

The case $n = 0$ is different. As we have noticed before, the spatial Fourier coefficient $\widetilde{f}_0(t)$ of the function $f_0(t, \theta)$ coincides with the function $f_0(t, \theta)$ itself. Its temporal Fourier transform is $\hat{\widetilde{f}}_0(j\omega) := \int_{-\infty}^{\infty}\widetilde{f}_0(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} F_0(t)\frac{1}{2}(e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)})e^{-j\omega t}dt = \frac{F_n(t)}{2}(e^{j\phi}\delta(\omega - \omega_0) + e^{-j\phi}\delta(\omega + \omega_0))$. One observes that the temporal Fourier transform of the spatial Fourier coefficient of the function $f_0(t, \theta)$ has two “spikes”: one at ω_0 and the other at $-\omega_0$.

While the flutter modes for $n \neq 0$ are represented by travelling waves, they can be excited by forcing inputs that are either travelling waves of the form $f_n(t, \theta) := F_n\cos(\omega_0 t - n\theta + \phi)$ or by the *standing waves* of the form

$$f_n(t, \theta) := F_n\cos(\omega_0 t + \phi)\cos(n\theta). \quad (5)$$

This is due to the fact that a standing wave can be represented as linear combination of two travelling waves: $F_n\cos(\omega_0 t + \phi)\cos(n\theta) = \frac{1}{2}F_n(\cos(\omega_0 t - n\theta + \phi) + \cos(\omega_0 t + n\theta + \phi))$.

The temporal Fourier transform of the standing wave (5) is $\hat{\widetilde{f}}_n(j\omega) := \int_{-\infty}^{\infty}\widetilde{f}_n(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} F_n(t)\frac{1}{2}(e^{j(\omega_n t + \phi)} + e^{-j(\omega_n t + \phi)})e^{-j\omega t}dt = \frac{F_n(t)}{2}(e^{j\phi}\delta(\omega - \omega_n) + e^{-j\phi}\delta(\omega + \omega_n))$. Note that the latter formula is valid for all integers n, including $n = 0$.

3. FLUTTER MODELS WITH CONTROL

We assume that we have continuum of actuators around the stator that influence flutter modes. We will control the n-th flutter mode with a control function $u(t, \theta)$ that, as a function of angle, has the same shape as the n-th flutter mode wave. The control magnitude and phase will be chosen appropriately as functions of the measured (or reconstructed using an observer) magnitude and phase of the n-th flutter mode. the angle in the stationary frame, previously denoted by θ_s.)

Similarly, for the identification purposes, one can force the n-th flutter mode with a using a wave of the with a constant magnitude and phase. More precisely, assume that the control input forcing function for the n-th mode is a *travelling wave* having some temporal frequency ω_0 and having the same shape as the n-th flutter mode:

$$\begin{aligned} u_n(t,\theta) = U_n\cos(\omega_0 t + \phi_{nu} - n\theta) = \\ U_n \cos(\omega_0 t + \phi_{nu})\cos(n\theta) + \\ U_n \sin(\omega_0 t + \phi_{nu})\sin(n\theta), \end{aligned} \tag{6}$$

for some constant U_n and ϕ_{nu}. The SFC of this forcing function is $\widetilde{u}_n(t) = U_n e^{j(\omega_0 t+\phi_{nu})}$. The corresponding temporal Fourier transform is $\hat{\widetilde{u}}_n(j\omega) = U_n e^{j\phi_{nu}}\delta(\omega-\omega_0)$.

We also assume that the steady-state n-th flutter mode component of the blade deflection response to the n-th nodal forcing of the form (6) is a travelling wave with the same spatial shape and temporal frequency, possibly shifted in phase by some angle ϕ_n relative to the forcing function:

$$\begin{aligned} \alpha_n(t,\theta) = A_n\cos(\omega_0 t - n\theta + \phi_n) = \\ A_n\cos(\omega_0 t + \phi_n)\cos(n\theta) + \\ A_n \sin(\omega_0 t + \phi_n)\sin(n\theta) \end{aligned} \tag{7}$$

for some constant A_n and ϕ_n that, for fixed U_n and ϕ_{nu}, are functions of ω_0.

