

Value of Information and Prevention in Insurance Markets.

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Abstract

This paper introduces prevention in a model of insurance and studies the equilibrium of a game, where the agent may by testing acquire an information about his loss probability. We point out that the value of information is a complex concept. Contrary to main papers that define at most two ways of measuring the value of information, five definitions make sense in our setting. This enlargement allows to precise the working mechanisms.

Keywords : genetic testing, insurance market, value of information, prevention.

1 Introduction

Without insurance, the willingness to take a particular test depends only on the cost of testing and the performance of prevention to decrease the probability to develop the disease. In the extreme case where testing is free but no treatment option against a particular disease exists, agents are indifferent between being informed or not, while nothing can affect the lottery they are faced. Likewise, if both prevention is very costly and only reduces marginally mortality, agents will prefer to stay uninformed, i.e. to stay behind the “veil of ignorance” (Milgrom and Stokey 1982 and Tabarrok 1994).

With health insurance, the decision to take or not the test depends in addition on the possibility and the way of insurers to use the additional information in pricing contracts. When no prevention exists, additional information is used in the market to tariffy the risk and to discriminate between agents. When information is symmetric, the additional information is never revealed, as long as testing creates a genetic risk (or a reclassification risk).

Asymmetric information (Doherty and Thistle (1996)) and prevention (Doherty and Posey (1998), Hoel and Iversen (2002)) are thus the sources of the value of information. Even if the idea of value of information appears in model of perfect information without prevention, this assertion is now common view. Most papers, however, analyse prevention by considering that prevention is a hidden action. In health markets, it is not unrealistic to assume that prevention is an observable expense for insurers.

We adopt an insurance model in which the agent can undertake some preventive observable expenses, which reduces his disease probability. Testing is economically relevant as long as preventive decisions take into account the revealed information, and may be irrelevant depending on the values of parameters. We gather and develop different existing results (and some new), around the following questions: do competitive markets provide the good incentives to obtain information and promote efficient testing, and why, consequently what kind of regulation can be designed for insurance ?

We focus on this problem through the concept of value of information. Three ways of measuring this value (definitions 2, 3 and 4) already exist in the literature. Five definitions of the value of information (some already appear in the literature and we propose the others) here emerge and make sense:

1. individual value of information: given that an uninformed agent is insured by a given contract, information has some value when being informed increases the agents' welfare (without change in contract);
2. contractual value of information: given that an uninformed agent is insured by a given contract, information has some value when testing allows to renegotiate the prior contract (increasing thus both the ex ante expected utility and profit);
3. social value of information: information has a social value when it increases the agent's ex post expected utility in a regulated market (where insurers propose efficient contracts);
4. private value of information in sense 1: in a competitive market, information has some value when being informed increases the agent's ex post expected utility;
5. private value of information in sense 2: in a competitive market, testing has some value when its existence creates an increase in the agent's expected utility.

The key point of this paper is that, depending on the informational context, these five definitions do not coincide, and more precisely, the concepts designed in definitions 3 and 4 differ. Indeed, the private value depends on how information is used in insurance markets. As a consequence, the private value depends on the informational status of both insurers and insureds. On contrary, we show that the social value of information is not affected by the informational status.

When information is symmetric, (insurers observe the result of testing), the insurance market provides poor incentives to purchase additional information, except when it efficiently deters the agent from testing, or in other words when forbidding the test is an obvious regulation measure. This result comes from the fact that insurers use information to discriminate between agents, whereas efficiency would involve that agents would be treated similarly in terms on insurance and differently in terms of prevention. This contractual inefficiency modifies the choice criterium of an insured faced to the opportunity of testing. He then takes the test on condition that the private value of information in sense 1 is positive, when faced to efficient contracts the agents decides testing using the social criterium based definition 3.

Such a problem already exists under symmetric information, and it is not solved, neither by asymmetric information about the result of testing, nor by hidden action. We show that asymmetric information creates other incentives for testing, leading the agent to take into account some effects of additional information which are independant from the social value of information: the reasons for which the agent takes the test wander from the straight ones.

After presenting the model (section 2), we solve the game under symmetric information (section 3). We then turn to the case of asymmetric information (concerning the information revealed by testing). We show in section 4 that our results are robust to the introduction of hidden action in the case CARA.

2 Model and assumptions

2.1 Notations, background and assumptions

We consider a heterogenous population of consumers assumed to be risk-averse. They own an identical endowment of wealth w and have a same Von Neumann Morgenstern utility function $U(\cdot)$ increasing and concave in wealth. However, they differ in their loss probabilities. More precisely, an agent may have a type θ , with $\theta \in \{\theta_L, \theta_M, \theta_H\}$, the loss probabilities depending on this type¹. Initially, individuals don't know their type, unless they take a test which perfectly reveals θ . However, individuals observe (without test) if they belong to a group of risk $K \in \{A, B\}$, for instance, depending on their genetic antecedents or their hygiene of life. Each individual belonging to a group K has beliefs λ^K to be a risk-type θ , with $\lambda^K = \{(\lambda_L^K(\theta), \theta \in \{\theta_L, \theta_M, \theta_H\})\}$. The subpopulation A will have has a probability of having the disease lower than the subpopulation B .

An individual of type θ after taking the test may be affected, with a probability $\rho(\theta)$, by a particular disease which restricts his quality of life and can worsen . In case of disease, an individual of type θ makes observable prevent expense e with $e \in E$, that reduces the probability to suffer a worsening of disease, $\sigma(e, \theta)$: with $\sigma_e < 0$ (for example, e is an anticholesterol or hypotensive medicament). In case of worsening disease (for example infarction or arteriosclerosis), he undertakes a treatment T to improve their quality of life (or to prevent the death). We consider a disease for which there always exists a treatment, having a financial fixed cost T which restores health. Moreover, the test is assumed to be free for insureds and insurers.

Consumers may insure their health risk by paying a premium P to an insurer against the gross indemnity I decomposable into two elements: I_e is the indemnity which reimburses the individual prevent expenses and I_T the indemnity which reimburses the expenses of treatment. A contract $C = (e, P, I_e, I_T)$ is then characterized by an amount of prevent expenses, a premium and two levels of indemnities.

Before taking the test, the expected utility of an agent K which subscribes a contract C is

$$E_K[V(C, \theta)] = \lambda^K(\theta_L)V(C, \theta_L) + \lambda^K(\theta_M)V(C, \theta_M) + \lambda^K(\theta_H)V(C, \theta_H) \quad (1)$$

with

$$V(C, \theta) = (1 - \rho(\theta))U(w - P) + \rho(\theta)\{(1 - \sigma(e, \theta))U(w - e - P + I_e) + \sigma(e, \theta)U(w - e - T - P + I_T + I_e)\} \quad (2)$$

Remark that $V(C, \theta)$ is also the expected utility of an agent θ with a contract C after taking the test.

If the insurer doesn't observe the result of the test, the expected profit realized by an insurer on an individual K who has subscribed a contract C is

$$E_K[\Pi(C, \theta)] = \lambda^K(\theta_L)\Pi(C, \theta_L) + \lambda^K(\theta_M)\Pi(C, \theta_M) + \lambda^K(\theta_H)\Pi(C, \theta_H) \quad (3)$$

with

$$\Pi(C, \theta) = P - \rho(\theta)\{I_e + \sigma(e, \theta)I_T\}. \quad (4)$$

¹The reason for which we consider three risk types rather than two types as usual will be justified later.

The latter expression also represents the expected profit earned by an insurer on an individual θ if the insurer does observe the result of the test.

