

Inventory Replenishment Planning of a Distribution System with Warehouses at the Locations of Producers and Minimum and Maximum Joint Replenishment Quantity Constraints

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Abstract: In this paper, an inventory replenishment planning problem in a three-echelon distribution system of Alibaba is studied. In addition to central distribution centers and front distribution centers, this system also has warehouses at the locations of producers. Multiple products are jointly replenished with minimum and maximum joint replenishment quantity constraints. Transshipments between distribution centers/warehouses are allowed. This problem, which is to determine the replenishment quantity of each product between any two inventory locations in the system, is formulated as a bi-objective optimization model that aims at finding a tradeoff between overall service level and total logistics cost of the system. This model is solved by applying an augmented ϵ -constraint method. The effectiveness of the model is demonstrated by numerical experiments generated from the data of Alibaba. The results show that having warehouses at the locations of producers can lead to lower logistics costs with a given customer service level.

1 INTRODUCTION

In today's society, e-commerce has entered the daily life of most people. To deliver goods to customers quickly at lower costs and increase market shares, e-commerce companies have to efficiently manage inventories in their distribution systems.

As a quickly emerged e-commerce giant with a very large market share in China, Alibaba is trying to improve the inventory management of its supply chain to gain its competitive advantages over other e-commerce companies. For this reason, we study a replenishment planning problem in a distribution system of Alibaba in this paper. Except for central distribution centers (CDCs) and front distribution centers (FDCs), the distribution system also has warehouses at the locations of producers which are the most upstream suppliers in the system. These suppliers produce goods and sometimes send them to the warehouses for temporary storage. The CDCs get products from the warehouses and distribute them to the FDCs which serve customers directly.

Hereafter, the warehouses are referred to as producers distribution centers or PDCs for short. Each PDC is located at the same location of its suppliers or near them. It collects products from the suppliers, and then sends the products to CDCs or FDCs. The introduction of PDCs can help to reduce logistics costs in the system that will be investigated in this paper.

In this study, we consider a single period inventory replenishment planning problem occurred in the three-echelon distribution system of Alibaba. Such inventory replenishment was usually made by Alibaba before its promotion activities, which is called "early product pushing down replenishment". For example, the well-known annual promotion activity called 'double 11 promotion' has been successfully held for nine years in Chinese e-commerce market, which was created by Alibaba in 2009. In 2018, the transaction volume of Alibaba in 'double 11 promotion' reached 213.5 billion RMB. To assure a high on-time delivery rate to customer orders in such a promotion with huge demand, e-

commerce companies like Alibaba adopt a strategy of early product pushing down replenishment, where products are sent to the stocks of a multi-echelon distribution system in advance for the sales of a single promotion period. Since the inventory replenishment is made in advance and for only one-period sales, the replenishment lead time can be neglected. The inventory replenishment of each stock in this distribution system has two important features: multiple products are replenished jointly, there are minimum and maximum replenishment quantity constraints for each replenishment, and transshipment between two stocks is allowed.

In the literature, both single-period inventory models (Khouja, 1999) with zero lead time like news boy model and multi-period inventory models (Aharon et al., 2009) with positive lead time are comprehensively studied. These two types of models have different application fields. Single-period models deal with one time ordering problems, whereas multi-period models deal with repetitive ordering problems. The latter models are usually more complex than the former ones. In this paper, we focus on the early product pushing down replenishment of e-commerce companies introduced above, so a single period model is adopted.

Most studies on inventory management of distribution systems deal with a single product (De Kok et al., 2018). The management of such systems has to address two issues, one is to choose an optimal inventory policy for each stock, and the other is to make an inventory allocation decision when the on-hand inventory of an upstream stock is not sufficient to satisfy all replenishment requirements of its immediate downstream stocks (Van der Heijden et al., 1997). These papers only consider single product, two-echelon distribution systems and do not take into account of any constraint on replenishment quantity of products in each stock. In this paper, we study multi-echelon multi-product joint replenishment planning problem with constraints on the replenishment quantity of each stock.

