Nonlinear Internal Model Controller Design for Wastegate Control of a Turbocharged Gasoline Engine

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Abstract—This work investigates the design of nonlinear internal model control (IMC) for wastegate control of a turbocharged gasoline engine. We extend the inverse-based IMC design for linear time-invariant (LTI) systems to nonlinear systems. To leverage the available tools for LTI IMC design, we have explored the quasi linear parameter varying (quasi-LPV) model. IMC design through transfer function inverse of the quasi-LPV model is ruled out due to parameter variability. A new approach for nonlinear inverse, referred to as the structured quasi-LPV model inverse, is developed and validated. A fourth-order nonlinear model which sufficiently describes the dynamic behavior of the turbocharged engine is implemented to serve as the model in the IMC structure. The controller based on structured quasi-LPV model inverse is designed to achieve boost pressure tracking. Finally, simulations on a validated high fidelity model are carried out to show the feasibility of the IMC. Its closed-loop performance and robustness are compared with a well-tuned PI controller with extensive feedforward and anti-windup built in.

I. INTRODUCTION

Internal model control (IMC), whose diagram is shown in Fig. 1, is a well-established control design methodology with an intuitive control structure [1]. It incorporates a system model as an explicit element in the controller so that the control actions are determined based on the error signal between the model output and the plant output. It has several desired features and closed-loop properties as established in [1], [2], such as dual stability criterion, zero offset, and perfect control. The design, analysis, and implementation of IMC for linear systems have been well developed. Rivera et al. showed that process industrial IMCs for many SISO models can lead to PID controllers, occasionally augmented with a first-order lag [3]. They also demonstrated the superiority of using IMC for PID tuning in terms of closed-loop performance and robustness.

Efficacy of IMC for nonlinear systems, however, has been investigated with limited comprehensive results. In 1986, Economou et al. presented an important result of nonlinear IMC [4], proving that the dual stability criterion, zero offset, and perfect control properties of LTI IMC would carry over to nonlinear cases. The IMC was implemented by finding a nonlinear dynamic inverse, which remained to be the key challenging part of extending the IMC design to solve control problems for nonlinear systems. While the invertibility condition, inverse structure and derivation for nonlinear dynamic system inverse were studied [5], the derivation of the nonlinear inverse involved higher-order derivatives and caused problems when noises and disturbances were present in the system. In [4], the nonlinear inverse was derived by exploiting the Hirschorn nonlinear inverse structure and solving it numerically using the contraction principle method or Newton’s method. Stability of the IMC structure was discussed under the ideal circumstance that the model was the same as the plant. Henson et al. also exploited the result of Hirschorn nonlinear inverse for nonlinear IMC design [6]. Several assumptions were made to calculate the higher-order derivatives. Nonlinear IMC was also investigated in the adaptive control framework. Hunt et al. used artificial neural networks for adaptive control of nonlinear IMC [7], [8]. Feasibility of identifying the nonlinear model and its inverse by a neural network was explored and demonstrated.

Another possible avenue to exploit the linear IMC design tools for nonlinear systems would be through the linear parameter varying (LPV) model. Mohammadpour et al. applied IMC on a quasi-LPV model in [9] with two approaches. In the first approach, the IMC controller parameters were scheduled based on the LPV model parameters which were assumed to be known in real time and not varying rapidly. In the second approach, the design problem was formulated in the $H_\infty$ framework as a set of linear matrix inequalities (LMI). Solving the associated LMI problem, however, was computationally intensive. Toivonen et al. derived the LPV model based on velocity-based linearization, then developed the IMC controller based on linear IMC theory [10]. It was much less computationally demanding, but it was only applicable when there was a small number of scheduling parameters.

The investigation pursued in this paper is motivated by the successful industrial applications of IMC and the need for developing robust and easy-to-calibrate powertrain control solutions. IMC has been successfully applied on chemical processes [11] and in the automotive area. Its simple architecture and intuitive design philosophy make it attractive for powertrain control system design and calibration. IMC has been applied to wastegate control for a turbocharged gasoline engine using a first-order model which was simplified from a fourth-order nonlinear model using singular perturbation [12]. While the simplicity of the first-order model based design was an advantage for implementation, its performance was limited by the linear approximation, as it is defined for a particular operating point.

