Pulse Design for Maximizing SIR in Partially Equalized OFDM/BFDM Systems

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Abstract—Mobile wave propagation undergoes severe distortions caused by inter symbol interference (ISI) and inter carrier interference (ICI), which are due to multipath and high mobility. OFDM/BFDM techniques have a good ability to overcome ISI limiting quality effects whereas ICI can be increased for severe Doppler spreading. In this paper, we study the use of well localized time-frequency pulse shaping to reduce the effect of these interferences. The linear combination of several of the first most localized Hermite waveforms enables the design of an optimal transmit/receive pulse. By subtracting the interferences coming from some neighborhood symbols that will be treated by a partial equalization scheme, considerable gain in the signal to interference ratio (SIR) is achieved. With a reduced complexity OFDM system, partial equalization of the interference coming from 2-frequency neighbor symbols results in a gain of about 20 dB in the SIR, at high spectral efficiency and severe channel spreading. More gain in the desired ratio is achieved by optimizing the time-frequency spacing of the underlying lattice.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) and Biorthogonal Frequency Division Multiplexing (BFDM) [1] are promising modulation schemes for high data rate wireless communication such as DVB-T, WIFI 802.11a/d and WiMAX 802.16d/e. In OFDM, the data stream is divided into parallel substreams that are transmitted on contiguous subcarriers with lower rate and longer symbol duration, thereby lowering ISI. In conventional OFDM systems [2], [3] rectangular pulses are used at the transmitter and the receiver, and a cyclic prefix is inserted at the transmitter in order to get rid of ISI and ICI in time dispersive channels. Unfortunately, cyclic prefix insertion reduces bandwidth efficiency and cannot effectively combat ICI in frequency dispersive channels. Future mobile communications will have to cope with these increased frequency dispersions, which are caused by the use of higher carrier frequencies and mobile velocities. Recently, several pulse shaping OFDM/BFDM techniques have been proposed as efficient ways for balancing ICI and ISI, and thereby offering an important reduction in overall interference, for both time and frequency dispersive channels [3], [4], [5]. The design of well localized time-frequency pulse shapes for OFDM/BFDM transmission is deemed to have several advantages over Cyclic Prefix OFDM: higher bandwidth efficiency, reduced sensitivity to carrier frequency offsets, lower ISI and ICI. In this work, we propose a pulse shaping for partially (turbo-)equalized OFDM/BFDM systems, operating on highly dispersive channels [6]. Although perfect orthogonality cannot be reached in this dispersive channel context, we still use here OFDM/BFDM to designate multicarrier systems with identical/different transmit and receive pulses. In partial equalization, those neighboring symbols leading to the highest interference contribution will not be considered in the pulse shaping optimization process, since they are assumed here to be perfectly estimated and subtracted by an appropriate partial equalization technique. The pulse design will be based on a truncated Hermite waveform expansion, thereby guaranteeing a good time-frequency localization of the transmit and receive pulse shapes. First used in the design of perfect orthogonal pulses for non-dispersive channels [7], Hermite waveforms are used here for non-orthogonal SIR optimization for high dispersions. For a reduced partial equalization complexity, one of the proposed subtracted symbol patterns is composed of 2 neighboring symbols in frequency. However, for an increased gain in performance, patterns composed of 4 or 8 symbols in time and frequency are also considered. For the 2 neighboring symbol pattern, an optimum distribution of time and frequency spacing in the underlying lattice should be found, because of the non symmetry between frequency and time. The paper is organized as follows. In section 2, we present the system model. Section 3 provides the SIR formulation. This ratio is given analytically in section 4. Optimal pulse design is described in section 5 using different optimization algorithms and simulation results are analyzed in section 6. Finally, we make our conclusions and give future perspectives.

II. SYSTEM MODEL

A. Modulator

We modulate the transmitted data by a pulse shape \( \phi(t) \) and denote its shifted versions by

\[
\phi_{mn}(t) = \phi(t - mT)e^{j2\pi nF(t - mT)}
\]  

(1)

where \( T \) is the symbols separation, \( F \) is the subcarrier frequency spacing, \( m \) and \( n \) are their indices. If the system has \( K \) subcarriers, then the transmitted signal can be written as

\[
s(t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{K-1} a_{mn} \phi_{mn}(t)
\]  

(2)
where the symbols \( a_{mn} \) are assumed to be i.i.d. with zero mean and mean power \( E\{|a_{mn}|^2\} = \sigma_a^2 \). Also for simplicity, we take \( K \) to be very large and \( FT \geq 1 \).