We assume that the timing of the game is the following

- Date 0 : Nature chooses the group K of any individual (A or B with probability ϕ_A or ϕ_B) and the risk-type θ . An agent only observes the group K and then believes that he is of type $\theta \in \{\theta_L, \theta_M, \theta_H\}$ with the probability $\lambda^K(\theta)$.
- Date 1: Each agent decides to take or not the test, which reveals the parameter θ .
- Date 2: Insurers offer contracts in the health market (like in the competitive process à la Rothschild and Stiglitz) and each individual chooses a (unique) contract among those offers.
- Date 3: Insureds choose a level of prevent expenses e (observable by insurers) in case of disease.
- Date 4: The state of nature health or illness occurs. In case of illness, the agents undertake a treatment T . Otherwise, no treatment is undertaken.
- Date 5: The insurer reimburses the damages claimed by insureds.

Our purpose is to study what happens when the agent obtains a more precise information concerning his risk. Several works are particular cases of our model, namely the studies of Bond and Crocker (1991), Crocker and Snow (1986, 1992), Doherty and Thistle (1996), Doherty and Posey (1998), Hoel and Iversen (2002) and Hoy (1989). Thus, we can reinterpret several results obtained by these authors.

Some assumptions in our analysis are comparable to those used in the mentioned-above literature while our work differs from the literature in other aspects. In Doherty and Posey (1998) the preventive expense e is hidden action (privately chosen by the agent and unobservable by the insurer) and $\rho(\theta) = 1$ (moreover the utility function is separable so $V(e, P, I_e, I_T, \theta) = (1 - \sigma(e, \theta))U(w - P + I_e) + \sigma(e, \theta)U(w - T - P + I_T + I_e) - e$ but this assumption surely plays no role in our problem).

In Crocker and Snow (1986, 1992), Doherty and Thistle (1996), Strohmenger and Wambach (2000), $\rho(\theta) = 1$ and e is always taken equal to zero (no prevention allows to diminish the accident risk, $E = \{0\}$) and the agent's utility function is thus $V(e, P, I_e, I_T, \theta) = (1 - \sigma(0, \theta))U(w - P) + \sigma(0, \theta)U(w - T - P + I_T)$. Crocker and Snow (1986, 1992), Doherty and Thistle (1996) implicitly assume that the agent always prefers buying insurance in the market rather than remaining self-insured. (We will adopt the same assumption). In Strohmenger and Wambach (2000), however, the agent can remain uninsured (so learning through the test the real value of his risk probability is worthwhile: in equilibrium, the agent may prefer being insured when his risk is high and being uninsured when his risk is low enough and the value of the loss T is small enough). When information concerning the probability revealed by testing is private, this phenomena may deter a pooling equilibrium to work, because once the test is made, the high risk prefers insurance and the low risk quits the insurance market. In separating equilibria, agents are worse when the test exists than when it does not.

In Bond and Crocker (1991), the analysis relies on an endogenous categorization based on the consumption by insureds of a 'hazardous good' (represented by e). In their model, the individual expected utility is $V(e, P, I_e, I_T, \theta) = (1 - \sigma(e))U(w - P + I_e) + \sigma(e)U(w - T - P + I_T + I_e) - c(e, \theta)$ and e is observable. They conclude that basing the insurance contracts on the observation of the expense e enhances the market efficiency. We assume this point always holds in our context. Finally note that Crocker and Snow (1986) consider a different informational framework: agents prior know their type. In a setting of exogenous categorization (based on immutable characteristics as sex, age or

race), companies observe or not (depending whether the categorization is free or costly) the group A or B in which they belong. The question they ask is then is it efficient to make contracts contingent on the observation of the immutable characteristic A or B . The objective of this paper is totally different because our purpose is to study what happens when the agent chooses to acquire or not a more precise information concerning his risk.

2.2 Additional assumptions and notations

In case of symmetric information, we denote by $C^{PI}(\theta)$ the contract offered by an insurer to an agent of type θ when information on θ is symmetrically known by the agent and his insurer, and C_K^{PI} the contract offered to an agent which belongs to a group K and is uninformed on his type, when the insurer observes the group K .

In a competitive context, $C^{PI}(\theta)$ is solution, for any θ , of

$$\begin{aligned} & \text{Max}_C V(C, \theta) && \text{(Program 1)} \\ \text{s.t. } & \Pi(C, \theta) \geq 0 \end{aligned}$$

and C_K^{PI} is solution, for any K , of

$$\begin{aligned} & \text{Max}_{C^K} E_K[V(C, \theta)] && \text{(Program 2)} \\ \text{s.t. } & E_K[\Pi(C, \theta)] \geq 0. \end{aligned}$$

When information is asymmetric, when no agent takes the test, additional incentive constraints are required to prevent each consumer K to subscribe the contract intended to the other group K' , $K, K' \in \{A, B\}$, $K \neq K'$. The optimal contracts verify Program 3 defined below:

$$\begin{aligned} & \text{Max}_{C_A, C_B} \phi E_A[V(C_A, \theta)] + (1 - \phi) E_B[V(C_B, \theta)] && \text{(Program 3)} \\ \text{s.t. } & E_A[V(C_A, \theta)] \geq E_A[V(C_B, \theta)] \\ & E_B[V(C_A, \theta)] \geq E_B[V(C_B, \theta)] \\ & E_A[\Pi(C_A, \theta)] \geq 0 \text{ and } E_B[\Pi(C_B, \theta)] \geq 0 \end{aligned}$$

with $\phi \in [0, 1]$ the weight associated to the group A in the objective function. It is easy to show that an insurer earns zero expected profit on each consumer (from group A or B), so that the set of optimal contracts are the Rothschild/Stiglitz contracts (RS hereafter) C_A^{RS} et C_B^{RS} for the groups A and B respectively.

When conversely insurers rationally expect that insureds have taken the test, the competitive market offers the contracts à la RS, noted $C^{RS}(\theta)$, $\theta \in \{\theta_H, \theta_M, \theta_L\}$ and solution of program 4 (where $\phi(\theta)$ are some positive weights) :

$$\begin{aligned} & \text{Max}_{\{C(\theta), \theta \in \{\theta_H, \theta_M, \theta_L\}\}} \sum_{\theta \in \{\theta_H, \theta_M, \theta_L\}} \phi(\theta) V(C(\theta), \theta) && \text{(Program 4)} \\ \text{s.t. } & V(C(\theta), \theta) \geq V(C(\theta'), \theta) \text{ for } \theta \neq \theta' \text{ in } \{\theta_H, \theta_M, \theta_L\} \\ & \Pi(C(\theta), \theta) \geq 0 \text{ for all } \theta \text{ in } \{\theta_H, \theta_M, \theta_L\} \end{aligned}$$

We assume that the RS equilibrium exists and the RS contract is separating.

The expected mean loss of an agent θ opting for a given prevent expense e is $\rho(\theta)(e + T\sigma(e, \theta))$. We assume this expression increases in θ whatever $T > 0$, which means that a higher type is associated to a riskier agent. Consequently, for a given contract, higher is the agent's type, smaller the insurer's expected profit. Moreover, the group B is more risky than A in the sense that whatever E and $T > 0$, the expected loss induced by an agent from group B is higher than when he belongs to group A, in other words,

$$E_B[\rho(\theta)(e + T\sigma(e, \theta))] > E_A[\rho(\theta)(e + T\sigma(e, \theta))] \quad (5)$$

Finally, in what follows, the efforts minimizing the expected loss play an important role. We define by $e^*(\theta)$ the optimal prevention when the test has revealed the type θ , and e_K^* the optimal one for an uninformed agent from group K, with

$$e^*(\theta) = \text{Min}_e \rho(\theta)(e + T\sigma(e, \theta)) \quad (6)$$

$$e_K^* = \text{Min}_e E[\rho(\theta)(e + T\sigma(e, \theta))]. \quad (7)$$

3 Symmetric information

Before presenting the game equilibrium, we want to precise on what conditions information has some value. The literature relative to testing has presented at least three ways of defining the value of information. We follow in a first time the definition given by Crocker and Snow (1992), the most general one, while it does not depend on the competitive context.