This work investigates the feasibility, performance, ad-
II. PROBLEM STATEMENT AND PRELIMINARIES

The turbocharged gasoline engine is a critical technology with potentials for improved fuel efficiency and power density. It also represents an exciting test bed for control technology demonstration given its control-intensive and control-critical nature [14]. The schematic of the turbocharged engine system is shown in Fig. 2. For this investigation, we will focus on the air control and boost pressure sub-system.

Boost pressure is a key variable in the engine. It affects the amount of air entering the engine and determines engine torque, so its control is critical and needs to be accurate. The wastegate is a main actuator for the boost pressure. It affects the engine operation by changing the rotational speed of the turbo/compressor. In this paper, nonlinear IMC is developed for boost pressure control with wastegate as the actuator.

A. Internal Model Control

IMC structure is shown in Fig. 1, where \( P, M, \) and \( C \) denote the plant, model, and controller respectively. The IMC controller contains two elements: the controller \( C \) and the model \( M \). The controller takes the reference input and the difference between the plant and the model as its input. Using model inverse as the controller, this system can be designed to track the reference. One can prove that even when \( M \neq P, C \) can be designed as the inverse of the model \( M \) to achieve the control objectives [1]. The sensitivity function of the IMC control system is

\[
S_{IMC} = \frac{1 - MC}{1 - MC + PC}.
\]  

When \( MC = 1 \), i.e., \( C = M^{-1} \), sensitivity function is 0, which means “perfect control” is achieved.

For IMC of LTI systems, the controller \( C \) can be designed as inverting the transfer function of \( M \) and appending a filter for causality. For both linear or nonlinear systems, once the model and its valid inverse are derived, IMC control design follows immediately. For a nonlinear model, therefore, we will focus our effort on deriving the model inverse.

B. A Nonlinear Model For IMC Design

Control-oriented models serve the IMC design and implementation in two different ways: first, the IMC incorporates a system model directly in its implementation as shown in Fig. 1; second, the standard IMC design procedure takes an inverse of the process model and augments it with a proper filter to avoid non-causal implementation to form the IMC controller.

The nonlinear model for the boost-pressure dynamics of a turbocharged engine presented in this paper is based on the work of [15]. The nonlinear model has the following four states and one input:

\[
\begin{align*}
x_1 & : P_b \quad \text{boost pressure (kPa)} \\
x_2 & : P_i \quad \text{intake pressure (kPa)} \\
x_3 & : P_e \quad \text{exhaust pressure (kPa)} \\
x_4 & : N_t \quad \text{turbocharger speed (rad/sec)} \\
u & : u_{w} \quad \text{wastegate (unit-less, fraction for opening, takes values in [0, 1])}
\end{align*}
\]
The dynamics of the pressures $P_b$, $P_i$, and $P_e$ are derived using mass conservation along with isothermal manifold assumptions, while the dynamics of the turbocharger speed $N_t$ are derived by a power balance between the turbine and the compressor as described in [12], [15]. The equations are summarized as follows:

\[
\begin{align*}
\frac{dP_b}{dt} &= \frac{RT_b}{V_b} (W_c - W_{th}), \\
\frac{dP_i}{dt} &= \frac{RT_i}{V_i} (W_{th} - W_{en}), \\
\frac{dP_e}{dt} &= \frac{RT_e}{V_e} (W_{en} \frac{1 + A/F}{A/F} - W_t - W_w), \\
\frac{dN_t}{dt} &= \frac{1}{I_t N_t} (P_t - P_e),
\end{align*}
\]

