This article is accepted for publication in *Developmental Psychology*.

This is the author's version of a work. Changes resulting from the publishing process, such as peer review, corrections, editing, and structural formatting, are not reflected in this document.

Young Children's Core Symbolic and Nonsymbolic Quantitative Knowledge in the Prediction of Later Mathematics Achievement

David C. Geary[1,2] and Kristy vanMarle[1]

University of Missouri

[1]Department of Psychological Sciences, University of Missouri

[2]Interdisciplinary Neuroscience Program, University of Missouri

*Correspondence addressed to GearyD@Missouri.edu

Corresponding author:

David C. Geary, Ph.D.
Curators' Professor
Department of Psychological Sciences
Interdisciplinary Neuroscience Program
210 McAlester Hall
University of Missouri
Columbia, MO 65211-2500
E-mail: GearyD@Missouri.edu
Ph: 573-882-6268

Acknowledgments

The study was supported by grants from the University of Missouri Research Board and DRL-1250359 from the National Science Foundation. We would like to thank Mary Rook for her help facilitating our assessments of the Title I preschool children. We are also grateful for the cooperation of Columbia Public Schools, and especially the children and parents involved in the study. We thank Felecia Chu, Mary Hoard, Lara Nugent, Tim Adams, Melissa Barton, Sam Belvin, Kaitlyn Bumberry, Lex Clarkson, Dillon Falk, Lauren Johnson-Hafenscher, Morgan Kotva, Bradley Lance, Kayla Legow, Natalie Miller, Lexi Mok, Molly O'Byrne, Rebecca Peick, Kelly Regan, Sara Schroeder, Claudia Tran, Hannah Weise and Grace Woessner, for help with data collection and other aspects of the project. We also thank several anonymous reviewers and Sara Gable for comments on an earlier version of this article.

Abstract

At the beginning of preschool (*M* = 46 months of age), 197 (94 boys) children were administered tasks that assessed a suite of nonsymbolic and symbolic quantitative competencies as well as their executive functions, verbal and nonverbal intelligence, preliteracy skills, and their parents' education level. The children's mathematics achievement was assessed at the end of preschool (*M* = 64 months). We used a series of Bayesian and standard regression analyses to winnow this broad set of competencies down to the core subset of quantitative skills that predict later mathematics achievement, controlling other factors. This knowledge included children's fluency in reciting the counting string, their understanding of the cardinal value of number words, and recognition of Arabic numerals, as well as their sensitivity to the relative quantity of two collections of objects. The results inform theoretical models of the foundations of children's early quantitative development and have practical implications for the design of early interventions for children at risk for poor long-term mathematics achievement.

Key words: quantitative knowledge, preschool, approximate number system, cardinal knowledge, mathematics learning, mathematics achievement

Young Children's Core Symbolic and Nonsymbolic Quantitative Knowledge in the Prediction of Later Mathematics Achievement

Children who score poorly on mathematics achievement tests at school-entry are at elevated risk for poor long-term mathematics outcomes (Duncan et al., 2007; Geary, Hoard, Nugent, & Bailey, 2013), and through this face compromised employment prospects in adulthood (Ritchie & Bates, 2013; Rivera-Batiz, 1992), as well as difficulties navigating the many now routine quantitative activities of daily life (Reyna, Nelson, Han, & Dieckmann, 2009). Identifying and remediating the preschool quantitative deficits that predict poor school-entry mathematical knowledge has the potential to substantially reduce these risks, and accordingly considerable resources have been devoted to these efforts in recent years. These efforts, however, have been quite diverse, ranging from a focus on domain-general abilities, such as executive functions (Bull, Espy, & Wiebe, 2008; Clark, Sheffield, Wiebe, & Espy, 2012; Clark, Pritchard, & Woodward, 2010) to the acuity of the inherent system for representing nonsymbolic quantities (Libertus, Halberda, & Feigenson, 2011, 2013; Star, Libertus, & Brannon, 2013) to early symbolic number knowledge, such as children's understanding of the cardinal value of number words (Chu, vanMarle, & Geary, 2015; vanMarle, Chu, Li, & Geary, 2014).

Each of these studies and many others like them has helped to identify competencies that are potentially foundational to children's school-entry mathematical knowledge. At the same time, the different foci across studies has resulted in a lack of coherence to the overall endeavor, a signal-to-noise ratio that has made it difficult to separate the quantitative knowledge critical to later achievement from less central knowledge. In an attempt to bring some coherence to these efforts, we conducted a broad assessment of potentially foundational domain-general and quantitative abilities identified in previous studies of preschool children and used Bayesian and standard statistical methods to determine the combination of competencies that best predicts their mathematics achievement just before entry into kindergarten.

Domain General Abilities

Measures of general intelligence are consistent predictors of academic achievement (Walberg, 1984), including mathematics (Deary, Strand, Smith, & Fernandes, 2007; Geary, 2011). Although there is some debate regarding the mechanisms underlying this relation, it is driven at least in part by ease of learning novel symbolic information (Carroll, 1993; Cattell, 1987), which is highly relevant to academic mathematics; that is, IQ measures index in part ease of symbolic learning (Geary, 2005; Geary & Moore, 2016).

Working memory is another commonly studied correlate of academic achievement, and the associated measures are consistently found to predict concurrent or prospective mathematics achievement (Bull & Lee, 2014; Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013). Much of the research is on older children's mathematical competencies and based on Baddeley and Hitch's (1978) working memory model, whereby a central executive orchestrates the manipulation of information represented in phonological and visuospatial short-term memory stores. However, the reliable measurement and differentiation of these subcomponents of working memory and the subcomponents of the central executive, such as updating and inhibition (Miyake et al., 2000), is difficult in preschool children (Allan & Lonigan, 2011; Garon, Bryson, & Smith, 2008; Wiebe, Espy, & Charak, 2008). As a result, most studies of preschoolers' working memory capacity use a single overall measure, typically called executive functions in this literature, and performance on these measures is a consistent predictor of later mathematics achievement (Blair & Razza, 2007; Bull et al., 2008; Clark et al., 2010).

Measures of reading ability are also consistently correlated with children's concurrent and prospective mathematics achievement, often above and beyond the influence of intelligence and working memory (e.g., Fuchs, Geary, Fuchs, Compton, & Hamlett, 2016; Grimm, 2008). The relation may reflect an underlying reliance on language competencies for many aspects of reading and some aspects of mathematics (LeFevre et al., 2010; Spelke, 2016), such as the early learning of number words (below). The relation may also reflect the ease with which children form arbitrary visual-verbal associations (Koponen, Salmi, Eklund, & Aro, 2013). This would explain the often-found relation between reading skills and memorization of arithmetic facts (e.g., Geary, 1993; Hecht, Torgesen, Wagner, & Rashotte, 2001), both of which may reflect the functional integrity of the domain-general hippocampal-dependent memory system (Qin et al., 2014). Whatever the underlying mechanism, it seemed prudent to include a preliteracy measure in this study.

Early Quantitative Competencies

Nonsymbolic

Infants' and young children's core nonsymbolic quantitative competencies emerge from the functioning of the approximate number system (ANS). The ANS is an evolutionarily ancient system for representing and comparing the relative quantity of collections of discrete objects and for performing simple arithmetical operations on these representations (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 2000; for recent reviews see Geary, Berch, & Mann Koepke, 2015a). The signature of ANS functioning is that discrimination accuracy depends on the ratio between the two quantities being compared and not their absolute difference. So, discriminating 8 objects from 4 objects (a 2:1 ratio) is easier than discriminating 28 objects from 24 objects (a 1.17 ratio). Acuity of the ANS is inferred based on the smallest ratios people can reliably discriminate, and the relation between this acuity and young children's mathematics achievement has been extensively studied (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2013; Libertus et al., 2011; Halberda, Mazzocco, & Feigenson, 2008; Mussolin, Nys, & Leybaert, 2012; Star et al., 2013; Wang, Odic, Halberda, & Feigenson, 2016). Although the results are mixed overall, several recent meta-analyses confirmed a consistent relation between ANS acuity and mathematics achievement in young children, *r*s ~ .2 to .4 (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2016).

The critical question, however, is whether ANS acuity is a pillar for later mathematics achievement once other competencies are controlled, and this issue is vigorously debated (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013; Fuhs & McNeil, 2013; Reeve, Reynolds, Humberstone, & Butterworth, 2012; Rousselle, Palmers, & Noël, 2004; Iuculano, Tang, Hall, & Butterworth, 2008). Accordingly, we included a standard computer-based measure of ANS acuity (Halberda et al., 2008). We also included a second measure of children's sensitivity to relative quantity but using actual and desirable objects, based on the assumption that comparing actual collections of objects might be a better measure of ANS acuity – in terms of mapping more transparently to the types of activities that drove the evolution of the ANS – than the standard computer-based measure. This ordinal choice task has been successfully used to assess sensitivity to more versus less with preverbal infants (Feigenson, Carey, & Hauser, 2002; vanMarle, 2013; vanMarle & Wynn, 2011) and with nonhuman primates (vanMarle, Aw, McCrink, & Santos, 2006).

