

# RECONSTRUCTION OF POLYHEDRON OBJECTS BY STRUCTURE GRAPHS INTEGRATION

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## SUMMARY

Reconstruction of polyhedron objects by structure graph integration was described. We assumed a graph representation of 3D scene. Reconstructed objects are polyhedrons and the recognized scene is static. Structure graphs are constructed for each view point. The process of structure graph integration consist in searching matchings between the graph of the observed view and the graph of the reconstructed 3D structure. After establishing graph matchings the reconstructed structure graphs are updated. The structure graphs are updated until the complete structure graphs are obtained. The presented algorithm is illustrated with examples of scene reconstructions.

## INTRODUCTION

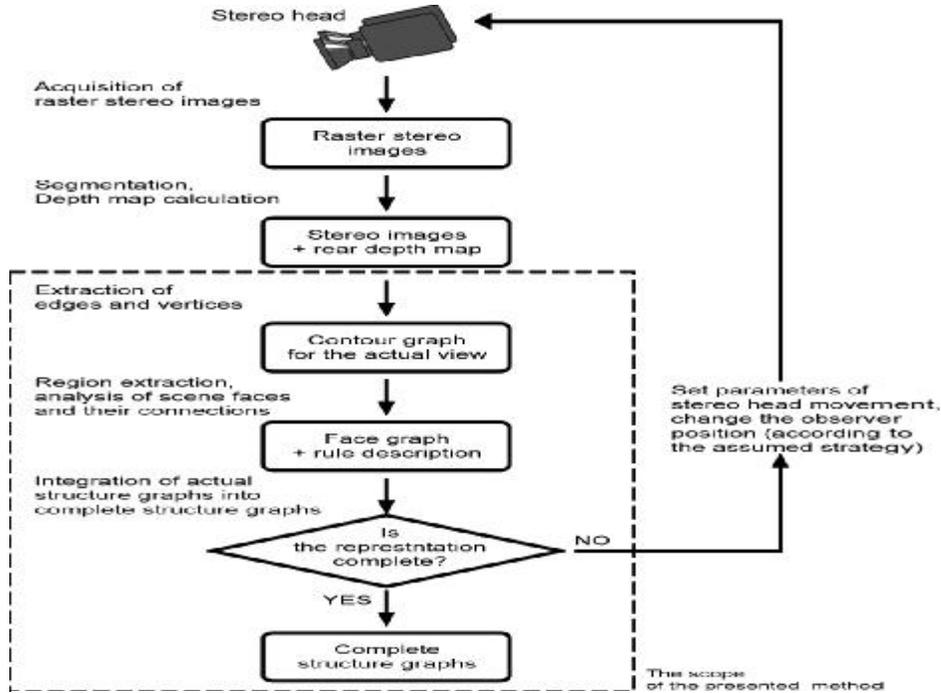
Many methods of 3D object recognition have been proposed. They have different segmentation methods, strategies of object matching, models of object representation and also algorithms of controlling the recognition process. There exist methods retrieving the 3D representation of the scene using geometric and rule dependencies between objects of 2D image (edges, vertices, intensity, etc.) [10]. Another methods make it possible to recognize scene objects as a set of primitives [11, 12].

We present the method of 3D scene reconstruction using stereo images from different view points located around the scene.

## GRAPH REPRESENTATION OF 3D SCENE

A scene structure can be defined as the location of each component in the scene and also selected geometric relations between them. Labeled graphs are used in the presented algorithms to represent the structure of objects.

As a natural representation for the scene structure defined above we assumed a structure graph [6] which is a four-tuple  $G_S = (V_S, E_S, m_S, x_S)$ , where:  $V_S$  – the set of vertices,  $E_S \subseteq V_S \times V_S$  – the set of edges,  $m_S : V_S \rightarrow L_{V_S}$  – a function assigning labels to vertices,  $x_S : V_S \rightarrow L_{E_S}$  – a function assigning labels to the edges.  $L_{V_S}$  and  $L_{E_S}$  denote the set of vertex and edge labels, respectively. Vertices in the structure graph determine relevant components of the scene structure and edges determine relations between these components.