First note that using the information revealed by the test enhances the set of feasible contracts. As long as information is costless, the agent always prefers acquiring the additional information rather than remaining uninformed. When the agent is insured, information has some value if a contract based on the additional information dominates in a Pareto sense any contract ignoring information revealed by testing. In other words, consider an agent from a group K who has not taken the test, and assume that he is insured by a contract C . Information has some value (ex ante) if, whatever the prior contract C , there exists a menu of contracts $\widehat{C}(\theta), \theta \in \{\theta_L, \theta_M, \theta_H\}$ such that both the insured and the insurer prefer (ex ante) that the agent first gets the additional information and in a second time insures with $\widehat{C}(\theta)$. That is, one of these inequalities being strict:

$$E_K[V(\widehat{C}(\theta), \theta)] \geq E_K[V(C, \theta)]$$

$$E_K[\Pi(\widehat{C}(\theta), \theta)] \geq E_K[\Pi(C, \theta)]$$

Proposition 1: Assume that information is symmetric. On condition that $e^*(\theta) \neq e^*$ for one θ in $\{\theta_H, \theta_L, \theta_M\}$, testing has some value.

Let us present an intuition of the proof. Assume that the agent has not acquired additional information. Let \overline{C} the starting contract and denote by \overline{e} the preventive expense and $\overline{\pi}$ the expected

profit of the insurance company. As the agent is risk averse, if \bar{C} is not a perfect insurance contract, it is possible to modify \bar{C} by offering a perfect insurance contract which gives an expected utility to the agent equal to $U(w - P)$ with a premium equal to $E[\rho(\bar{e} + \sigma(\bar{e}, \theta)T) + \bar{\pi}]$. Furthermore, since the effort differs from e^* (recall that by definition e^* is the preventive expense which minimizes $E[\rho(e + \sigma(e, \theta)T)]$), changing the contract level \bar{e} and replacing it by e^* increases the agent's expected utility without decreasing the insurance company's expected profit. This new contract then dominates in the Pareto sense any contract based on the fact that the agent has not taken the test.

Now assume that the agent takes the test and consider a menu of perfect insurance contracts $C^*(\theta, \bar{\pi}) = (P(\theta), e(\theta), I_e(\theta), I_T(\theta))$ such that:

$$I_T(\theta) = T, I_e(\theta) = e(\theta), P(\theta) = \hat{P} = E_K[\rho(e + \sigma(e, \theta)T)] + \bar{\pi} \quad (8)$$

Note that this contract entails perfect coverage both with respect to accident and to type. Indeed, whatever the state of nature, the agent's wealth will be equal to $w - P$. Note that this menu of contracts gives an expected profit equal to $\bar{\pi}$. Finally, the best level of prevention is such that the premium \hat{P} is the smallest possible, that is $e(\theta) = e^*(\theta)$ (which by definition minimizes the premium $\rho(e + \sigma(e, \theta)T)$ for any given type).

Comparing the contract using information to that ignoring it allows to understand how information is used in this model, from an efficiency point of view. Both the optimal contract without testing and the contract based on the information revealed by the test are perfect insurance contract. This notion of perfect insurance has now two dimensions: the risk incurred by the agent is not only an accident risk, but also a genetic risk. This genetic risk comes from the fact that from an ex ante point of view the type discovered by the test is variable. An efficient contract thus insures the agent against the two dimensions of random variables. In that way, information could be useless.

Information, however, enhances the set of feasible contract by allowing prevention to depend on type. Comparing the two previous premia leads straightforwardly to the conclusion: when information about type is not revealed the premium is $E[\rho(e^* + \sigma(e^*, \theta)T) + \bar{\pi}]$ when it equates $E[\rho(e^*(\theta) + \sigma(e^*(\theta), \theta)T) + \bar{\pi}]$ when testing occurs. Using information thus dominates in a Pareto sense only when the information allows a reduction in the optimal premium, or in other words, in the expected loss. That is when:

$$E[\rho(e^*(\theta) + \sigma(e^*(\theta), \theta)T)] < E[\rho(e^* + \sigma(e^*, \theta)T)]. \quad (9)$$

Such a condition obviously holds so long that a type θ exists such that $e^*(\theta) \neq e^*$ (a more technical conclusion could be $\sigma_{e\theta}(e^*(\theta), \theta) \neq 0$ and $e^*(\theta)$ is an interior solution in E). As conclusion, using information dominates ignoring it on condition that the preventive efficient decision effectively depends on the revealed information.

The condition is necessary and sufficient. To see why, assume for instance that the insurance company uses information to design contracts when information has no value (that means in this context $e^*(\theta) = e^*$ for all θ). Denote by $\{\bar{C}(\theta), \theta \in \Theta\}$ the starting menu of (different) contracts. From what is preceding, we know that the optimal menu $\{C^*(\theta, \bar{\pi}), \theta \in \Theta\}$ is pooling, or, in other words, the contracts $C^*(\theta, \bar{\pi})$ are all identical. As a consequence, the optimal menu induces less variation, and then dominates in a Pareto sense the prior menu as long as the agent is risk averse. Ignoring information is then better.

Information has no value under the assumptions made by Crocker and Snow (1992) and Doherty and Thistle (1996). Indeed, assume there is no prevention. For example, the set of feasible actions is a singleton, $E = \{0\}$. An insurance contract is thus reduced to the couple (premium, indemnity), $C = (P, I)$, and the insured's expected utility and the insurer's profit can be rewritten as follows:

$$V(C, \theta) = (1 - \sigma(0, \theta))U(w - P) + \sigma(0, \theta)U(w - P - T + I)$$

$$\Pi(C, \theta) = P - \sigma(0, \theta)I$$

Using the concavity of the agent's VNM utility function, we obtain:

$$V(C(\theta), \theta) \leq U(w - P(\theta) + \sigma(0, \theta)(I(\theta) - T)) \text{ which implies}$$

$$E[V(C(\theta), \theta)] \leq U(w - E[P(\theta) + \sigma(0, \theta)(I(\theta) - T)]).$$

Whenever the prior menu of contract differs from the full insurance pooling contract, $C = (E[P(\theta) + \sigma(0, \theta)(I(\theta) - T)], T)$ the inequality above is strict. The best contract from an ex ante point of view makes the ex post contract independent of the revealed information.

The reason is rather simple. The ex post optimal contract always offers a full coverage. From an ex ante point of view, testing only implies that agents bear a reclassification risk, as long as the equilibrium premium depends on the revealed information. As no prevention exists, acquiring information reveals the probability of accident but cannot reduce the risk. Consequently, testing cannot enhance the set of feasible contracts and is thus Pareto dominated. In what follows, such an assumption is referred by "information has no value".

We turn now to determine the competitive equilibrium of the game presented in section 2. The first result is well known and reflects the fact that the competitive market uses information to discriminate between agents.

Proposition 2 Assume that insureds take the test revealing thus all the information. The competitive equilibrium is efficient ex post but is not efficient ex ante.

The proof is very simple so we present it in the main text.

Proof : Consider the case where all the agents take the test. All the players of the game are thus fully informed, they know that the agent is either θ_L, θ_M or θ_H . As the competitive process works like in Bertrand, an agent θ will be proposed the contract $C^{PI}(\theta)$ as defined in program 1, which maximizes his expected utility $V(C, \theta)$ subject to the constraint that the expected profit is positive or equal to zero, that is $\Pi(C, \theta) \geq 0$. The optimal contract is finally:

$$I_T^{PI}(\theta) = T, I_e^{PI}(\theta) = e^*(\theta), P^{PI}(\theta) = \rho(\theta)\{e^*(\theta) + \sigma(e^*(\theta), \theta)T\}. \quad (10)$$

Clearly a contract which increases the agent's expected utility whatever his type θ and ensures a positive expected profit for the company does not exist and the contract $C^{PI}(\theta)$ is ex post efficient.