As noted, the ANS is also thought to embody an intuitive understanding of simple arithmetic, such as knowing that adding objects to a collection results in a larger collection whereas removing objects results in a smaller collection (Barth, La Mont, Lipton, & Spelke, 2005; McCrink, 2015; Slaughter, Kamppi, & Paynter, 2006). This intuitive knowledge may guide young children's learning of how to use counting to solve symbolic arithmetic problems (Gelman, 2006), and their understanding of the relations among numerals; e.g., that '5' is composed of '3 and 2' or '4 and 1' (Moore, vanMarle, & Geary, 2016). A thorough assessment of potential foundational quantitative knowledge should therefore include a measure of nonverbal calculation abilities (Levine, Huttenlocher, & Jordan, 1992).

The focus on discrete quantity discriminations or manipulations in all of these studies makes theoretical sense in terms of the core functions of the ANS, but children can also discriminate continuous quantities, such as area (Defever, Reynvoet, & Gebuis, 2013; Lourenco, 2015; Mix, Huttenlocher, & Levine, 2002). In fact, this approximate magnitude system (AMS) may be the evolutionary precursor to the ANS (Geary, Berch, & Mann Koepke, 2015b), and there is evidence that acuity of these discriminations may be related to some aspects of mathematical performance above and beyond sensitivity to discrete quantities (Bonny & Lourenco, 2015; Lourenco & Bonny, 2016; Lourenco, Bonny, Fernandez, & Rao, 2012). To assess this possibility, we included a measure of sensitivity to relative area. The final nonsymbolic measure assessed children's ability to track changes in the quantities of sets of three or fewer objects (vanMarle & Wynn, 2001), because this ability may be dependent on the object tracking system (OTS) rather than the ANS (Feigenson et al., 2002; vanMarle, 2013). Although the OTS is not an evolved quantitative system per se, it may contribute to children's initial learning of the cardinal values of the first three number words (Le Corre & Carey, 2007; Mou & vanMarle, 2014; Pylyshyn & Storm, 1988; vanMarle et al., in press).

Symbolic

Young children's symbolic quantitative abilities have been studied for decades and are well understood (Fuson, 1988; Geary, 1994; Gelman & Gallistel, 1978; Le Corre & Carey, 2007; Sarnecka & Carey, 2008; Siegler & Robinson, 1982; Wynn, 1990), but surprisingly only a handful of these studies have attempted to forge a link between these abilities and children's later mathematics achievement (Chu et al., 2015; Kolkman, Kroesbergen, & Leseman, 2013; Mussolin et al., 2012; vanMarle et al., 2014); the majority of such studies have focused on kindergarten or older children (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008).

Number words are young children's first symbolic quantitative knowledge, including the memorization of the standard count list, 'one, two, three…", and an understanding that the sequence of number words can be used to enumerate collections of items; that is, number words are applied in sequence as items are counted (Fuson, 1988; Gelman & Gallistel, 1978). However, the recitation of number words, even during enumeration tasks, does not necessarily indicate that children know the quantities these words represent; that is their cardinal values. In fact, coming to understand the cardinal values of the words in their count list is a protracted process, but once achieved represents children's first explicit understanding of a formal mathematical concept (Carey, 2004; Le Corre & Carey, 2007; Wynn, 1992). The give-a-number task can be used to illustrate children's progression to this insight (Wynn, 1992). Children are asked to provide x number of objects from a pile of objects. One-knowers provide one object when asked to do so but random numbers of objects for other number words. Similarly, two- to four-knowers learn the relation between the associated number words and quantity, but do not generalize cardinality to larger counting words (Le Corre & Carey, 2007). Children who score above this generalize to all number words and are considered cardinal-principle (CP) knowers.

Accordingly, a broad assessment of early symbolic quantitative abilities must, at the very least, include children's knowledge of the standard count list (verbal counting), their use of this list in a counting context (enumeration), and their understanding of the cardinal value of number words (give-a-number, point-to-*x*). During this time, many children are also learning to associate number words with the corresponding Arabic numerals (Siegler & Robinson, 1982), and thus we also included a numeral recognition measure.

Current Study

The children participating in this study were enrolled in Title 1 preschool, a federally funded two-year program for three- to five-year-old children at risk for school failure. We focused on the relation between their just described nonsymbolic and symbolic quantitative competencies assessed during the first semester of the program; their domain-general abilities assessed mid-year; and, their standardized mathematics achievement at the end of the second year of preschool. The combination resulted in a unique study with both practical and theoretical implications. Identifying the most critical early quantitative abilities that predict at risk children's later mathematics achievement has clear implications for the design of preschool school interventions. Theoretically, the results address current and often acrimonious debate regarding which early quantitative abilities form the foundation for children's long-term mathematical development by discriminating central from less central early quantitative competencies.

Method

Participants

Two hundred and thirty-two children were recruited from the Title I preschool program within the public school system in Columbia, MO. Fourteen children were excluded from the analyses due to low IQ scores (IQ<70), and another 21 children were excluded because they did not complete the majority of tasks (e.g., due to inattention or moving out of district). The final sample included 197 (94 boys) children of average intelligence (*M* = 97, *SD* = 15) and average mathematics achievement (*M* = 95, *SD* = 14) at the end of preschool. The children were on average 3 years and 10 months of age (range 3y2m to 4y4m) at the time of the first assessment and 5 years and 4 months of age (range 4y9m to 5y10m) when the mathematics achievement test was administered.

Demographic information was obtained through parent survey for a subset (*n*=157) of the sample. The ethnic composition was 85% non-Hispanic, 10% Hispanic/Latino, and the remaining unknown. The racial composition was 55% White, 24% Black, 8% Asian, and 13% more than one race. The self-reported total household income was: $0-$25k (38%), $25k -$50k (23%), $50k-$75k (23%), $75k-$100k (12%), $100k-$150k (3%), $150k or more (<1%). Forty-two percent of respondents reported receiving food stamps, and 9% reported receiving housing assistance. Mother's education and father's education were highly correlated (r = .67, p<.001), and thus these variables were collapsed into parental education categories based on the highest attainment of the two parents (no information; high school degree or less; at least a college degree). Forty-five percent of the children had at least one parent with a high school degree or less and 35% had a parent with at least a college degree; the educational attainment of the remaining parents was unknown. To control for parental education, we created a three-class variable (college, high school, no information) that contrasted the mathematics achievement of children from college households to children in high school households and to children in households in which parents did not report educational level.

Quantitative tasks

The symbolic tasks required the recitation, recognition, or understanding of number words or Arabic numerals; specifically, *enumeration* (ability to apply the count list during the act of counting), *verbal counting* (memorization of standard count list), *give-a-number* (knowledge of the cardinal value of number words), *numeral recognition* (ability to verbally name numerals) and *point-to-x* (knowledge of the cardinal value of number words, and the ability to discriminate one quantity from another). The nonsymbolic tasks involved the discrimination or manipulation of quantitative information but could be done without the use of number words; specifically, *ordinal comparison* (ability to track and determine the set size of collections of discrete quantities), *discrete quantity discrimination* (a measure of ANS acuity), *continuous quantity discrimination* (a measure of AMS acuity), *magic box* (tracking small quantity manipulations within the limits of the OTS), and *nonverbal calculation* (implicit understanding of simple addition and subtraction).

Symbolic. For the *enumeration* task, children were shown an array of 20 stickers and asked to count them, pointing to each one. The score was the highest number counted before committing an error. The *verbal counting* task involved the child reciting the count list, starting from "one" and counting as high as they could without an error, or until they reached 100. In the *give-a-number* task (Wynn, 1990), children were asked to give the experimenter exactly 1, 2, 3, 4, 5, and 6 objects from a pile. Children began at set size 1 and advanced to the next set size after a correct response; if they were incorrect, they went down one set size. The highest number of objects they correctly gave the experimenter on at least 2 of 3 attempts was taken as the highest set size for which the child understood cardinality (LeCorre & Carey, 2007).

For the *numeral recognition* task, children were shown the Arabic numerals 1 to 15 one-at-a-time and in a random order. Children were asked to name each one, and the score was the total number of numerals correctly named. The *point-to-x* (Wynn, 1990) task required children to "point to the picture that has *x* objects". Children received two blocks of 6 trials with ratios ranging from .5 to .67 (1v2, 5v10, 2v3, 6v9, 4v7, and 5v8), with both exclusively large and small sets represented. On each trial, children saw two sets of pictured objects on a laptop display (one on the left and one on the right). The smaller number was the target on half of the trials, and the side on which the smaller set was displayed was counterbalanced across trials. Because the comparisons varied in difficulty, we generated a single score that was weighted for the difficulty of the comparison. The score was determined by multiplying each trial's score (incorrect=0 or correct =1) by the ratio of the comparison (e.g. 5v10=.5). Products were summed across trials to produce a single score weighted for the difficulty of the comparison (maximum score = 3.535), and the outcome was the percentage of the maximum score.

