Analytical Evaluation of an IMT-Advanced Compliant LTE System Level Simulator

Maciej Mühleisen, Daniel Bültmann, Rainer Schoenen
ComNets Research Group
Faculty 6, RWTH Aachen University, Germany
Email: {mue|dbn|rs}@comnets.rwth-aachen.de

Abstract—LTE-Advanced (LTE-A) proposed by the 3rd Generation Partnership Project (3GPP) has successfully applied to the International Telecommunication Union (ITU) in 2010 to be accepted as an IMT-Advanced (IMT-A) compliant 4G mobile radio system. As part of this application 3GPP has provided LTE-A system level simulation results for cell spectral efficiency and cell edge user spectral efficiency. The ITU-R has encouraged independent evaluation groups to mutually verify their simulation results using their own simulators. Any simulator implementations is credible only if it is verified with regard to correct behaviour and correct assumptions.

In this work we present a method to analytically derive the spectral efficiency, throughput distribution, and cell edge user spectral efficiency results for LTE-A in an IMT-A scenario. The method can be used to estimate performance gains of protocol improvements and to validate system level simulators for IMT-A compliant LTE-A evaluation.

I. INTRODUCTION

In March 2008 the International Telecommunication Union Radiocommunication Sector (ITU-R) published a circular letter calling for proposals for next generation radio communication systems. In response the 3GPP proposed a set of radio interface technologies (SRIT) in June 2009 named Long Term Evolution Advanced (LTE-A). Along with the description of the proposed system, a self-evaluation of LTE-A performance was submitted to ITU-R.

Multiple external evaluation groups worldwide support the IMT-Advanced (IMT-A) process to verify the self-evaluation results of IMT-A candidate systems performance. Besides others, system level simulation is an important method to evaluate the performance of candidate systems. Emerging challenges are the high run times of the simulations and the problem to assure an error free implementation and valid modelling assumptions with known impact. To guarantee comparable results and agree on common assumptions, organisations involved in the evaluation process had the need to verify their own simulation tools. The baseline reference configuration for LTE Release 8 calibration [1] served as a starting point for the organisations involved to calibrate their simulators against each other and against the 3GPP simulator. In the European WINNER+ project partners worked together to evaluate LTE performance and calibrate their simulators as described in [2].

The channel model calibration results of [2] show that all simulator implementations used by WINNER+ partners produce identical results. The system level performance calibration results, however, show significant differences among the partners. Especially, downlink throughput distribution results for the so-called Indoor Hotspot scenario show significant differences.

It is not clear whether the deviating results come from wrong assumptions or errors in simulator implementations. In the following we present an analytical model to calculate the Signal to Interference and Noise Ratio (SINR) and throughput distribution in the downlink and from there the cell- and cell edge user spectral efficiency. With this results we verify the correct implementation of the Open Source Wireless Network Simulator (openWNS) [3] used to evaluate LTE-A performance within the WINNER+ project. The model proves the correct implementation of the simulator. It does not allow any conclusions regarding the simulation model assumptions.

The remainder of this work is organised as follows: After introducing the IMT-A evaluation methodology for cell spectral efficiency (CSE) and cell edge user spectral efficiency, in Section II we introduce an analytical model, aiming at yielding the same results as the system level calibration presented in Section III. We conclude in Section IV and give an outlook on possible future extensions.

A. IMT-Advanced Evaluation Methodology

The IMT-A Evaluation Methodology document [4] defines four test environments in which candidate radio interface technologies (RITs) have to prove their performance. Defined methods for performance evaluation are system and link level simulation, mathematical analysis, and inspection. The last one means checking the specifications of the radio interface technology (RIT) for compliance with IMT-A requirements. The cell spectral efficiency, cell edge user spectral efficiency and Voice over IP (VoIP) capacity can only be evaluated by system level simulation.

The test environments for evaluation are Indoor, Microcellular, Base coverage urban, and High speed. Each environment has a specific geometric deployment scenario namely Indoor Hotspot (InH), Urban Micro (UMi), Urban Macro (UMa), and Rural Macro (RMa), defined by the cell size and Inter-site distance (ISD) for the last three. The InH scenario is formed by a rectangular floor spanning 120 m by 50 m with two Base Station (BS) sites as shown in Figure 1. The BSs in the InH scenario are equipped with omnidirectional antennas, while the cellular scenarios define three sector BS sites. For each scenario the carrier frequency, transmission bandwidth,
maximum transmission power, transceiver height, and number of antenna elements is specified. The channel model comprises a large- and a small scale fading component with individual parameters for each scenario.