(2)

where $R$ is the universal gas constant, $A/F$ is the air fuel ratio, $T, V, W$, and $P$ are temperature, volume, flow, and power respectively, and the subscript indicates the physical location of the variable as in Fig. 2. Modeling of $W$ (flow) and $P$ (power) are described in detail in [12], [15], and the resulting functional expressions are summarized as follows:

\[
\begin{align*}
W_c &= f_c \left( \frac{P_b}{P_a}, N_t \right), \\
W_{th} &= \frac{\text{sat}(0, u_{th}, 1)}{\sqrt{RT_b}} \gamma P_b \phi \left( \frac{P_i}{P_b} \right), \\
W_{en} &= \frac{P_i \eta_{en} V_{en}}{\sqrt{RT_e} N_t} \frac{P_e}{2}, \\
W_w &= \frac{\text{sat}(0, u_{w}, 1)}{\sqrt{RT_e}} \gamma P_e \phi \left( \frac{P_x}{P_e} \right), \\
W_t &= f_t \left( \frac{P_x}{P_e}, \frac{N_t}{\sqrt{RT_e}}, \frac{P_c}{\sqrt{RT_e}} \right), \\
\text{Power}_i &= c_{p,i} T_i W_i \eta_t \psi_t, \\
\text{Power}_e &= c_{p,e} T_a W_e \frac{1}{\eta_e} \psi_e,
\end{align*}
\]

(3)

where $P_a$ is the ambient pressure, $\phi(\cdot)$ is a function of pressure ratio across the component, $\psi(\cdot)$ is a mass flow parameter, $\gamma$ is the specific heat ratio for air, $c_{p,(\cdot)}$ is the coefficient of power, $\eta_{(\cdot)}$ is the isentropic efficiency, $u_{th}$ is the throttle opening, and $N_{en}$ is the engine speed. We consider $u_{th}$ and $N_{en}$ as exogenous inputs in this work.

The nonlinear model is evaluated by comparing its responses with those of the virtual “plant”, which is a high fidelity Ford proprietary model that has been validated extensively. Responses to a step change in the wastegate setting from 0.25 to 0.75 at $t = 5 \text{sec}$ for the nonlinear model and the virtual “plant” are shown in Fig. 3, confirming that the control-oriented nonlinear model and the “plant” have very similar dynamic responses. Control objectives can be achieved if $C$ in Fig. 1 is designed as the inverse of this nonlinear model.

C. Quasi Linear Parameter Varying Model

Quasi-LPV models are LPV models for nonlinear models where nonlinearities are hidden through state-dependent parameters, so that a nonlinear model can be represented by an LPV model and treated by LPV design techniques [13].

In general, a nonlinear model in the form of

\[
\dot{x} = f(x, u)
\]

(4)

can be expressed as an LPV model in the form of

\[
\dot{x} = A(p)x + B(p)u
\]

(5)

if the model (4) is affine in $u$ and the time varying parameter vector $p$ in (5) is allowed to be state-dependent to disguise the nonlinearities. For example, the nonlinear model $\dot{x} = x^2 + xu$ can be expressed in quasi-LPV form as $\dot{x} = A(p)x + B(p)u = [x]x + [x]u, p = [x]$.

In the next section, we exploit the quasi-LPV approach for developing a nonlinear IMC for the turbocharged gasoline engine.

III. QUASI-LPV MODEL AND ITS INVERSION

A. Quasi-LPV Turbocharged Gasoline Engine Model

Note that there are infinite number of quasi-LPV models in the form of (5) that can match (4). For the turbocharged system, we consider the physical couplings of state variables and choose the following structure that leads to the most sparse $A, B$ matrices:

\[
A = \begin{bmatrix}
a_{11} & 0 & 0 & a_{14} \\
a_{21} & a_{22} & 0 & 0 \\
0 & a_{32} & a_{33} & 0 \\
0 & 0 & a_{43} & a_{44}
\end{bmatrix}, B = \begin{bmatrix}
0 \\
0 \\
b_3 \\
0
\end{bmatrix},
\]

(6)

\[
x = [P_b, P_i, P_e, N_t]^T, u = u_w.
\]