The SFC of the n-th component of the blade deflection is $\widetilde{\alpha}_n(t) = A_n e^{j(\omega_0 t+\phi_n)}$. The temporal Fourier transform of the SFC of the n-th component of the blade deflection is $\hat{\widetilde{\alpha}}_n(j\omega) = A_n e^{j\phi_n}\delta(\omega-\omega_0)$. We assume that we measure the blade displacement at finite number of locations on the stator. (This is going to be accomplished with eddy current sensors.) At a fixed angle θ_y the measured blade dispalcement is going to be

$$\begin{aligned} y_{n\theta_y}(t) := \alpha_n(t,\theta_y) = \\ A_n\cos(\omega_0 t - n\theta_y + \phi_n) \\ = A_n\cos(\omega_0 t + \phi_n)\cos(n\theta_y) + \\ A_n\sin(\omega_0 t + \phi_n)\sin(n\theta_y). \end{aligned} \tag{8}$$

The temporal Fourier transform of the output function is $\hat{y}_n(j\omega) = \frac{A_n}{2}(e^{j(\phi_n - n\theta_y)}\delta(\omega-\omega_0) + e^{-j(\phi_n-n\theta_y)}\delta(\omega+\omega_0))$.

Now we present dynamic system models for the evolution of the n-th flutter mode subject to control. The description adapts an approach to model rotating stall from (Paduano, 1992).

One can obtain a low order model describing the dynamics of the n-th flutter suitable for control purposes in the following three steps.

1. Conduct an experiment to obtain the transfer function between the n-th SFC of the forcing function given by (6) and the corresponding n-th SFC of the blade deflection function given by (7).

2. Fit a low-order transfer function to the one obtained experimentally.

3. Obtain a state-space realization of the low-order transfer function obtained in step 2.

We assume that the uncontrolled n-th flutter mode behaves like a lightly damped harmonic oscillator with individual blades moving in the stationary frame according to the formula (3). Thus, we expect the mode to have a significant response to forcing only at a narrow band of frequencies of interest around the mode's natural frequency $\omega_{ns} := \omega_{nr} + n\omega_r$. The control goal is to add damping to the mode by a feedback control only at this narrow band of frequencies. Therefore, it is sufficient to have an approximate low order model describing the dynamics of the n-th mode at this narrow frequency range. Even if the frequency response of the n-th flutter mode were that of a low pass, rather than a band pass filter and the actuator dynamics cannot be neglected over a wide band of frequencies, so that a narrow band frequency model will not be accurate at low frequencies, the inaccuracy of the model will not significantly impact control performance. The controllers will have a band pass characteristic, so that the unmodelled dynamics at both low and high frequencies will not be destabilized.

The transfer function between the n-th SFC's of the forcing function and the corresponding blade deflection response is defined by

$$G_n(j\omega) := \frac{\hat{\widetilde{\alpha}}_n(j\omega)}{\hat{\widetilde{u}}_n(j\omega)} = \frac{A_n}{U_n}e^{j(\phi_n - \phi_{nu})}. \tag{9}$$

Both A_n and ϕ_n are, in general, functions of the frequency ω.

To obtain the transfer function $G_n(j\omega)$ from a sine sweep experiment, one has to access the function $\widetilde{\alpha}_n(t)$. To obtain an approximation to $\widetilde{\alpha}_n(t)$ one would have to simultaneously measure the blade displacement $\alpha_n(t,\theta)$ at some finite number of angles around the annulus and use a discrete approximation of the integral defining the spatial Fourier transform. A reasonable approximation would require at least $2n+1$ blade displacement sensors around the annulus.

