1 Introduction

Photoacoustic imaging (PAI) is an emerging imaging technique with the unique capability of quantitatively imaging of optical absorption properties of endogenous tissue chromophores as well as extrinsic contrast agents (Zhang et al., 2006; Razansky et al., 2009; Agarwal et al., 2007). Therefore, tremendous clinical applications are envisioned including, for example, skin cancers (Oh et al., 2006), inflammatory joint diseases (Wang, Chamberland, and Jamadar, 2007), brain disorders (Yang et al., 2007), and eye diseases (Jiao et al., 2009). In PAI, biological tissue is usually irradiated by short laser pulses (a few ns). The locally absorbed optical energy is then converted into heat, which generates an ultrasonic wave via thermoelastic vibration. The out-propagating ultrasonic wave, which carries information of tissue’s optical absorption property, is then detected by ultrasonic transducer for imaging.

In reconstruction-based PAI with uniform illumination of the sample, many algorithms have been developed to exactly or approximately reconstruct the image when the ultrasonic transducer collects signals along a sphere, cylinder, or plane in the 3D case or along a circle or line in the 2D case (Kruger et al., 1995; Köstli et al., 2001; Xu and Wang, 2002; Xu, Feng and Wang, 2002; Xu, Xu and Wang, 2002; Köstli and Beard, 2003; Xu and Wang, 2005; Haltmeier, Schuster and Scherzer, 2005; Jiang, Yuan and Gu, 2005). For exact reconstruction, a single-element ultrasonic transducer usually needs to scan around the subject, or an ultrasound array is used to simultaneously acquire data from a full view, i.e. 4π steradians in the 3D case or 2π radians in the 2D case. However, in many potential applications of PAI, such as ophthalmic imaging and breast imaging, the object is only accessible from limited angles and, thus, only a limited-view signal acquisition is possible (i.e., the coverage of all view angles are less than 4π steradians in the 3D case or 2π radians in the 2D case). Although it is known theoretically that data collected from limited-view are sufficient for a complete reconstruction (Agranovsky and Quinto, 1996), this claim neither provides numerical reconstruction algorithms, nor guarantees that the reconstruction is practically stable. There is no exact reconstruction formula for the limited-view PAI yet; some features of the object to be imaged are impossible to reconstruct practically due to lack of information from only limited-view data.

To address the issue of insufficient information, numerous reconstruction algorithms have been proposed to incorporate some prior information about the object or missing data such that artifacts are
reduced. However, their results are still unsatisfactory with visible artifacts and loss of details. In this paper, a novel method is proposed to address the issue of artifacts in limited-view PAI by changing the physical setup of the imaging system.

The new method is based on the compressed sensing (CS) theory (Candès, Romberg, and Tao, 2006; Donoho, 2006), a new mathematical framework for data acquisition and signal recovery, and inspired by the idea of “single-pixel” camera using the CS theory (Duarte et al., 2008). Although recent work (Provost and Lesage, 2009) has applied CS reconstruction algorithms to reduce the number of view angles in conventional photoacoustic tomography (PAT), the acquisition still has to cover a full view. The reconstruction from limited-view acquisition still suffers from artifacts even if CS reconstruction algorithm is used. The proposed method employs spatially and temporally varying random illuminations for PAT acquisition and CS for image reconstruction such that the image from limited-view acquisition is free from artifacts. Simulation results using both phantom and in vivo images demonstrate that the image can be recovered using a few acquisitions from only two view angles.

2 Background

2.1 Photoacoustic Tomography and Limited-view Problem

The advantage of PAI lies in its capability of combining the spatial resolution of ultrasound with the contrast of optical absorption for biological imaging. Most reported PAT systems use an unfocused ultrasonic transducer to scan around the tissue and then reconstructs a high-resolution image of the tissue. With conventional uniform illumination, the forward model for PAT in Fourier domain is given by (Xu and Wang, 2005):

\[
\tilde{p}(r_0, k) = -ik \int dS' \tilde{G}_k^{(\text{out})}(r', r_0)p_0(r') ,
\]

where \( \tilde{G}_k^{(\text{out})}(r', r_0) = \exp(ik |r' - r_0|) / (4\pi |r' - r_0|) \), \( p_0(r') \) denotes the acoustic source, \( \tilde{p}(r_0, k) \) denotes the pressure detected at \( r_0 \) on a surface \( S' \), and \( k \) is the amplitude of the acoustic wave vector. When data are acquired from sufficient angles in a full view, high-quality images \( p_0(r') \) representing the acoustic source can be exactly reconstructed by back-projection method (Xu and Wang, 2005):

\[
p_0(r) = \frac{1}{\pi} \int dS_0 \int dk \tilde{p}(r_0, k)(n_0 \cdot \nabla \tilde{G}_k^{(\text{out})}(r, r_0)) ,
\]

where \( \tilde{G}_k^{(\text{in})}(r, r_0) = \exp(-ik |r - r_0|) / (4\pi |r - r_0|) \).