The large scale channel model defines fixed, distance dependent, path-loss and additionally log-normally distributed shadowing loss with standard deviation \( \sigma_c \). Multiple simulation runs, each with different uniformly distributed random positions of User Terminals (UTs), need to be executed to obtain statistically relevant results. For each run the path-loss is fixed due to the fixed UT positions. The realisation of the random shadow loss is drawn once for each link and remains constant for the entire simulation run. The path-loss is calculated from \( PL = \beta_c + \gamma_c \log_{10} d \), where \( d \) is the distance between the communicating nodes. The index \( c \) indicates that the standard deviation \( \sigma_c \), the fixed offset \( \beta_c \), and the slope \( \gamma_c \) depend on the channel conditions. The channel conditions may be either line-of-sight (LoS) or non line-of-sight (NLoS). For each simulation run the channel conditions for each link are determined randomly following a distance dependent distribution specified in [4].

B. Related Work

Results for upper and lower bound CSE for cellular systems with reuse distance equal or greater two are presented in [5]. The authors of [6] present a model to obtain results for reuse-1 scenarios without shadowing and also provide lower and upper bound approximations for CSE. In [7] we present a method to derive the uplink capacity in scenarios with assumptions similar to the IMT-A evaluation methodology under reuse-1 and Fractional Frequency Reuse (FFR). The authors of [8] provide CSE results for cellular IMT-A scenarios with relays but do not include random shadow fading in their model.

The system level simulator openWNS [3] calibrated in the WINNER+ project will be validated in this work with regard to LTE-A system level performance results in IMT-A compliant scenarios.

II. ANALYTIC MODEL

A. SINR Distribution

To derive the overall downlink SINR distribution of the InH scenario we first derive it for a single UT at position \([x, y]\). We choose the position of the left BS in Figure 1 as the origin of the coordinate system. The distance of a UT to the left BS is \( d_L = \sqrt{x^2 + y^2} \) and \( d_R = \sqrt{(x - 60)^2 + y^2} \) is the distance to the right BS. From Eq. (1) taken from [4] the LoS probability for the links to both BSs can be calculated. For now we assume the UT has a LoS link to both BSs. The probability for LoS channel conditions on both links is \( P(c = LoS|d_L)p(c = LoS|d_R) \), since the link conditions are stochastically independent.

\[
P(c = LoS|d) = \begin{cases} 
1, & d \leq 18 \\
\exp(-\frac{d-18}{27}), & 18 < d \leq 37 \\
0.5, & d > 37
\end{cases}
\]  

(1)

The path-loss to each BS is then normally distributed with mean value \( \mu_{LoS}(d) = \beta_{LoS} + \gamma_{LoS} \log_{10}(d) \) since the random shadowing component of the path-loss is normally distributed. The probability density functions (PDFs) are \( p(PL_L) = N(\mu_{LoS}(d_L), \sigma_{LoS}) \) and \( p(PL_R) = N(\mu_{LoS}(d_R), \sigma_{LoS}) \) for the path-loss to the left and right BS, respectively. Example results for a UT located at \([20 m; 15 m]\) are shown in Figure 2.

\[
p(PL_R - PL_L) = N(\mu_{LoS}(d_R) - \mu_{LoS}(d_L), \sqrt{\sigma_{LoS}^2 + \sigma_{LoS}^2}).
\]  

(2)

which results in

\[
P(a = L) = P(PL_R - PL_L > 0) = 1 - \frac{1}{2} \text{erf}\left(\frac{\mu_{LoS}(d_L) - \mu_{LoS}(d_R)}{\sqrt{2(\sigma_{LoS}^2 + \sigma_{LoS}^2)}}\right)
\]  

(3)

Since both BSs transmit with the same power, the resulting SINR in dB can be calculated as \( PL_R - PL_L \). The distribution of the path-loss difference \( p(PL_R - PL_L) \) is again normally distributed. The SINR distribution of a UT served by the left BS with LoS channel conditions on both, the serving and the interfering link, is then:

\[
p(SINR|a = L) = \frac{p(PL_R - PL_L)1_{\{SINR \geq 0\}}}{P(a = L)}.
\]  

(4)

The indicator function \( 1_{\{SINR \geq 0\}} \) assures that only SINR values greater zero are possible. Values below zero are not possible since in this case the right BS would be serving the UT. The resulting PDF is shown in Figure 2.

