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A set of novel correlation tests for nonlinear system variables

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A set of novel correlation tests using omni-directional cross-correlation functions (ODCCFs), which are based on the first order cross-correlation functions (CCF), are proposed in the present study to comprehensively detect nonlinear relationships between variables. Then the ODCCFs are combined into a set of concise formulations to provide better illustration of detected correlations and reduce the number of correlation plots. Compared to the other approaches, the new methodology brings much more power in detection of nonlinear correlations. The efficiency and effectiveness of the new algorithm are demonstrated through simulation studies and comparisons with other linear and nonlinear correlation tests. The results can be widely applied in many relevant fields.

Keywords: Nonlinear correlation tests; CCF; MIMO models; Higher order CCF; NARMAX models

1. Introduction

Linear correlation tests are frequently used to provide as a statistical means of measuring the extent to which two random variables are linearly associated (Lange 1967). For linear model identification and parameter estimation, auto-correlation function (ACF), cross-correlation function (CCF), and other correlation functions have been widely applied to select significant independent model variables and validate estimated models (Bohlin 1971, Box and Jenkins 1976, Söderström and Stoica 1990, Kleinbaum *et al*. 1998, Hall 2000, Jouan-Rimbaud *et al*. 1996, Bohlin 1978). When a model under analysis is linear, ACF and CCF are comprehensively available for detecting the associations between variables.

However these linear correlation tests provide an incorrect detection whenever nonlinear effects are presented in data. Accordingly several methods have been developed to cope with nonlinearities, such as multi-dimensional correlation tests (Billings and Zhu 1994, Aguirre 1997, Maddess *et al*. 2004), higher order CCF (Aguirre 1995, Zhu and Billings 1997), a combination of five first and second order CCFs (Billings and Voon 1986), higher order CCFs between outputs, inputs and residuals (Billings and Zhu 1994, 1995), and multi-directional correlation tests (Mao and Billings 2000). It has been observed that most nonlinear correlation tests are not as straightforward as linear correlation tests (Billings and Voon 1983). All these nonlinear correlation tests have their individual drawbacks. Due to the complexity of nonlinear relationships, multidimensional correlation tests cause an enormous increase in computation (Billings and Zhu 1994). They are clearly unrealistic in tackling complex models. Although several higher order CCF based approaches have been developed to enhance the detections, unfortunately there still exists a number of unsatisfactory issues, typically listed below.

1. They still cannot fully and effectively detect nonlinear correlations in a satisfactory manner for every model.
2. Higher order CCF can misdiagnose some special nonlinear relationships since it only detects the dependence between amplitudes of variables.
3. The results obtained from combined correlation tests are not comparable because they are obtained by using different order correlation functions. The correlation tests between outputs, inputs and residuals (Billings and Zhu 1994, 1995) only can be applied in model validation. However this method

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cannot directly detect the correlations between any two variables.

4. The multi-directional correlation tests (Mao and Billings 2000), expanded from the studies of (Billings and Voon 1986, Billings and Zhu 1994, 1995), also include seven different order CCFs and involve input, output and residual simultaneously. Therefore, they cannot provide a comparable detection between any two variables.
5. Some of the methods are not able to indicate whether the relationships are positively or negatively correlated.
6. Normally higher order CCF can sometimes exhibit less detection power when variable variances are small (Billings and Zhu 1994).

Research in this area has been undertaken, particularly, during the last decade. However due to the difficulty of making further progress the studies have been stopped recently, and consequently there have been no closely related publications issued for about two years. It is hoped this study will provide a new impetus to the research progress in the field of nonlinear signal and model correlation tests.

In the present study, the new CCF based correlation tests are developed to overcome these problems in general. These tests provide a comprehensive and effective method of detecting nonlinear correlations between variables for any type of nonlinear models. Some simulation examples are provided to illustrate the efficiency and effectiveness of computational detections. The rest of the study is organised as follows. In section 2, fundamentals of linear and nonlinear models, CCF and higher order CCF are briefly explained to lay a basis for further developments. In sections 3 and 4, ODCCFs and their combined formulations are proposed to detect different types of nonlinear associations. Simulation results are presented to demonstrate the theoretical developments. In section 5 conclusions are drawn to summarize the study.

2. Correlation functions (CCF)

In order to understand the development of the new correlation tests, both relevant linear and nonlinear correlation tests are initially introduced.

Consider a general MIMO (multi-input and multi-output) discrete time model representation (Billings and Zhu 1995).

$$\mathbf{y}(n) = \mathbf{f}(\mathbf{y}^{n-1}, \mathbf{x}^{n-1}, \mathbf{e}^{n-1}) + \mathbf{e}(n) \tag{1}$$

where n $(n = 1, 2, \ldots, N)$ is a time index and $\mathbf{y}(n)$, $\mathbf{x}(n)$, and $\mathbf{e}(n)$ denote the dependent variable, independent variable and residual vectors respectively. $\mathbf{f}(\cdot)$ is the vector valued linear or nonlinear function so that

$$\left.\begin{aligned} \mathbf{y}(n) &= [y_1(n), \ldots, y_q(n)]^T \\ \mathbf{x}(n) &= [x_1(n), \ldots, x_r(n)]^T \\ \mathbf{e}(n) &= [e_1(n), \ldots, e_q(n)]^T \\ \mathbf{f} &= [f_1, \ldots, f_q]^T \end{aligned}\right\}, \tag{2}$$

where q is the number of dependent variables and r is the number of independent variables. Define

$$\left.\begin{aligned} \mathbf{y}^{n-1} &= [\mathbf{y}_1^{n-1}, \ldots, \mathbf{y}_q^{n-1}]^T \\ \mathbf{x}^{n-1} &= [\mathbf{x}_1^{n-1}, \ldots, \mathbf{x}_r^{n-1}]^T \\ \mathbf{e}^{n-1} &= [\mathbf{e}_1^{n-1}, \ldots, \mathbf{e}_q^{n-1}]^T \end{aligned}\right\}, \tag{3}$$

where

$$\left.\begin{aligned} \mathbf{y}_i^{n-1} &= [y_i(n-1), \ldots, y_i(n-n_y)] \\ \mathbf{x}_i^{n-1} &= [x_i(n-1), \ldots, x_i(n-n_x)] \\ \mathbf{e}_i^{n-1} &= [e_i(n-1), \ldots, e_i(n-n_e)] \end{aligned}\right\}. \tag{4}$$

Linear case: Here $f_j(\cdot)$ is a linear function to satisfy the following superposition and homogeneity principles

$$\begin{aligned} &f_j(\mathbf{y}^{n-1} + \mathbf{y}^{n-2}, \mathbf{x}^{n-1} + \mathbf{x}^{n-2}, \mathbf{e}^{n-1} + \mathbf{e}^{n-2}) \\ &\quad = f_j(\mathbf{y}^{n-1}, \mathbf{x}^{n-1}, \mathbf{e}^{n-1}) + f_j(\mathbf{y}^{n-2}, \mathbf{x}^{n-2}, \mathbf{e}^{n-2}) \end{aligned} \tag{5}$$

$$f_j(\alpha\mathbf{y}^{n-1}, \alpha\mathbf{x}^{n-1}, \alpha\mathbf{e}^{n-1}) = \alpha f_j(\mathbf{y}^{n-1}, \mathbf{x}^{n-1}, \mathbf{e}^{n-1}). \tag{6}$$