Nonsymbolic. During the *ordinal choice* task, children watched an experimenter sequentially hide two different numbers of objects (e.g., small toy fish) in two opaque cups; items were dropped into the cups one at a time. The children were then asked to pick the cup that contained more objects. There were 6 different comparisons (1v2, 2v3, 3v4, 4v5, 5v6, and 6v7). In order to successfully identify the larger quantity, children had to mentally sum the items in each set and then mentally compare the two sets. Because the comparisons varied in difficulty (i.e., ratios varied from .5 to .86), we generated a single score that was weighted for the difficulty of the comparison. This was done by first multiplying each trial's score (incorrect=0, correct=1) by the ratio of the comparison (e.g. 2v3=.67) and then summing the products across trials (maximum score = 4.41); the outcome was the percentage of the maximum score.

The *discrete quantity discrimination* task assessed the precision with which children represent discrete quantities of objects, and is a commonly used measure of ANS acuity. Using the Panamath program (Halberda et al., 2008), the first group of children (n = 70) we assessed received 24 test trials on a laptop computer; based on results for this group, we added 6 relatively easy trials for the remaining children. Each trial contained two sets of dots (blue and yellow); each set was contained within a rectangle, and children identified which set "had more dots". The color and side of the larger set was counterbalanced across trials. All dot arrays consisted of 5 to 21 dots and were displayed for only 2533 ms in order to discourage verbal counting. Ratios of blue:yellow dots were randomly selected for each trial and varied between 1.29 and 3.38; the ratio for the six added items ranged between 3.5 and 4.0. The ratios used were based on empirical findings and are standard for 3-year-olds (Halberda & Feigenson, 2008). We used accuracy (percent correct) as the outcome measure, because for children this is more reliable than the Weber fraction obtained from this task (Inglis & Gilmore, 2014). For ½ of the items, total surface area was controlled across the two sets of dots, and there was no difference in accuracy across trials with and without control of surface area; M = 67% (SD = 20) for items without this control, and M = 67% (SD = 19) for items with the control, p = .599. Because the ratio of dots for each trial was drawn randomly from binned ranges, individual children did not experience the exact same ratios, making it inappropriate to directly compare weighted scores. Indeed, when we computed weighted percent maximum scores, we found they were very highly correlated with percent correct scores ($r_{180} = .99$, $p < .0001$), thus we chose to use the standard percent correct as our dependent measure.

The *continuous quantity discrimination* task was conceptually similar, but children were asked to discriminate a continuous quantity, surface area. Children were presented with 24 test trials; in each trial, they were presented with a rectangle made of a blue and red proportion (4 trials at each of 6 red:blue ratios – 1:4, 1:3, 1:2, 2:3, 3:4, and 4:5). For each trial, children reported whether there was "more red" or "more blue" in the picture. As with discrete quantity discrimination, we used accuracy as the outcome measure.

The *magic box* task (vanMarle & Wynn, 2001) is a variant of Starkey's (1992) search-box task and was designed to assess children's implicit understanding of addition and subtraction. Children were first introduced to a puppet that they were told would sometimes perform a magic trick on items hidden in a box. Unbeknownst to the child, a false floor inside the box could be manipulated to create the illusion that an object had appeared or disappeared when the lid was closed. On each trial, the child watched an experimenter hide 0, 1, or 2 objects in the box. The experimenter then added or removed an object from the hidden set as the child watched. Children were not allowed to see the resulting set. The lid was closed and then opened to reveal either the correct result of the operation, or the incorrect result. There were 8 trials in this task, with a correct and an incorrect result for each of four problems: 0+1=1 or 0, 1+1=2 or 1, 1-1=0 or 1, 2-1=1 or 2. When the result was revealed, children were asked whether the puppet had done a magic trick. In order to correctly identify the incorrect outcomes as magical and the correct outcomes as not magical, the child needed to understand the effects of addition or subtraction of an object on set size. This task only required children to detect whether the operation was correct or incorrect, and did not require them to predict the exact result of the operation. Percent correct was the outcome measure.

For the *nonverbal calculation* task children were shown addition or subtraction of one or more discs from a hidden set of discs and then asked to predict the exact numerical result (Levine et al., 1992). Children watched an experimenter place a number of plastic discs in a line; the experimenter then covered the disks with a plate and added or removed some from under the plate. Children were asked to create a set of discs equal in number to the hidden set, but could also report the answer verbally. After four familiarization trials in which the children simply matched a hidden set, there were 12 test trials, presented in random order: 3-1, 2+2, 4-2, 1+3, 4-1 4+1, 3+2, 1+4, 5-2, 5-3, 2+4, and 6-4. Percent correct was the outcome measure.

Cognitive and Achievement Measures

Intelligence. The children completed the Receptive Vocabulary, Block Design, and Information subscales of the *Wechsler Preschool and Primary Scale of Intelligence – III* (Wechsler, 2002). Following standard procedures, scores were scaled and prorated to generate an estimate of Full Scale IQ (FSIQ), as reported above. As a contrast to FSIQ, the Block Design subscale was used as a measure of nonverbal IQ and the mean of the Receptive Vocabulary and Information subscales as a measure of verbal IQ; the national means are 10 for these subscales ($SD = 3$). Preliminary analyses revealed that nonverbal IQ and verbal IQ had different patterns of relations to later mathematics achievement (Results) and thus these separate measures were used in the analyses rather than FSIQ; the estimated reliability of the subscale scores ranges from .89 to .94; .96 for FSIQ.

Executive functions. Executive functions were assessed using the Conflict EF scale developed for children from 2 to 6 years of age (Beck, Schaefer, Pang, & Carlson, 2011). This scale consists of 6 levels; the first four included 2 subsections (5 trials each), while Levels 5, 6A, and 6B included 10 trials each. All children began on Level 2 following age-based procedures.

The Conflict EF scale consisted of a card-sorting task. Children were presented with two black plastic index card boxes with slots cut into the top of each; each box had a target card affixed to the front. Children were given a rule and asked to place a card in the appropriate box. Each level consisted of normal sorting trials, followed by conflict trials. For example, children placed the card in the corresponding box depending on whether the card was a “big kitty” or a “little kitty”; in conflict trials, children were asked to switch the rule, i.e., a “big kitty” would go in the “little kitty” box. In subsequent levels, children sorted the cards depending on shape or color of the card (again, the rule was reversed to create a conflict trial). More advanced levels required children to sort cards according to either shape or color depending on whether a black border was present or absent on the card. In order to move on to the next level, children had to complete four out of five trials correctly; in levels with ten trials, children had to complete four shape trials and 4 color trials correctly in order to proceed to the next level. The score was the total number of correct conflict trials (maximum score = 70). Beck et al. (2011) report intraclass correlations for test-retest reliability ranging from .75 to .80.

Preliteracy. To assess children’s preliteracy skills, one subtest of the *Phonological Awareness Literacy Screening-PreK* (PALS; Invernizzi, Sullivan, Meier, & Swank, 2004), Upper-Case Alphabet Recognition, was administered. This task was chosen because it is a reliable indicator of later reading ability (Blatchford, Burke, Farquhar, Plewis, & Tizard, 1987). Children were presented with capital letters in the alphabet (a few at a time) and asked to identify each letter. The score was the total number of letters correctly identified.

Mathematics achievement. The *Test of Early Mathematical Ability-3* (TEMA-3; Ginsburg & Baroody, 2003) was used to assess end of preschool mathematics achievement. This test is a nationally normed (*M*=100, *SD*=15) measure of young children’s mathematical competencies, and includes items such as producing finger displays to represent different quantities, counting, making numerical comparisons, and some arithmetic and number line items. All children started on the first item of the test and continued until they failed five consecutive items. Reliabilities based on internal consistency are >.92 and alternate form and test-retest reliabilities are > .80.

Overlap of some of the TEMA-3 items with the quantitative tasks creates a potential confound, that is, the relation between early quantitative performance and later mathematics achievement could be due to item similarity. To examine this, we compared the content of the mean number of items (*n* = 19, *SD* = 8) passed on the TEMA-3 (Form B) to the four quantitative tasks that emerged as predictors of later achievement (below), give-a-number, verbal counting, ordinal comparison, and numeral recognition. In the first 19 TEMA-3 items, there are six that overlap with these tasks, and seven that overlap the first 27 items. Although there is some overlap, performance on the TEMA-3 provides a broader assessment of mathematics achievement than provided by these four quantitative tasks.

Procedure

The quantitative, intelligence, executive function, and preliteracy measures were assessed across three ~35 min testing sessions during the first semester and early second semester of the first year of preschool year (see vanMarle et al., 2014). The TEMA-3 was administered at the end of the second year of preschool.