Joint replenishment was usually studied for a single stock with only few exceptions. A two-echelon inventory system with a central warehouse and multiple identical retailers was investigated by Axsäter and Zhang (1999). In this system, if the sum of the inventory positions of all retailers reaches a joint reorder point, the retailer with the lowest inventory position orders a batch quantity. They assumed that the inventory position of each stock was supplied infinitely by the warehouse. Wang and Axsäter (2013) studied a distribution system with a

central warehouse and multiple retailers with stochastic demands. They developed a time based joint replenishment policy. However, they did not consider replenishment quantity constraints and transshipments. Zhou et al. (2012) considered a multi-product multi-echelon inventory system with multiple suppliers, one producer, and multiple distributors and buyers. A joint replenishment and (T, S) inventory control strategy was proposed, which orders multiple products in one order cycle.

Besides, most papers studying lateral transshipments consider stocks at the same echelon (level) (De Kok et al., 2018). Kukreja and Schmidt (2005) studied multiple stocks in a single echelon inventory system with Poisson demand, where lateral transshipments among stocks are allowed. Yang et al. (2013) investigated a customer-oriented service measure which takes into account pipeline stocks and lateral transshipments in a single-echelon inventory system. Fattahi et al., (2015) studied a multiple period inventory system with one manufacturer and one retailer. The systems studied in above cited papers involve only a single product without considering joint replenishment and are simpler than the multiple echelon distribution system studied in this paper.

To the best of our knowledge, no paper has studied an inventory replenishment problem with all of the following features we consider.

- a. The joint inventory replenishment of multiple products is considered for a multi-echelon distribution system.
- b. A three-echelon distribution system with warehouses at the locations of producers is investigated.
- c. The minimum and maximum joint replenishment quantity constraints are considered for each replenishment.
- d. Both vertical and horizontal inventory replenishments are considered.
- e. Two objectives, service level and cost, are considered in the inventory replenishment planning.

The rest of this paper is organized as follows. Section 2 describes the optimization problem studied. Section 3 proposes relevant mathematical models. Section 4 presents a model reformulation of the bi-objective problem. Section 5 evaluates the performances of the models by numerical experiments. Section 6 concludes this paper with remarks for future works.

2 PROBLEM DESCRIPTION

A three-echelon distribution system operated by Alibaba is considered. As shown in Figure 1, this system is composed of multiple stocks with multiple suppliers, one PDC (Producers Distribution Center) and multiple CDCs (Central Distribution Centers) and multiple FDCs (Front Distribution Centers). Each stock holds multiple products which are fast moving goods. The demand of each stock is assumed to be subject to a normal distribution.

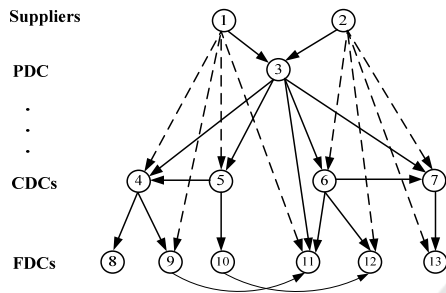


Figure 1: A three-echelon distribution system with warehouses at the locations of producers.

Figure 1 provides an illustrative example for the studied system. Previously, the CDCs were supplied directly by external suppliers (stock 1 and stock 2). In recent years, a producers distribution center (PDC, stock 3) was introduced in the system. The PDC is located near its suppliers (producers) geographically where suppliers' goods can be transported to the PDC very quickly. In contrast, the PDC is far away from the CDCs (stocks 4-7) but it can provide frequent replenishments to CDCs (as indicated by the solid line from stock 3 to stock 4) in small batches, which can help to increase the service level, shorten the replenishment lead time of its successors, and reduce the logistics cost of the distribution system. This effect of cost reduction due to the introduction of PDCs will be further investigated in section 5. In the system, each FDC (stocks 8-13) can be supplied directly by the suppliers (as indicated by the dashed line from stock 1 to stock 9), by the PDC (as indicated by the solid line from stock 3 to stock 11), or by the CDCs (as indicated by the solid line from stock 5 to stock 10) that supply the FDCs directly. Lost sales may happen at the FDCs. Both vertical replenishment (as indicated by the solid line from stock 4 to stock 9) and horizontal replenishment (as indicated by the solid line from stock 10 to stock 12) are possible, whereas reverse replenishment from any stock to any other stock at a higher level (echelon) is not allowed.