Cross-correlation function (CCF): Correlation is a measure of linear dependence between two variable sequences on a scale from -1 to 1. Consider the situation where data of interest are measurements of two discrete random sequences $\mathbf{x}_i$ and $\mathbf{y}_j$, which are assumed to be stationary and ergodic. Hence they can be represented by individual time history records $x_i(n)$ and $y_j(n)$. Therefore an additional variable is introduced, namely a time delay τ between $x_i(n-\tau)$ and $y_j(n)$. The normalised cross-correlation function between $x_i(n)$ and $y_j(n)$ for any time delay τ can be expressed as follows:

$$r_{x_i y_j}(\tau) = \frac{\sum_{n=\tau+1}^{N} (y'_j(n))(x'_i(n-\tau))}{\left[\left(\sum_{n=1}^{N} (y'_j(n))^2\right)\left(\sum_{n=1}^{N} (x'_i(n))^2\right)\right]^{1/2}} \tag{7}$$

where the dash $'$ in (7) denotes that the mean level has been removed from the corresponding data sequences

$$\left.\begin{aligned}(x_i(n))' &= x_i(n) - \frac{1}{N}\sum_{1}^{N} x_i(n)\\ (y_j(n))' &= y_j(n) - \frac{1}{N}\sum_{1}^{N} y_j(n)\end{aligned}\right\}. \tag{8}$$

A correlation value of 0 indicates a random or independent relationship between the two variables, and the correlation values of 1 and -1 denote positive and negative perfect linear associations between the two variables. For a special case $y_j(n) = x_i(n)$, formulation (7) is called normalized auto-correlation function.

Nonlinear case: Here $f_j(\cdot)$ is a nonlinear function. It will not, in general, satisfy the superposition and homogeneity principles. A typical parametric expression is the polynomial Nonlinear AutoRegressive Moving Average with eXogenous input (NARMAX) model (Leontaritis and Billings 1985), which is formulated as

$$y_j(n) = \sum_{i=1}^{r} \alpha_i p_i(n) + e_j(n), \tag{9}$$

where $p_i(n)$ denotes nonlinear terms such as $p_1(n) = y_i(n-1)x_m(n-1)$, $p_2(n) = x_j^2(n)x_k(n-1)$, and etc.

As the ordinary CCF and ACF are no longer sufficient (Billings and Voon 1983) for nonlinear cases, several higher order CCF based tests have been developed to detect the nonlinear associations. Full fundamentals of these tests can be found in Billings and Voon (1986), Billings and Zhu (1994, 1995) and Mao and Billings (2000). Here a brief description of Zhu and Billings (1997) is described as a typical solution.

The normalized higher order CCF $r_{(x_i^2)'(y_j^2)'}(\tau)$ is derived as follows:

$$r_{(x_i^2)'(y_j^2)'}(\tau) = \frac{\sum_{n=\tau+1}^{N}\left(((y_j^2(n))')((x_i^2(n-\tau))')\right)}{\left[\left(\sum_{n=1}^{N}((y_j^2(n))')^2\right)\left(\sum_{n=1}^{N}((x_i^2(n))')^2\right)\right]^{1/2}}, \tag{10}$$

where

$$\left.\begin{aligned}(x_i^2(n))' &= x_i^2(n) - \frac{1}{N}\sum_{1}^{N} x_i^2(n)\\ (y_j^2(n))' &= y_j^2(n) - \frac{1}{N}\sum_{1}^{N} y_j^2(n)\end{aligned}\right\}. \tag{11}$$

It should be noticed that $r_{(x_i^2)'(y_j^2)'}(\tau) = 1$ represents perfect dependence and $r_{(x_i^2)'(y_j^2)'}(\tau) = 0$ represents complete independence between two variables.

Higher order CCF can indicate the nonlinear dependences between the amplitudes of $y_j(n)$ and $x_i(n)$ since it can be regarded as a measure of the correlation between $y_j^2(n)$ and $x_i^2(n)$. In nonlinear models, however, variables are always correlated with others in very complicated relationships, which possibly relate to not only the amplitudes but also the signs (positive or negative) of each variable. For example, both the amplitude and the sign of $y_j(n)$ depend on the amplitude of $x_i(n)$ when the nonlinear relationship between $y_j(n)$ and $x_i(n)$ is characterized as $y_j(n) = \sin(x_i^2(n))$. The higher order CCF, hence, cannot be used to deal with all nonlinear models and may fail to detect the nonlinear associations under some special conditions.

An example is selected here to illustrate the inefficiency of CCF and higher order CCF. Consider a simulated nonlinear discrete dynamic model (model 1)

$$\begin{aligned}y(n) = {} & x_1^2(n-1) - 2x_3^2(n-5)\\ & + 2x_2(n-3)x_4(n-7) - x_2^3(n) + e(n)\end{aligned} \tag{12}$$

where each of the four independent variables of $x_1(n)$, $x_2(n)$, $x_3(n)$ and $x_4(n)$ has a length $N = 1000$. Each variable is uniformly distributed random sequence with zero mean and amplitude from -1 to 1. Noise $e(n)$ is selected as an uncorrelated normally distributed (zero mean and variance of 0.01) data sequence.

For large N, both CCF and the higher order CCF estimates of the random relationship are asymptotically normal with zero mean and finite variance according to centre limit theorem (Bowker and Lieberman 1972), and the confidence limits are approximately $\pm 1.95/\sqrt{N} = \pm 0.062$. Figures 1, 2 and 3 show the measured dependent variable sequence $y(n)$, the results obtained from using CCF, and higher order CCF tests respectively.

Figure 2 shows that only $r_{x_2y}(\tau)$ is outside the confidence intervals at $\tau = 0$. CCF misdiagnoses all the other associated independent variables. In figure 3, $r_{(x_2^2)'(y^2)'}(\tau)$, $r_{(x_3^2)'(y^2)'}(\tau)$ and $r_{(x_4^2)'(y^2)'}(\tau)$ are significantly outside confidence intervals at $\tau = 3$, 5 and 7 respectively. Inspection of model (12), $x_1(n-1)$ is obviously nonlinear associated with $y(n)$ and $x_2(n)$ should display a more significant negative correlation to $y(n)$. In conclusion, CCF is inadequate to detect the nonlinear relationships. The higher order CCF also cannot track all the complex nonlinear associations even though it enhances the detection power compared to CCF. All the CCF and higher order CCF based nonlinear correlation tests (Billings and Voon 1986, Billings and Zhu 1994, 1995, Aguirre 1995, Zhu and Billings 1997, Mao and Billings 2000) have a similar problem to some extent. To overcome this problem, a new solution is proposed in the following section.

3. Omni-directional cross-correlation functions (ODCCFs)

The nonlinear associations are not as simple as the linear associations and they cannot be simply described as that one variable increases/decreases as another variable increases/decreases. To comprehensively and systematically investigate the nonlinear relationships, the nonlinear associations are classified into four types in this study. First of all two symmetrical properties are proposed here to lay a basis for the classifications.

Consider the general MIMO nonlinear model given by (1), the two symmetries for representing relationship between independent variable $x_i(n-\tau)$ and dependent variable $y_j(n)$ are defined as follows.

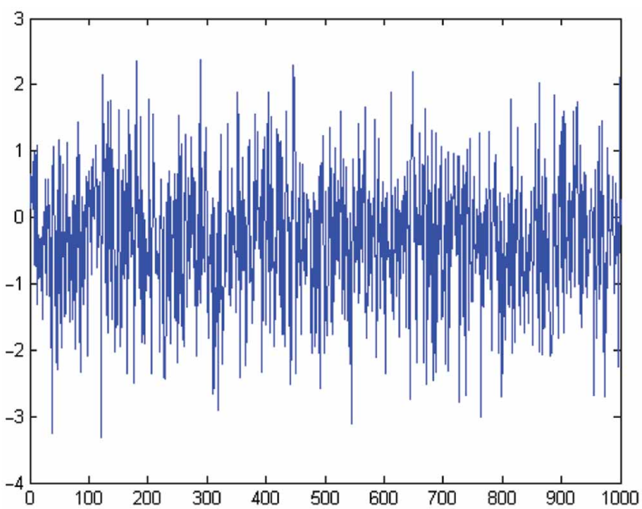

Figure 1. The dependent variable $y(n)$ data sequence of model 1.