Analyses

The 6.6% of missing data were estimated using the multiple imputations procedure of SAS (2004). With the exception of sex and parental education, all variables were standardized (*M* = 0, *SD* = 1). We used Bayes factors and standard regression techniques to make decisions about the best domain general and quantitative predictors of end of preschool mathematics achievement (Dienes, 2014; Gallistel, 2009; Raftery, 1995; Rouder & Morey, 2011; 2012; Rouder, Speckman, Sun, Morey, & Iverson, 2009; Wagenmakers, 2007). The Bayes factors were computed for regression models (Liang, Paulo, Molina, Clyde, & Berger, 2008; Rouder & Morey, 2012) using the BayesFactor package (version 0.9.12-2) for R (Morey & Rouder, 2015).

Alternative models are presented for the sets of domain general variables, symbolic quantitative variables, nonsymbolic quantitative variables, combined quantitative variables, and the combination of the best domain general and quantitative variables from the initial analyses.

The first set of analyses Bayes factors are noted as MDG_m, where m = the specific set of domain general (DG) predictors in the model, and comparisons as BDG_{mn}, with B representing Bayes factor and m and n representing the compared models. BDG_{m0} represents a contrast of the null model, with no predictors in the analysis, and alternative models. These analyses assess the likelihood of the data for alternative models. The approach allowed us to observe the degree of evidence in favor of one model versus another as contrasted with observing evidence in relation to the null model as in traditional regression analyses (Rouder & Morey, 2012). As an example, model MDG_1 included all four domain general predictors and MDG_2 dropped verbal IQ from this set. The comparison of the models (BDG_{12}) is an odds ratio that represents how probable the data of MDG_2 are in explaining mathematics achievement as compared to MDG_1. In this case (below) the ratio was .8739, meaning the data are 87.39% as probable without verbal IQ as with it. The same notations are used for the other sets of variables, but with S (symbolic), NS (nonsymbolic), C (combined quantitative), and A (all) substituted for DG in the model names. We elected to include a step that combined the symbolic and nonsymbolic measures to account for potentially high collinearity between them, but the final model (below) is the same whether or not we include this step. The sequence of analyses allowed us to first identify the most important predictors among a conceptually similar set of predictors before proceeding to the final analyses.

Generally, ratios less than .33 are considered suggestive evidence for the importance of the variable and those less than .10 are considered strong evidence (Jeffreys, 1961; Raftery, 1995). Or stated differently, models that are less than 33% as probable without the variable provide evidence for retaining it in the model, and models that are less than 10% as probable provide strong evidence for retaining the variable. In addition to readily interpretable evidence for or against the inclusion of a variable, Bayes factors are more robust than standard linear regression in the selection of models that contain collinear predictors, as with our variables (see Table 1). The Bayes factor is higher when one of two highly correlated variables are included in relation to models containing both or none, providing the ability to compare the relative contribution of the predictors to the outcome measure (as odds ratios). In selecting models, we first compared all possible combinations of the sets of predictors noted above, and identified the best combination of predictors as the initial model. The approach is susceptible to type-I errors, and thus we also tested alternative models in which each predictor in the best model was dropped, one at a time, and the resulting model evaluated in terms of odds ratios. The best set of predictors in the model that included the best domain general and quantitative predictors was then regressed, along with demographic factors, on mathematics achievement using standard regression analyses.

For the Bayes analyses, give-a-number was included as a continuous variable, because of difficulties in modeling knower level within the BayesFactor package. For the regression analyses, five knower-level categories were created (e.g., Le Corre & Carey, 2007; Lee & Sarnecka, 2011); due to small *n*s, pre-knowers ($n = 12$) were included with one-knowers, and five-knowers ($n = 7$) were included with four-knowers. In all, there were 39 pre- and one-knowers (hereafter one-knowers), 53 two-knowers, 25 three-knowers, 31 four- and five-knowers (hereafter four-knowers), and 49 CP-knowers.

Results

Bayes Factors

As shown in the top section of Table 2, the best Bayes model included all four domain general variables in the prediction of later mathematics achievement. The BDG_{m0} is very large for this first model and all alternative models, providing strong evidence in support of some combination of domain general predictors relative to the null. As noted, dropping verbal IQ resulted in a model that was 87.39% as probable as the first model, and based on Jeffreys' (1961) and Raftery's (1995) criteria suggest that verbal IQ does not substantively add to the prediction of mathematics achievement above and beyond the other variables. The remaining models, however, provide strong evidence for the importance of each of the three other variables, and thus these were retained for the final analysis.

As shown in Table 2, the best set of symbolic predictors of later mathematics achievement included given-a-number, verbal counting, and numeral recognition. Adding enumeration to this set resulted in a model that was 53.58% as probable as the best model (not shown in Table 2), and adding point-to-x resulted in a model that was 14.21% as probable, providing positive evidence for excluding these variables; that is, the data are more likely without these variables than with them. Dropping each variable one at a time, resulted in models that were .01% (numeral recognition), .7% (give-a-number) and 6.8% (verbal counting) as probable as the model that included them, thus providing strong evidence for the importance of each of these variables.

The best set of nonsymbolic predictors included ordinal comparison, continuous quantity discrimination, and nonverbal calculation. Adding discrete quantity discrimination to this set resulted in a model that was 51.13% as probable as the best model (not shown in Table 2), and adding magic box resulted in a model that was 23.44% as probable, providing positive evidence for excluding these variables. Dropping each variable one at a time resulted in models that were < .00001% (nonverbal calculation), 86.61% (continuous quantity discrimination) and 2.25% (ordinal comparison) as probable as the model that included them. The models thus provide strong evidence for retaining nonverbal calculation and ordinal comparison and suggest that continuous quantity discrimination does not substantively add to the prediction of later mathematics achievement, above and beyond these two other variables.

The combined quantitative analyses included give-a-number, verbal counting, numeral recognition, ordinal comparison, and nonverbal calculation, and each of these was included in the best fitting model. As shown in Table 2, there is strong evidence for retaining each of these variables, except for nonverbal calculation (see MC_5). The latter was therefore dropped for the final analysis.

The final (all) models thus included nonverbal IQ, preliteracy, executive functions, give-a-number, verbal counting, numeral recognition, and ordinal comparison. The best model included nonverbal IQ, preliteracy, given-a-number, verbal counting, and ordinal comparison. Adding executive functions to this set resulted in a model that was 15.63% as probable as the best model (not shown in Table 2), and adding numeral recognition resulted in a model that was 6.63% as probable, providing positive evidence for excluding these variables. Dropping variables one at a time provided strong evidence for the importance of verbal counting (the data were .72% as probable without this variable), give-a-number (3.37% as probable), and preliteracy (<.00001% as probable), and suggestive evidence for nonverbal IQ (18.93% as probable). In contrast, dropping ordinal comparison resulted in a model was only 50.27% as probable as the model that included this variable. The latter suggests ordinal comparison is not critical to the prediction of later mathematics achievement above and beyond the other variables in the best model. Nevertheless, we decided to retain it for the standard regression analyses, because it was the best nonsymbolic quantitative variable to emerge in the final models.

Regression Analyses

The regression results for the demographic factors (age, sex, parental education contrast) and the key variables to emerge in the Bayes factor analyses are shown in the first set of results in Table 3, $R^2 = .56$, $F(12,184) = 19.91$, $p<.0001$; for the record, a regression model that included all variables is in the appendix (Table A1). None of the demographic factors were significant (*p*s > .05) but all of the domain general and quantitative variables were, consistent with the Bayes factor results. Of these variables, the evidence that emerged from the Bayes analyses was the weakest for ordinal comparison, but the regression results suggest retention of this variable ($\beta = .11$, $p = .0343$). As noted, give-a-number was included as a categorical variable in these analyses and the overall effect was significant, $F(4,184) = 2.57$, $p = .0396$. The critical results are the contrasts of CP knowers with children at the various knower levels. These contrasts revealed significantly lower end of preschool mathematics achievement for one knowers ($\beta = -.58$, $p < .0022$) and two knowers ($\beta = -.41$, $p < .0134$) relative to CP knowers, controlling all other variables in the model.

In a post-hoc analysis we substituted numeral recognition for preliteracy in the regression equation; second set of results in Table 3, $R^2 = .52$, $F(12,184) = 16.38$, $p<.0001$. We did this because of the substantial correlation between these two variables, $r = .71$, $p < .001$ (Table 1). In fact, these variables showed the highest correlation among all of the predictors used in this study and as a result one or the other would almost certainly be excluded from the same Bayes factor models, meaning the importance of the excluded variable may be underestimated. As can be seen in Table 3, all of basic results from the first regression analysis are the same in this new analysis, and the effect for numerical recognition is highly significant ($\beta = .30$, $p < .0001$).