Consider now the menu of contracts $\{C^*(\theta, 0), \theta \in \Theta\}$. Note that the two menus $\{C^*(\theta, 0), \theta \in \Theta\}$ and $\{C^{PI}(\theta), \theta \in \Theta\}$ are identical, except that the premium is now constant and equal to $E[P^{PI}(\theta)]$.

The menu $\{C^*(\theta, 0), \theta \in \Theta\}$ gives an expected profit equal to 0, and an ex ante expected utility to the agent:

$$U(w - E[P^{FB}(\theta)]) > E[U(w - P^{FB}(\theta))]$$

A contract thus exists which dominates the equilibrium menu in a Pareto sense. ■

Competition pushes companies to fix the price of the risk at his real (known) value. Ex ante, insureds are thus faced to a reclassification risk, that increases the premium of the high risk and reduces that of the low risk. From an ex ante point of view, insureds would prefer to be proposed a contract that insures him with respect to that risk. This well known point is developed in Tabarrok (1994), who suggests that agents could insure before testing against such a premium risk. Hoel and Iversen (2002) propose to use a compulsory insurance to solve this problem. Note that in our context, a compulsory insurance destroy the incentives of providing prevention, so is never optimal. Indeed, assume that agents are reimbursed a level of loss x , and pay an additional premium which balances the accounts, $[\sum \lambda^K(\theta)\rho(\theta)\sigma(e(\theta), \theta)]x$, set before the competitive process occurs. In equilibrium, insurers propose a full coverage contract associated with the prevention expenses that minimize the competitive premium which is now:

$$P = \rho(\theta)\{e(\theta) + \sigma(e(\theta), \theta)(T - x)\}$$

So the prevention decision will be as smaller as the level of compulsory coverage, x , is high.

Proposition 3 : When information is symmetric and has no value, the agents remain uninformed in equilibrium. When information has some value, either agents get additional information or remain uninformed, depending on their risk aversion.

The insured's decision of testing rests on the comparison between his expected utility in equilibrium when he is informed and his expected utility when he is uninformed. The difference between these two terms is usually called the private value on information, given in our context by:

$$V_P = E_K[V(C^{PI}(\theta), \theta)] - E_K[V(C_K^{PI}, \theta)]. \quad (11)$$

Using equations (6) and (7), we obtain :

$$V_P = E_K[U(w - \rho(\theta)\{e^*(\theta) + \sigma(e^*(\theta), \theta)T\})] - U(w - E_K[\rho(\theta)\{e^* + \sigma(e^*, \theta)T\}]). \quad (12)$$

When information has no value ($e^*(\theta) = e^*$ whatever θ), V_P is clearly negative and the agent does not purchase additional information. Testing only creates risk reclassification, so the informed agent is faced to a risky lottery (from an ex ante point of view). This point is well known. Relative to testing, the insured's decision is thus efficient: the competitive market gives no incentives to reveal an useless information. Finally the competitive process leads to an efficient equilibrium from a double point of view. As far as information is concerned, competition leads to an efficient decision of not testing. As far as insurance is concerned, competition propose a full coverage contract.

On contrary, when information has some value ($e^*(\theta) \neq e^*$ for one θ), the private value of information can be positive or negative, depending on the insured's risk aversion. The competitive process pushes the firms to propose full insurance contract and to rate for the risk at its fair price, relative to the available information. As a consequence, when the agent becomes informed, he bears an additional risk, a reclassification risk, but he benefits from a better decision concerning prevention. Indeed, when the agent is informed, the competitive contract discriminates with respect to type, and both the insurance premium and the preventive expenses depend on the type. So from an ex ante point of view, testing makes the agent facing the lottery induced by the heterogeneity of premia, whereas his premium is certain when he does not take the test. Thus, as the agent is risk averse, he would prefer not to take the test in order to avoid an increase in premia. However, the information allows to adapt prevention in a finer way, so finally testing makes the expected premium to fall. The agent thus faces a riskier lottery, but his expected wealth is higher, so the trade-off. When the agent is almost risk neutral, he will take the test, while the opposite occurs when his risk aversion is high enough.

Once again, however, the insured's decision concerning testing is efficient taking into account the competitive offers: he takes the test when this maximizes his well-being. Does such a conclusion mean that regulation is not likely in this market? If the regulator could organize the insurance market, he would impose testing on condition that the social value of information is positive. Another design of value of information thus emerges. It reflects the difference between the informed 's expected utility and the uninformed's expected utility when the market is regulated:

$$\begin{aligned} V_S &= E_K[V(C^*(\theta, 0), \theta)] - E_K[V(C^*(0), \theta)]. \\ &= U(w - E_K[\rho(\theta)\{e^*(\theta) + \sigma(e^*(\theta), \theta)T\}]) - U(w - E_K[\rho(\theta)\{e^* + \sigma(e^*, \theta)T\}]) \end{aligned} \quad (13)$$

The market would then offer the best full insurance contract, which pools with respect to premia but discriminates with respect to prevention when testing has occurred. When information has no value, V_S equates zero. The market is efficient and testing is never used. When information has some value, however, V_S is positive and $V_P < V_S$. We have already seen that the competitive market fails to propose the efficient contract (from an ex ante point of view) to the agent, since efficiency ex post would lead insurers to offer a full coverage contract that pools with respect to premia and separates with respect to preventive expenses. This time, the competitive equilibrium presents two sources of inefficiency. On the one hand, when information is revealed, competition implies a reclassification risk, which is not efficient from a social point of view. On the other hand, when information is not revealed, testing would improve welfare, allowing to adapt preventive decisions to the real risk.

These results are robust to the introduction of a costly test. Indeed, assume that taking the test generates an expense γ . A straightforward extension of proposition 1 gives that information has some value when :

$$E[\rho(\theta)\{e^*(\theta) + T\sigma(e^*(\theta), \theta)\}] + \gamma > E[\rho(\theta)\{e^* + T\sigma(e^*, \theta)\}].$$

This condition is exactly equivalent to the fact that the social value of information is positive. The private value may be now written:

$$V_P = E[U(w - \rho(\theta)\{e^*(\theta) + T\sigma(e^*(\theta), \theta)\} - \gamma)] - U(w - E[\rho(\theta)\{e^* + T\sigma(e^*, \theta)\}]).$$

and once again, $V_P < V_S$.

Depending on the value taken by γ , information has some value when the decrease in the expected loss induced by an appropriate prevention compensates the testing cost. Note that the parameter γ could be negative, reflecting the fact that insureds prefer being informed than uninformed (as in Hoel and al (2002)). In such a case, even if no prevention is feasible, information can have some value per se under symmetric information.

- Optimal regulation ?

When information has no value a good regulation measure would be to forbid testing. When conversely information has some value, the issues are much more imbricated. Is it possible to provide insurance with respect to a premium variation when economic conditions (managing cost, wealth and so on) are likely to change?

- Crocker and Snow (1986, 1992), Doherty and Thistle (1996) note that under competition and symmetric information, testing never occurs in equilibrium (information has no value under their assumptions). They conclude that asymmetric information is the key for creating the value of information, in two directions, either insurers do not observe the type revealed by testing (so testing is the source of adverse selection) (Doherty and Thistle (1996), Hoel et al. (2002)), or insureds opt for preventive unobservable measures (so prevention is associated to hidden action) (Doherty and Posey (1998), Hoel and Iversen (2002), Hoy and Polborn (2000)).

4 Asymmetric information

Assume now that only insureds are informed about their type (after testing) and group. This implies that not only insurers ignore the agents' prior group but also cannot observe (or verify) the information revealed by testing.