Definition 1 (Symmetry along dependent variable axis): A constant time k exists that for any time m, there is a time p so that

$$f_j(\mathbf{y}^{n-m}, \mathbf{x}_1^{n-m}, \ldots, \mathbf{x}_i^{n-m}, \ldots, \mathbf{x}_q^{n-m}, \mathbf{e}^{n-m}) = f_j(\mathbf{y}^{n-p}, \mathbf{x}_1^{n-p}, \ldots, (\mathbf{x}_i^{n-p})^*, \ldots, \mathbf{x}_q^{n-p}, \mathbf{e}^{n-p}), \quad (13)$$

where $(\mathbf{x}_i^{n-p})^* = [x_i(n-p), \ldots, x_i(n-p-\tau+1), 2x_i(k) - x_i(n-m-\tau), x_i(n-p-\tau-1), \ldots, x_i(n-p-r)]$.

Definition 2 (Symmetry along independent variable axis): A constant h exists that for any time m, there is a time p so that

$$f_j(\mathbf{y}^{n-m}, \mathbf{x}_1^{n-m}, \ldots, \mathbf{x}_i^{n-m}, \ldots, \mathbf{x}_q^{n-m}, \mathbf{e}^{n-m}) - f_j(\mathbf{y}^{n-h}, \mathbf{x}_1^{n-h}, \ldots, \mathbf{x}_i^{n-h}, \ldots, \mathbf{x}_q^{n-h}, \mathbf{e}^{n-h}) = f_j(\mathbf{y}^{n-h}, \mathbf{x}_1^{n-h}, \ldots, \mathbf{x}_i^{n-h}, \ldots, \mathbf{x}_q^{n-h}, \mathbf{e}^{n-h}) - f_j(\mathbf{y}^{n-p}, \mathbf{x}_1^{n-p}, \ldots, (\mathbf{x}_i^{n-p})^*, \ldots, \mathbf{x}_q^{n-p}, \mathbf{e}^{n-p}) \quad (14)$$

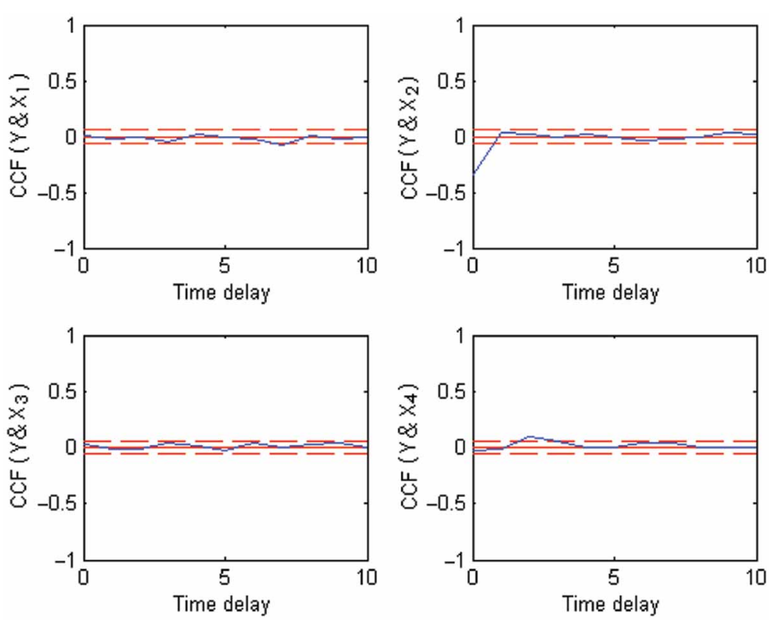

Figure 2. CCF tests for model 1.

where $(\mathbf{x}_i^{n-p})^* = [x_i(n-p), \ldots, x_i(n-p-\tau+1), x_i(n-m-\tau),\ x_i(n-p-\tau-1), \ldots, x_i(n-p-r)]$.

The two symmetrical properties are shown in figures 4 and 5 by using two simple examples with $x(k)=0$ and $y(h)=f(x(h))=0$ respectively.

Four types of nonlinear associations, which include all possible nonlinear effects with both amplitude and sign, are defined as follows.

Type 1: Amplitude of dependent variable varies with the amplitude variation of independent variable; of which both symmetrical properties are satisfied.

Type 2: Both sign and amplitude of dependent variable vary with the amplitude variation of independent variable, of which only the first symmetrical property is satisfied.

Type 3: Both sign and amplitude of dependent variable vary with variations of both sign and amplitude of independent variable, of which both symmetrical properties are not satisfied.

Type 4: The amplitude of the dependent variable varies as both the sign and the amplitude of the independent variable vary; which only satisfies Principle 2.

In addition, linear association is a special case of Type 3. In order to fully and precisely detect the linear and nonlinear correlations, a series of first order CCF, named normalized omni-directional cross-correlation functions (ODCCFs), are proposed for detecting the above four types of nonlinear associations separately. They are formulated as

For type 1

$$r_{\alpha\beta}(\tau) = \frac{\sum_{n=\tau+1}^{N} (\beta(n)-\bar{\beta})(\alpha(n-\tau)-\bar{\alpha})}{\left[\left(\sum_{n=1}^{N} (\beta(n)-\bar{\beta})^2\right)\left(\sum_{n=1}^{N} (\alpha(n)-\bar{\alpha})^2\right)\right]^{1/2}}. \tag{15}$$

For type 2

$$r_{\alpha y_j'}(\tau) = \frac{\sum_{n=\tau+1}^{N} y_j'(n)(\alpha(n-\tau)-\bar{\alpha})}{\left[\left(\sum_{n=1}^{N} (y_j'(n))^2\right)\left(\sum_{n=1}^{N} (\alpha(n)-\alpha)^2\right)\right]^{1/2}}. \tag{16}$$

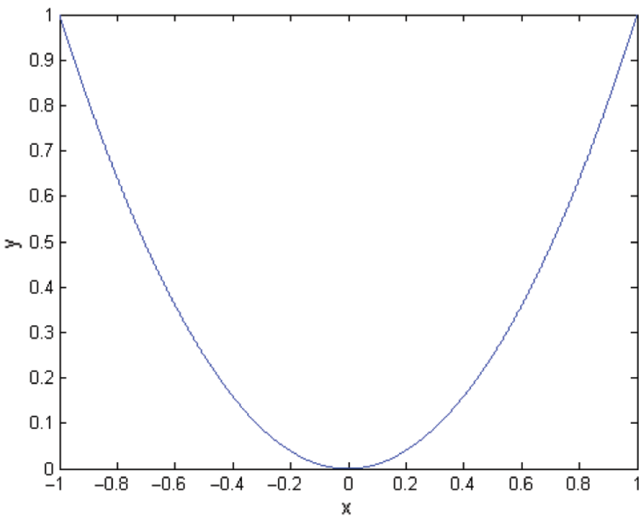

Figure 4. The shape of scatter plot symmetry along dependent variable axis.