However, even with one sensor one can measure the transfer function $G_n(j\omega)$ because of the following simple observation. Assume that we have a blade displacement sensor at some angle θ_y at the stationary frame. The measured output function $y_{n\theta_y}(t) := \alpha_n(t,\theta_y)$ is given by the equation 8. Assume also that we measure the value of the actuation function $u_n(t,\theta)$ at a fixed angle θ_u. Let $u_{n\theta_u}(t) := u_n(t,\theta_u)$. Note that $y_{n\theta_y}(t) = A_n\cos(\omega_0 t - n\theta_y + \phi_n)$ and $u_{n\theta_u}(t) = U_n\cos(\omega_0 t + \phi_{nu} - n\theta_u)$ have relative phase shift of $\phi_n - \phi_{nu} - n(\theta_y - \theta_u)$. Hence, the measured transfer function between them during a sine sweep experiment is $G_{n\theta_u\theta_y}(j\omega) := \frac{\hat{y}_{n\theta_y}(j\omega)}{\hat{u}_{n\theta_u}(j\omega)} = \frac{A_n}{U_n} e^{j(\phi_n - \phi_{nu} - n(\theta_y - \theta_u))} = e^{-jn(\theta_y - \theta_u)} G_n(j\omega)$. Therefore, $G_n(j\omega)$ can be obtained from $G_n(j\omega) = e^{jn(\theta_y - \theta_u)} G_{n\theta_u\theta_y}(j\omega)$.

One can observe that, except for the case $n = 0$, any description of transfer functions $G_n(j\omega)$ as a rational function of $j\omega$ valid in a wide frequency band must have *complex* rather than real coefficients. To see that, note that a transfer function $G(j\omega)$ with real coefficients has the property $G(-j\omega) = G(j\omega)^*$, i.e., it has a Nyquist diagram symmeric with respect to the real axis. We know from experiments that the response of the n-th flutter mode to the forward or backward travelling forcing wave with the same temporal frequency is not symmetric. Thus, for $n \neq 0$, one has $G_n(-j\omega) \neq G_n(j\omega)^*$. However, we do expect the reponse to be symmetric for $n = 0$, so that we have $G_0(j\omega) = G_0(-j\omega)^*$. Therefore, we expect the transfer function $G_0(j\omega)$ considered as a rational function of $j\omega$ to have real coefficients. Because of this difference between the cases $n \neq 0$ and $n = 0$, we are going to derive the corresponding models separately.

A low order, narrow band model for the transfer function $G_n(j\omega)$ between the n-th SFC of the forcing function given by (6) and the corresponding output function given by (8) for $n \neq 0$ is a *first order* transfer function with *complex* coefficients

$$G_n(j\omega) = \frac{b_{nR} + jb_{nI}}{\zeta_n + j(\omega - \omega_{ns})}. \qquad (10)$$

A complex-valued state-space realization of this transfer function is

$$\dot{\tilde{\alpha}}_n(t) = (-\zeta_n + j\omega_{ns})\tilde{\alpha}_n(t) + (b_{nR} + jb_{nI})\tilde{u}_n(t). \qquad (11)$$

Note that both $\tilde{\alpha}_n(t)$ and $\tilde{u}_n(t)$ are *complex* valued functions of time. Observe also that the unforced response of (11) is $\tilde{\alpha}_n(t) = e^{(-\zeta_n + j\omega_{ns})t}\tilde{\alpha}_n(0)$, which agrees with postulated unforced evolution of the n-th flutter mode given by (3).

Let us emphasize again that the simple transfer function model (10) and its state-space realization (11) are valid only for a narrow range of frequencies around the flutter frequency ω_{ns}. The actuator characteristic over that frequency range is simply represented by the magnitude and phase of the n-th mode of the actuator disk at the flutter frequency and incorporated into the complex number $b_{nR} + jb_{nI}$. This approximation is reasonable, as long as the actuator frequency response does not change significantly over the frequency interval of interest and a feedback controller characteristic will be that of a sufficiently narrow band-pass filter. If this is not the case, the actuator dynamics should be incorporated in the model.