A second post-hoc analysis assessed whether accuracy on the discrete quantity discrimination and magic box tasks predicted ordinal choice based on the potential contributions of the ANS and OTS to performance on these tasks (Barth et al., 2005; Feigenson et al., 2002; vanMarle, 2013), controlling age, parental education, preliteracy skills, nonverbal intelligence, and the three-counting related tasks (i.e., give-a-number, verbal counting, enumeration). We conducted this analysis, because the ordinal choice measure included comparisons of both smaller (< 4) and larger (> 3) sets and thus could also engage the OTS for some trials (Feigenson et al., 2002), and trials of any size might be solved using verbal counting. Accuracy on the discrete quantity discrimination task was the only significant quantitative predictor of ordinal choice performance ($\beta = .22$, $p = .0043$).

Discussion

The current study provides a more comprehensive analysis of the relation between young children's quantitative competencies and their later mathematics achievement than is typical in this literature and sought to identify both the core nonsymbolic and symbolic competencies that contribute to this achievement, controlling demographic factors, as well as domain general abilities and preliteracy skills. We begin with brief consideration of the domain general effects and then move to nonsymbolic and symbolic quantitative competencies, and finally the implications of our results.

Domain General Abilities

We discuss the results related to preliteracy below, and consider here implications for the emergence of nonverbal IQ as an important predictor of early mathematics achievement, and why verbal IQ and executive functions did not provided additional predictive utility. The finding for executive functions in particular is surprising given the consistent relation between these measures and young children's concurrent or later mathematics achievement in previous studies (Blair & Razza, 2007; Bull et al., 2008; Clark et al., 2010). Indeed, executive functions are significantly correlated with later mathematics achievement for our sample ($r = .25$, $p < .001$; Table 1), but did not emerge in the final set of domain general predictors. Many previous studies did not include simultaneous control of intelligence (Bull et al., 2008; Clark et al., 2010), and one study that did assessed preschool verbal IQ as a predictor of kindergarten mathematics achievement and concurrent kindergarten nonverbal IQ (Blair & Razza, 2007). Preschool executive functions but not verbal IQ predicted later mathematics achievement, consistent with our finding that executive functions was more strongly correlated with mathematics achievement than verbal IQ ($r = .16$, $p < .05$; Table 1). Kindergarten nonverbal IQ predicted concurrent mathematics achievement, but preschool executive functions, specifically inhibitory control as in our study, remained significant.

It is not clear whether the differences are due to the timing of the administration of the nonverbal IQ measure, but the across-study results for young children are consistent with the literature on the mathematical development of older children; specifically, both intelligence and executive functions (or the central executive component of working memory) contribute to ease of learning mathematics, but the relative contributions of one or the other can vary depending on the mathematical content being assessed and students' level of domain specific mathematical knowledge (Geary, 2011; Geary, Nicholas, Li, & Sun, 2016; Lee & Bull, 2016). These patterns however do not address the question of why nonverbal IQ emerged among our domain general predictors. The measure used here involved determining analogical relations among sets of pictures of concrete objects or abstract shapes and thus is a good measure of fluid intelligence (Holyoak, 2012). The latter in turn is a good predictor of ease of learning evolutionarily novel information (Geary, 2005), which includes most of symbolic mathematics. The implication here is that the foundations of children's mathematical competence include intelligence, in addition to any contributions of inherent number knowledge, number-related natural language competencies, and early domain-specific symbolic quantitative knowledge.

As noted, preschoolers' verbal IQ in our study and that of Blair and Razza (2007) was not predictive of later mathematics achievement, once other factors were controlled. In older children it is typically the case the both verbal IQ and nonverbal IQ or a composite measure that includes both (e.g. full scale IQ) predicts later mathematics achievement (Deary et al., 2007; Geary, 2011; Siegler et al., 2012). The reason for the age-related differences in the predictive utility of verbal IQ are unclear, but one possibility is that verbal IQ in older children, as reflected in vocabulary or general knowledge, is a better indicator of their earlier fluid intelligence than it is in younger children. As Cattell argued, " … this year's crystallized ability level is a function of last year's fluid ability level – and last year's interest in school work" (Cattell, 1987, p. 139). In other words, individual differences in the accumulated knowledge assessed by many verbal IQ tests reflect in part differences in fluid abilities, as measured by our verbal IQ test, but in younger children there has been less opportunity to accumulate the experiences that would eventually result in a correlation between verbal IQ and academic learning; both of which depending in part on fluid intelligence.

Nonsymbolic Quantitative Competencies

As we noted in the introduction, the majority of studies in this area have focused on the relation between the acuity of preschoolers' ANS and their concurrent or later mathematics achievement (Bonny & Lourenco, 2013; Libertus et al., 2013; Libertus et al., 2011; Mussolin et al., 2012; Star et al., 2013). In keeping with other studies, we too found a significant correlation between accuracy on the ANS task (i.e., discrete quantity discrimination) and later achievement (r = .22, p = .0019, Table 1), and the magnitude of this correlation was in the range (r ~ .2 to .4) estimated in several recent meta-analyses (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., in press). However, with control of demographic information, domain general measures, and other nonsymbolic quantitative competencies, the relation between this measure of ANS acuity and mathematics achievement was no longer significant.

Although ANS acuity may contribute to some aspects of children's emerging understanding of symbolic mathematics, such as their cardinal knowledge (Chu et al., 2015; vanMarle et al., 2014; vanMarle et al., in press), our results suggest that it is not broadly related to mathematics achievement. At the same time, we note that a recent study showed that presenting ANS items from easiest to hardest resulted in better performance on a subset of items on the TEMA-3 than presenting them from hardest to easiest (Wang et al., 2016). Random presentations, as in our study, resulted in intermediate TEMA-3 performance and thus our study might have under-estimated the contributions of the ANS.

In any event, a similar pattern emerged for the continuous quantity discrimination task; that is, it was correlated with mathematics achievement (r = .14, p = .052, Table 1) but was not significant once other variables were controlled. These findings are inconsistent with Lourenco and Bonny's (2016) recent finding that a combination of discrete and continuous quantity discrimination measures predicted performance across several number- and geometry-based mathematics achievement tests, controlling age and verbal IQ. Our Bayes results, however, suggest that verbal IQ might not be the best domain-general control variable for predicting young children's mathematics achievement, and this may have contributed to the different pattern of results. At the same time, given the focus on standardized mathematics achievement, our results do not address whether acuity of continuous quantity discriminations (area in this case) contributes to skill development in more specific symbolic quantitative areas, such as ease of learning about geometric shapes, an important part of preschool mathematics programs (e.g., Clements, Sarama, & Liu, 2008; Weiland et al., 2012). Analogously, as noted, there is some evidence that acuity of the ANS contributes to children's learning of cardinal value (Chu et al., 2015; vanMarle et al., 2014; vanMarle et al., in press), even if it is not consistently related to overall mathematics achievement, once other factors are controlled.

With control of no other variables, the Bayes analyses of only nonsymbolic measures provided evidence for the importance of ordinal comparison and nonverbal calculation in the prediction of later mathematical achievement, both of which could in theory engage the ANS (Barth et al., 2005; McCrink, 2015; vanMarle, 2013). However, nonverbal calculation was no longer critical with inclusion of the symbolic variables and ordinal comparison was no longer critical with inclusion of the domain general variables. We nevertheless retained ordinal comparison for the regression analyses, because it was the most important nonsymbolic quantitative variable to emerge from the Bayes analyses, and its significance in the regression analyses suggests a potential nonsymbolic contribution to later mathematics achievement, with control of many other factors. And, our post-hoc analysis revealed discrete quantity discriminations predicted ordinal choice performance, consistent with some level of ANS engagement. This raises the question of why our ANS acuity (i.e., discrete quantity discrimination) measure did not predict later mathematics achievement, controlling other factors. As mentioned in the introduction, one possibility is that for young children, the use of actual and desirable objects might be a better, more ecologically valid assessment of ANS acuity than the computer based dot comparison task, but this remains to be determined. The take away message here is that the search for contributions of the ANS to young children's learning of symbolic mathematics might include a broader assessment of ANS functioning.

Symbolic Quantitative Competencies

Children's performance on each of the five symbolic quantitative measures was significantly correlated with later mathematics achievement (Table 1), but only three of these – children's memorization of the count list, their recognition of Arabic numerals, and their understanding of the cardinal value of number words (Fuson, 1988; Wynn 1990) – emerged as important predictors with simultaneous consideration of all symbolic measures in the Bayes analyses. Each of these variables remained significant with inclusion of the nonsymbolic predictors but numeral recognition was no longer important with control of the domain general predictors, likely due to its high correlation with preliteracy skills (below). These findings speak to the central importance – above and beyond the influence of nonverbal IQ – of children's early learning of number words, and the quantities they represent (Fuson, 1988; Spelke, 2016), but also that some aspects of this knowledge are more critical early on than other aspects. The finding that children's skill at counting sets of objects (enumeration) did not predict mathematics achievement above and beyond their memorization of the count list and cardinal knowledge suggests the well-documented importance of counting for symbolic mathematics emerges later in development, that is, only after children learn number words and their meaning.