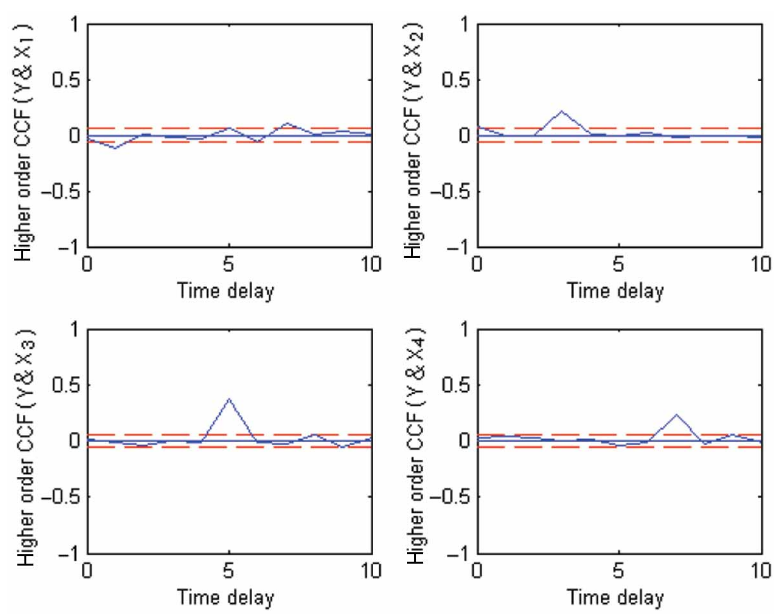

Figure 3. Higher order CCF tests for model 1.

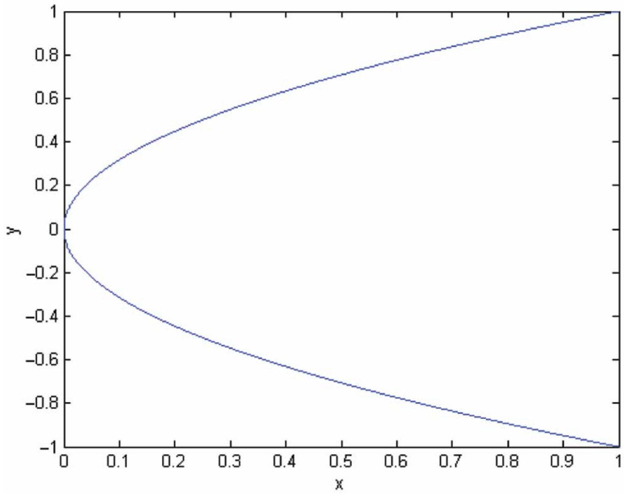

Figure 5. The shape of scatter plot symmetry along independent variable axis.

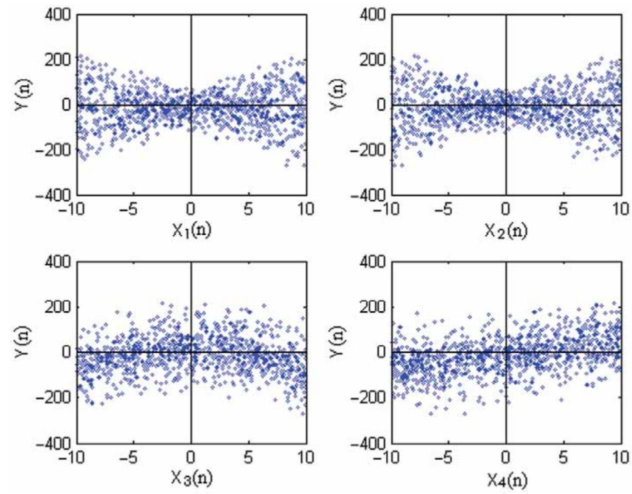

Figure 6. The scatter plots of $y(n)-x_1(n)$, $y(n)-x_2(n)$, $y(n)-x_3(n)$ and $y(n)-x_4(n)$ of model 2.

For type 3

$$r_{x_i'y_j'}(\tau)=\frac{\sum_{n=\tau+1}^{N} y_j'(n)x_i'(n-\tau)}{\left[\left(\sum_{n=1}^{N}(y_j'(n))^2\right)\left(\sum_{n=1}^{N}(x_i'(n))^2\right)\right]^{1/2}}. \qquad (17)$$

For type 4

$$r_{x_i'\beta}(\tau)=\frac{\sum_{n=\tau+1}^{N}(\beta(n)-\bar{\beta})x_i'(n-\tau)}{\left[\left(\sum_{n=1}^{N}(\beta(n)-\bar{\beta})^2\right)\left(\sum_{n=1}^{N}(x_i'(n))^2\right)\right]^{1/2}}, \qquad (18)$$

where the overbar denotes the time average operation and

$$\left.\begin{aligned}\alpha(n)&=|x_i'(n)|\\ \beta(n)&=|y_j'(n)|\end{aligned}\right\}. \qquad (19)$$

These ODCCFs have considered both absolute values and signs of the analysed variables. For whichever nonlinear association exists, there should be one or more functions that are able to detect it properly. In addition, ODCCFs avoid a weakness of other approaches in that all the higher order CCF based nonlinear correlation tests can sometimes exhibit less detection power when the variables variances are small because fourth and higher moments become small (Billings and Zhu 1994). For a special case $y_j(n)=x_i(n)$, the functions (15) to (18) are called normalized omni-directional auto-correlation functions (ODACFs).

The capability of each function to detect four types of nonlinear associations individually can be affirmed in terms of functional analysis and numerical simulations. To make the main context of the study concise, the analytical proof is moved to an appendix. Here some visual appreciation of the individual detection capability of each ODCCF is initially illustrated by testing two simple nonlinear models. The independent variable data sequences used in these models are random and uniformly distributed with zero mean and amplitude from −10 to 10. Each data sequence is with length of 1000. Consider the first nonlinear model (model 2) given as

$$\begin{aligned}y(n)&=f(x_1(n),x_2(n),x_3(n))\\&=2x_1(n)x_2(n)-x_3^2(n)+x_4(n)+20.\end{aligned} \qquad (20)$$

Figure 6 shows the scatterplots for the variables $y(n)-x_1(n)$ ($y(n)$ versus $x_1(n)$), $y(n)-x_2(n)$ ($y(n)$ versus $x_2(n)$), $y(n)-x_3(n)$ ($y(n)$ versus $x_3(n)$) and $y(n)-x_4(n)$ ($y(n)$ versus $x_4(n)$)) and figure 7 shows the results obtained from using ODCFs respectively.

As illustrated in figure 6, the shapes of the scatter plots $y(n)-x_1(n)$ and $y(n)-x_2(n)$ are symmetric with respect to both x (independent variable)-axis and the y (dependent variable)-axis. This indicates that for any time m, there is a time p that exists so that

$$\left.\begin{aligned}&f(x_1(m),x_2(m),x_3(m))\\&\quad=f(2x_1(k_1)-x_1(m),x_2(q),x_3(q))\\&f(x_1(m),x_2(m),x_3(m))\\&\quad=f(x_1(q),2x_2(k_2)-x_2(m),x_3(q)),\end{aligned}\right\} \qquad (21)$$

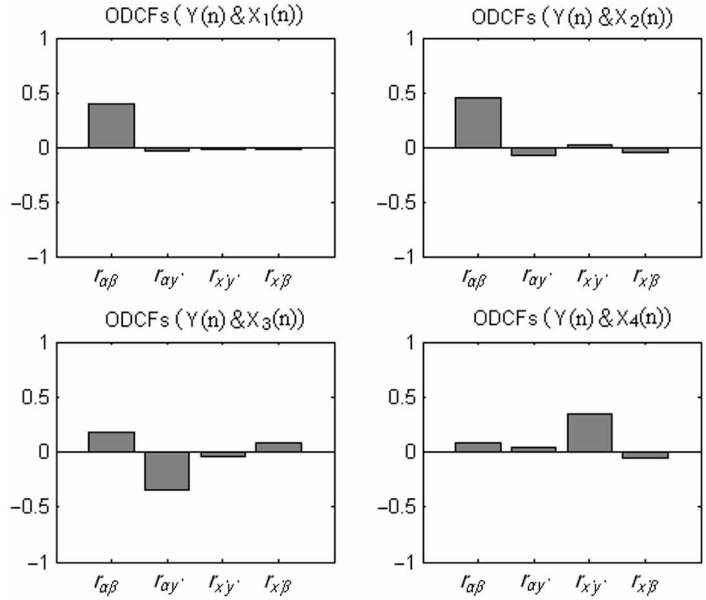

Figure 7. ODCFs tests for model 2.

