Keywords: Feature extraction, Support vector machines, Face recognition, Principal component analysis, Independent components analysis, Linear discriminant analysis.

Abstract: In this paper, we present an approach that unifies sub-space feature extraction and support vector classification for face recognition. Linear discriminant, independent component and principal component analyses are used for dimensionality reduction prior to introducing feature vectors to a support vector machine. The performance of the developed methods in reducing classification error and providing better generalization for high dimensional face recognition application is demonstrated.

1 INTRODUCTION

Choosing an appropriate set of features is critical when designing pattern classification systems under the frame-work of supervised learning. Ideally, we would like to use only features having high separability power while ignoring or paying less attention to the rest. Recently, there has been an increased interest in deploying feature selection in applications such as face and gesture recognition (Sun et al., 2004). Most efforts in the literature have been focused mainly on developing feature extraction methods (Jain et al., 2000, Belhumeur, et al., 1997) and employing powerful classifiers such as probabilistic (Moghaddam, 2002), hidden Markov models (HMMs) (Othman and Aboulnasr, 2003), neural networks (NNs) (Er et al., 2002) and support vector machine (SVM) (Lee et al., 2002).

The main trend in feature extraction has been representing the data in a lower dimensional space computed through a linear or non-linear transformation satisfying certain properties. Principal component analysis (PCA) (Turk and Pentland, 1991) selects features which are maximally variant across the data. With independent components analysis (ICA) (Liu and Wechsler, 2003) statistically independent features result. Linear discriminant analysis (LDA) (Yu and Yang, 2001) encodes the discriminatory information in a linear separable space by maximizing the ratio of between-class to within-class variances.

SVM have shown to be very effective classifiers for face recognition applications and provide the ability to generalize over imaging variants (Heisele et al., 2001). SVM provide an optimal decision hyperplane by employing kernel learning, projecting the data into a high-dimensional space (Vapnik, 1995). Some authors used PCA and ICA for dimensionality reduction before using SVM for face recognition (Wang et al., 2002, Qi, et al., 2001). Without using effective schemes to select an appropriate subset of features in the computed subspaces, these methods rely mostly on classification algorithms to deal with the issues of redundant and irrelevant features. This might be problematic, especially when the number of training examples is small compared to the number of features. Fortuna and Capson (Fortuna and Capson, 2004) proposed an iterative component algorithm for feature selection by combining PCA and ICA methods and SVM classifier.

In this paper, we present an approach that uses SVM to classify PCA, ICA and LDA extracted features and a hybrid iterative method for improving the generalization of the classifier. Application of the developed algorithm to a facial image database
demonstrates the improvement in correctness, margin and number of support vectors of the classifier. The rest of the paper is organized as follows: Section 2 provides a brief review of feature extraction algorithms including PCA, ICA and LDA. In section 3, we present the classification algorithm using SVM and the iterative method for improving the generalization of the classifier. Section 4 is devoted to experimental results and discussion. Finally, concluding remarks and plans for future works are given in section 5.

2 FEATURE EXTRACTION

Given a set of centred input vectors \( \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n \) of \( n \) variables, a data matrix \( \mathbf{X} \) is defined with each vector forming a column of \( \mathbf{X} \). The goal of feature extraction algorithms is to construct a decomposition of the data such that a set of basis vectors for the data which are maximally decorrelated can be found. In other words, we look for a matrix \( \mathbf{A} \) such that:

\[
\mathbf{S} = \mathbf{A}^T \mathbf{X}
\]

where the columns of \( \mathbf{S} \) are decorrelated. For pattern recognition, the decorrelated space \( \mathbf{S} \) is used for dimensionality reduction.

2.1 Principal Component Analysis

Finding the principal components from \( N \) observations of \( \mathbf{X} \) creates an \( n \times n \) covariance matrix \( \Sigma = \mathbf{X}^T \mathbf{X} \). When \( N > n \), this is a convenient form of the covariance matrix to use. An \( N \times N \) covariance matrix results from \( \mathbf{X} \mathbf{X}^T \) and is useful when \( n < N \). This is typically the case when an image forms an observation and \( n \) is very large. If the SVD is used to decompose \( \mathbf{X} \) as, \( \mathbf{X} = \mathbf{U} \Sigma^{1/2} \mathbf{V}^T \) the \( n \times n \) covariance matrix is found by:

\[
\Sigma = \mathbf{U} \Sigma^{1/2} \mathbf{V}^T \mathbf{V} \Sigma^{1/2} \mathbf{U}^T = \mathbf{U} \Sigma \mathbf{U}^T
\]