**B. Mobile Channel**

The multipath fading channel is assumed to be noiseless, underspread and satisfying the wide sense stationary uncorrelated scattering (WSSUS) property. It is characterized by its time varying impulse response \( h(\tau; t) \) and transfer function \( H(f; t) \). In this work, we consider the scattering function of the channel \( \Phi_h(\tau, \nu) \) to be constant over the rectangular region defined by \([-T_a/2, T_a/2] \times [-B_d/2, B_d/2]\) of area \( B_d T_a \ll 1 \).

![Fig. 1. Lattice structures used for demodulation: ‘●’ Demodulated data symbol; ‘○’ Partially equalized interfering symbols; ‘X’ Non equalized interfering symbols](image)

**C. Demodulator**

For BFDM systems, the estimated symbols \( \hat{a}_{mn} \) are found by calculating the inner product of the received signal

\[
r(t) = \int h(\tau; t)s(t−\tau)d\tau
\]

and the demodulating pulse shape \( \psi_{mn}(t) \). Then the estimated symbol is calculated as

\[
\hat{a}_{mn} = \langle r(t), \psi_{mn}(t) \rangle = \int r(t)\psi_{mn}^*(t)dt
\]

When an OFDM system is considered, both, the modulating and demodulating pulse shapes will be identical. Interference-free demodulation is achieved when perfect orthogonality is not lost. Two conditions for this to hold are \( FT \geq 1 \) and \( B_d T_a = 0 \). Unfortunately, these conditions can never be met in time-frequency dispersive channels, where \( B_d T_a \) can never be null. However it should be kept in mind that spectral efficiency is inversely proportional to \( FT \). Hence this product should be kept close to 1. Previous works outlined in [3] took \( FT \) in the range 1.03 to 1.25.

**III. SIGNAL TO INTERFERENCE RATIO FORMULATION**

The decision variable corresponding to the transmitted symbol \( a_{mn} \) is written as:

\[
\hat{a}_{mn} = H_{mn}^{\text{mn}}a_{mn} + \sum_{(m',n') \neq (m,n)} H_{mn}^{mn'}a_{m'n'}
\]

where \( H_{mn}^{mn'}(t,\psi_{mn}(t)) > 0 \) and \( H\phi_{mn'}(t) \) is the transformation of \( \phi_{mn'}(t) \) by the channel. The first right hand side term in (5) is the useful part of the decision variable and the second one accounts for the ISI and ICI introduced by the channel.

If the mean ISI/ICI power is \( \sigma_i^2 \) and that of the interest symbol is \( \sigma_I^2 \) then the SIR will be defined by

\[
\text{SIR} = \frac{\sigma_I^2}{\sigma_I^2}
\]

**A. Non equalized receiver**

This corresponds to the model shown in Fig1a. It was shown in [3] that the corresponding mean power of the useful symbol is written as

\[
\sigma_D^2 = \sigma_a^2 \int |A\phi\psi(\tau, \nu)|^2 \Phi_h(\tau, \nu)d\tau d\nu
\]

and the mean power of ISI/ICI as

\[
\sigma_I^2 = \sigma_a^2 \int \Gamma_{\psi\psi}(\tau, \nu)\Phi_h(\tau, \nu)d\tau d\nu
\]

where

\[
\Gamma_{\psi\psi}(\tau, \nu) = \sum_{(k,l) \neq (0,0)} |A\phi\psi(\tau − kT, \nu − lF)|^2
\]

and

\[
A\phi\psi(\tau, \nu) = \int \phi(t − \frac{\tau}{2})\psi^*(t + \frac{\tau}{2})e^{2\pi i \nu t}dt
\]

is the cross ambiguity function of \( \phi(t) \) and \( \psi(t) \). The SIR will be calculated by replacing (7) and (8) in (6).