Our results also indicate that 3- to 4-year olds do not have to be CP knowers (Carey, 2004; Le Corre & Carey, 2007; Wynn, 1992) to score well on mathematics achievement tests when they are 5-years-old, but they do need to have taken the first few critical steps toward this end. In comparison to CP knowers and with control of other factors, one-knowers (20% of our sample) scored more than ½ of a standard deviation lower and two-knowers (27% of our sample) scored .4 standard deviations lower on the mathematics achievement test just months before entering kindergarten, putting them at heightened risk for longer term difficulties with mathematics (Duncan et al., 2007; Jordan et al., 2009). On top of these practical considerations, these results also speak to the importance of research on the mechanisms that support children's learning of cardinality, and this is where nonsymbolic quantitative abilities (i.e., the ANS and OTS), along with children's natural language abilities, may contribute to children's learning of symbolic mathematics (Carey, 2004; Chu et al., 2015; Le Corre & Carey, 2007; Spelke, 2016; vanMarle et al., 2014; vanMarle et al., in press).

The strong correlation between alphabet recognition (our preliteracy measure) and numeral recognition not only resulted in the exclusion of the latter in our final Bayes analyses; it suggests similar cognitive mechanisms or experiences contribute to individual differences on both measures. As we mentioned in the introduction, both alphabet recognition and numeral recognition could reflect ease of forming visual-verbal associations that contribute to some aspects of symbolic mathematics (e.g., memorizing basic arithmetic facts) and the often found correlation between reading achievement and mathematics achievement (e.g., Fuchs et al., 2016; Koponen et al., 2013); ease of associative learning in turn likley reflects, at least in part, the functional integrity of the frontal-hippocampal memory network (Qin et al., 2014). It might also be that parents who foster alphabet learning also foster numeral learning (LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010) in ways not strongly correlated with our parental education contrasts. Whatever the source of the relation, our results suggest that young children's recognition of numerals is a key precursor to later mathematics achievement.

Implications and Limitations

Our results suggest that preschool assessments and interventions for children at risk for poor long-term outcomes in mathematics should focus first on the children's acquisition of the count list, their understanding of these number words, and their recognition of Arabic numerals. This might seem like a common sense recommendation that does not need empirical justification, but needs to be considered in the context of typical preschool mathematics curricula that cover a much broader array of skills (e.g., Clements & Sarama, 2007). We are not advocating that other competencies, such as counting, simple arithmetic, and learning shapes, be dropped from any such curricula, but rather some skills may be more critical and foundational than others, and that these skills should be prioritized, especially for at-risk children.

The implications for improving young children's intuitive, nonsymbolic quantitative competencies are not as clear. There have been several successful interventions targeting adults' skill at making discrete quantity discrimination as related to their symbolic arithmetic skills (Park & Brannon, 2013, 2014). And, a recent study suggesting that the ordering of items within ANS tasks may prime young children's performance on symbolic mathematics tasks (Wang et al., 2016), but it is not clear from our results that improving young children's performance on these types of dot comparison tasks will yield long-term gains across all types of symbolic mathematics once other factors are controlled; the one critical exception, as we noted, appears to be for the initial learning of the cardinal value of number words (Chu et al., 2015; vanMarle et al., 2014; vanMarle et al., in press). This is not to say that targeting nonsymbolic competencies is not a potentially useful component of early interventions, but that for young children any such targeting might focus on more/less quantity comparisons using collections of real-world attention grabbing objects, as in our ordinal comparison task, rather than more abstracted collections of dots. At this time, however, firm conclusions cannot be drawn either way without experimental tests.

Of course, this is the case for all of our conclusions, given the correlational nature of our data, which is the strongest limitation of this study. To address this limitation, we attempted to control most known potential confounds and alternative explanations, but this does not mean that we identified all of them. For instance, we did not assess individual differences in children's spontaneous focus on numerosity during their engagement in non-quantitative tasks (Hannula, Lepola, & Lehtinen, 2010), given our already extensive array of assessments. Despite these limitations, the study provides a relatively large scale and broad longitudinal assessment that helps to reduce the signal-to-noise ratio in this literature by winnowing a broad range of young children's core nonsymbolic and symbolic quantitative competencies to a few competencies that are most central to their later mathematics achievement. In doing so, we provide straightforward directions for future theoretical and applied research in this area.

References

Allan, N. P., & Lonigan, C. J. (2011). Examining the dimensionality of effortful control in preschool children and its relation to academic and socioemotional indicators. *Developmental Psychology, 47*, 905-915. http://dx.doi.org/10.1037/a0023748

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*, *96*, 699-713. http://dx.doi: 10.1037/0022-0663.96.4.699

Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. H. Bower (Ed.), *The psychology of learning and motivation: Advances in research and theory* (Vol. 8, pp. 47-90). New York: Academic Press.

Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences USA, 102*, 14116-14121. http://dx.doi: 10.1073/pnas.0505512102

Beck, D. M., Schaefer, C., Pang, K., & Carlson, S. M. (2011). Executive function in preschool children: Test-retest reliability. *Journal of Cognition and Development, 12*, 169-193. http://dx.doi: 10.1080/15248372.2011.563485

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development, 78*, 647-663. http://dx.doi: 10.1111/j.1467-8624.2007.01019.x

Blatchford, R., Burke, J., Farquhar, C., Plewis, I., & Tizard, B. (1987). Associations between pre-school reading related skills and later reading achievement. *British Educational Research Journal, 13*, 15-23. http://dx.doi: 10.1080/0141192870130102

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology, 114*, 375-388. http://dx.doi: 10.1016/j.jecp.2012.09.015

Bonny, J. W., & Lourenco, S. F. (2015). Individual differences in children's approximations of area correlate with competence in basic geometry. *Learning and Individual Differences*, *44*, 16-24. http://dx.doi: 10.1016/j.lindif.2015.11.001

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functions in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology, 33*, 205-228. http://dx.doi: 10.1080/87565640801982312

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, *8*, 36-41. http://dx.doi:10.1111/cdep.12059

Carey, S. (2004). Bootstrapping and the origins of concepts. *Daedalus*, 59-68. http://dx.doi: 10.1162/001152604772746701

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. New York: Cambridge University Press.

Cattell, R. B. (1987). *Intelligence: Its structure, growth, and action*. Amsterdam: North -Holland.

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica, 148*, 163-172. http://dx.doi: 10.1016/j.actpsy.2014.01.016

Chu, F. W., vanMarle, K., & Geary, D. C. (2015). Early numerical foundations of young children's mathematical development. *Journal of Experimental Child Psychology, 132*, 205-212. http://dx.doi: 10.1016/j.jecp.2015.01.006

Clark, C. A., Sheffield, T. D., Wiebe, S. A., & Espy, K. A. (2013). Longitudinal associations between executive control and developing mathematical competence in preschool boys and girls. *Child Development, 84*, 662-677. http://dx.doi: 10.1111/j.1467-8624.2012.01854.x

Clark, C. A. C., Pritchard, V. E., & Woodward, L. J. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental Psychology, 46*, 1176-1191. http://dx.doi: 10.1037/a0019672

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks project. *Journal for Research in Mathematics Education, 38*, 136-163. http://dx.doi: 10.3102/0002831207312908

Clements, D. H., Sarama, J. H., & Liu, X. H. (2008). Development of a measure of early mathematics achievement using the Rasch model: the Research-Based Early Maths Assessment. *Educational Psychology*, *28*, 457-482. http://dx.doi: 10.1080/01443410701777272

Deary, I. J., Strand, S., Smith, P., & Fernandes, C. (2007). Intelligence and educational achievement. *Intelligence, 35*, 13-21. http://dx.doi: 10.1016/j.intell.2006.02.001

Defever, E., Reynvoet, B., & Gebuis, T. (2013). Task-and age-dependent effects of visual stimulus properties on children's explicit numerosity judgments. *Journal of Experimental Child Psychology*, *116*, 216-233. http://dx.doi.org/10.1016/j.jecp.2013.04.006

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non

-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*, 48-55. http://dx.doi: 10.1016/j.tine.2013.06.001

Dienes, Z. (2014). Using Bayes to get the most out of non-significant results. *Frontiers in Psychology*. http://dx.doi: 10.3389/fpsyg.2014.00781

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P. … & Sexton, H. (2007). School readiness and later achievement. *Developmental Psychology, 43,* 1428-1446. http://dx.doi: 10.1037/0012-1649.44.1.217

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123*, 53-72. http://dx.doi: 10.1016/j.jecp.2014.01.013

Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: Object-files versus analog magnitudes. *Psychological Science, 13,* 150-156. http://dx.doi: 10.1111/1467-9280.00427

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi: 10.1016/j.tics.2004.05.002

Friso-van den Bos, I., van der Ven, S. H., Kroesbergen, E. H., & van Luit, J. E. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review*, *10*, 29-44. http://dx.doi:10.1016/j.edurev.2013.05.003