In this paper, we assume that the inventory replenishment of each stock is made in advance for the sales of a single period, with minimum and maximum joint replenishment quantity constraints. These minimum and maximum joint replenishment quantities may be different for different distribution channels, as the suppliers of some products such as fruits are located in isolated agricultural areas where transportation capacity is much smaller than that in economically well developed industrial areas. Furthermore, multiple products may be replenished simultaneously. Because the inventory replenishment of each stock is made in advance with a quite short lead time, we assume in our model that all replenishments are carried out immediately with zero lead time. In addition, for a stock which may both receive and deliver goods, it is assumed that goods are received first from its supplier stocks before the goods can be delivered to its customer stocks.

The replenishment decision is made based on demand forecast and historical demand data especially historical demand forecast errors. Before the replenishment, each stock holds a certain on-hand inventory of each product. The shipping costs, maximum and minimum joint replenishment quantity between any two stocks are given. During the replenishment, the products of suppliers will be transported to FDCs directly or through PDCs and CDCs with possible transshipments between them at the same level (echelon). The objectives of this distribution system are to maximize the service levels at the FDCs and minimize total replenishment cost. All products at all FDCs are expected to have the same service level and inventory holding costs are not considered. As the above two objectives are in conflict with each other, so we formulate this replenishment planning problem as a bi-objective optimization problem, which aims at finding a tradeoff between the two objectives by providing a set of Pareto optimal replenishment plans for the distribution system.

3 PROBLEM FORMULATION

Before presenting the model, we first introduce the following notations.

Indices

i, j : stock index, $i, j \in N$, where N is the set of all stocks in the distribution system

k : product index, $k \in K$, where K is the set of all products in the distribution system

Parameters

SS : set of stocks at the supplier echelon of the distribution system

SO : set of stocks at the PDC echelon of the distribution system

SC : set of stocks at the CDC echelon of the distribution system

SF : set of stocks in the FDC echelon of the distribution system

N : set of all stocks in the distribution system, $N = SS \cup SO \cup SC \cup SF$

K : set of all products in the distribution system

I_{ki}^0 : initial on-hand inventory of product k at stock i at the beginning of replenishment, $k \in K, i \in N$

μ_{ki} : demand forecast of product k at stock i

σ_{ki} : standard deviation of demand forecast of product k at stock i

d_{ki} : real demand of product k at stock i , d_{ki} is a random variable. It is assumed that d_{ki} is subject to a normal distribution with mean value μ_{ki} and standard deviation σ_{ki}

sc_{ij} : shipping cost from stock i to stock j , where $sc_{ij} = sc_{ji}$ and the triangle inequality,

$sc_{in} + sc_{nj} \geq sc_{ij}$, holds for any i, j, n with $n \neq i, n \neq j$

C_{ij}^{\max} : maximum joint replenishment quantity from stock i to stock j ($i, j \in N$) for each replenishment

C_{ij}^{\min} : minimum joint replenishment quantity from stock i to stock j ($i, j \in N$) for each replenishment

M : a big positive number

Decision Variables

I_{ki} : on-hand inventory of product k at stock i after replenishment

α : common service level of each product at each stock

z_α : z -value corresponding to the service level α

x_{ij}^k : replenishment quantity of product k from stock i to stock j

y_{ij} : if the replenishment of products happens from stock i to stock j , $y_{ij} = 1$, otherwise $y_{ij} = 0$

With the above notations, the single period replenishment planning problem of the three-echelon distribution system can be formulated as the following mixed-integer programming model *SPRPP*.

Model *SPRPP*:

$$Z1^{SPRPP} = \text{Min} \left\{ \sum_{k \in K} \sum_{i \in N} \sum_{j \in N, j \neq i} x_{ij}^k \cdot sc_{ij} \right\} \quad (1)$$

$$Z2^{SPRPP} = \text{Min} \{ 1 - \alpha \} \quad (2)$$

Subject to:

$$I_{ki} = I_{ki}^0 + \sum_{j \in N, j \neq i} x_{ji}^k - \sum_{j \in N, j \neq i} x_{ij}^k, i \in N, k \in K \quad (3)$$

$$I_{ki} \geq \mu_{ki} + z_\alpha \cdot \sigma_{ki}, i \in SF, k \in K \quad (4)$$