An equivalent description to (10) is possible with a real-valued model of real dimension two. In the sequel the subscripts $(\cdot)_R$ and $(\cdot)_I$ will denote the real and imaginary part of a complex number. One can easily check (Paduano, 1992) that the real and imaginary part of the SFC's of blade displacement and forcing function satisfy the following set of two differential equations

$$\begin{bmatrix} \dot{\tilde{\alpha}}_{nR}(t) \\ \dot{\tilde{\alpha}}_{nI}(t) \end{bmatrix} = \begin{bmatrix} -\zeta_n & -\omega_{ns} \\ \omega_{ns} & -\zeta_n \end{bmatrix} \begin{bmatrix} \tilde{\alpha}_{nR}(t) \\ \tilde{\alpha}_{nI}(t) \end{bmatrix} + \begin{bmatrix} b_{nR} & -b_{nI} \\ b_{nI} & b_{nR} \end{bmatrix} \begin{bmatrix} \tilde{u}_{nR}(t) \\ \tilde{u}_{nI}(t) \end{bmatrix}. \qquad (12)$$

The corresponding transfer function desription is

$$\begin{bmatrix} \hat{\tilde{\alpha}}_{nR}(j\omega) \\ \hat{\tilde{\alpha}}_{nI}(j\omega) \end{bmatrix} = \begin{bmatrix} G_{nr}(j\omega) & -G_{ni}(j\omega) \\ G_{ni}(j\omega) & G_{nr}(j\omega) \end{bmatrix} \begin{bmatrix} \hat{\tilde{u}}_{nR}(j\omega) \\ \hat{\tilde{u}}_{nI}(j\omega) \end{bmatrix}. \qquad (13)$$

One can verify that

$$G_{nr}(j\omega) = \frac{b_{nR}(j\omega + \zeta_n) - b_{nI}\omega_{ns}}{(j\omega + \zeta_n)^2 + \omega_{ns}^2} \qquad (14)$$

$$G_{ni}(j\omega) = \frac{-b_{nI}(j\omega + \zeta_n) - b_{nR}\omega_{ns}}{(j\omega + \zeta_n)^2 + \omega_{ns}^2} \qquad (15)$$

and

$$G_n(j\omega) = G_{nr}(j\omega) + jG_{ni}(j\omega). \qquad (16)$$

Assume that a blade displacement sensor is located at some angle θ at the stationary frame. The measured output function $y_{n\theta}(t) := \alpha_n(t,\theta)$ can be expressed in terms of the real and imaginary parts of the SFC of $\alpha_n(t,\theta)$ via the inverse spatial Fourier transform (4) as $y_{n\theta}(t) = Re(\tilde{\alpha}_n(t)e^{-jn\theta}) = Re(\tilde{\alpha}_{nR}(t) + j\tilde{\alpha}_{nI}(t))(\cos(n\theta) - j\sin(n\theta)) = \cos(n\theta)\tilde{\alpha}_{nR}(t) + \sin(n\theta)\tilde{\alpha}_{nI}(t)$. Let

$$x_n(t) := \begin{bmatrix} \tilde{\alpha}_{nR}(t) \\ \tilde{\alpha}_{nI}(t) \end{bmatrix}, v_n(t) := \begin{bmatrix} \tilde{u}_{nR}(t) \\ \tilde{u}_{nI}(t) \end{bmatrix}, \qquad (17)$$

$$A_n := \begin{bmatrix} -\zeta_n & -\omega_{ns} \\ \omega_{ns} & -\zeta_n \end{bmatrix}, B_n := \begin{bmatrix} b_{nR} & -b_{nI} \\ b_{nI} & b_{nR} \end{bmatrix}, \quad C_{n\theta} := [\cos(n\theta)\,\sin(n\theta)]. \qquad (18)$$

The state and the output equation for the n-th nodal flutter mode can be concisely written as

$$\begin{aligned} \dot{x}_n(t) &= A_n x_n(t) + B_n v_n(t) \\ y_{n\theta}(t) &= C_{n\theta} x_n(t). \end{aligned} \qquad (19)$$

If there is only one sensor at some fixed angle θ, we will skip the subscript θ in the description of $y_{n\theta}(t)$ and $C_{n\theta}$.

Observe that all the quantities in the equation (19) are real. One can identify the parameters in the model using travelling wave excitation, as described in the previous section. Alternatively, one can exploit the skew-symmetric structure of the matrices A_n and B_n and use only one of the inputs of $v_n(t)$ for excitation. This amounts to forcing the system with a standing wave, rather than travelling wave pattern.