The exact methods treat PAI as an analytical problem and aim at obtaining the analytical relation between the tissue’s optical absorption property and detected photoacoustic signal. However, in limited-view acquisition where the detection path is not closed around the object to be imaged, reconstruction from incomplete data usually needs operations like Fourier filtrations with fast growing filters to compensate for the loss of high-frequency information (Xu et al., 2004). As a result, reconstruction from limited-view data can be unstable due to high-frequency amplification (Patch, 2004) or suffer from blurry edges and loss of details.

The sufficient conditions for exact reconstruction from limited-view PAT acquisition have been extensively studied in literature (Xu et al., 2004; Patch, 2004; Gamelin et al., 2008). Xu and Wang (2004) theoretically studied “detection region” where the image can be stably recovered from the limited-view acquisition. As shown in Figure 1, the detection region (hatched area) is defined as the envelope of the detection curve and all straight lines going through each point in this region should intersect the detection curve at least once. When an object (e.g., object A) is fully enclosed in the detection region, stable recovery is possible from limited-view data. If an object (e.g., object B) is partially in the region, only the parts within the detection region can be stably reconstructed. When an object is outside of the region, only the boundaries of the object represented as a solid line can be stably recovered.

![Figure 1 Detection region (hatched area) of an open arc.](image)

Objects A, B, and C represent three different scenarios where stable reconstruction can be achieved for the entire object, part of the object, and the boundaries of the object, respectively.
Although theoretically possible, reconstruction of objects within the detection region using practical algorithms still suffers from artifacts. Methods have been proposed to reduce artifacts by introducing a weight factor depending on the covered view angle, (Xu et al., 2004; Paltauf, Nuster and Burgholzer, 2009) or using iterative reconstruction techniques (Paltauf et al., 2007). Other attempts include regaining missing information using data completion (Patch 2004, Patrickeyev and Oraevsky 2004) or precomputing the filter functions for certain detector geometries (Kunyansky, 2008). It has also been shown that incorporating prior information about the object interfaces can reconstruct the parts of the object that are theoretically invisible (Wang and Yang, 2007). When an object is not entirely in the detection region, exact analytical methods are not suitable, and the image is better reconstructed by solving the discretized problem, for example, using the deconvolution reconstruction algorithm (Zhang and Wang, 2008).

2.2 Compressed Sensing and Its Application to PAT

CS (Candès, Romberg, and Tao, 2006a; Donoho, 2006) is a newly discovered sensing/sampling paradigm that goes against the common wisdom in data acquisition. Based on the CS theory, an image can be recovered from far fewer measurements than what the Shannon sampling theory requires if the following conditions hold (Candès and Romberg, 2007): (1) the image to be recovered is sparse or sparse after certain transformations \( \Psi \), which means there are very few non-zero elements in the image or the transformed image, but the non-zero locations are unknown \textit{a priori}; (2) the measurement matrix \( \Phi \) is incoherent with the sparsifying transform matrix \( \Psi \), which means the image should be spread out in the domain of measurement. For example, an independently and identically distributed random matrix is known to have the maximal incoherence with any sparsifying basis; (3) the image is reconstructed using a nonlinear method enforcing both sparsity and data consistency. For example, the constrained \( \ell_1 \) minimization can be used to reconstruct the desired image vector \( x \):

\[
\min_{x \in \mathbb{R}^n} \| \Psi x \|_1 \quad \text{subject to} \quad y = \Phi x, \tag{3}
\]

where \( \| x \|_1 = \sum_i |x_i| \) is the \( \ell_1 \) norm of the image, \( y \) is the measurement vector.