where $x_1(k_1)=0$ and $x_2(k_2)=0$, and for any time m, there is a time p that exists that

$$\begin{cases} f(x_1(m), x_2(m), x_3(m)) - f(x_1(h_1), x_2(h_1), x_3(h_1)) \\ \quad = f(x_1(h_1), x_2(h_1), x_3(h_1)) - f(x_1(m), x_2(p), x_3(p)) \\ f(x_1(m), x_2(m), x_3(m)) - f(x_1(h_2), x_2(h_2), x_3(h_2)) \\ \quad = f(x_1(h_2), x_2(h_2), x_3(h_2)) - f(x_1(m), x_2(p), x_3(p)), \end{cases} \tag{22}$$

where $f(x_1(h_1), x_2(h_1), x_3(h_1))=0$ and $f(x_1(h_2), x_2(h_2), x_3(h_2)) = 0$. The dependence, therefore, only exists between the amplitude of $y(n)$ and the amplitudes of $x_1(n)$ and $x_2(n)$, which accords with the Type 1 of nonlinear association. In addition, the shape of the scatterplot $y(n)-x_3(n)$ is symmetric with respect to the y-axis that For any time m, there is a time p that exists so that

$$\begin{aligned} &f(x_1(m), x_2(m), x_3(m)) \\ &\quad = f(x_1(q), x_2(q), 2x_3(k_3) - x_3(m)), \end{aligned} \tag{23}$$

where $x_3(k_3)=0$. It denotes that the variance of y depends on the variance of the amplitude of x_3, which means that the relationship between $y(n)$ and $x_3(n)$ accords with the Type 2 of nonlinear association. According to figure 6, linear association exists between $y(n)$ and $x_4(n)$, which is a special condition of the Type 3 of nonlinear association.

Figure 7 shows that $r_{\alpha\beta}(\tau)$, $r_{\alpha y'}(\tau)$ and $r_{x'y'}(\tau)$ can be used to properly detect these nonlinear and linear correlations respectively.

Consider another nonlinear model (model 3)

$$y = f(x_1, x_2) = x_2(x_1 + 10). \tag{24}$$

Figures 8 and 9 show the scatterplots for the variables ($y(n)-x_1(n)$, $y(n)-x_2(n)$) and the results obtained from using CDCFs respectively.

Figure 8 shows that the shape of the scatterplot $y(n)-x_1(n)$ is symmetric with respect to the x-axis which accords with the Type 4 of nonlinear association, where for any time m, there is a time p that exists so that

$$\begin{aligned} &f(x_1(m), x_2(m), x_3(m)) - f(x_1(h_1), x_2(h_1), x_3(h_1)) \\ &\quad = f(x_1(h_1), x_2(h_1), x_3(h_1)) - f(x_1(m), x_2(p), x_3(p)) \end{aligned} \tag{25}$$

where $f(x_1(h_1), x_2(h_1), x_3(h_1))=0$. Furthermore, the shape of the scatterplot $y(n)-x_2(n)$ is not symmetric with respect to both the x-axis and the y-axis, which accords with the Type 3 of nonlinear association.

As shown in figure 9, $r_{x'\beta}(\tau)$ and $r_{x'y'}(\tau)$ can be used to properly detect these nonlinear correlations.

4. Combined ODCCFs

The magnitudes of each ODCCF are comparable since they are all computed using the first order CCF. This means that a higher value of a particular ODCCF denotes a more significant corresponding nonlinear association. Accordingly the results obtained from

ODCCFs can be effectively combined to constitute a much condensed combination.

Definition 3 Combined ODCCFs $\rho_{xy}(\tau)$**:** If, $|\max(r_{\alpha\beta}(\tau), r_{\alpha y'_j}(\tau), r_{x'_i y'_j}(\tau), r_{x'_i\beta}(\tau))| > |\min(r_{\alpha\beta}(\tau), r_{\alpha y'_j}(\tau), r_{x'_i y'_j}(\tau), r_{x'_i\beta}(\tau))|$. Then,

$$\rho_{xy}(\tau) = \max(r_{\alpha\beta}(\tau), r_{\alpha y'_j}(\tau), r_{x'_i y'_j}(\tau), r_{x'_i\beta}(\tau)). \tag{26}$$

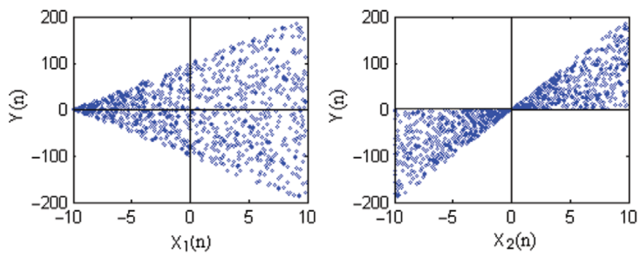

Figure 8. The scatterplots of $y(n) - x_1(n)$ and $y(n) - x_2(n)$ of model 3.

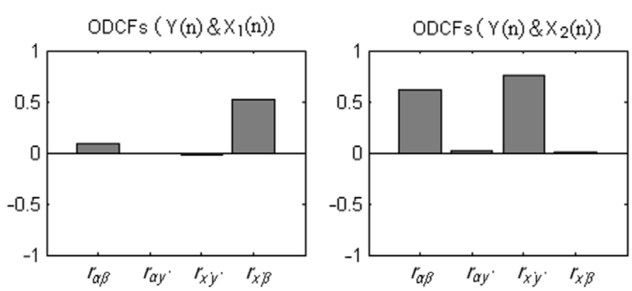

Figure 9. ODCFs tests for model 3.

If, $|\max(r_{\alpha\beta}(\tau), r_{\alpha y'_j}(\tau), r_{x'_i y'_j}(\tau), r_{x'_i\beta}(\tau))| \leq |\min(r_{\alpha\beta}(\tau), r_{\alpha y'_j}(\tau), r_{x'_i y'_j}(\tau), r_{x'_i\beta}(\tau))|$. Then,

$$\rho_{xy}(\tau) = \min(r_{\alpha\beta}(\tau), r_{\alpha y'_j}(\tau), r_{x'_i y'_j}(\tau), r_{x'_i\beta}(\tau)). \tag{27}$$

Inspection of the combined formulations shows a single quantity has been combined from four formulations for each correlation test. This will provide a better illustration for detected correlations and reduce the number of correlation plots.

To illustrate the combined ODCCFs, use the same data to test model (12).

As shown in figure 10, $\rho_{x_1y}(\tau)$, $\rho_{x_2y}(\tau)$, $\rho_{x_3y}(\tau)$ and $\rho_{x_4y}(\tau)$ are significantly outside confidence intervals at $\tau = 1$, 0 and 3, 5 and 7 respectively, and furthermore, both $x_2(n)$ and $x_3(n-5)$ are revealed as significant negative association with $y(n)$. All the relevant independent variables, hence, are clearly detected with proper correlation values and signs. The comparison of figure 10 and figure 3 (obtained by higher order CCFs) shows much better illustrative results achieved from the combined ODCCFs.