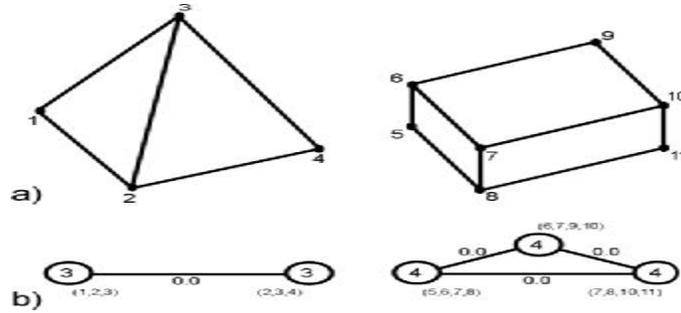
**Fig. 1 Stages of the 3D scene reconstruction**

Two kinds of structure graphs were used in the presented method. The first one is a *contour graph*  $G_C = (V_C, E_C, m_C, x_C)$ . Its vertices  $V_C$  correspond to vertices of objects in the scene. The set  $L_{V_C}$  consists of scene coordinates  $(x, y, z)$  of the scene vertices. Edges  $E_C$  of  $G_C$  describe connections among vertices (edges in the scene). The second kind of structure graphs is a *face graph*  $G_F = (V_F, E_F, m_F, x_F)$ . Its vertices  $V_F$  correspond to faces in the scene. The set  $L_{V_F}$  consists of numbers of edges in the respective face. Edges  $E_F$  of  $G_F$  correspond to connections among the scene faces. Two vertices of the face graph are connected if there exists a common edge of the faces represented by these vertices. The set  $L_{E_F}$  consists of the values of the solid angle between each pair of connected faces.

### STAGES OF 3D SCENE RECONSTRUCTION

The main purpose of the presented method is to acquire a complete 3D scene representation on the basis of stereo sequences from different points of view placed around the scene. The process of 3D scene reconstruction are presented in Fig. 1 and consists of the following stages: 1) Acquisition of raster stereo images, 2) Segmentation and depth map calculation, 3) Construction of the contour graph, 4) Region extraction and face graph construction, 5) Analysis of scene faces and relations between them, 6) Integration of the current face graph into the complete face graph, 7) Checking the completeness of the representation.

Data structures change their form after every stage of the reconstruction process: starting with raster stereo images which after combining with a depth map assumes the shape of a stereo impression of the scene. Its can be described as  $2\frac{1}{2}D$ . After the segmentation of the stereo impression a structure graph can be extracted and after the re-



**Fig. 2 Structure graphs: a) contour graph, b) face graph**

gion extraction a face graph is obtained. It is supplemented with properties of faces and relations among them. Finally a complete face graph together with the corresponding structure graph are constructed.

The scope of this article encloses stages from construction of structure graphs to the obtaining of the complete structure graph. The tasks connected with acquisition, segmentation and depth map calculation are described in separate articles [3, 4, 5]

At the stage of preprocessing a pair of raster images is segmented; vertices and edges are located in the images [4]. Separated in this way, contours are further thinned and discontinuities are closed. Additionally the depth map for crucial points is calculated [5]. As a result the contour graph is constructed. An example of contour graph is presented in Fig. 2a.

### THE CONSTRUCTION OF THE FACE GRAPH

The following stage is converting the contour graph into the face graph. The conversion algorithm finds all minimal coplanar cycles in the contour graph. The algorithm works on a copy of the contour graph which is a directed graph with both directions filled for each edge. The research is conducted starting with any graph vertex  $v_0$  through the following connected vertices until the vertex  $v_0$  is encountered again. The analysis of the actual sequence is interrupted and the examination of the next path begins in case when the same vertex is encountered in the considered sequence (for a vertex other than  $v_0$ ) or when the number of vertices in the considered sequence is less than 3 (even if the encountered vertex is  $v_0$ ). The same reaction occurs when the following vertex being examined in the sequence is not coplanar with the previous ones. The cycles found in the examined copy of the contour graph are eliminated and a new vertex is inserted into the constructed face graph. The search for coplanar cycles is conducted until all paths in the contour graph have been followed.

The structure of a face graph constructed in this way is presented in Fig. 2b.

At the stage of face graph construction, graph parameters are calculated. They increase the effectiveness of the graph consolidation process. The parameter describing face properties is the number of vertices. The relations between faces are described by the values of solid angles.