Now we propose a real-valued model for control of the 0-th nodal flutter of dimension two. Assume that a blade displacement sensor is located at some angle θ at the stationary frame. The measured output function is $y_{0\theta}(t) := \alpha_0(t,\theta) = \widetilde{\alpha}_0(t)$. Let

$$G_0(j\omega) := \frac{\hat{\widetilde{\alpha}}_0(j\omega)}{\hat{\widetilde{u}}_0(j\omega)} = \frac{\hat{\alpha}_0(j\omega)}{\hat{u}_0(j\omega)} = \frac{A_0}{U_0} e^{j(\phi_0 - \phi_{0u})}. \tag{20}$$

A simplest model for $G_0(j\omega)$ with real coeffcients that exhibits a behavior of a lightly damped oscillator is

$$G_0(j\omega) = \frac{b_1(j\omega) + b_0}{(j\omega + \zeta_0)^2 + \omega_0^2}, \tag{21}$$

for some real b_1, b_0, ζ_0, and ω_0. The corresponding state-space description (in the observer canonical form) is

$$\begin{aligned} \dot{x}_0(t) &= A_0 x_0(t) + B_0 v_0(t) \\ y_{0\theta}(t) &= C_{0\theta} x_0(t), \end{aligned} \tag{22}$$

where

$$v_0(t) := u_0(t), \tag{23}$$

$$A_0 := \begin{bmatrix} -\zeta_0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix}, B_0 := \begin{bmatrix} b_1 \\ b_0 \end{bmatrix}, C_{0\theta} := [1\ 0]. \tag{24}$$

Let consider a finite number k_f of flutter modes with nodal numbers n_1, n_2, $\ldots$, n_{k_f}. Assume that as the measurement outputs we use m blade displacement sensors located at the angles θ_1, θ_2, $\ldots$, θ_m, respectively. We assume that the blade displacement y_θ measured at some angle θ at the stationary frame is the sum of the diplacements due to particular flutter modes:

$$y_\theta(t) = \sum_{k=1}^{k_f} \alpha_{n_k}(t,\theta). \tag{25}$$

We can write down the following state-space model describing the dynamics of the k_f most active flutter modes:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) \\ y(t) &= Cx(t), \end{aligned} \tag{26}$$

where $x(t) := [x_{n_1}(t) \ldots x_{n_{k_f}}(t)]^T$, $v(t) := [u_{n_1}(t) \ldots u_{n_{k_f}}(t)]^T$, $y(t) := [y_{\theta_1}(t) \ldots y_{\theta_m}(t)]^T$, A and B are block diagonal matrices containing A_{n_j} and B_{n_j} blocks, respectively, and C is a matrix composed of $C_{n\theta}$ blocks.

The dimension of the output variable $y(t)$ is equal to m, which is the number of blade displacement sensors (e.g., eddy current sensors) used for measurement.

One may be tempted to place many sensors to make the C matrix invertible and use a full-state static feedback to arbitrarily place the damping of the flutter modes. This strategy might be succesfull for flutter modes with the nodal number $n_k \neq 0$ if the variations of the actuator dynamics with frequency can be neglected. However, note that C is never invertible if one includes the 0-th nodal flutter dynamics, as $C_{0\theta} = [1\ 0]$ for all θ, and hence C has a column of zeros. Moreover, a strong output noise component, which includes all unmodelled sources of blade displacement, such as periodic forcing due to rotor and blades assymetry, neglected flutter modes, rotating stall dynamics, an inlet distorsion, etc., would make reconstructing the state of the flutter modes by inverting the C matrix problematic.

To circumvent the problems with direct inversion of the C matrix and the output noise, and at the same time reduce the number of blade displacement sensors, we are going to reconstruct the state of the system using an observer. As we will see, in principle, just one blade displacement sensor is sufficient for this purpose.

CONCLUSION

We described a linear control-oriented model for fan or compressor blades flutter in gas turbine engines. The details of observer-based control algorithm design are described in (Banaszuk *et al.*, 2002) and the demonstration of the flutter control in a transsonic fan rig operating at 9000 RPM is described in (Banaszuk *et al.*, 2003).

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