We show here that asymmetric information provides different incentives to acquire additional information. In equilibrium, agents are proposed a menu of separating contracts. Being informed thus allows to optimize their choice in this set. As a consequence, the private value of information (defined by the difference between the informed agent's expected utility and the uninformed agent's one given the insurers' equilibrium offers) is always strictly positive (when insurers ignore whenever the agent has taken the test or not).

Testing now creates asymmetric information. One could expect this affects the value of information (as defined in proposition 1). Proposition 4 states that the condition under which an additional asymmetric information has some value is not modified: testing enhances the contracting set on condition that the optimal preventive expenses effectively depend on the revealed information.

Proposition 4 : When insurers do not observe the information provided by testing, information has some positive value under the same conditions than proposition 1 . This holds whether the insurance company observes the prior agent's group or not.

The complete proof is reported in appendix. We present here only an intuition by focusing on the case where the agent's group is known.

Assume the company knows that the insured belongs to group K. Let C an offered contract when the test is not taken. Such a contract gives to the agent an expected utility smaller than $C^*(\bar{\pi})$ defined in equation (8). Now assume that the agent takes the test, and the insurer proposes the menu of contract $C^*(\theta, \bar{\pi}), \theta \in \{\theta_H, \theta_L, \theta_M\}$ defined in (9). Every contract of this menu gives to the agent the same expected utility equal to $U(w - P)$. So ex post, once he has obtained his information, the agent is indifferent between announcing his right type or lying, and finally, he will tell the truth to the company. On condition that the optimal efforts effectively depend on type, the contract then differs from $C^*(\bar{\pi})$, which is the best contract ignoring information because it involves a smaller premium. This reduction of premium is due to the fact that the effort is better adapted to the nature of risk: as already seen, the preventive option creates the value of information.

The fact that the information revealed by testing now creates adverse selection does not matter to determine on what conditions information has some value. In other words, surprisingly, the incentive constraints play no role.

This is due to the fact that we reason ex ante. As a pooling contract is preferred in the model of Crocker and Snow (1992) where the insurer can observe the results of testing, information has no value ex ante. This is obviously not modified when incentive constraints are introduced: every pooling contract is incentive compatible while no revelation is required to implement the contract.

More interesting is the fact that this holds in our context whereas our optimal contract $C^*(\theta, \bar{\pi}), \theta \in \{\theta_H, \theta_L, \theta_M\}$ (defined by equation (9)) is no longer pooling. Indeed, such a contract pools with respect to premium and perfectly insures agents but proposes different levels of prevention. Ex post, however, the agent's utility does not depend on his type. So anytime such a property holds, this contract is incentive compatible, leading the agents to reveal the truth.

Information asymmetry does not affect optimal contracts, so his value. That point is new, and contradicts the common view.

Consider now the competitive game presented in section 2 when insurers do not observe if agents has tested. Proposition 5 presents the game equilibria.

Proposition 5 : Assume insurers can observe neither the occurrence of testing nor the information revealed by the testing. When they know the insured's prior group, two Bayes perfect equilibria exist. In the first one, the insured from group K does not take the test and is proposed C_K^{PI} . In the second one, the agent takes the test and companies propose in the second step the menu of contracts à la RS $\{C^{RS}(\theta_L), C^{RS}(\theta_M), C^{RS}(\theta_H)\}$. Conversely, when insurers ignore the agent's group, an unique Bayes perfect equilibrium exists. The insured takes the test whatever his group and is offered the menu of contracts à la RS $\{C^{RS}(\theta_L), C^{RS}(\theta_M), C^{RS}(\theta_H)\}$.

Proof

Insurance companies can not observe whether the agent has taken the test. Recall that they do not observe the type revealed by the test. So a priori, the companies can face agents who are either informed or not. They offer the contract à la RS relative to their beliefs, so four cases are feasible:

1. all the agents have taken the test, any agent's type belongs to $\{\theta_L, \theta_M, \theta_H\}$;
2. nobody has taken the test so agents remain uninformed, they belongs to group A or group B;
3. only agents from group A have taken the test. This time an agent can either belong to group B and being uninformed or have a type in $\{\theta_L, \theta_M, \theta_H\}$;
4. conversely only agents of group B have taken the test. This time an agent can either belong to group B and being uninformed or have a type in $\{\theta_L, \theta_M, \theta_H\}$.

Now assume we are in one of the four preceding cases and denote by S^* the set of contracts which are both offered by insurers and chosen by an agent in equilibrium. The agent thus picks a contract from this set in the third step of the game. The expected utility of an agent from group K (evaluated before he decides for testing or not) is thus $E_K[\text{Max}_{C \in S^*} V(C, \theta)]$ and the following inequality obviously holds for each group:

$$E_K[\text{Max}_{C \in S^*} V(C, \theta)] \geq \text{Max}_{C \in S^*} E_K[V(C, \theta)] \quad (14)$$

For the sake of the proof, denote by $\bar{C}_K = \text{Argmax}_{C \in S^*} E_K[V(C, \theta)]$ the contract chosen by an agent in group K when he decides not to take the test. For all type θ in Θ , we have

$$\text{Max}_{C \in S^*} V(C, \theta) \geq V(\bar{C}_K, \theta), \quad K = A, B. \quad (15)$$

When one type θ , say θ_L , exists such that (15) holds as a strict inequality, (15) holds as a strict inequality for both groups A and B, and the agent, whatever his group, will prefer testing. So only case 1 occurs. Such a remark implies that the occurrence of cases 2, 3 and 4 is feasible only on condition that the equilibrium contracts set is such that (14) and (15) hold as an equality whatever the type and the group, so we have:

$$\text{Max}_{C \in S^*} V(C, \theta) = V(\bar{C}_B, \theta) = V(\bar{C}_A, \theta) \quad (16)$$

In other words, the information revealed by the test is useless to choose in the contract equilibrium set. Under our assumptions, however, the agent which is indifferent between testing or not the test will not take the test. Only case 2 can finally exist.

Note that when only one group of agent a priori exists, in case 2 the equilibrium contract is unique, then (16) evidently holds. The game has thus an equilibrium in which no agent takes the test. Increasing the number of groups creates an heterogeneity ex ante and deters the existence of such an informative equilibrium. Indeed, assume we have two prior groups A and B. In case 2, the equilibrium set is $\{C_A^{RS}, C_B^{RS}\}$ and we have

$$V(C_B^{RS}, \theta) = V(C_A^{RS}, \theta) \text{ for all } \theta \text{ in } \Theta$$

We prove in appendix that these equalities do not hold in our model, because we have

$$E_A[V(C_A^{RS}, \theta)] > E_B[V(C_B^*, \theta)] = E_B[V(C_B^{RS}, \theta)].$$

In case 1, firms anticipate the agents' behavior in equilibrium, so they know that all the agents are informed. As a consequence, the competitive process entails they offer the RS contracts, given in section 2. Each agent anticipates this offer, so he prefers being informed if

$$E_K[V(C^{RS}(\theta), \theta)] > \text{Max}_C E_K[V(C, \theta)] \text{ whatever } C \text{ in } \{C^{RS}(\theta_L), C^{RS}(\theta_M), C^{RS}(\theta_H)\} \quad (17)$$

The agent will get the information because it allows him to choose in the equilibria set. Assuming that $\rho\sigma$ increases with respect to θ allows to show that:

$$\begin{aligned} V(C^{RS}(\theta_L), \theta_L) &> V(C^{RS}(\theta_M), \theta_L) \\ V(C^{RS}(\theta_L), \theta_L) &> V(C^{RS}(\theta_H), \theta_L) \end{aligned}$$

Hence, equation (17) holds provided that one of the following inequalities is strict:

$$\begin{aligned} V(C^{RS}(\theta_M), \theta_M) &\geq V(C^{RS}(\theta_H), \theta_H) \\ V(C^{RS}(\theta_H), \theta_H) &\geq V(C^{RS}(\theta_L), \theta_H) \end{aligned}$$

In other words, the information is useful for choosing in the RS set. ■

In equilibrium, the decision of being informed follows the private value of information defined in sense 1. Taking into account the companies' quotes, let S^* the set of equilibrium contracts. The insured compares the best contract in S^* when he is informed to his best choice when he is uninformed. The private value of information is thus :

$$V_P^1 = E_K[Max_{C \in S^*} V(C, \theta)] - Max_{C \in S^*} [E_K[V(C, \theta)]] \quad (18)$$

This expression is always positive, and strictly positive when there exist more than two types (in Doherty and Thistle (1996) with two types, the private value of information equates zero). Furthermore, the sign of the private value of information does not depend on the value of information defined in proposition 4. As a consequence, the decision of being informed rests on the diversity of quotes offered by the market and never takes into account any social efficiency criterium.