The non-zero elements in (6) are defined as follows:

\[
\begin{align*}
o_{11} &= -\frac{RT_b W_{th}}{V_b} = -\frac{\sqrt{RT_b}}{V_b} \text{sat}(0, u_{th}, 1) \gamma \phi(\frac{P_i}{P_b}), \\
o_{14} &= \frac{RT_i W_c}{V_i} \frac{N_t}{V_b N_i} f_1(\frac{P_b}{P_a}, N_i), \\
o_{21} &= \frac{RT_i W_{th}}{V_i} = \frac{RT_i}{V_i} \text{sat}(0, u_{th}, 1) A_{max} \phi(\frac{P_i}{P_b}) \gamma, \\
o_{22} &= -\frac{RT_i W_{en}}{V_i} = -\frac{RT_i}{V_i} \text{sat}(0, u_{th}, 1) A_{max} \phi(\frac{P_i}{P_b}) \gamma, \\
o_{32} &= \frac{1 + A/F RT_e W_{en}}{A/F V_e T_i} = \frac{1 + A/F T_e \eta_{en} V_{en} N_{en}}{2V_e T_i},
\end{align*}
\]

(7)
The quasi-LPV model (6-7) is implemented in Simulink and validated through simulations. Responses of the nonlinear model and the quasi-LPV model are matched exactly. It can be observed from (7) that the scheduling signals for these parameters are states ($x_1$-$x_3$), temperatures ($T_b, T_i, T_e$), throttle opening ($u_{th}$), and engine speed ($N_{en}$).

**B. Structured Quasi-LPV Inverse**

Given that the parameters defined by (7) in the quasi-LPV model (6) are varying extensively during transients, deriving the transfer function of (6) and treating the parameters as frozen will not be effective for deriving the inverse. Indeed, this is confirmed by numerical simulation. In this section, we explore the special form of the quasi-LPV structure of (6) and derive its inverse model in an effort to minimize the approximation error.

We express the quasi-LPV model as an integration of several first-order submodels. Given the sparse matrices $A, B$ in the form of (6), we define the first-order submodels $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4$ as

\[
\begin{align*}
\Sigma_1 : \quad & \dot{x}_1 = a_{11} x_1 + a_{14} x_4 \quad \Rightarrow \quad x_1 = \Sigma_1(x_4), \\
\Sigma_2 : \quad & \dot{x}_2 = a_{22} x_2 + a_{21} x_1 \quad \Rightarrow \quad x_2 = \Sigma_2(x_1), \\
\Sigma_3 : \quad & \dot{x}_3 = a_{32} x_2 + a_{33} x_3 + b_3 u \quad \Rightarrow \quad x_3 = \Sigma_3(x_2, u), \\
\Sigma_4 : \quad & \dot{x}_4 = a_{42} x_2 + a_{43} x_3 \quad \Rightarrow \quad x_4 = \Sigma_4(x_3).
\end{align*}
\]

The input-output relationship of the quasi-LPV model is then expressed as a composition of these submodels:

\[
x_1 = \Sigma_1(\Sigma_3(\Sigma_2(x_1, u))),
\]

\[
x_2 = \Sigma_1(\Sigma_3(\Sigma_2(x_1))),
\]

\[
x_3 = \Sigma_3(\Sigma_2(x_1) + \Sigma_2(u)),
\]

\[
x_4 = \Sigma_4(x_2, u) = \Sigma_3(\Sigma_2(x_1)) + \Sigma_3(\Sigma_2(u)),
\]

(9)

whose block diagram representation is shown in Fig. 4. Expressing the input $u$ in terms of the output $x_1$ based on (9) without any approximation, we have

\[
u = \Sigma_{32}^{-1}(\Sigma_4^{-1}(\Sigma_1^{-1}(x_1)) - \Sigma_{31}(\Sigma_2(x_1))).
\]

(10)

It is an inverse model of (9), which is equivalent to the nonlinear model in (2). Fig. 5 shows the block diagram representation of (10) through the composition of several inverse models of first-order blocks.