The angle between two faces is calculated using the formula:

$$\cos(j) = \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

where:  $A_1, A_2, B_1, B_2, C_1, C_2$  – parameters of the plane equations.

Properties of faces and relations between them are stored in the face graph as parameters of vertices and weights of edges, respectively.

An example of face graph parameters is presented in Fig. 2b.

### THE ALGORITHM OF STRUCTURE GRAPH CONSOLIDATION

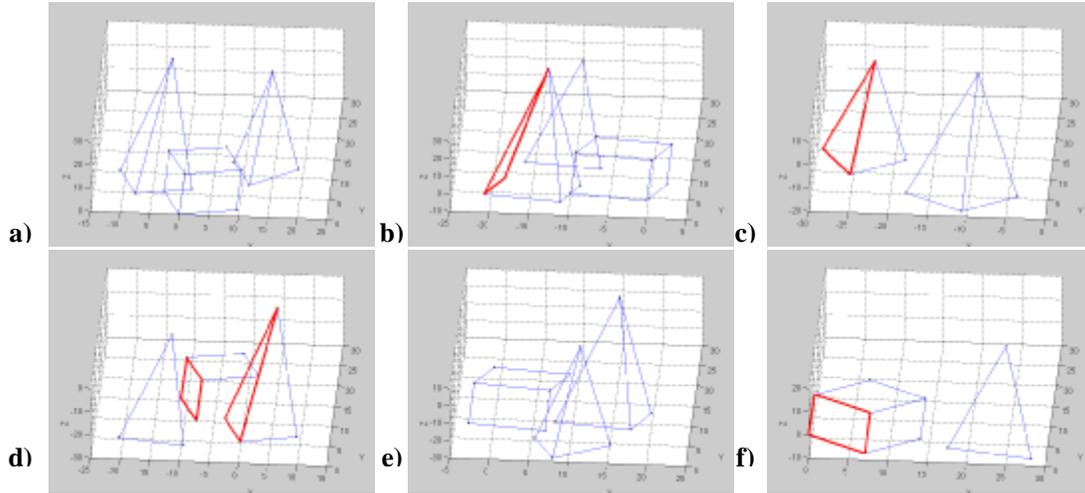
We assumed the following structures as input data to the algorithm of structure graph consolidation:  $GK_i$  – the contour graph obtained in the acquisition process for  $i$ th view;  $GS_i$  – the face graph for  $i$ th view created using  $GK_i$ . We assumed the following output structures:  $GKM$  – the integrated contour graph;  $GSM$  – the integrated face graph. Additional structures are used during the reconstruction:  $WK_i$  – a set of corresponding vertices in  $GK_i$  and  $GKM$ ;  $WS_i$  – a set of corresponding vertices in  $GS_i$  and  $GSM$ .

Scene reconstruction takes  $n$  iterations of the main algorithm sequence, where  $n$  is the number of view points. The main algorithm sequence consists of the following stages:

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1. Building  $GK_i$  and  $GS_i$  structures for  $i$ th view point.
  2. Searching graphs  $GS_i$  and  $GSM$  for subgraph matchings; storing the matched vertices in  $WS_i$ .
  3. Finding the correspondences between groups of vertices in  $GK_i$  and the matched vertices from  $WS_i$ .
  4. Calculation of the translation vector  $T_i$  and the rotation matrix  $R_i$  for the transformation of coordinate system for  $i$ th scene view into the coordinate system for the first scene view; we use coordinates included in structures  $WK_i$  and  $GKM$ , respectively.
  5. Subtraction of unmatched vertices from the structure  $GK_i$ ; the unmatched vertices are stored in  $NGK_i$ .
  6. Calculation of coordinates for the vertices from  $NGK_i$  in the coordinate system of the first view point; we use  $T_i$  and  $R_i$  for this purpose.
  7. Consolidation of the integrated structure graphs  $GKM$  and  $GSM$  with the structure graphs for  $i$ th view point:  $GK_i$  and  $GS_i$ , respectively.
  8. Checking the completeness of the reconstruction; if it is not complete then move to a new view point (increment  $i$ ) and go back to step 1; otherwise the algorithm is complete.
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Before entering the main loop of the algorithm we assume the first structures  $GK_1$  and  $GS_1$  as the consolidated graphs  $GKM$  and  $GSM$ , respectively. The main algorithm loop presented above is processed for the individual scene views until the complete scene representation is obtained.