To obtain a unique equilibrium, we thus need at least two prior groups and three different types.

Indeed, assume first that there exists only an initial group (that is $A = B$ or more precisely $\lambda^A = \lambda^B$). If companies expect that nobody has got the additional information, then they offer only one contract, the contract of symmetric information relative to the existing group (denoted previously by C_K^{IP}). This time, because the market does not propose a menu of contract, obtaining an additional information has no value since it does not allow to enhance the choice between the offers. As a consequence, either the agent is indifferent between testing or not when the test is free, or he strictly prefers not to take the test when this latter is costly. Moreover, such a result also holds when the companies know the agent's prior group, for the same reason. In that case we obtain two types of equilibria, the one where everybody becomes informed, and another in which information is ignored.

In a different way, when ex post two types only exist, for instance $\lambda_M^A = \lambda_M^B = 0$, on condition that an incentive constraint of the RS program is binding, the equilibrium game in which the agent becomes informed disappears, whenever the information is costly, even if such a costs tends to zero. Indeed, we know from the RS equilibrium that the RS contract is such that:

$$\begin{aligned} V(C^{RS}(\theta_H), \theta_H) &= V(C^{RS}(\theta_L), \theta_H) \\ V(C^{RS}(\theta_L), \theta_L) &\geq V(C^{RS}(\theta_H), \theta_L). \end{aligned}$$

In such a case, however, the agent will prefer not to test and choose $C^{RS}(\theta_L)$ in the second step. Since an agent indifferent between being informed or not remains uninformed, insureds do not take the test in equilibrium. This result is noted by Doherty and Thistle (1996) in a model where information has no value, but they assume that agents get the information in case of free testing.

The key point thus relies on the fact agents get information whenever they use it to modify their contract choice between the proposed contracts. When adverse selection leads the market to quotes different offers, it gives the agent a reason to obtain additional information, even if such an information has no value to design contracts (in sense of proposition 1). The incentive to being informed rests in the diversity of quotes coming from the market.

As a first conclusion, like in symmetric information, testing thus occurs when information has a private positive value. Does it mean that the insurance market gives the good incentives to purchase additional information, as suggested by Doherty and Thistle (1996) ?

In equilibrium, insureds may prefer to commit to remain uninformed, or they may prefer that the test is unavailable or very costly. So we could compare the agent's expected utility when testing is feasible to his expected utility when testing is prohibited. This second definition of value of information is:

$$V_P^2 = E_K[V(C^{RS}(\theta), \theta)] - E_K[V(C^{RS}(K), \theta)].$$

When V_P^2 is positive, the existence of testing increases the welfare, even if insurance is provided in unregulated markets. When on contrary V_P^2 is negative, regulators must clearly prohibit testing if they do not regulate insurance contracts. Obviously the two definitions of private values V_P^1 and V_P^2 are identical in case of symmetric information. Under asymmetric information, however, V_P^1 and V_P^2 differ and V_P^2 may be negative. In such a case, the market gives the incentives to testing whereas prohibiting testing increases social welfare. This point is new and is due to asymmetric information.

Moreover, like under symmetric information, the social value does not coincide with the private value of information (VP_2 or VP_1). This comes from the fact that competition pushes companies to discriminate between agents. Note that asymmetric information does not affect in our model the social value of information defined in equation (13). The private value of information may be positive, when the best regulation measures would be to impose a pooling contract and no testing.

Doherty and Thistle (1996) and Crocker and Snow (1992) argued that the informational status play an important role for understanding testing. We turn now to study that case, and assume that companies observe whenever agents have taken or not the test.

When only one prior group, say B, exists, the conclusion is very strong: insurers offer an informed agent the menu of contracts à la RS, $\{C^{RS}(\theta), \theta \in \Theta\}$. On contrary, an uninformed agent is offered C_B^{PI} . The agent thus evaluates the private value (and here the two definitions give the same expression) before opting for testing:

$$V_P = E_K[V(C^{RS}(\theta), \theta)] - E_K[V(C_B^{PI}, \theta)]$$

The conclusion of proposition 3 thus holds, applied to the case where the revealed type is private information. When information has no value, it will not be revealed in equilibrium, and the market is

efficient, in providing both the good incentive relative to information, and a first best contract. When information has some value, because the market discriminates in premia between the different types, $V_P < V_S$, and sometimes an efficient information from a social point of view may remain unknown.

Assume now two prior groups, A and B, exist. This time the game equilibria look very different, indeed, we can classify the equilibria as follows:

- In equilibrium 1, the agent whatever his group prefers testing. An informed agent is proposed the RS menu $\{C^{RS}(\theta), \theta \in \Theta\}$, and the offer made to uninformed insureds depends on the insurers's beliefs. The worst for insureds is that only B does not take the test, so such an equilibrium exists on condition that:

$$E_K[V(C^{RS}(\theta), \theta)] > E_K[V(C_B^{IP}, \theta)] \text{ pour } K = A, B. \quad (19)$$

- In equilibrium 2, no agent is informed. They are proposed the RS equilibrium relative two the information structure A, B, and they expect that if any of them would be informed, he will be proposed $\{C^{RS}(\theta), \theta \in \Theta\}$. Such a situation is an equilibrium as long as any agent prefers to remain uninformed, that is when equation hereafter holds:

$$E_K[V(C^{RS}(K), \theta)] \geq E_K[V(C^{RS}(\theta), \theta)] \text{ pour } K = A, B$$

- In equilibrium 3, only the agents from group A are informed and are offered the menu of RS contracts relative to their information $\{C^{RS}(\theta), \theta \in \Theta\}$. Companies anticipates the agents' equilibrium behavior, and know that when the test is not observed, the agent belongs to group B: consequently they offer the perfect insurance contract C_B^{PI} . This situation holds as an equilibrium when the value of parameters of the models are such that:

$$E_A[V(C^{RS}(\theta), \theta)] > E_A[V(C_B^{PI}, \theta)] = E_B[V(C_B^{PI}, \theta)] \geq E_B[V(C^{RS}(\theta), \theta)].$$

- Equilibrium 4 is the symmetric case: only agents from group B are informed, and we have:

$$E_A[V(C^{RS}(\theta), \theta)] \leq E_A[V(C_A^{PI}, \theta)] = E_B[V(C_A^{PI}, \theta)] < E_B[V(C^{RS}(\theta), \theta)].$$

Once again except in equilibrium 2, agents opt for information without considering the private value of information defined in sense 2, but use the first definition.

When information has no value, only equilibrium 2 is feasible, and the market is efficient. This result supports the conclusion due to Doherty and Thistle (1996) according to a public information informational status allows to solve the issue.

This is no longer true when information has some value. In that case, information may be revealed whereas its private value (in sense 2) is negative (equilibrium 1 or 3). Finally, note that depending on the parameters only equilibria 1 or 2 and 3 or 4 are feasible so we have at most two equilibria (1 and 3 or 2 and 4) in that game.