**B. Partially equalized receiver**

When considering perfect partial equalization, few neighboring symbols are subtracted (as shown in Fig1b, c and d). Notice here that 2-symbol partial equalization in time is not considered because of the extra buffering complexity with respect to frequency equalization. Generally, partial equalized interference mean power can be written as

\[
\sigma_{eq}^2 = \sigma_a^2 \sum_{(k,l) \neq \zeta} \int |A\phi\psi(\tau − kT, \nu − lF)|^2 \Phi_h(\tau, \nu)d\tau d\nu
\]

where \( \zeta \) is the set of symbols to be subtracted in the partial equalization process. For Fig1b, Fig1c and Fig1d, \( \zeta \) is given respectively by \( \{(0, ±1)\}, \{(0, ±1)\} \), \( \{(0, ±1), (±1, 0)\} \). Again, the desired SIR can be calculated by replacing (7) and (11) in (6).

**IV. OPTIMAL PULSE SHAPE CHARATERISTICS**

The proposed pulse shapes are expressed as linear combinations of a finite number of the first most localized Hermite waveforms. Hermite waveforms form an orthonormal basis for the Hilbertian space of square summable functions and show,
in decreasing index order, the best localization in the time frequency domain. They are defined as
\[ u_k(t) = \frac{\sqrt{2}H_k(2\sqrt{t})e^{-\pi t^2}}{\sqrt{k!}}; k \geq 0, \]  
where
\[ H_k(t) = (-1)^k e^{x_2} \frac{d^k}{dt^k} (e^{-x^2}). \]  
We have to use the even waveforms to guarantee an energy concentration at the origin of the time-frequency plane. Then the designed pulses will be expressed as \( \phi(t) = \sum_{k=0}^{N-1} \alpha_{2k}u_{2k}(t) \) for the modulator and \( \psi(t) = \sum_{k=0}^{N-1} \beta_{2k}u_{2k}(t) \) for the demodulator, where \( N \) is the number of contributing Hermite waveforms. According to these expressions, the cross ambiguity function of \( \phi(t) \) and \( \psi(t) \) can be expressed as
\[ A_{\phi\psi}(\tau, \nu) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} \alpha_k \beta_l^* A_{kl}(\tau, \gamma), \]  
where \( A_{kl}(\tau, \nu) \), the cross ambiguity of \( u_k(t) \) and \( u_l(t) \), is given by [7]
\[ A_{kl}(\tau, \nu) = (-1)^{k+l} \sqrt{\frac{l!}{k!}} e^{-\frac{\pi}{4}(\tau^2 + \nu^2)} \times \]  
\[ \sqrt{\pi}^{k-l} (\tau + i\nu)^{k-l} L_{k-l}^l(\pi(\tau^2 + \nu^2)), \]  
where
\[ L_{\alpha}^l(x) = \sum_{i=0}^{l} \frac{(-x)^i}{i!(l-i)!} \prod_{j=i+1}^{l} (j + \alpha). \]  
Finally inserting (15) in equation (14) and using equations (7), (8), (9) and (11), the desired ratio can be calculated by (6).

V. PULSE SHAPE OPTIMIZATION

The maximization of the multi-variable SIR with respect to the coefficients \( \{\alpha_{2k}, \beta_{2k}\} \) may easily trap in a local maximum and convergence to the global maximum cannot be guaranteed in practice. To increase the chance of getting the global maximum, we proceed in two steps by hybridizing global search algorithms with local search ones. The SIR, being the ratio of 2 homogeneous functions of degree 2 in \( \alpha_{2k} \) and \( \beta_{2k} \), can be normalized by choosing \( \alpha_0 = \beta_0 = 1 \). Then, due to the good time-frequency localization of \( \phi(t) \) and \( \psi(t) \), it is expected that the following coefficients will decrease in value while being less than 1. For global search, we consider the Genetic Continuous Algorithm (GCA) [8], as well as the Simulated Annealing Algorithm (SAA) [9] and the Tabu Search Algorithm (TSA) [9], [10]. The GCA starts from an initial population and makes repeated crossovers and mutations to improve the solution. The TSA differs from the SAA mainly by its non return to already explored regions, however they both start from an initial solution. Afterwards, we carry on with the local search, which has the object of refining the global solution for which we use the following algorithms: the Nelder Mead Algorithm (NMA) [11], the Direct Search Algorithm (DSA) [12] and the gradient algorithm (GRA), for OFDM systems, and the Generalized EigenValue Decomposition (GEVD), for BFDM systems. The NMA performs a reflection, expansion and shrinkage to replace the worst point in a simplex lying in all directions of the objective function. The DSA uses repeatedly the worst accepted solutions until it replaces the worst one, otherwise it stagnates. The gradient algorithm moves in the directions where it gets steeper. Finally the EVD finds the best coefficients that maximize the matrix power of the system. It should be mentioned that optimization difficulty doesn’t depend on the number of variables in the objective function but mainly on the density of its local maxima.