In the first step of the algorithm  $GK_i$  and  $GS_i$  are constructed on the basis of segmented stereo images and a depth map. Next, the matching sets of vertices in  $GS_i$  and  $GSM$  that fulfill the given criteria are selected. Further analysis is performed for the sets with the greatest number of vertices. The selected sets of vertices from  $GS_i$  are detected in  $GSM$  using an algorithm for detection of subgraph isomorphisms [7]. Heu-



**Fig. 3 Stages of the scene reconstruction process**

ristic cutting of the search tree was introduced. The examination of a search branch is skipped for the branches with the number of vertices not exceeding the most numerous set of vertices found so far. This operation increases the effectiveness of the computations without losing the significant solutions. The chosen vertices compose the structure  $WS_i$ .

In the next step the corresponding vertices from  $GK_i$  are found for each vertex of  $WS_i$ ; the vertices are stored in  $WK_i$ . The determined correspondence of vertices is the basis to calculate the translation vector  $T_i$  and the rotation matrix  $R_i$  for the transformation between  $i$ th view and the model view.

Consolidation of structure graphs resolves itself into updating the integrated structure graphs:  $GSM$  and  $GKM$  and into calculating new locations of inserted vertices in coordinates of the model view.

The function for breaking the computations and checking the completeness of the representation plays the important role in the process of scene reconstruction. The function for appointing the next view point is very important here [8].

The following conditions describe the completeness of the representation: the process is terminated when the total rotation angle in all iterations exceeds  $360^\circ$  or after passing a given number (e.g. 3) of iterations without encountering new scene objects. The total rotation angle describes the degree of completeness of the reconstruction.

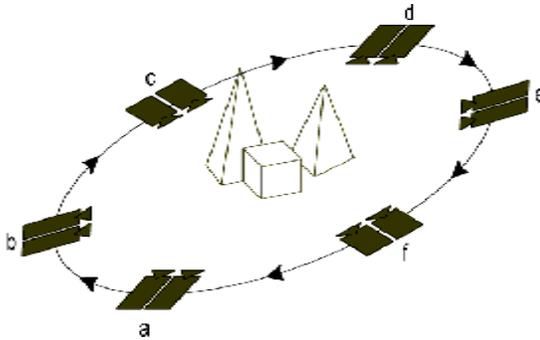
### EXAMPLES OF SCENE RECONSTRUCTION

In order to illustrate and verify our reconstruction method we tested views of simulated scenes from different points around the given scene.

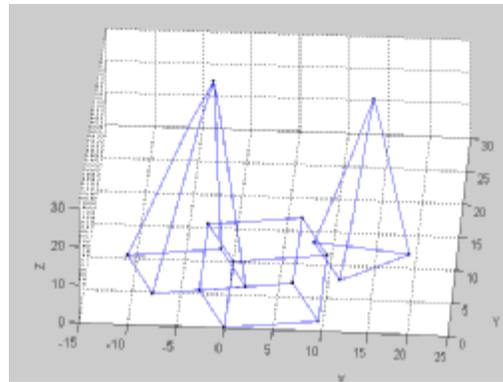
We determined 6 evenly distributed view points on the observation circle (Fig. 4). The visible faces were determined using a view simulator [9]. Next we used these data to obtain contour graphs and then process them in the way presented above.

The structure graphs for the first view point were assumed as the initial integrated structure graphs. The next structure graphs were processed relative to the first ones.

During the following iterations new faces, not visible earlier, were discovered. In Fig. 3 the newly visible faces were highlighted. The integrated face graph and the integrated contour graph were updated after each iteration step.



**Fig. 4** Location of view points on the observation circle



**Fig. 5** The complete contour graph

When the cameras had completed the circle around the scene and the whole reconstruction process had been completed the integrated contour graph had the shape presented in Fig. 5.

### CONCLUSIONS

The method makes it possible to reconstruct 3D scenes consisting of objects of unknown polyhedra. This is an important advantage during object recovery.

The main drawback of this method is high worst-case computational complexity of the algorithm for subgraph isomorphism detection. However, the assumed heuristics significantly increase the average time of computations.

Analysis of curved objects could be a valuable extension of the method.

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