5 Hidden action

Assume now that prevention is privately chosen by the agent. The insurance contract thus is given by a premium P and an indemnity I_T . Define for this section, $C = (P, I_e, I_T)$ and define

$$V(C, e, \theta) = (1 - \rho(\theta))U(w - P) + \rho(\theta)\{(1 - \sigma(e, \theta))U(w - P - e - I_e) + \sigma(e, \theta)U(w - P - e - I_T - I_e)\}$$

$$\Pi(C, e, \theta) = P - \rho(\theta)\{I_e + \sigma(e, \theta)I_T\}$$

and denote by $e^*(C, \theta)$ the informed agent's decision and $e_K^*(C)$ the choice of an uninformed agent from group K, that is:

$$e^*(C, \theta) = \text{Arg max}_e V(C, e, \theta)$$

$$e_K^*(C) = \text{Arg max}_e E_K[V(C, e, \theta)]$$

• In the preceding, we have shown that four definitions of value of information make sense. When hidden action, one could consider another one: an agent faced to a given contract may increase his expected utility by testing. Indeed, once he knows his type, he can adapt his preventive expenses to his needs. Information then for a contract C has some value whenever the informed insured's effort, $e^*(C, \theta)$, differs (for at least one type) of the uninformed insured's effort, $e^*(C)$. In technical words, we have

$$E[V(C, e^*(C, \theta), \theta)] \geq E[V(C, e^*(C), \theta)]$$

and the inequality strictly holds whenever $e^*(C, \theta) \neq e^*(C)$ for at least one θ .

This new definition means that information has some value under hidden action per se. But this notion of value of information can be compared to the definition of private value V_P^1 given in equation (17). Indeed, a contract $C = (P, I_e, I_T)$ generates for the agent the set of contracts $\{C = (P, I_e, I_T, e), \text{whatever } e \text{ in } E\}$, in which the insured has to choose. So our fifth definition is exactly in the same spirit that the private value under asymmetric information. It is always positive and is strictly positive on condition that the agent's effort choice really depends on type.

to be finished

6 Conclusion

to be made.

7 Appendix

7.1 Proof of proposition 1

Consider first the situation in which an agent from a group K has not acquired additional information. Let \bar{C} the prior contract. We know that, according to the group, the best contract between the company and the agent is solution of :

$$\text{Max}_C E_K[V(C, \theta)] \text{ subject to the constraint that } E_K[\Pi(C, \theta)] \geq E_K[\Pi(\bar{C}, \theta)] = \pi$$

Such a contract involves perfect insurance so that

$$I_T = T, I_e = e, P = e + E_K[\sigma(e, \theta)]T + \pi \quad (20)$$

The agent is perfectly insured and gets an expected utility $U(w - P) = U(w - e - E_K[\sigma(e, \theta)]T - \pi)$ so the best effort minimizes the insurance premium in E , that is $e = e^*$.

Now assume that the agent obtains the additional information. The best contract taking into account the information is solution of

$$\text{Max}_C E_K[V(C(\theta), \theta)] \text{ subject to the constraint that } E_K[\Pi(C(\theta), \theta)] \geq E_K[\Pi(\bar{C}, \theta)] = \pi$$

The solution of this program is presented at the end of the proof. It entails a perfect insurance contract, with respect to type θ and to accident that is

$$I_T(\theta) = T, I_e(\theta) = e(\theta), P(\theta) = P = E[e(\theta) + \sigma(e(\theta), \theta)T] + \pi$$

The agent obtains whatever his type an expected utility equal to $U(w - E[e(\theta) + \sigma(e(\theta), \theta)T] - \pi)$ and obviously the optimal effort minimizes $E[e(\theta) + \sigma(e(\theta), \theta)T]$ in E , that is $e^*(\theta)$. Information has thus some values on condition that

$$\text{Min}_{e(\theta) \in E} E[e(\theta) + \sigma(e(\theta), \theta)T] < \text{Min}_{e \in E} \{e + E_K[\sigma(e, \theta)]T\} \quad (21)$$

This occurs since the effort $e(\theta)$ effectively depends on θ , or in other words at least two types make different efforts. A sufficient condition is that $\sigma_{e\theta}(e, \theta) \neq 0$ and the solution is interior in E .

Let us present the solution of the following program, for a given $e(\theta)$:

$$\text{Max}_{C(\theta)} E_K[V(C(\theta), \theta)] \text{ subject to } E_K[\Pi(C(\theta), \theta)] \geq \pi$$

Define L the Lagrangian of this program:

$$L = E_K[V(C(\theta), \theta)] + \mu E_K[\Pi(C(\theta), \theta)] \quad (22)$$

The CNO are given by equations hereafter:

$$\frac{\partial L}{\partial P(\theta)} = -\lambda(\theta)\{(1 - \rho)U'_S + \rho(1 - \sigma)U'_E + \sigma p U'_T\} + \mu\lambda(\theta) = 0 \quad (23)$$

$$\frac{\partial L}{\partial I_e(\theta)} = \lambda(\theta)\rho\{-(1 - \sigma)U'_E - \sigma U'_I\} - \mu\lambda(\theta)\rho = 0 \quad (24)$$

$$\frac{\partial L}{\partial I_T(\theta)} = \lambda(\theta)\rho\sigma U'_I - \mu\lambda(\theta)\rho\sigma = 0 \quad (25)$$

Rearranging these equations finally gives:

$$\{(1-p)U'_S + p(1-\sigma)U'_E + \sigma pU'_I\} = \mu = \{(1-\sigma)U'_E + \sigma U'_I\} = U'_I \quad (26)$$

$$\text{and } U'_S = U'_E = U'_I = \mu. \quad (27)$$

The optimal solution is a perfect insurance contract, with $P(\theta) = P$, $I_e(\theta) = e(\theta)$ and $I_T(\theta) = T$. As $\mu > 0$, we obtain from the constraint the definition of premium P :

$$P = E_K[\rho(\theta)\{e(\theta) + \sigma(e(\theta), \theta)T\}] - \pi$$

7.2 Proof of proposition 3

Assume that the insurance companies observe the result of the test and know the agent's group. If an agent of group K decides not to take the test, he is proposed C_K^{PI} which entails a full coverage ($I_T = T$ and $I_e = e$) and a fair premium $P = E_K[\rho(\theta)(e + T\sigma(e, \theta))]$. When conversely he takes the test, the competitive contract involves a full coverage ($I_T(\theta) = T$ and $I_e(\theta) = e(\theta)$) and a fair premium relative to the revealed type that is $P(\theta) = \rho(\theta)(e(\theta) + T\sigma(e(\theta), \theta))$. So the optimal effort minimizes the premium $E_K[\rho(\theta)(e + T\sigma(e, \theta))]$ when the agent is uninformed ($e = e^*$) and $\rho(\theta)(e(\theta) + T\sigma(e(\theta), \theta))$ when he is ($e(\theta) = e^*(\theta)$).

Now consider the first stage. The agent has to decide whether to take or not the test and expects the market equilibrium offers presented above. So finally he decides to become informed if:

$$E_K[U(w - \rho(\theta)Min_e(e + \sigma(e, \theta)T))] > U(w - Min_e E_K[\rho(\theta)\{e + \sigma(e, \theta)T\}])$$

Assume first that information has no value, i.e. $e^*(\theta) = e^*$. Comparing the expected utilities gives $U(w - E_K[\rho(\theta)(e + T\sigma(e, \theta))])$ when the agent is uninformed and $E_K[U(w - \rho(\theta)(e + T\sigma(e, \theta)))]$ when informed gives, as long as the agent is risk averse :

$$E_K[U(w - \rho(\theta)(e + T\sigma(e, \theta))] < U(w - E_K[\rho(\theta)(e + T\sigma(e, \theta))]).$$

As his expected utility is smaller when he obtains the additional information, he always chooses not to take the test.