For each first-order quasi-LPV submodel of the form

\[
\dot{x} = ax + bu,
\]

(11)

its inverse can be derived as follows. Introducing a filter whose transfer function is $\frac{1}{\tau s+1}$ with $\tau$ being the time constant, we have

\[
u \approx \begin{pmatrix} \frac{1}{\tau s+1} \end{pmatrix} u = \begin{pmatrix} \frac{1}{\tau s+1} \end{pmatrix} \begin{pmatrix} \dot{x} \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{\tau s+1} \end{pmatrix} \begin{pmatrix} d x \\ \frac{b x}{b^2} - \frac{a x}{b} \end{pmatrix} + \begin{pmatrix} \frac{1}{\tau s+1} \end{pmatrix} \begin{pmatrix} \frac{1}{\tau b} - \frac{b}{b^2} + \frac{a}{b} \end{pmatrix} x.
\]

Note that $\begin{pmatrix} \frac{1}{\tau s+1} \end{pmatrix}$ here denotes the time-domain dynamic operator with a transfer function $\frac{1}{\tau s+1}$. Let

\[
z = \begin{pmatrix} \frac{1}{\tau s+1} \end{pmatrix} \left( \begin{pmatrix} \frac{1}{\tau b} - \frac{b}{b^2} + \frac{a}{b} \end{pmatrix} x \right),
\]

(13)

and the inverse of the first-order system can be represented by a first-order LPV model

\[
u = \frac{1}{\tau b} x - z,
\]

(14)

\[
\dot{z} = \frac{1}{\tau} \left( -z + \begin{pmatrix} \frac{1}{\tau b} - \frac{b}{b^2} + \frac{a}{b} \end{pmatrix} x \right).
\]

For simplicity, the $\frac{b}{b^2}$ term can be dropped if the parameter variation is substantially slower than the system dynamics. A reference signal is fed into a first-order model inverse derived from (14), whose output is then fed into this first order model in (11). The model output should be close to the reference signal if the inverse represents the inverse dynamics of the model. An analysis of the inverse model performance is shown in Fig. 6. It is obvious that the inverse incorporating the $\frac{b}{b^2}$ term with smaller time constant $\tau$ has better accuracy. Therefore, the inverse incorporating $\frac{b}{b^2}$ is adopted in the subsequent derivation. The time constant $\tau$ is the tuning parameter for the IMC design. Using (14) to
represent each first-order model inverse in (10), an inverse model for the model in (9) is derived.

To incorporate the quasi-LPV inverse model in the IMC structure, two implementable configurations are possible as shown in Fig. 7 and Fig. 8. First, one can use the states in the nonlinear model \( \Sigma \) to schedule the parameters in the inverse model \( \Sigma^{-1} \) (as in Fig. 7). Since the states used for parameter scheduling are external to \( \Sigma^{-1} \), we refer to Fig. 7 as the external scheduled quasi-LPV inverse. Secondly, one can derive parameters used in \( A \) and \( B \) for \( \Sigma^{-1} \) based on the internal states in \( \Sigma^{-1} \) (as in Fig. 8), which we refer to as the internal scheduled quasi-LPV inverse. The other scheduling signals such as temperatures, are not vital, yet their steady-state values still affect the steady-states of the inverse. Therefore, \( \Sigma^{-1} \) uses quasi steady-state temperatures. Throttle opening and engine speed are external.

\[
\begin{align*}
\begin{array}{ccc}
\Sigma^{-1} & \rightarrow & y \\
\downarrow & & \downarrow \\
A, B & \text{matrices in } \Sigma^{-1} & \text{are scheduled by states from } \Sigma
\end{array}
\end{align*}
\]

Fig. 7. External scheduled quasi-LPV inverse validation structure.

\[
\begin{align*}
\begin{array}{ccc}
\Sigma^{-1} & \rightarrow & y \\
\downarrow & & \downarrow \\
A, B & \text{matrices in } \Sigma^{-1} & \text{are scheduled by states in } \Sigma^{-1}
\end{array}
\end{align*}
\]

Fig. 8. Internal scheduled quasi-LPV inverse validation structure.