Fuchs, L. S., Geary, D. C., Fuchs, D., Compton, D. L., & Hamlett, C. L. (2016). Pathways to third-grade calculation versus word-reading competence: Are they more alike or different? *Child Development*, *87*, 558-567. http://dx.doi: 10.1111/cdev.12474

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from Low-income homes: contributions of inhibitory control. *Developmental Science*, *16*, 136-148. http://dx.doi: 10.1111/desc.12013

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag. http://dx.doi: 10.1007/978-1-4612-3754-9

Gallistel, C.R. (2009). The importance of proving the null. *Psychological Review*, *116*, 439-453. http://dx.doi.org/10.1037/a0015251

Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, 4, 59-65. http://dx.doi: 10.1016/s1364-6613(99)01424-2

Garon, N., Bryson, S. E., & Smith, I. M. (2008). Executive function in preschoolers: A review using an integrated framework. *Psychological Bulletin, 134*, 31-60. http://dx.doi.org/10.1037/0033-2909.134.1.31

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345-362. http://dx.doi.org/10.1037/0033-2909.114.2.345

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association. http://dx.doi.org/10.1037/10163-000

Geary, D. C. (2005). *The origin of mind: Evolution of brain, cognition, and general intelligence*. Washington, DC: American Psychological Association. http://dx.doi.org/10.1037/10871-000

Geary, D. C. (2011). Cognitive predictors of individual differences in achievement growth in mathematics: A five year longitudinal study. *Developmental Psychology, 47*, 1539-1552. http://dx.doi: 10.1037/a0025510

Geary, D. C., Berch, D. B., & Mann Koepke, K. (Eds.) (2015a). *Evolutionary origins and early development of number processing* (Vol. 1, *Mathematical Cognition and Learning*). San Diego, CA: Elsevier Academic Press

Geary, D. C., Berch, D. B., & Mann Koepke, K. (2015b). The evolution of number systems. In D. C. Geary, D. B. Berch, & K. Mann Koepke (Eds.), *Evolutionary origins and early development of number processing* (pp. 337-355). San Diego, CA: Academic Press/Elsevier. http://dx.doi.org/10.1016/B978-0-12-420133-0.00013-2

Geary, D.C., Hoard, M. K., Nugent, L., & Bailey, H. D. (2013). Adolescents' functional Numeracy is predicted by their school entry number system knowledge. *PLoS ONE, 8*(1): e54651. http://dx.doi: 10.1371/journal.pone.0054651

Geary, D. C., & Moore, A. M. (2016). Cognitive and brain systems underlying early mathematical development. In M. Cappelletti and W. Fias (Eds.), *Progress in Brain Research: The Mathematical Brain Across the Lifespan* (Vol. 226, pp. xx-xx). Amsterdam: Elsevier. http://dx.doi:10.1016/bs.pbr.2016.03.008

Geary, D. C., Nicholas, A., Li, Y. & Sun, J. (2016) Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: An eight-year longitudinal study. (submitted).

Gelman, R. (2006). Young natural-number arithmeticians. *Current Directions in Psychological Science*, *15*, 193-197. http://dx.doi: 10.1111/j.1467-8721.2006.00434.

Gelman, R. & Gallistel, C. R. (1978). *The child's understanding of number.* Cambridge, Mass: Harvard University Press. http://dx.doi: 10.2307/1175067

Ginsburg, H. P., & Baroody, A. J. (2003). *Test of early mathematical ability* (3rd edition). Austin, TX: Pro-ed. http://dx.doi: 10.1177/073724770503000409

Grimm, K. J. (2008). Longitudinal associations between reading and mathematics achievement. *Developmental Neuropsychology, 33*(3), 410-426. http://dx.doi: 10.1080/87565640801982486

Halberda, J. & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*, 1457-1465. http://dx.doi: 10.1037/a0012682

Halberda, J., Mazzocco, M., & Feigenson L. (2008). Individual differences in nonverbal number acuity correlate with maths achievement. Nature, 455, 665-669. http://dx.doi: 10.1038/nature07246

Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of Experimental Child Psychology, 107*, 394-406. http://dx.doi: 10.1016/j.jecp.2010.06.004

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology*, *79*, 192-227. http://dx.doi: 10.1006/jecp.2000.2586

Holyoak, K. J. (2012). Analogy and relational reasoning. In K. J. Holyoak & R. G. Morrison (Eds.), *The Oxford handbook of thinking and reasoning* (pp. 234-259). New York: Oxford University Press. http://dx.doi: 10.1093/oxfordhb/9780199734689.013.0013

Huntley-Fenner, G., & Cannon, E. (2000). Preschoolers' magnitude comparisons are mediated by a preverbal analog mechanism. *Psychological Science*, *11*, 147-152. http://dx.doi: 10.1111/1467- 9280.00230

Inglis, M., & Gilmore, C. (2014). Indexing the approximate number system. *Acta Psychologica, 145*, 147-155. http://dx.doi: 10.1016/j.actpsy.2013.11.009

Invernizzi, M., Sullivan, A., Meier, J., & Swank, L. (2004). *PreK teacher's manual: Phonological awareness and literacy screening*. Charlottesville, VA: University of Virginia. http://dx.doi: 10.1037/t27727-000

Iuculano, T., Tang, J., Hall, C. W., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental science*, *11*, 669-680. http://dx.doi: 10.1111/j.1467-7687.2008.00716.x

Jeffreys, H. (1961). *Theory of probability*. New York: Oxford University Press.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*, 850-867. http://dx.doi: 10.1037/a0014939

Keller, L., & Libertus, M. (2015). Inhibitory control may not explain the link between approximation and math abilities in kindergarteners from middle class families. *Frontiers in Psychology*, *6*, 685. http://dx.doi: 10.3389/fpsyg.2015.00685

Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and Instruction*, *25*, 95-103. http://dx.doi: 10.1016/j.learninstruc.2012.12.001

Koponen, T., Salmi, P., Eklund, K., & Aro, T. (2013). Counting and RAN: Predictors of arithmetic calculation and reading fluency. *Journal of Educational Psychology, 105*(1), 162-175. http://dx.doi: 10.1037/a0029285

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395-438. http://dx.doi: 10.1016/j.cognition.2006.10.005

Lee, K., & Bull, R. (2016). Developmental changes in working memory, updating, and math achievement. *Journal of Educational Psychology*. http://dx.doi: 10.1037/edu0000090

Lee M.D. & Sarnecka, B.W. (2011). Number-knower levels in young children: Insights from Bayesian Modeling. *Cognition*, *120*, 391-402. http://dx.doi:10.1016/j.cognition.2010.10.003

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*, 1753-1767. http://dx.doi: 10.1111/j.1467-8624.2010.01508.x

LeFevre, J. A., Polyzoi, E., Skwarchuk, S. L., Fast, L., & Sowinski, C. (2010). Do home Numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education*, *18*, 55-70. http://dx.doi: 10.1080/09669761003693926

LeFevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science, 41*, 55-66. http://dx.doi: 10.1037/a0014532

Levine, S. C., Huttenlocher, J., and Jordan, N. (1992). Development of calculation abilities in young children. *Journal of Experimental Child Psychology, 53*, 72-93. http://dx.doi: 10.1016/s0022-0965(05)80005-0

Liang, F., Paulo, R., Molina, G., Clyde, M.A. & Berger, J.O. (2008). Mixtures of g-priors for Bayesian variable selection. *Journal of the American Statistical Association*, *103*, 410-423. http://dx.doi: 10.1198/016214507000001337

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Numerical approximation abilities correlate with and predict informal but not formal mathematics abilities. *Journal of Experimental Child Psychology*, *116*, 829-838. http://dx.doi: 10.1016/j.jecp.2013.08.003

Libertus, M. E. Halberda, J. & Feigenson, L. (2011). Preschool acuity of the Approximate Number System correlates with math abilities. *Developmental Science, 14*, 1292-1300. http://dx.doi: 10.1111/j.1467-7687.2011.01080.x

Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, *41*, 451-459. http://dx.doi: 10.1177/0022219408321126

Lourenco, S. F. (2015). On the relation between numerical and non-numerical magnitudes: Evidence for a general magnitude system. In D. C. Geary, D. B. Berch, & K. Mann Koepke, (Eds.), *Evolutionary origins and early development of number processing* (Vol. 1, *Mathematical Cognition and Learning*, pp. 145-174). San Diego, CA: Elsevier Academic Press. http://dx.doi: 10.1016/b978-0-12-420133-0.00006-5

Lourenco, S. F., & Bonny, J. W. (2016). Representations of numerical and non-numerical magnitude both contribute to mathematical competence in children. *Developmental Science*. http://dx.doi: 10.1111/desc.12418

Lourenco, S. F., Bonny, J. W., Fernandez, E. P., & Rao, S. (2012). Nonsymbolic number and cumulative area representations contribute shared and unique variance to symbolic math competence. *Proceedings of the National Academy of Sciences USA*, *109*, 18737-18742. http://dx.doi: 10.1073/pnas.1207212109

McCrink, K. (2015). Intuitive nonsymbolic arithmetic. In D. C. Geary, D. B. Berch, & K. Mann Koepke, (Eds.), *Evolutionary origins and early development of number processing* (Vol. 1, *Mathematical Cognition and Learning*, pp. 201-223). San Diego, CA: Elsevier Academic Press. http://dx.doi: 10.1016/b978-0-12-420133-0.00008-9

Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). Multiple cues for quantification in infancy: Is number one of them? *Psychological Bulletin*, *128*, 278-294. http://dx.doi: 10.1037/0033- 2909.128.2.278

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex “frontal lobe” tasks: A latent variable analysis. *Cognitive Psychology*, *41*, 49-100. http://dx.doi:10.1006/cogp.1999.0734

Morey, R.D., & Rouder, J.N. (2015). BayesFactor 0.9.12-2, Comprehensive R Archive Network.