Although these conditions are in general not satisfied in practical sampling and reconstruction problems, the signal can still be recovered from few incoherent measurements with good fidelity under relaxed conditions. For example, the signals may not be strictly sparse but only compressible instead (i.e., sparse after thresholding the transform coefficients), and the measurements \( y \) may contain some noise \( e \) whose energy is bounded by a constant \( \varepsilon \), i.e.,

\[
y = \Phi x + e, \quad \| e \|_1 \leq \varepsilon. \tag{4}
\]

Under this circumstance, the signal can be recovered by solving:

\[
x^* = \arg \min_{x} \| \Psi x \|_1 \quad \text{s.t.} \quad \| \Phi x - y \|_2 \leq \varepsilon, \tag{5}
\]

and the reconstruction error is proved to be bounded by (Candès , Romberg, and Tao, 2006b)

\[
\| x^* - x \|_2 \leq c_1 \frac{\| x - x^\varepsilon \|_1}{\sqrt{S}} + c_2 \varepsilon, \tag{6}
\]

which is proportional to the noise level \( \varepsilon \) and the approximation error between the signal \( x \) and its closest \( S \)-sparse signal \( x^\varepsilon \). The \( c_1 \) and \( c_2 \) are the constants whose values decrease with a higher level of incoherence and a larger number of measurements. In summary, under practical conditions, the CS reconstruction quality depends on the level of incoherence, number of measurements, measurement noise, and compressibility of the image.

The property of reduced measurement with CS is desirable in PAT because it leads to reduced acquisition time or reduced number of elements in ultrasound transducer array. Existing work (Provost and Lesage, 2009) has studied the feasibility to replace conventional back-projection reconstruction by CS reconstruction algorithm to reduce the acquisition time of full-view PAT. Specifically, with the same experimental setup as conventional PAT, the data are acquired from a reduced number of angles that are uniformly distributed in \( 2\pi \) radians. In image reconstruction, the CS reconstruction in equation (3) is used in replace of the conventional back-projection reconstruction, where \( x \) is the vector form of acoustic source \( p_0(r) \), \( y \) is that of measured pressure in Fourier domain \( \tilde{p}(r_0, k) \), \( \Phi \) is the matrix representation of the forward operator, all after discretization, and \( \Psi \) can be an identity or wavelet transform matrix. Although there is no theoretical proof, the incoherence of sensing matrix \( \Phi \) can be assumed based on the empirical success of a similar CS matrix employed in (Candès, Romberg, and Tao, 2006a). The experimental results demonstrated that CS can achieve good image quality from reduced acquisitions with a full-view setup. However, as will be shown in the results and discussion section, this CS-PAT method still cannot completely remove the artifacts from limited-view acquisition because the incoherent condition of CS is not satisfied when the acquisitions are from a limited view. Application of CS reconstruction algorithm
alone without changing the measurement setup cannot yet address the issue of artifacts in limited-view PAT.

3 Proposed methods

The proposed method applies CS to PAI from a broader perspective where both the measurement setup is changed to satisfy the incoherent condition and the CS-reconstruction algorithm is employed to incorporate prior information. The method introduces a new degree of freedom – non-uniform illumination – in PAI data acquisition and utilizes the sparsity of the PA images as the prior information in reconstruction. The proposed method is inspired by the idea of single-pixel camera using CS (Duarte et al., 2008), where non-uniform, random illumination are used to ‘encode’ an image into a single pixel.

Figure 2 Detailed illustration of proposed scheme.

Figure 2 shows the diagram of data acquisition in our method with only two transducers. A mask is placed between the light source and the sample to realize the non-uniform illumination. Without the mask, the setup is the same as conventional PAT with two view angles. Apparently, the object is outside the detection region of conventional PAT and thus cannot be reconstructed using the existing algorithms. The inclusion of a mask transfers the diversity of measurements required for exact recovery from detector to source, i.e., from multiple detectors and a single image source to two detectors but multiple modulations of the image source, where the modulations are realized by the time-varying non-uniform illumination. Therefore, two view angles might be sufficient to reconstruct a high-quality image.

To achieve random illumination of the object, the mask changes its optical absorption distribution pattern randomly for each light pulse. For each mask pattern, two time-resolved PA signals are recorded by two single-element transducers. Then the mask changes its pattern and two new PA waves are recorded until sufficient data are acquired for high-quality reconstruction. In practice, the mask shown in Figure 2 can be implemented by a spin disk as used in modern optical confocal microscope (Murphy, 2001). The optical density of the spin disk varies spatially such that the object is non-uniformly illuminated. Changing of the mask pattern can be realized by rotating the disk to create temporally varying illuminations on the object.