This can be recognized as an eigen-decomposition on \( \mathbf{XX}^T \) where \( \mathbf{U} \) is an \( n \times n \) matrix whose columns are the eigenvectors of \( \mathbf{XX}^T \), \( \mathbf{V} \) is an \( N \times N \) matrix whose columns are the eigenvectors of \( \mathbf{X}^T \mathbf{X} \) and \( \Sigma \) is an \( n \times N \) matrix whose first \( r \) diagonal elements correspond to non-zero eigenvalues of the covariance matrix in descending order. Thus the \( r \) dimensional subspace is formed by selecting the first \( r \) rows of the transformed data matrix \( \mathbf{X}_{id} \):

\[
\mathbf{X}_{id} = \mathbf{U}^T \mathbf{X}
\]

The \( N \times N \) covariance matrix \( \mathbf{XX}^T \) gives:

\[
\mathbf{XX}^T = \mathbf{V} \mathbf{\Sigma}^{1/2} \mathbf{U}^T \mathbf{U} \mathbf{\Sigma}^{1/2} \mathbf{V}^T = \mathbf{V} \mathbf{\Sigma} \mathbf{V}^T
\]

and the following relation may be used for dimensionality reduction when \( n > N \) (Fortuna and Capson, 2004):

\[
\mathbf{X}_{id} = \mathbf{XV}
\]

2.2 Independent Component Analysis

ICA is originally developed for blind source separation whose goal is to recover mutually independent but unknown source signals from their linear mixture without knowing the mixing coefficients. ICA decorrelates \( \mathbf{X} \) by finding a matrix \( \mathbf{A} \) such that \( \mathbf{S} \) is not just decorrelated but statistically independent. The degree of independence is measured by the mutual information between the components of the random variable \( \mathbf{S} \):

\[
I(s) = \int p(s) \log \left( \prod \frac{p(s_i)}{p_i(s_i)} \right) ds
\]

where \( p(s) \) is the joint probability of \( \mathbf{S} \) and \( p_i(s_i) \) are the marginal densities. If a nonlinear mapping \( \mathbf{y} = \mathbf{g}(\mathbf{s}) \) is applied such that \( \mathbf{y} \) has uniform marginal densities, it has been shown that mutual information is obtained by (Bartlett and Sejnowski, 1997):

\[
I(y) = -H(y) = \int p(y) \log p(y) dy
\]

\( I(y) \) can then be minimized with:

\[
\frac{\partial I}{\partial \mathbf{A}} = \frac{\ln |\mathbf{A}|}{\mathbf{A}} + \mathbb{E} \frac{\partial \ln \mathbf{g}(\mathbf{A}^T \mathbf{x})}{\partial \mathbf{A}} \mathbf{A}
\]

where \( \mathbb{E}[\ ] \) denotes expected value. Multiplying by \( \mathbf{A}^T \) leads to the natural gradient algorithm (Shi et al., 2004):

\[
\Delta \mathbf{A} \propto (\mathbf{I} + \mathbb{E}[\mathbf{g}(\mathbf{A}^T \mathbf{x}) \mathbf{x}^T])\mathbf{A}
\]

2.3 Linear Discriminant Analysis

LDA criteria are mainly based on a family of functions of scatter matrices. For example, the maximization of \( tr(\Sigma_r \Sigma^-) \) or \( \|E[L]\| \) is used, where \( \Sigma_r, \Sigma^- \) are within and between-class scatter matrices, respectively. In LDA, the optimum linear transform is composed of \( r(\leq n) \) eigenvectors of \( \Sigma^- \Sigma_r \) corresponding to its \( r \) largest eigenvalues. Alternatively, \( \Sigma_r \Sigma^- \) can be used for LDA, where \( \Sigma_r \) and \( \Sigma^- \) are respectively the within-class and between-class scatter matrices.
represents the mixture scatter matrix \( \Sigma = \Sigma + \Sigma \).

A simple analysis shows that both \( \Sigma \Sigma - 1 \) and \( \Sigma \Sigma - 1 \) has the same eigenvector matrix \( \phi \). In general, \( \Sigma \) is not full rank, hence \( \Sigma \) is used in place of \( \Sigma \). The computation of the eigenvector matrix \( \phi \) from \( \Sigma \Sigma - 1 \) is equivalent to the solution of the generalized eigenvalue problem \( \Sigma \phi = \Sigma \phi \), where \( \Lambda \) is the eigenvalue matrix (Fukunaga, 1990).