The capability of the combined ODCCFs to detect the nonlinear associations for polynomial models (such as NARMAX models) can be demonstrated through the simulation studies. However any infinitely differentiable function can be expanded in polynomial form (Taylor series expansion) around some nominal points (Farlow *et al.* 2002). In theory, therefore ODCCFs and the combination can be used to test the nonlinear correlations between variables for all analytic nonlinear systems whatever form of model is used to characterize the mapping of the dependent/independent variables.

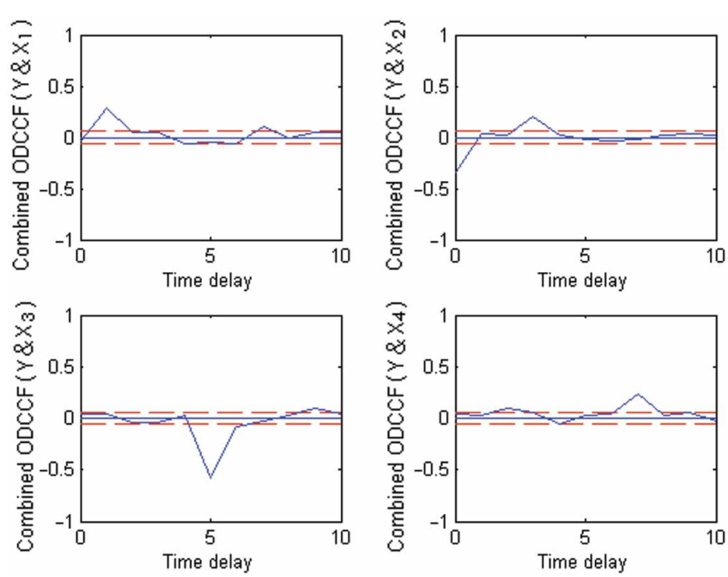

Figure 10. Combined ODCCFs tests for model 1.

Here is an example. Consider a complex simulated nonlinear discrete dynamic model (model 4), which is a fractional function and involves sinusoidal term, quadratic term, cross product term, and exponential term. It is expressed as follows:

$$y(n) = \frac{\sin(2x_2(n-3)x_3(n-5) + x_1^2(n-1))}{\exp(x_4(n-7))} + e(n), \tag{28}$$

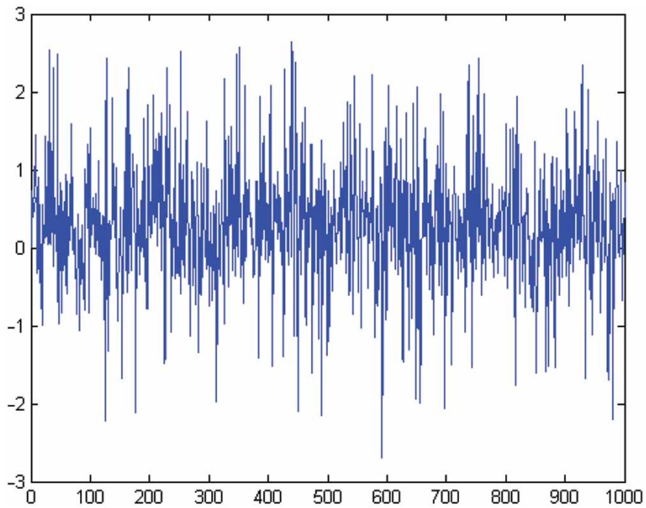

Figure 11. The dependent variable $y(n)$ data sequence of model 4.

where variables $x_1(n)$, $x_2(n)$, $x_3(n)$ and $x_4(n)$ and noise $e(n)$ were selected as the same as the data sequences employed in Model 1. Figure 11 shows the measured dependent variable data sequence.

Figure 12 shows that the results obtained using the combined ODCCFs, in which all nonlinear associations have been properly detected.

5. Conclusions

A set of novel correlation tests, which is particularly efficient in dealing with nonlinear signals and models, has been developed in this study. The interpretation and performance of the methodology have been proved in theory and demonstrated through numerical simulations. The integrated test procedure provides a more comprehensive and effective detection of nonlinear associations for a wider class of nonlinear models than other nonlinear correlation tests. For applications the test procedure can be widely used to provide solutions for model regression term selection, model validation, and other relevant issues in nonlinear modelling and signal processing.

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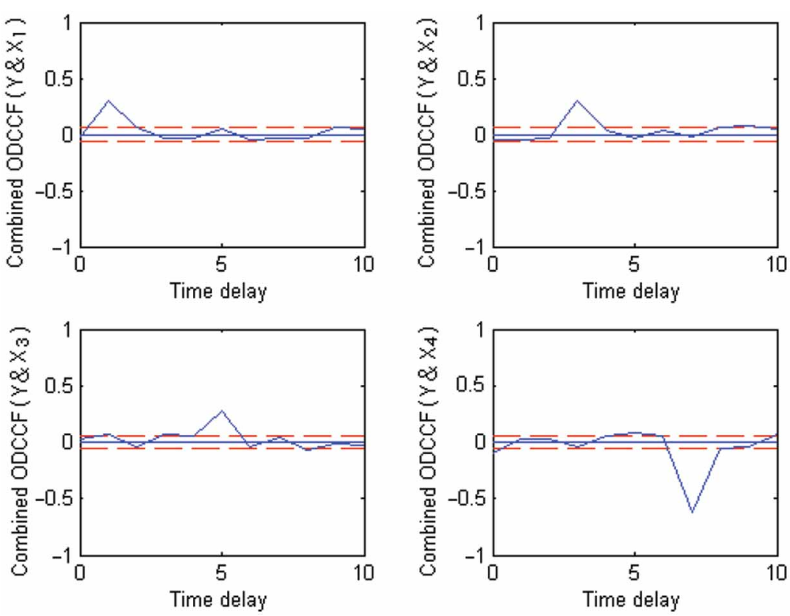

Figure 12. Combined ODCCFs tests for model 4.

Appendix

The capability of each function to detect four types of nonlinear associations individually can be proved as follows. Consider a single input and single output (SISO) stationary and ergodic model and independent variable $x(n)$ is uniformed distributed random sequence and the number of data samples N is large enough to satisfy the relevant statistical properties.

$$y(n) = f(x(n)). \tag{A1}$$

Consider a general correlation formula,

$$r_{xy}(\tau) = E[y\ (n)x\ (n)] = \frac{1}{N}\sum_{n=1}^{N} y\ (n)x\ (n), \tag{A2}$$

where $E[\cdot]$ is the expectation operator and

$$\left.\begin{aligned} y(n) &= \frac{y(n) - E[y(n)]}{[E[[y(n) - E[y(n)]]^2]]^{1/2}} \\ x(n) &= \frac{x(n) - E[x(n)]}{[E[[x(n) - E[x(n)]]^2]]^{1/2}}. \end{aligned}\right\} \tag{A3}$$

ODCCF (15) can detect Type 1 of nonlinear associations.