Based on equation (1), the forward model for the proposed method becomes

\[
\tilde{\rho}(r_i, k, m) = -ik \int_S dS' G^{(\text{out})}(r_i, r_j) I_m(r_j) \rho_0(r) \quad (7)
\]

where \( I_m(r_j) \) denotes the random illumination on the object at spatial location \( r_j \) from the \( m \)-th mask. After discretization, equation (7) can be represented as a linear equation:

\[
d = Ax \quad (8)
\]

where \( d \) denotes the vector for all pressure measurements in Fourier domain and is a stack of measurements from two transducers, matrix \( A \) represents the linear forward operator including both transducers. In general, as long as sufficient data are acquired to make equation (8) to be an over-determined linear equation, the image vector \( x \) can be reconstructed by finding the least-squares solution to equation (8).

Apparently, to acquire sufficient data for an over-determined equation (8) requires many different illumination patterns of the mask and thus prolongs the acquisition time. It is desirable to significantly reduce the acquisition time using CS. To use CS, we confirm the three conditions of CS: (1) Most PA images are sparse either by themselves or after a certain transform such as wavelet transform (Provost and Lesage, 2009); (2) The matrix \( A \) is a random matrix due to the random illuminations and is thereby maximally incoherent with any sparsifying transform (Candès and Romberg, 2007). For example, whether the light pass through the mask or not at a certain spatial location is pseudorandom and can be modelled as random 1/0 with Bernoulli distribution. (3) The image \( x \) is reconstructed by incorporating the prior information that the image is sparse after transformation \( \Psi \):

\[
\min_{x \in \mathbb{R}^N} ||\Psi x||_1 + TV(x) \quad \text{subject to} \quad d = Ax, \quad (9)
\]

where \( TV(.) \) denotes the total variation of the image (Rudin, Osher and Fatemi, 1992) and is included to enforce additional constraint of piecewise smoothness.

4 Results

Computer simulations were conducted to demonstrate the effectiveness of the aforementioned method. The
simulation was in 2-D where the imaged sources are approximately located within a thin slab, though it is straightforward to extend the conclusion in 2-D to 3-D. All reconstruction methods were implemented in MATLAB (MathWorks, Natick, MA). SPGL1 toolbox (Van den Berg and Friedlander, 2008) was used to solve CS reconstruction.

A sparse phantom with 128×128 pixels shown in Figure 3 (a) was used in our simulation. Different 128×128 random masks were generated from 0/1 Bernoulli distribution and used for different acquisitions. Two transducers were assumed to be located at the virtual circular detection curve with 90-degree apart to acquire the PA signals of the non-uniformly illuminated object simultaneously. The geometry is the same as the one for object C in Figure 1, where only two detectors are located at the two ends of the 90-degree detection curve. The bandwidth limitation from the ultrasonic detection was ignored and it was assumed that no ultrasonic impedance mismatch exists during the photoacoustic wave propagation. For this setup, acquisitions with 64 masks are sufficient to generate a square matrix A and thus the image can be reconstructed by a least-squares solution. When fewer masks are used, CS reconstruction algorithm in equation (9) was used to reconstruct the image with \( \Psi \) the identity transform matrix. Figure 3 (d)-(e) show the reconstruction results of the proposed method using 43, 32, and 21 masks, respectively.

For comparison, the conventional PAT acquisition was also simulated with a similar 90-degree view but uniform illumination and multiple transducers uniformly covering the 90-degree view. To make a fair comparison, the total number of acquisitions was assumed to be the same as the proposed method. Namely, the number of view angles is equal to twice the number of masks. For example, if 43 masks are used in the proposed method, there are 43×2=86 acquisitions in total. Then 86 angles uniformly covering the 90-degree view are used for conventional PAT. Both back-projection and CS reconstruction algorithms were used for reconstruction and the results are shown in Figure 3 (b) and (c) respectively.

It is seen that with traditional PAT, the limited-view reconstructions by back-projection and CS both have artifacts. The CS reconstruction shows some improvement due to the prior information incorporated but loses some details. In contrast, the proposed method is able to recover the image exactly from only two views using the CS reconstruction algorithm. Even if the number of masks is reduced to 21 (equivalent to 42 acquisitions), the reconstruction still has reasonable quality with slight artifacts and loss of details.