$$\sum_{j \in N, j \neq i} x_{ij}^k \leq I_{ki}^0, i \in SS, k \in K \quad (5)$$

$$x_{ji}^k = 0, i \in SS, j \in N, k \in K \quad (6)$$

$$C_{ij}^{\min} \cdot y_{ij} \leq \sum_{k \in K} x_{ij}^k \leq C_{ij}^{\max}, i, j \in N \quad (7)$$

$$\sum_{k \in K} x_{ij}^k \leq y_{ij} \cdot M, i, j \in N \quad (8)$$

$$x_{ii}^k = 0, i \in N, k \in K \quad (9)$$

$$I_{ki}, x_{ij}^k \geq 0, 0 \leq \alpha \leq 1, y_{ij} \in \{0, 1\}, i, j \in N, k \in K \quad (10)$$

The objective function (1) seeks to minimize the total replenishment cost. The objective function (2) aims to maximize the common service level α of all products at all FDCs in the distribution system. Constraints (3) are the inventory balance constraints of each product at each stock. Constraints (4) ensure that the same service level α of each product at each stock can be achieved after replenishment. These constraints are derived from the probability constraints $P\{I_{ki} \geq d_{ki}\} \geq \alpha, i \in SF, k \in K$, where z_α is the z -value corresponding to the service level α , with the relationship between them given by $P\{x \leq z_\alpha\} = \alpha, x \sim N(0, 1)$, i.e., $\Phi(z_\alpha) = \alpha$, where $\Phi(\cdot)$ is the cumulative distribution function of $N(0, 1)$. Constraints (5) ensure that the total replenishment quantity of product k from supplier i to all stocks in the distribution system does not exceed the on-hand inventory of the product available in the supplier. Constraints (6) ensure that no product will be replenished between supplier stocks or send back to a supplier. Constraints (7) guarantee that the replenishment quantity of all products from one stock to another is subject to

minimum and maximum joint replenishment quantity constraints. Constraints (8) indicate the relationship between x_{ij}^k and y_{ij} . Constraints (9) mean that each stock is not replenished by itself. Constraints (10) indicate the types and the domains of all decision variables.

As the z -value z_α is a monotone increasing function of α , we can replace objective function (2) of the above model by an equivalent objective function (11) below.

$$Z2^{SPRPP} = \text{Min}\{-z_\alpha\} \quad (11)$$

4 SOLUTION APPROACH

To solve the bi-objective model *SPRPP*, an augmented ε -constraint method (Mavrotas, 2009) is employed. This method is a revised version of ε -constraint method (Chankong and Haimes, 1983), which can avoid the generation of weakly Pareto optimal solutions and accelerates the whole computation process without redundant iterations. Firstly, we introduce some new parameters and variables required for the description of this method as follows.

Parameters

- $f_1(x)$: objective function (1) of model *SPRPP*
- $f_2(\alpha)$: objective function (2) of model *SPRPP*
- lb_2 : upper bound of the objective function $f_2(\alpha)$, which is obtained by solving the model with single objective function $f_2(\alpha)$ and constraints $f_1(x) = f_1^*$, where f_1^* is obtained by optimally solving the model with single objective function $f_1(x)$.
- r_2 : range of the objective function value $f_2(\alpha)$, which is the difference between its best value and upper bound. The best value can be obtained by optimally solving the model with single objective function $f_2(\alpha)$.
- Ng_2 : number of grid points in the range of objective function value $f_2(\alpha)$
- gi_2 : grid point index, $gi_2 = 0, 1, \dots, Ng_2$
- eps : a small positive number, which is usually taken from 10^{-6} to 10^{-3}
- ε_2 : a variable parameter depending on gi_2 , $\varepsilon_2 = lb_2 - (gi_2 \cdot r_2) / Ng_2$

Decision Variables

- s_2 : slack variable of the objective function $f_2(\alpha)$

With the above notations, a model *SPRPP2* modified from model *SPRPP* can be formulated as following.

$$Z^{SPRPP2} = \text{Min}(f_1(x) - eps \cdot s_2 / r_2) \quad (12)$$

Subject to constraints (3) to (10) and:

$$f_2(u) + s_2 = \varepsilon_2 \quad (13)$$

$$s_2 \geq 0 \quad (14)$$

By taking Ng_2 sufficiently large and iteratively solving model *SPRPP2* for different values of ε_2 generated by taking the grid point index gi_2 from 0 to Ng_2 , representative Pareto optimal solutions of the original model *SPRPP* can be found. These solutions provide multiple choices for the decision-maker of replenishment planning of the distribution system under different customer service levels.