Assume now that the information has some value and $e^*(\theta)$ really depends on type. The agent must compare two lotteries, the second one being certain but having a smaller expectation. Equation () can be rewritten as where Γ designs the risk premium:

with

$$E_K[U(w - \rho(\theta)Min_e(e + \sigma(e, \theta)T))] = U(w - E_K[\rho(\theta)Min_e(e + \sigma(e, \theta)T)]) - \Gamma$$

$$E_K[\rho(\theta)Min_e(e + \sigma(e, \theta)T)] + \Gamma < Min_e E_K[\rho(\theta)\{e + \sigma(e, \theta)T\}]$$

When $\Gamma = 0$ the inequality always holds, and never for Γ high enough.

Whenever the agent is risk neutral, he will become informed, when he become more risk averse, he prefers ignoring the additional information. ■

7.3 Proof of proposition 4

Assume in a first step that the company ignores the agent's group. Let C_A and C_B the contract proposed by an insurance company, and denote by $\bar{\pi}$ its expected profit.

Note now that the best solution for the agent from an ex ante point of view, ignoring the incentive constraints is the solution of the program:

$$\text{Max } \lambda_A E_A[V(C_A(\theta), \theta)] + \lambda_B E_B[V(C_B(\theta), \theta)]$$

$$\text{subject to the constraint } \lambda_A E_A[V(C_A(\theta), \theta)] + \lambda_B E_B[V(C_B(\theta), \theta)] \geq \bar{\pi}$$

Define by $\{\widehat{C}_A(\theta), \widehat{C}_B(\theta), \theta \in \Theta\}$ the solution of this program. At the optimum, the information about the group does not matter, $\widehat{C}_A(\theta) = \widehat{C}_B(\theta) = \widehat{C}(\theta)$. $\widehat{C}(\theta)$ is a perfect insurance contract (with $I_T(\theta) = T$, $I_e(\theta) = e(\theta)$) with a premium

$$\widehat{P} = E_K[\rho(\theta)\{e^*(\theta) + \sigma(e^*(\theta), \theta)T\}] + \bar{\pi} \quad (28)$$

and the efforts are $e(\theta) = e^*(\theta)$ for $\theta \in \Theta$.

So the agent whatever his type has an ex post expected utility equal to

$$\widehat{U} = U(w - E[\rho(\theta)\{e^*(\theta) + \sigma(e^*(\theta), \theta)T\}] - \bar{\pi}). \quad (29)$$

Note that the solution is unique, and obviously incentive compatible. Indeed, any agent θ from one given group prefers to reveal both his group and his type, once testing has occurred..

Furthermore, any menu of contract acceptable by the insurance company (and different from $\{\widehat{C}_A(\theta), \widehat{C}_B(\theta), \theta \in \Theta\}$) is such that $\lambda_A E_A[V(C_A(\theta), \theta)] + \lambda_B E_B[V(C_B(\theta), \theta)] < U^*$. Two cases are thus possible.

In the first one, the prior contract gives an expected utility smaller than U^* (or $E_K[V(C_K, \theta)] \leq U^*$ for $K = A, B$, the inequality being strict for at least one group). When this occurs, offering the optimal menu $\{\widehat{C}_A(\theta), \widehat{C}_B(\theta), \theta \in \Theta\}$ allows to increase the expected utility of both groups (and this increase is strict for one of the group).

In the second case, one group of agent prefers the prior contract, $E_A[V(C_A, \theta)] > U^*$, on the contrary to the other group, $E_B[V(C_B, \theta)] < U^*$. Note that the prior menu of contract $\{C_A, C_B\}$ satisfies the incentive constraints

$$E_A[V(C_A, \theta)] \geq E_A[V(C_B, \theta)] \quad (30)$$

$$E_B[V(C_B, \theta)] \geq E_B[V(C_A, \theta)] \quad (31)$$

Let π_B the expected profit made when the agent's group is B. And assume that the insurance company proposes the menu $\{C_A, \widetilde{C}_B(\theta)\}$. $\widetilde{C}_B(\theta)$ is a perfect insurance contract with the effort $e^*(\theta)$ and the premium $\widetilde{P} = E_B[\rho(\theta)\{e^*(\theta) + \sigma(e^*(\theta), \theta)T\}] + \pi_B$. Such a menu increases the expected utility of agents in group B (who now obtains $U(w - \widetilde{P})$), who then prefer announcing they are B, pass the test and insure under the contract $\widetilde{C}_B(\theta)$. Moreover, $\widetilde{C}_B(\theta)$ gives to the agent B an expected utility smaller than $E_B[V(C_B, \theta)] < E_B[V(\widetilde{C}_B(\theta), \theta)] < U^*$. Then the menu satisfies the incentive constraints under which an agent from group A prefers C_A to pass the test and have $\widetilde{C}_B(\theta)$. Indeed, we have:

$$E_A[V(C_A, \theta)] \geq U^* > U(w - \widetilde{P}) = E_A[V(\widetilde{C}_B, \theta)] = \quad (32)$$

$$E_B[V(\widetilde{C}_B(\theta), \theta)] > E_B[V(C_B, \theta)] \geq E_B[V(C_A, \theta)] \quad (33)$$

Finally, whatever the prior contract, we have found a contract which dominates it in the Pareto sense. Such a contract uses the information on condition that $e^*(\theta)$ really depends on θ . The conditions under which the information has some value are thus identical, the information revealed by the test being private or not.

When the insurance company knows the group to which the agent prior belongs, the reasoning is the same, and can be easily deduced from the preceding by assuming only one group exists. ■

7.4 Proof of proposition 5

We present here the properties of the RS contract when testing has not occurred. The program 3 can be simplified in the following

$$\text{Max } E_A[V(C, \theta)] \text{ subject to } E_A[\Pi(C, \theta)] \geq 0 \text{ and } E_B[V(C, \theta)] \leq U^*$$

Let L the Lagrangian of the program:

$$L = E_A[V(C, \theta)] + \mu E_A[\Pi(C, \theta)] - \lambda E_B[V(C, \theta)]$$

The optimality conditions give:

$$\{E_A[\sigma\rho] - \lambda E_B[\sigma\rho]\}U'_T = \mu E_A[\rho\sigma] \quad (34)$$

$$\begin{aligned} E_A[\rho\{(1-\sigma)U'_E + \sigma U'_T\}] - \lambda E_B[\rho\{(1-\sigma)U'_E + \sigma U'_T\}] &= \mu E_A[\rho] \\ \text{or } \{E_A[\rho(1-\sigma)] - \lambda E_B[\rho(1-\sigma)]\}U'_E &= \mu E_A[\rho(1-\sigma)] \end{aligned} \quad (35)$$

$$E_A[-\{(1-\rho)U'_S + \rho(1-\sigma)U'_E + \sigma\rho U'_T\}] - \lambda E_B[-\{(1-\rho)U'_S + \rho(1-\sigma)U'_E + \sigma\rho U'_T\}] = -\mu \quad (36)$$

These conditions imply that:

$$\{E_A[1-\rho] - \lambda E_B[1-\rho]\}U'_S = \mu \quad (37)$$

Thus, $\mu = 0$ leads to $E_A[1-\rho] = \lambda E_B[1-\rho]$, $E_A[\rho(1-\sigma)] = \lambda E_B[\rho(1-\sigma)]$

and $E_A[\sigma\rho] = \lambda E_B[\sigma\rho]$ implying than $\lambda = 1$ and as a consequence

$E_A[P - \rho(\theta)\{e + \sigma(e, \theta)T\}] = E_B[P - \rho(\theta)\{e + \sigma(e, \theta)T\}]$ which contradicts our assumptions. . As a consequence, C_B^{PI} is not a solution of the program, so we have

$$E_A[V(C_A^{RS}, \theta)] > E_A[V(C_B^*, \theta)].$$

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