VI. SIMULATION RESULTS

We made the simulations with Matlab using the algorithms mentioned previously and considered the linear combination of the 10 first even Hermite waveforms \( \{u_0, u_2, ..., u_{18}\} \). We found that the difference in SIR performance between all considered optimization algorithms is within 0.01 dB for two-neighbor partial equalization and within 1 dB for 8-neighbor equalization. We carried the simulations for different channel spreading factors in the case of OFDM and BFDM systems using a square lattice structure network with densities between 1.03 and 1.24. We considered the three patterns shown in figures 1.b, 1.c and 1.d for partially equalized system. We also considered both, perfect partial equalization, where partial interference is completely subtracted by the equalizer, and imperfect equalization, where residual interference remains because of unrecovered decision errors.

A. Perfectly equalized OFDM and BFDM systems

In figures 2 and 3, which are presented for a channel spreading factor \( B_d T_m = 0.01 \), we see that a non equalized BFDM system performs about 2 dB better than a non equalized OFDM system at low values of \( F' \). However the difference is more than 7 dB for 2 and 4 equalization neighbors. On the one hand, we notice a small increase in performance when 4-symbol equalization is used instead of 2-symbol equalization. This surprising result is due to the good time localization of the prototype transmit/receive pulses, the bad frequency localization of which is tackled by partial equalization. On the other hand, we notice a dramatic increase in performance of about 20 dB when an 8-neighbor partial equalization is considered. This gain increases even more at high \( F' \) because of the extra freedom in pulse design. Comparing figures 2 and 3 to 4 and 5 respectively, the observed gain decreases for higher channel spreading factors. For instance, at low values of the network density, a decrease of about 7 dB in SIR is observed when the channel spreading is increased 10 times.

B. Optimal lattice structure for a perfect two-symbol partially equalized OFDM system

The non time-frequency symmetry of the two-symbol equalization pattern shown in figure 1.b requires an optimization
of the $F/T$ ratio for a given value of the ratio $B_d/T_m$. From figure 6, we notice that there is an optimal lattice network structure corresponding to a non-trivial optimum ratio $\eta_{opt} = (B_d/T_m)/(F/T)$, leading to the maximization of the SIR gain. This optimal ratio is about 0.7 when $B_dT_m = 0.1$ and around 1 when $B_dT_m = 0.01$, independently of the value of $FT$.

C. Imperfect 2-symbol partial equalization with optimized lattice

In figure 7, we assumed an imperfect 2-symbol partially equalized OFDM system, based on the optimum lattice structure derived previously for perfect equalization. It is clearly seen that the SIR decreases abruptly when the residual interference is just above 0%, independantly of the channel spreading. This observed sharp decrease in SIR should be seriously taken into account for practical systems employing partial equalization.

D. Prototype transmit/receive pulses for 2-symbol perfectly equalized OFDM and BFDM systems

In figure 8, we see that the OFDM prototype functions for $B_dT_m = 0.01$ and 0.1 are very close. They have some resemblance with the Gaussian function which is known to be the best localized function in time and frequency. This ensures that the optimal used transmit/receive pulses will not strongly depend on the channel statistics. For BFDM, the optimized transmit/receive pulses $\phi(t)$ and $\psi(t)$ can be interchangeably used in the transmitter and receiver sides, as can be seen from equations (7) and (8). From figure 9, we confirm that BFDM transmit/receive pulses are completely different from
VII. Conclusion

In this paper, we tackled the design of optimal pulse shapes, with partial equalization at the receiver, for OFDM and BFDM systems. We considered different channel spreading factors and used different algorithms to ensure the finding of the optimal pulse shapes. We showed that BFDM performs much better than OFDM. Partial equalization brings a considerable gain of several 10s of dBs for the 8 partial equalization of surrounding symbols and less for 2 and 4 neighbors. The surprising result was the small gain difference between 2- and 4-symbol equalization, which will be in favor of the 2-symbol equalization in frequency for its reduced implementation complexity. We have also found that imperfect partial equalization has a considerable effect on SIR performance and remarkably for residual interference just above 0% of the unequalized interference. Our work can be extended by including additive noise in the optimization process.

REFERENCES