5 NUMERICAL EXPERIMENTS

In this section, we report and analyze the results of our numerical experiments conducted to evaluate the models proposed in this paper. Twenty instances generated partially based on Alibaba's data were used to validate the models and evaluate the performances of distribution systems with PDCs. For the sake of confidentiality, some data of the instances will be not presented hereafter.

The initial inventory of each PDC, CDCs, and FDC stock is set to zero, and the initial inventory of each supplier is randomly generated and is high enough to ensure that an expected service level at FDCs can be achieved.

Based on the data of Alibaba, the maximum joint replenishment quantity is set as a multiple of the minimum joint replenishment quantity for the replenishment between any two stocks. After the coordinates of all nodes are given, the Euclidean distance c_{ij} (shipping cost) between any two stocks i and j is calculated.

In addition, the demand forecast and its standard deviation of each product at each FDC are also generated based on data of Alibaba. For all instances, the number of products is set to 3 ($K = 3$).

For the first ten instances, the number of stocks is set to 10 with 4 suppliers, 1 PDC, 1 CDC, and 4 FDCs. For the second ten instances, the number of stocks is set to 18 with 8 suppliers, 1 PDC, 1 CDC, and 8 FDCs. All models involved in the instances were solved by using the solver of Cplex 12.8 on a personal PC with i7-8650U CPU and 16GB RAM. The computation time of each instance is very small, which is usually in several seconds.

The computational results are given in Table 1 to

Table 12. To evaluate the influence of the minimum and maximum joint replenishment quantity constraints on the replenishment plan, we tested three scenarios denoted by MR1, MR2, and MR3 respectively for each instance. For scenario MR1, the minimum and maximum joint replenishment quantities are set based on real data of Alibaba. For scenario MR2 and MR3, the minimum and maximum joint replenishment quantities are set as two and four times of those in the first scenario respectively.

For each instance, we consider three cases with different service levels to examine different situations of the distribution system. In Case 1 (C1) the service level α is set to 0.92 with z_α is equal to 1.41, in Case 2 (C2) the service level α is set to 0.95 with z_α is equal to 1.65, and in Case 3 (C3) the service level α is set to 0.98 with z_α is equal to 2.06.

Furthermore, in the following tables, CA represents the replenishment cost of the distribution system without PDCs, CB represents the replenishment cost of the system with PDCs, and CR indicates the cost reduction in percentage of CB with respect to CA, i.e., $CR = (CA - CB)/CA$.

Table 1: Computational results of the instances 1 to 5 (scenario MR1).

Instance	1	2	3	4	5	
C1	CA	946378	855362	879649	869401	865940
	CB	887305	799543	822589	812915	807978
	CR	6.24%	6.53%	6.49%	6.5%	6.69%
C2	CA	1038930	939450	964502	946854	939234
	CB	979322	881588	905372	889462	880951
	CR	5.74%	6.16%	6.13%	6.06%	6.21%
C3	CA	1197490	1084290	1109760	1079190	1064470
	CB	1137910	1026420	1050630	1021780	1006170
	CR	4.98%	5.34%	5.33%	5.32%	5.48%

Table 2: Computational results of the instances 1 to 5 (scenario MR2).

Instance	1	2	3	4	5	
C1	CA	682239	613529	630274	623977	611243
	CB	617991	562506	576774	570291	566934
	CR	9.42%	8.32%	8.49%	8.6%	7.25%
C2	CA	768727	688948	707926	694107	680734
	CB	669265	608242	619727	609836	604172
	CR	12.94%	11.71%	12.46%	12.14%	11.25%
C3	CA	921745	828128	847817	821704	803534
	CB	809473	716824	731208	710968	690256
	CR	12.18%	13.44%	13.75%	13.48%	14.1%

Table 3: Computational results of the instances 1 to 5 (scenario MR3).