Proof: Consider Type 1 of nonlinear associations exists between $x(n)$ and $y(n)$. From (A2) it has

$$\begin{aligned} r_{xy}(\tau) &= \frac{1}{N}\sum_{n=1}^{N}\Big(y(1)x(1), \ldots, y(N)x(N)\Big) \\ &= \frac{1}{N}\left\{\sum_{n=n_{a1}}^{N_a}\Big(y(n_{a1})x(n_{a1}), \ldots, y(n_{aN})x(n_{aN})\Big)\right. \\ &\quad + \sum_{n=n_{b1}}^{N_b}\Big(y(n_{b1})x(n_{b1}), \ldots, y(n_{bN})x(n_{bN})\Big) \\ &\quad + \sum_{n=n_{c1}}^{N_c}\Big(y(n_{c1})x(n_{c1}), \ldots, y(n_{cN})x(n_{cN})\Big) \\ &\quad + \sum_{n=n_{d1}}^{N_d}\Big(y(n_{d1})x(n_{d1}), \ldots, y(n_{dN})x(n_{dN})\Big) \\ &\quad + \sum_{n=n_{e1}}^{N_e}(y(n_{e1})x(n_{e1}), \ldots, y(n_{eN})x(n_{eN})) \\ &\quad \left. + \sum_{n=n_{f1}}^{N_f}\Big(y(n_{f1})x(n_{f1}), \ldots, y(n_{fN})x(n_{fN})\Big)\right\} \\ &= \frac{1}{N}\left\{\sum_{n=n_{a1}}^{N_a}\Big(y(n_{a1})x(n_{a1}), \ldots, y(n_{aN})x(n_{aN})\Big)\right. \\ &\quad + \sum_{n=n_{b1}}^{N_b}(y(n_{b1})x(n_{b1}), \ldots, y(n_{bN})x(n_{bN})) \\ &\quad + \sum_{n=n_{c1}}^{N_c}(y(n_{c1})x(n_{c1}), \ldots, y(n_{cN})x(n_{cN})) \\ &\quad \left. + \sum_{n=n_{d1}}^{N_d}(y(n_{d1})x(n_{d1}), \ldots, y(n_{dN})x(n_{dN}))\right\}, \end{aligned} \tag{A4}$$

where

$$\left.\begin{aligned} \{n\} &= \{1, \ldots, N\} = \{n_a\} + \{n_b\} + \{n_c\} + \{n_d\} + \{n_e\} + \{n_f\} \\ N &= N_a + N_b + N_c + N_d + N_e + N_f \end{aligned}\right\} \tag{A5}$$

and

$$\left.\begin{aligned} &x(n_{ai}) < 0 \forall n_{ai} \in \{n_a\} \\ &y(n_{ai}) > 0 \forall n_{ai} \in \{n_a\} \\ &x(n_{bi}) < 0 \forall n_{bi} \in \{n_b\} \\ &y(n_{bi}) > 0 \forall n_{bi} \in \{n_b\} \\ &x(n_{ci}) < 0 \forall n_{ci} \in \{n_c\} \\ &y(n_{ci}) > 0 \forall n_{ci} \in \{n_c\} \\ &x(n_{di}) < 0 \forall n_{di} \in \{n_d\} \\ &y(n_{di}) > 0 \forall n_{di} \in \{n_d\} \\ &x(n_{ei}) = 0 \forall n_{ei} \in \{n_e\} \\ &y(n_{fi}) = 0 \forall n_{fi} \in \{n_f\}. \end{aligned}\right\} \tag{A6}$$

Since N is large, for any $x(n_{ai}) \in \{x(n_a)\}$ there should be $x(n_{bj}) \in \{x(n_b)\}$, $x(n_{cm}) \in \{x(n_{cm})\}$ and $x(n_{dp}) \in \{x(n_{dp})\}$ exist and satisfy

$$x(n_{ai}) = -x(n_{bj}) = x(n_{cm}) = -x(n_{dp}). \tag{A7}$$

Therefore,

$$\left.\begin{aligned} \{x(n_b)\} &= \{x(n_{b1}), \ldots, x(n_{bN})\} \\ &= \{-x(n_{a1}), \ldots, -x(n_{aN})\} = \{-x(n_a)\} \\ \{x(n_c)\} &= \{x(n_{c1}), \ldots, x(n_{cN})\} \\ &= \{x(n_{a1}), \ldots, x(n_{aN})\} = \{x(n_a)\} \\ \{x(n_d)\} &= \{x(n_{d1}), \ldots, x(n_{dN})\} \\ &= \{-x(n_{a1}), \ldots, -x(n_{aN})\} = \{-x(n_a)\}. \end{aligned}\right\} \tag{A8}$$

In addition, for any $y(n_{ai}) \in \{y(n_a)\}$ there should be $y(n_{bj}) \in \{y(n_b)\}$, $y(n_{cm}) \in \{y(n_{cm})\}$ and $y(n_{dp}) \in \{y(n_{dp})\}$ exist and satisfy

$$y(n_{ai}) = -y(n_{bj}) = -y(n_{cm}) = y(n_{dp}) \tag{A9}$$

therefore

$$\left.\begin{array}{l}\{y(n_b)\} = \{-y(n_a)\}\\ \{y(n_c)\} = \{-y(n_a)\}\\ \{y(n_d)\} = \{y(n_a)\}.\end{array}\right\} \tag{A10}$$

Since $y(n)=f(x(n))$ satisfies (13), that

$$f(x(n_i)) = f(2x(k) - x(n_i)) = f(-x(n_i)), \tag{A11}$$

where $x(k)$ is the mean value of $x(n)$ which is zero, so that if

$$x(n_{ai}) = -x(n_{bj}) = x(n_{cm}) = -x(n_{dp}) \tag{A12}$$

then

$$f(x(n_{ai})) = f(x(n_{bj})) = f(x(n_{cm})) = f(x(n_{dp})). \tag{A13}$$

In addition, $y(n)=f(x(n))$ satisfies (14) so that

$$f(x(n_i)) - f(x(h)) = f(x(h)) - f(x(n_i)) \tag{A14}$$

which means

$$f^{-1}(y(n_i)) = f^{-1}(-y(n_i)), \tag{A15}$$

where $n_i \in \{n\}$ and $f(x(h))$ is the mean value of $y(n)$ which is zero. According to (A15), if

$$y(n_{ai}) = -y(n_{bj}) = -y(n_{cm}) = y(n_{dp}) \tag{A16}$$

then

$$x(n_{ai}) = x(n_{cj}) = x(n_{bm}) = x(n_{dp}) \tag{A17}$$

using $\alpha(n)$ and $\beta(n)$ as the substitutes for $x(n)$ and $y(n)$ as (19)

$$\left\{\begin{array}{l}\alpha(n)\\ \beta(n)\end{array}\right. = \left\{\begin{array}{l}|x(n)|\\ |y(n)|.\end{array}\right. \tag{A18}$$

It is shown in the following derivation that (15) can be used to detect Type 1 nonlinear associations

$$\begin{aligned}r_{\alpha\beta}(\tau) &= \frac{1}{N}\sum_{n=1}^{N}|y(n)||x(n)|\\ &= \frac{1}{N}\left\{\sum_{n_a=n_{a1}}^{N_a}(y(n_a))(x(n_a)) + \sum_{n_b=n_{b1}}^{N_b}(-y(n_b))(-x(n_b))\right.\\ &\quad\left. + \sum_{n_c=n_{c1}}^{N_c}(-y(n_c))(x(n_c)) + \sum_{n_a=n_{d1}}^{N_d}(y(n_d))(-x(n_d))\right\}\\ &= \frac{1}{N}\left\{\sum_{n_a=n_{a1}}^{N_a}y(n_a)x(n_a) + \sum_{n_a=n_{a1}}^{N_a}(y(n_a))(x(n_a))\right.\\ &\quad\left. + \sum_{n_a=n_{a1}}^{N_a}(y(n_a))x(n_a) + \sum_{n_a=n_{a1}}^{N_a}y(n_a)(x(n_a))\right\}\\ &= \frac{4}{N}\sum_{n_a=n_{a1}}^{N_a}(y(n_a))(x(n_a)).\end{aligned} \tag{A19}$$

When Type 1 of nonlinear associations exists between the two variables, therefore, $r_{\alpha\beta}(\tau)$ should be a nonzero numeral. Otherwise it is a zero for independent variables.