While the external scheduled quasi-LPV inverse looks appealing, its utility is ruled out after more in-depth analysis and simulation. The internal scheduled quasi-LPV inverse, however, also shows tendency to diverge in simulations. After extensive investigation of the model structure and state behavior during the transient, it is found that by replacing the scheduling signals \( x_3 \) by the steady-state exhaust pressure and \( x_4 \) by the steady-state turbo speed, we achieve the desired inverse performance. This steady-state mapping is generated with respect to different engine speed and throttle opening. The tuning parameters are the time constants as in (12) in each first-order LPV submodel inverse \( \Sigma_1^{-1} \), \( \Sigma_4^{-1} \), and \( \Sigma_3^{-1} \), respectively. Inverse 1 has the time constants 0.1s, 0.04s, and 0.02s in \( \Sigma_1^{-1} \), \( \Sigma_4^{-1} \), and \( \Sigma_3^{-1} \), respectively. Inverse 2 has the time constants 0.05s, 0.04s, 0.02s in \( \Sigma_1^{-1} \), \( \Sigma_4^{-1} \), and \( \Sigma_3^{-1} \), respectively. The inverse model with larger time constants is more damped than the inverse with smaller time constants, and has less overshoot and less aggressive control input.

IV. IMC DESIGN AND ITS PERFORMANCE EVALUATION

IMC can be constructed with the fourth-order nonlinear model and the model inverse developed as in Fig. 1. The IMC is applied to the virtual “plant”, which is a validated Ford proprietary model. The structure of the resulting nonlinear IMC system is shown in Fig. 10.

To evaluate the performance of IMC, it is compared with a well-tuned PI controller with extensive feedforward and anti-windup built in, which is referred to as PI control later. The system response and control input are compared in two cases: constant engine speed without disturbance (as in Fig. 11(a)); varying engine speed without disturbance (as in Fig. 11(b)). Here, varying engine speed rises from 1500 to 3000 rpm gradually. We also do robustness analysis for IMC by adding a disturbance (as in Fig. 11(c)). The disturbance is a step change in throttle opening from 45 to 30 degrees at \( t = 12 \text{ sec} \).

The tuning parameters are the time constants as in (12) in each first-order LPV submodel inverse \( \Sigma_1^{-1} \), \( \Sigma_4^{-1} \) and \( \Sigma_3^{-1} \), respectively.

Fig. 6. Analysis of first-order inverse with and without \( \dot{b}, \tau = 0.01 \).

Fig. 7. External scheduled quasi-LPV inverse validation structure.

Fig. 8. Internal scheduled quasi-LPV inverse validation structure.

Fig. 9. Validation of structured quasi-LPV inverse.

Fig. 10. Internal model structure with structured quasi-LPV inverse.
as shown in Fig. 5. The time constants are 0.1s, 0.04s, 0.02s, which is the same as in Section III inverse 1.

In Fig. 11(a), IMC achieved a faster reference tracking than PI with less overshoot. In Fig. 11(b), the IMC response does not overshoot as the PI does, even without the anti-windup design. In Fig. 11(c), IMC rejects the disturbance of the throttle opening step change. Overall, the nonlinear IMC for the wastegate control of a turbocharged gasoline engine is shown to be valid. It shows some novel features, good reference tracking, no steady-state error, no need for separate anti-windup design, and intuitive tuning. Its performance matches, and in cases exceeds that of, a well-tuned PI control with extensive feedforward and anti-windup built in.

V. CONCLUSIONS

A nonlinear IMC design for the turbocharged gasoline engine is presented. A nonlinear fourth-order dynamics model is adopted in the controller. The challenges for inverting the nonlinear model are addressed by: (1) representing the nonlinear dynamics with a quasi-LPV model, and (2) exploring the special quasi-LPV model structure. The simulation results demonstrate the validity of the proposed approach and established the performance of the closed-loop system.

REFERENCES