3 SUPPORT VECTOR MACHINES

To perform classification with a linear SVM, a labelled set of features \( \{x_i, y_i\} \) is constructed for all \( r \) features in the training data set. The class of feature \( c \) is defined by \( y_i = \{-1, 1\} \). If the data are assumed to be linearly separable, the SVM attempts to find a separating hyperplane with the largest margin. The margin is defined as the shortest distance from the separating hyperplane to the closest data point. If the training data follow:

\[
y(x, w + b) - 1 \geq 0 \quad \forall i
\]

Then the points for which the above equality holds lie on the hyperplanes \( x, w + b = 1 \) and \( x, w + b = -1 \). The margin can be shown to be (Cristianini and Shawe-Taylor, 2000):

\[
\text{Margin} = \frac{2}{\|w\|}
\]

The SVM attempts to find the pair of hyperplanes which give the maximum margin by minimizing \( \|w\| \) subject to constraints on \( w \). Reformulating the problem using the Lagrangian, the expression to optimize for a nonlinear SVM can be written as:

\[
L(\alpha) = \sum_{i=1}^{l} \alpha_i - \frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} \alpha_i \alpha_j y_i y_j K(x_i, x_j)
\]

\( K(x, x') \) is a kernel function satisfying Mercer’s conditions. An example kernel function is the Gaussian radial basis function:

\[
K(x, x') = \exp \left( -\frac{\|x - x'\|^2}{2\sigma^2} \right)
\]

where \( \sigma \) is the standard deviation of the kernel’s exponential function. The decision function of the SVM can be described by:

\[
f(x) = \text{sgn} \left( \sum_{i=1}^{l} y_i \alpha_i K(x_i, x) + b \right)
\]

For data points which lie closest to the optimal hyperplane the corresponding \( \alpha_i \) are non-zero, and these are called support vectors. All other parameters \( \alpha_i \) are zero. As such, any modification of the data points which are not support vectors will have no effect on the solution. This indicates that the support vectors contain all the necessary information to reconstruct the hyperplane.

3.1 General Subspace Classification

An SVM can be used to classify subspace features (including PCA, ICA and LDA extracted features) as described below:

i) The Transformation matrix \( A \) is determined using the training data set \( X^{\text{train}} \).

ii) The training and test data sets in the reduced dimension subspace are determined as follows:

\( S^{\text{train}} = A X^{\text{train}} \), \( S^{\text{test}} = A X^{\text{test}} \)

iii) Define data pairs \((S^{\text{train}}, y_i)\) and apply a support vector classifier to classify \( S^{\text{test}} \).

3.2 Iterative Subspace Classification

In order to improve the generalization of the classifier, an iterative algorithm which moves outlier feature vectors toward their class mean and modifies the basis vectors \( S \) to fit the new features has been proposed (Fortuna and Capson, 2004). We used this algorithm with all three feature extraction methods as follows:

i) find \( A \) from \( X^{\text{train}} \).

ii) initialize:

\( S^{\text{train}} = A X^{\text{train}} \), \( S^{\text{test}} = A X^{\text{test}} \)

iii) initialize the support vector coefficient matrix \( \Gamma \) to the identity matrix.

iv) repeat

v) move the support vectors toward the mean by an amount proportional to the support vector \( \alpha \) by:

\( S^{\text{train}} = S^{\text{train}} - \Gamma (S^{\text{train}} - S^{\text{mean}}) \)

vi) recalculate \( A \) by:

\( A = X^{\text{train}} S^{\text{train}} \)

where + denotes pseudo-inverse.

vii) calculate:

\( S^{\text{train}} = A X^{\text{train}} \), \( S^{\text{test}} = A X^{\text{test}} \)

viii) define data pairs \((S^{\text{train}}, y_i)\) and apply a support vector classifier to classify \( S^{\text{test}} \).
4 EXPERIMENTAL RESULTS

4.1 Gaussian Mixture Data

An example of two classes, each comprising a mixture of three Gaussian random variables, is used to illustrate the relationship between PCA, ICA and LDA extracted features classified by SVM. The mixture of Gaussian data points $X = [x_{c1}, x_{c2}]$ are defined by:

$$X_{c1} = \sum_{n=1}^{3} \frac{1}{\sigma_{n}} \exp \{ k - \mu_n \} \Sigma_n \{ x - \mu_n \}^T,$$

$$X_{c2} = \sum_{n=4}^{6} \frac{1}{\sigma_{n}} \exp \{ k - \mu_n \} \Sigma_n \{ x - \mu_n \}^T,$$

where:

$$\Sigma_1 = \begin{bmatrix} 38 & 17 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 64 & 16 \end{bmatrix}, \quad \Sigma_3 = \begin{bmatrix} 90 & 19 \end{bmatrix}, \quad \Sigma_4 = \begin{bmatrix} 125 & 9 \end{bmatrix}, \quad \Sigma_5 = \begin{bmatrix} 125 & 9 \end{bmatrix}, \quad \Sigma_6 = \begin{bmatrix} 125 & 9 \end{bmatrix}.$$

Figure (1.a) illustrates the distribution of the original data points in two dimensional space. Figures (1.b-d) illustrate the transformed data by PCA, ICA and LDA, respectively. Table (1) shows the classification results using direct and iterative implementation of each method. As shown, LDA extracted features provide slightly improved recognition performance compared to PCA and ICA features.