Instance	1	2	3	4	5	
C1	CA	613015	556239	571134	564595	564530
	CB	612830	556229	571051	564518	564530
	CR	0.03%	0.002%	0.01%	0.01%	0%
C2	CA	659466	598809	613673	603500	601206

C3	CB	659273	598761	613590	603359	601206
	CR	0.03%	0.01%	0.01%	0.02%	0%
	CA	738924	671868	686366	670040	663870
	CB	738689	671568	686272	669744	663870
CR	0.03%	0.04%	0.01%	0.04%	0%	

Table 4: Computational results of the instances 6 to 10 (scenario MR1).

Instance	6	7	8	9	10	
C1	CA	798457	888181	872951	898750	931109
	CB	740557	833167	816825	841345	875136
	CR	7.25%	6.19%	6.43%	6.39%	6.01%
C2	CA	869174	969267	954169	964632	1019340
	CB	810531	913780	897866	907227	962056
	CR	6.75%	5.72%	5.9%	5.95%	5.62%
C3	CA	990081	1107880	1092920	1077190	1170140
	CB	931514	1052460	1036650	1019780	1112760
	CR	5.92%	5%	5.15%	5.33%	4.9%

Table 5: Computational results of the instances 6 to 10 (scenario MR2).

Instance	6	7	8	9	10	
C1	CA	555609	647823	632874	651628	685687
	CB	529090	576119	566325	576504	598705
	CR	4.77%	11.07%	10.52%	11.53%	12.69%
C2	CA	619808	724730	710581	716832	769287
	CB	564730	623285	610943	613549	658509
	CR	8.89%	14%	14.02%	14.41%	14.4%
C3	CA	737443	859343	847674	828682	917948
	CB	629748	754111	737566	715203	803412
	CR	14.6%	12.25%	12.99%	13.69%	12.48%

Table 6: Computational results of the instances 6 to 10 (scenario MR3).

Instance	6	7	8	9	10	
C1	CA	527254	569731	559990	574277	592346
	CB	527101	569701	559713	574136	592316
	CR	0.03%	0.01%	0.05%	0.02%	0.01%
C2	CA	562685	610437	600679	607250	636521
	CB	562527	610394	600427	607100	636490
	CR	0.03%	0.01%	0.04%	0.02%	0.005%
C3	CA	623223	680031	670280	663613	712023
	CB	623068	679948	670083	663447	711962
	CR	0.02%	0.01%	0.03%	0.03%	0.01%

Table 7: Computational results of the instances 11 to 15 (scenario MR1).

Instance	11	12	13	14	15	
C1	CA	1013390	1102040	1158610	1155320	1279270
	CB	957531	1026230	1071180	1060360	1181500
	CR	5.51%	6.88%	7.55%	8.22%	7.64%
C2	CA	1136720	1250040	1313900	1293050	1429550
	CB	1044870	1149870	1216400	1194000	1330450
	CR	8.08%	8.01%	7.42%	7.66%	6.93%
C3	CA	1360530	1512160	1587120	1540520	1694240
	CB	1261720	1406960	1486670	1436560	1591650
	CR	7.26%	6.96%	6.33%	6.75%	6.06%

Table 8: Computational results of the instances 11 to 15 (scenario MR2).

Instance		11	12	13	14	15
C1	CA	944804	1017380	1025400	1026850	1084460
	CB	944729	1017200	1025160	1026650	1084360
	CR	0.01%	0.02%	0.02%	0.02%	0.01%
C2	CA	1013540	1096050	1106860	1101450	1163300
	CB	1013490	1095840	1106540	1101160	1163240
	CR	0.005%	0.02%	0.03%	0.03%	0.01%
C3	CA	1135130	1232300	1249180	1230980	1301260
	CB	1135050	1232050	1248830	1230670	1301150
	CR	0.01%	0.02%	0.03%	0.03%	0.01%

Table 9: Computational results of the instances 11 to 15 (scenario MR3).

Instance		11	12	13	14	15
C1	CA	944362	1016430	1024750	1025310	1082640
	CB	944362	1016430	1024670	1025300	1082600
	CR	0%	0%	0.01%	0.001%	0.004%
C2	CA	1012950	1094910	1106020	1099720	1161050
	CB	1012950	1094910	1105940	1099700	1161020
	CR	0%	0%	0.01%	0.002%	0.003%
C3	CA	1134400	1230870	1248220	1229090	1298520
	CB	1134380	1230850	1248140	1229070	1298460
	CR	0.002%	0.002%	0.01%	0.002%	0.005%

Table 10: Computational results of the instances 16 to 20 (scenario MR1).