Besides numerical phantom, an in vivo PA image of microvasculature in mice ears acquired by the system reported in Xie et al., (2009) was also used for simulation. The setup was similar to the phantom simulation with two transducers 90 degree apart. The original image and the reconstructions from the proposed method are shown in Figure 4. Full acquisition with 64 masks and reduced acquisition with 43 masks were simulated. With 64 masks, the original image can be recovered exactly. When the number of masks decreases to 43, some features with a low contrast are lost. This is because the in vivo image is not as sparse as the numerical phantom when \( \Psi \) is the identity transform matrix, and the number of measurements needed for exact recovery is thus increased. With the same number of masks, the reconstruction of the in vivo image is not as good as that of the phantom.

**Figure 3** (a) Original phantom. (b) Back-projection reconstruction and (c) CS reconstruction from conventional PAT acquisition from 86 views uniformly covering a 90-degree view. CS reconstructions from the proposed method using (d) 43, (e) 32 and (f) 21 masks with 2 view angles that are 90 degrees apart.

**Figure 4** An in vivo PA image of mice ear (left column) and the reconstructions using the proposed method with 64 (middle column) and 43(right column) masks.

### 5 Discussion

It has been demonstrated that 90-degree separation between the two transducers is sufficient for high-quality reconstruction. In some practical applications, only a smaller separation angle is allowed between the
two transducers. To study the effects of the separation angle on the reconstruction quality for the proposed method, acquisitions were simulated when the two view angles are separated by 60 degrees and 30 degrees. The reconstructions are shown in Figure 5. It is seen that a high quality reconstruction can still be achieved with 43 or 32 masks if the two view angles are 60 degrees apart. However, when two view angles are as close as 30 degrees, artifacts will show up in the reconstruction which becomes severer with decreased number of masks. This is because the information collected by only two transducers becomes more redundant such that the measurement matrix $A$ becomes more coherent.

In actual PAI experiments, the acquired PA signals are usually contaminated by noise. To study the robustness of the proposed method to noise, different levels of Gaussian noise were added to the measurements with average SNRs of 20, 15 and 10. Figure 6 shows the reconstruction results with different SNRs. Similarly, 43 and 32 masks were used and the two views are 90 degrees apart. We can see that the larger the measurement noise is, the larger the reconstruction noise is. The results demonstrate that the increased measurement noise only increases the reconstruction noise, but does not bring in additional artifacts or distortion. This observation agrees with the CS reconstruction error bound in equation (6), which increases with the measurement noise level when other conditions remain the same.

In the case of insufficient measurements, there are two ways for the proposed method to increase measurements and thus improve reconstruction. One is to increase the number of masks as demonstrated in Figure 3. The other is to increase the number of view angles using more transducers. We investigate the effect of the number of view angles on the reconstruction quality. Figure 6 shows the reconstructions with three transducers uniformly covering 90 degrees, 60 degrees and 30 degrees. The number of masks was chosen to be 21 for comparison with the results in Figure 3 and Figure 5, where only two transducers were used. An obvious improvement over the two-view-angle simulations can be seen for the 90-degree and 60-degree cases. The comparison demonstrates that increasing the number of view angles can reduce the number of masks needed for high-quality reconstruction. The same reconstruction quality can be achieved using many masks and a few transducers or many transducers and a few masks. Fewer masks lead to reduced acquisition time, while fewer transducers result in reduced cost of a PAI system. The user has the option to balance the trade-off between reducing the acquisition time with fewer masks and reducing the cost of the PAI system with fewer transducers. In the case of 30 degrees separation, the improvement is still significant but artifacts are present in reconstruction. Although increasing the number of view angles can partly improve the image quality, it cannot avoid artifacts completely; the view range needs to be sufficiently large to ensure the incoherence of the measurement matrix $A$.

It worth noting that although the single-pixel camera requires only a single photodiode as the detector, the proposed PAI method performs poorly with a single transducer (except full data are acquired). This is due to the fact that the acquired PAI signals are 1-D projection data of the modulated image along a curve, while the single-pixel camera captures the 2-D projection data. The incoherence condition of CS is hardly satisfied for the measurement matrix $A$ if a single transducer is used to acquire the 1-D projection data.
Conclusion

We report a new method for PAI to address the issue of artifacts in the limited-view imaging. The essential difference of the proposed method from the existing CS-PAT methods is that spatially and temporally varying random illuminations are employed. CS is also used to reduce the number of random illuminations for data acquisition. Simulation results suggest that images can be exactly recovered from limited-view acquisitions with merely two viewing angles. The proposed method is expected to be useful in a variety of PAI applications after the masks for random illumination can be efficiently implemented physically.

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