ODCCF (16) can detect Type 2 of nonlinear associations.

Proof: Consider Type 2 of nonlinear association exists between $x(n)$ and $y(n)$. The general correlation function (A2) is expanded as

$$\begin{aligned}r_{xy}(\tau) &== \frac{1}{N}\sum_{n=1}^{N}(y(1)x(1),\ldots,y(N)x(N))\\ &= \frac{1}{N}\left\{\sum_{n=n_{b1}}^{N_b}(y(n_{a1})x(n_{a1}),\ldots,y(n_{aN})x(n_{aN}))\right.\\ &\quad + \sum_{n=n_{b1}}^{N_b}(y(n_{b1})x(n_{b1}),\ldots,y(n_{bN})x(n_{bN}))\\ &\quad\left. + \sum_{n=n_{c1}}^{N_c}(y(n_{c1})x(n_{c1}),\ldots,y(n_{cN})x(n_{cN}))\right\}\\ &= \frac{1}{N}\left\{\sum_{n=n_{a1}}^{N_a}(y(n_{a1})x(n_{a1}),\ldots,y(n_{aN})x(n_{aN}))\right.\\ &\quad\left. + \sum_{n=n_{b1}}^{N_b}(y(n_{b1})x(n_{b1}),\ldots,y(n_{bN})x(n_{bN}))\right\},\end{aligned} \tag{A20}$$

where

$$\left.\begin{aligned}&\{n\} = \{1, \ldots, N\} = \{n_a\} + \{n_b\} + \{n_c\}\\&N = N_a + N_b + N_c\end{aligned}\right\} \tag{A21}$$

and

$$\left.\begin{aligned}&x(n_{ai}) > 0 \forall n_{ai} \in \{n_a\}\\&x(n_{bi}) < 0 \forall n_{bi} \in \{n_b\}\\&x(n_{ci}) = 0 \forall n_{ci} \in \{n_c\}.\end{aligned}\right\} \tag{A22}$$

Since N is large, for any $x(n_{ai}) \in \{x(n_a)\}$ there should be a $x(n_{bj}) \in \{x(n_b)\}$ satisfies $-x(n_{ai}) = \mathrm{x}(n_{bj})$ so that

$$\begin{aligned}\{x(n_b)\} &= \{x(n_{b1}), \ldots, x(n_{bN})\}\\&= \{-x(n_{a1}), \ldots, -x(n_{aN})\} = \{-x(n_a)\}.\end{aligned} \tag{A23}$$

Since the dependence between $x(n)$ and $y(n)$ accords to Type 2 of nonlinear associations,

$$\begin{aligned}f(x(n_{ai})) &= f(2x(k) - x(n_{ai})) = f(-x(n_{ai}))\\&= f(x(n_{bj})),\end{aligned} \tag{A24}$$

where $x(k)$ is the zero mean of $x(n)$ using $\alpha(n)$ as a substitute for $x(n)$ as (19).

$$\alpha(n) = |x(n)|. \tag{A25}$$

Therefore, using (16) to detect the dependence can be derived as follows:

$$\begin{aligned}r_{\alpha y'}(\tau) &= \frac{1}{N}\sum_{n=1}^{N} y(n)|x(n)|\\&= \frac{1}{N}\left\{\sum_{n=n_{a1}}^{N_a} (y(n_{a1})(x(n_{a1})), \ldots, y(n_{aN})(x(n_{aN})))\right.\\&\quad\left. + \sum_{n=n_{b1}}^{N_b} (y(n_{b1})(-x(n_{b1})), \ldots, y(n_{bN})(-x(n_{bN})))\right\}\\&= \frac{1}{N}\left\{2\sum_{n=n_{b1}}^{N_b} (y(n_{a1})(x(n_{a1})), \ldots, y(n_{aN})(x(n_{aN})))\right\}\\&= \frac{2}{N}\sum_{n=n_{a1}}^{N_a} y(n_a)(x(n_a)).\end{aligned} \tag{A26}$$

$r_{\alpha\beta}(\tau)$ should be a non-zero numeral when Type 2 of nonlinear association exists between the two variables.

ODCCF (17) corresponds to Type 3 of nonlinear associations.

Since Type 3 of associations including linear association, (17) can be properly used to detect the dependence and it has been proved in many literatures (Bendat and Piersol 1993).

ODCCF (18) can detect Type 4 of nonlinear associations.

The proof of (18) to detect Type 4 of nonlinear associations can be proved similarly as the proof of (15) to detect Type 1 of nonlinear associations.

Finally, the magnitude boundaries of the ODCCFs are limited within a unit range, that is

$$\left.\begin{aligned}&-1 \le r_{\alpha\beta}(\tau) \le 1\\&-1 \le r_{x'\beta}(\tau) \le 1\\&-1 \le r_{x'y'}(\tau) \le 1\\&-1 \le r_{\alpha y'}(\tau) \le 1.\end{aligned}\right\} \tag{A27}$$

Proof: Assume any two data sequences z_1, z_2, and a real constant a. Their weighted covariance is expressed as

$$\begin{aligned}&E[(a(z_1(n) - E[z_1(n)]))\\&\quad + (z_2(n) - E[z_2(n)]))^2] \ge 0.\end{aligned} \tag{A28}$$

Expand the squared mean function to yield

$$\left.\begin{aligned}&a^2E\Big[(z_1(n) - \overline{z_1})^2 + 2a(z_1(n) - \overline{z_1})(z_2(n) - \overline{z_2})\\&\quad + (z_2(n) - \overline{z_2})^2\Big] \ge 0\\&a^2E\Big[(z_1(n) - \overline{z_1})^2\Big] + 2aE[(z_1(n) - \overline{z_1})(z_2(n) - \overline{z_2})]\\&\quad + E\Big[(z_2(n) - \overline{z_2})^2\Big] \ge 0.\end{aligned}\right\} \tag{A29}$$

Solving the above equation for the unknown constant a, it will produce two complex roots since the equation is non-negative. Then the discriminant is non-positive and

$$\begin{aligned}&4\big\{E[(z_1(n) - \overline{z_1})(z_2(n) - \overline{z_2})]\big\}^2\\&\quad - 4\big\{E\big[(z_1(n) - \overline{z_1})^2\big]E\big[(z_2(n) - \overline{z_2})^2\big]\big\} \le 0.\end{aligned} \tag{A30}$$

It follows that

$$\left.\begin{aligned}&\{E[(z_1(n)-\overline{z_1})(z_2(n)-\overline{z_2})]\}^2\\&\le\{E[(z_1(n)-\overline{z_1})^2]E[(z_2(n)-\overline{z_2})^2]\}\\&\left\{\frac{1}{N}\sum_1^N(z_1(n)-\overline{z_1})(z_2(n)-\overline{z_2})\right\}^2\\&\le\frac{1}{N^2}\sum_1^N(z_1(n)-\overline{z_1})^2\sum_1^N(z_2(n)-\overline{z_2})^2\\&\frac{\left\{\sum_1^N(z_1(n)-\overline{z_1})(z_2(n)-\overline{z_2})\right\}^2}{\sum_1^N(z_1(n)-\overline{z_1})^2\sum_1^N(z_2(n)-\overline{z_2})^2}\\&\le 1.\end{aligned}\right\} \quad \text{(A31)}$$

In general $z_1(n)$ can be selected as $y'(n)$ or $\beta(n)$ and $z_2(n)$ can be selected as $x'(n-\tau)$ or $\alpha(n-\tau)$.

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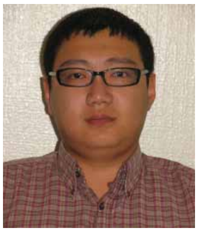

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