Instance		16	17	18	19	20
C1	CA	1310400	1143130	1263490	1276570	1181300
	CB	1214280	1049400	1168190	1178210	1093050
	CR	7.34%	8.20%	7.54%	7.71%	7.47%
C2	CA	1459840	1285920	1411980	1425210	1325780
	CB	1360470	1187470	1309290	1322040	1231210
	CR	6.81%	7.66%	7.27%	7.24%	7.13%
C3	CA	1731150	1536620	1673730	1688930	1587940
	CB	1627500	1434010	1567610	1582810	1488360
	CR	5.99%	6.68%	6.34%	6.28%	6.27%

Table 11: Computational results of the instances 16 to 20 (scenario MR2).

Instance		16	17	18	19	20
C1	CA	1106090	1017200	1100580	1091480	1028570
	CB	1105810	1017030	1100560	1091440	1028510
	CR	0.03%	0.02%	0.002%	0.004%	0.01%
C2	CA	1186440	1092170	1178760	1169920	1107760
	CB	1186180	1091990	1178710	1169860	1107730
	CR	0.02%	0.02%	0.004%	0.01%	0.003%
C3	CA	1326790	1221760	1312800	1304250	1243910
	CB	1326510	1221540	1312690	1304200	1243770
	CR	0.02%	0.02%	0.01%	0.004%	0.011%

Table 12: Computational results of the instances 16 to 20 (scenario MR3).

Instance		16	17	18	19	20
C1	CA	1104940	1015320	1099930	1090210	1027320
	CB	1104920	1015320	1099900	1090210	1027310
	CR	0.002%	0%	0.003%	0%	0.001%
C2	CA	1185280	1090030	1178060	1168640	1106480
	CB	1185260	1090030	1178050	1168610	1106480
	CR	0.002%	0%	0.001%	0.003%	0%

C3	CA	1325600	1219060	1311900	1302920	1242460
	CB	1325540	1219050	1311900	1302890	1242430
	CR	0.005%	0.001%	0%	0.002%	0.002%

From the above tables, we can see, for each given service level of FDCs, the replenishment plan of the distribution system with PDCs can lead to a smaller replenishment cost than that of the system without PDCs for almost all instances and cases. For the cases with smaller maximum joint replenishment quantity (Table 1, 2, 4, 5, 7 and 10), the replenishment cost of the system with PDCs is much lower than that of the system without PDCs for all instances. The reason is that some products are consolidated in PDCs before they are transported to FDCs in these cases. When the maximum joint replenishment quantity is set larger (Table 3, 6, 8, 9, 11 and 12), the cost reduction of the system with PDCs with respect to the system without PDCs becomes smaller for all instances. The reason is that more products are directly transported from suppliers to FDCs due to a larger and almost unconstrained maximum joint replenishment quantity. Since the replenishment quantity between two stocks is usually constrained by the maximum joint replenishment quantity because of limited transportation capacity, our numerical experiment results show the introduction of PDCs in a distribution system can significantly reduce inventory replenishment costs.

Today, PDCs have been introduced by Alibaba in its distribution system for some fresh products (fruits), where a novel project called ‘Shen Nong Plan of Alibaba’ is in the process of implementation. The inventory replenishment via PDCs brings both economic and social benefits to Alibaba. On the one hand, it can lead to lower replenishment costs for a given service level of FDCs. On the other hand, as PDCs are located in the areas of producers that are usually less developed, the project of implementation of PDCs will provide jobs for habitants in such areas, which can both reduce the poverty in these areas and increase their local industry incomes.

6 CONCLUSIONS

An inventory replenishment planning problem in a three echelon distribution system with warehouses at the locations of producers is studied in this paper. This problem is formulated as a bi-objective optimization problem. Numerical experiments on instances generated based on the data of Alibaba validate the proposed model and demonstrate the

advantage of having such warehouses in the system. Our future work is to study a multi-period replenishment planning problem of the system.

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