Moore, A. M., vanMarle, K., & Geary, D. C. (2016). Kindergartners’ fluent processing of symbolic numerical magnitude is predicted by their cardinal knowledge and intuitive understanding of arithmetic 2 years earlier. *Journal of Experimental Child Psychology, 150*, 31-47. http://dx.doi.org/10.1016/j.jecp.2016.05.003

Mou, Y., & vanMarle, K. (2014). Two core systems of numerical representation in infants. *Developmental Review, 34*, 1-25. DOI: 10.1016/j.dr.2013.11.001

Mussolin, C., Nys, J., & Leybaert, J. (2012). Relationships between approximate number system acuity and early symbolic number abilities. *Trends in Neuroscience and Education*, *1*, 21-31. http://dx.doi: 10.1016/j.tine.2012.09.003

Park, J., & Brannon, E. M. (2013). Training the approximate number system improves math proficiency. *Psychological Science, 24*, 2013-2019. http://dx.doi: 10.1177/0956797613482944

Park, J., & Brannon, E. M. (2014). Improving arithmetic performance with number sense training: An investigation of underlying mechanism. *Cognition*, *133*, 188-200. http://dx.doi: 10.1016/j.cognition.2014.06.011

Pylyshyn, Z. W., & Storm, R. W. (1988). Tracking multiple independent targets: Evidence for a parallel tracking mechanism. *Spatial Vision, 3,* 179-197. http://dx.doi: 10.1163/156856888x00122

Qin, S., Cho, S., Chen, T., Rosenberg-Lee, M., Geary, D. C., & Menon, V. (2014). Hippocampal -neocortical functional reorganization underlies children's cognitive development. *Nature Neuroscience, 17*, 1263-1269. http://dx.doi:10.1038/nn.3788

Raftery, A.E. (1995). Bayesian model selection in social research. *Sociological Methodology*, *25*, 111-163. http://dx.doi: 10.2307/271063

Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and change in markers of core numerical competencies. *Journal of Experimental Psychology: General*, *141*, 649-666. http://dx.doi: 10.1037/a0027520

Reyna, V. F., Nelson, W., Han, P., & Dieckmann, N. F. (2009). How numeracy influences risk comprehension and medical decision making. *Psychological Bulletin, 135*, 943-973. http://dx.doi: 10.1037/a0017327

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*, 1301-1308. http://dx.doi: 10.1177/0956797612466268

Rivera-Batiz, F. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. *Journal of Human Resources, 27*, 313-328. http://dx.doi: 10.2307/145737

Rouder, J.N. & Morey, R.D. (2011). A Bayes factor meta-analysis of Bem's ESP claim. *Psychonomic Bulletin & Review*, *18*, 682-689. http://dx.doi: 10.3758/s13423-011-0088-7

Rouder, J.N. & Morey, R.D. (2012). Default Bayes factors for model selection in regression. *Multivariate Behavioral Research*, *47*, 877-903. http://dx.doi: 10.1080/00273171.2012.734737

Rouder, J.N., Speckman, P.L., Sun, D., Morey, R.D., & Iverson, G. (2009). Bayesian t-tests for accepting and rejecting the null hypothesis. *Psychonomic Bulletin and Review*, *16*, 225-237. http://dx.doi :10.3758/PBR.16.2.225

Rousselle, L., Palmers, E., & Noël, M. P. (2004). Magnitude comparison in preschoolers: What counts? Influence of perceptual variables. *Journal of Experimental Child Psychology*, *87*, 57-84. http://dx.doi: 10.1016/j.jecp.2003.10.005

Sarnecka, B. W. & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674. http://dx.doi: 10.1016/j.cognition.2008.05.007

Schneider, M., Beeres, K., Coban, L., Merz, S., Schmidt, S. S., Stricker, J., & De Smedt, B. (2016). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: a meta-analysis. *Developmental Science*. http://dx.doi: 10.1111/desc.12372

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., ... & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological science*, *23*, 691-697. http://dx.doi: 10.1177/0956797612440101

Siegler, R. S., & Robinson, M. (1982). The development of numerical understandings. In H. W. Reese & L. P. Lipsett (Eds.), *Advances in child development and behavior: Vol. 16* (pp. 242-312). New York: Academic Press. http://dx.doi: 10.1016/s0065-2407(08)60072-5

Slaughter, V., Kamppi, D., & Paynter, J. (2006). Toddler subtraction with large sets: Further evidence for an analog-magnitude representation of number. *Developmental Science*, *9*, 33-39. http://dx.doi: 10.1111/j.1467-7687.2005.00460.x

Spelke, E. S. (2016). *Core knowledge, language and number*. (submitted).

Star, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences USA, 110*, 18116–18120. http://dx.doi: 10.1073/pnas.1302751110

Starkey, P. (1992). The early development of numerical reasoning. *Cognition, 43*, 93-126. http://dx.doi: 10.1016/0010-0277(92)90034-f

vanMarle, K. (2013). Different mechanisms underlie infants' representation of small and large numbers in an ordinal choice task. *Journal of Experimental Child Psychology, 114*, 102-110. http://dx.doi: 10.1016/j.jecp.2012.04.007

vanMarle, K., Aw, J., McCrink, K., & Santos, L. R. (2006). How capuchin monkeys (*Cebus apella*) quantify objects and substances. *Journal of Comparative Psychology*, 120(4), 416-426. http://dx.doi: 10.1037/0735-7036.120.4.416

vanMarle, K., Chu, F.W., Li, Y., & Geary, D. C. (2014). Acuity of the approximate number system and preschoolers' quantitative development. *Developmental Science*, *17*, 492-505. http://dx.doi: 10.1111/desc.12143

vanMarle, K., Chu, F. W., Mou, Y., Seok, J. H., Rouder, J. N., &. Geary, D. C. (in press). Attaching meaning to the number words: Contributions of the object tracking and approximate number systems. *Developmental Science*.

vanMarle, K. & Wynn, K. (2001, April). Preschoolers' detection of magical events. Poster presented at the biannual conference of the Society for Research in Child Development (SRCD) in Minneapolis, MN.

vanMarle, K., & Wynn, K. (2011). Tracking and quantifying objects and non-cohesive substances. *Developmental Science*, 302-316. http://dx.doi: 10.1111/j.1467-7687.2010.00998.x

Wagenmakers, E.-J. (2007). A practical solution to the pervasive problem of p values. *Psychonomic Bulletin and Review*, *14*, 779-804. http://dx.doi: 10.3758/BF03194105

Walberg, H. J. (1984). Improving the productivity of America's schools. *Educational Leadership*, *41*, 19-27.

Wang, J. J., Odic, D., Halberda, J., & Feigenson, L. (2016). Changing the precision of preschoolers' approximate number system representations changes their symbolic math performance. *Journal of Experimental Child Psychology*, *147*, 82-99. http://dx.doi.org/10.1016/j.jecp.2016.03.002

Wechsler, D. (2002). *Wechsler Preschool and Primary Scale of Intelligence-III*. San Antonio, TX: The Psychological Corporation, Harcourt Brace & Co. http://dx.doi: 10.1037/t15177-000

Weiland, C., Wolfe, C. B., Hurwitz, M. D., Clements, D. H., Sarama, J. H., & Yoshikawa, H. (2012). Early mathematics assessment: Validation of the short form of a prekindergarten and kindergarten mathematics measure. *Educational Psychology*, *32*, 311-333. http://dx.doi: 10.1080/01443410.2011.654190

Wiebe, S. A., Espy, K. A., & Charak, D. (2008). Using confirmatory factor analysis to understand executive control in preschool children: I. Latent structure. *Developmental Psychology, 44*, 575-587. http://dx.doi.org/10.1037/0012-1649.44.2.575

Wynn, K. (1990). Children's understanding of counting. Cognition, 36(2), 155-193. http://dx.doi: 10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Children’s acquisition of the number words and the counting system. *Cognitive Psychology, 24,* 220-251. http://dx.doi: 10.1016/0010-0285(92)90008-p