Table 1: Classification results for mixture of Gaussian data set

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean of Margin</th>
<th>Mean of SV</th>
<th>Mean of Recognition Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>No subspace</td>
<td>0.0030</td>
<td>11.2720</td>
<td>99.4867</td>
</tr>
<tr>
<td>PCA</td>
<td>0.0030</td>
<td>11.2680</td>
<td>99.4800</td>
</tr>
<tr>
<td>ICA</td>
<td>0.0030</td>
<td>11.2720</td>
<td>99.4867</td>
</tr>
<tr>
<td>LDA</td>
<td>0.0029</td>
<td>11.0080</td>
<td>99.4600</td>
</tr>
<tr>
<td>PCA iterative</td>
<td>0.0298</td>
<td>7.8800</td>
<td>99.3397</td>
</tr>
<tr>
<td>ICA iterative</td>
<td>0.0260</td>
<td>8.1200</td>
<td>99.5107</td>
</tr>
<tr>
<td>LDA iterative</td>
<td>0.0295</td>
<td>6.9480</td>
<td>99.2941</td>
</tr>
</tbody>
</table>

4.2 Facial Image Database

The developed algorithms were also applied to Yale face database B (Georghiades, et al., 2001). For this experiment, 2 class recognition experiments are performed over 36 pairs of subjects. For each pair of subjects, a training data set is constructed from the first 32 lighting positions for pose 1 and 2 of each subject. The test data set comprised the same pair of subjects imaged under the last 32 lighting position from pose 7 and 8. The training and test images were histogram equalized and mean centred before subspace calculation and classification. For this example $n > N$, so we used $XX^{T}$ to compute the eigenvectors. Recognition performance (margin, number of support vectors and error rate) was tested for each subject pair for kernel $\sigma$ ranging from 1 to 5. The dimensionality of the training subspace is reduced to 25 prior to recognition. Figures (2.a) and (2.b) show the training images for two faces (selected randomly) from the data set. Figs. 2(c) and (d) show the test images for the same two faces. Fig. 3 shows the resulting principal, independent and linear basis images for the training images shown in Figs. 2(a) and (b). Table (2) shows the average number of support vectors, margin and recognition rate for the entire data set. As illustrated in Table (2), iterative algorithms provide better generalization of the SVM classifier. The improvement in generalization the iterative techniques illustrated by...
improved margin and reduced number of support vectors is statistically significant for all results on the face database.

Figure 2: Example of training and test images: (a) class 1 training, (b) class 2 training, (c) class 1 test, (d) class 2 test.

Moreover, among three feature extraction algorithms, LDA component representation exhibits higher performance with respect to margin, number of support vectors and recognition rate. In all of the experiments, ICA consistently increased the margin and the number of support vectors compared to raw data and PCA component representations.

Table 2: Classification results for Yale face

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean of Margin</th>
<th>Mean of SV</th>
<th>Mean of Recognition Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>No subspace</td>
<td>0.2027</td>
<td>126.8</td>
<td>95.33</td>
</tr>
<tr>
<td>PCA</td>
<td>0.2201</td>
<td>120.9</td>
<td>96.50</td>
</tr>
<tr>
<td>ICA</td>
<td>0.2247</td>
<td>122.0</td>
<td>96.52</td>
</tr>
<tr>
<td>LDA</td>
<td>0.2745</td>
<td>120.0</td>
<td>96.94</td>
</tr>
<tr>
<td>PCA iterative</td>
<td>1.2032</td>
<td>10.1</td>
<td>98.5</td>
</tr>
<tr>
<td>ICA iterative</td>
<td>1.1547</td>
<td>10.6</td>
<td>97.5</td>
</tr>
<tr>
<td>LDA iterative</td>
<td>1.3521</td>
<td>9.5</td>
<td>98.9</td>
</tr>
</tbody>
</table>

5 CONCLUDING REMARKS

In this paper, we used three feature extraction methods including PCA, ICA and LDA to reduce the dimensionality of the training space. An iterative algorithm was utilized to further enhance the generalization ability of the feature extraction methods by producing compact classes. Our experimental results on simulated data illustrated that the proposed methods improve the performance of the SVM classifier both in terms of accuracy and complexity. These results also illustrated that LDA provides slightly improved generalization compared to PCA and ICA. Experimental results on a facial database demonstrated the same improvement in classification performance using LDA extracted features. In our future work, we plan to evaluate the performance of adaptive PCA and LDA algorithms for feature extraction in facial data.

REFERENCES