B. Mapping of SINR on Data Rate

The SINR distribution for the given position and channel condition can be mapped to a date rate distribution. For that purpose a mapping of SINR to modulation and coding scheme (MCS) must be performed. A subset of 13 MCSs defined by the Long Term Evolution (LTE) standard is used. The mapping is shown in Figure 3 for a channel bandwidth 20 MHz and further described in [9]. The MCSs have been used by
some WINNER+ partners to create system level simulation results. Three out of 14 symbols are used for the Downlink Control Channel (DLCCH) [1]. Overhead introduced by the Broadcast Control Channel (BCH) is neglected since it is only transmitted every tenth frame. The rate is further reduced by 8 bit fixed Radio Link Control (RLC) header and 32 bit Media Access Control (MAC) header in each frame. The code rate has been reduced to model 8 symbols per Resource Block (RB) used as reference symbols not available for user data traffic.

The channel model, especially the SINR distribution, has significant impact on performance results. Equation (4) provides for all combinations of channel conditions and serving BS, after integration and normalisation to the scenario area, the

$$P(a = A, c_L = C_L, c_R = C_R) = P(a = A)P(c_L = C_L|d_L) P(c_R = C_R|d_R).$$

The overall data rate distribution for a given position $[x; y]$ is the weighted superposition of the probability for each conditional rate distribution:

$$P(r_i|x, y) = \sum_{\forall a, c_L, c_R} P(a, c_L, c_R) P(r_i|x, y, a, c_L, c_R)$$

C. Data Rate Distribution

Based on Figure 3 switching points between MCSs can be chosen that guarantee a maximum Packet Error Rate (PER) of 1 %. The minimum SINR for an MCSs of data rate $r_i$ is denoted $SINR_{r_i, Min}$. Using Eq. (4) the probability $P(r_i|x, y, a, c_L, c_R) = P(SINR_{r_i, Min} < SINR < SINR_{r_{(i+1)}, Min}$ for each MCS can be calculated.

Equation (4) applies only to UTs served by the left BS under LoS channel conditions ($C_L = C_R = LoS$). If the UT is served by the right BS, the mean values $\mu_{LoS}(d_L)$ and $\mu_{LoS}(d_R)$ in Eq. (2) are to switch to obtain $P(a = R)$ and $P(PL_L - PL_R)$. This result is also shown in Figure 2. Besides LoS channel conditions, the UT could also have an NLoS link to either one or both BSs. In this case the first two moments of the path-loss distributions have to be replaced by the ones for NLoS conditions.

In total there are eight possible combinations: Each link has two possible channel conditions and either of the two BSs can be serving the UT. The probability for each combination is

$$P(a = A, c_L = C_L, c_R = C_R) = P(a = A)P(c_L = C_L|d_L) P(c_R = C_R|d_R).$$

The IMT-A evaluation methodology [4] specifies that 20 user terminals are placed randomly in the scenario. Each terminal associates to the BS serving it with highest SINR. Since the scenario is symmetric, the probability to be served by either one of the BSs equals 0.5. Repeating the experiment for all 20 randomly placed UTs results in the number $a$ of associated UTs to be Binomially distributed $B(a|n, p)$ with $n = 20$ trials and $p = 0.5$ success probability.

Each UT receives the same amount of radio resources according to 3GPP LTE calibration assumptions [1]. The achievable throughput capacity therefore depends on how many UTs are served by a BS. It has to be shared by the number $a$ of associated UTs. Each MCS with rate $r_i$ results in some throughput capacity. If a UT shares the channel with $a$ other stations it only reaches the throughput $r_i/a$. The probability of a UT to achieve throughput $r_i/a$ is $P(r_i/a|x, y) = B(a|20, 0.5)P(r_i|x, y)$.

$$P(r_i/a) = \int_A P(r_i/a|x, y) \, A$$

$$\eta = \int_A P(r_i|x, y) \, A$$

To achieve the overall throughput distribution integration over the entire scenario and normalisation to the area size $A = 50\cdot120$ m$^2$ is required, see Eq. (7). As far as we know there is no closed form solution for the integral. For that we numerically sum up the data rate distributions at each position and normalise the result to the number of sampling intervals.

The cell spectral efficiency $\eta$ does not depend on the number of associated stations $a$ since the throughput of all stations is summed up. It is obtained by numerically solving Eq. (8).

III. RESULTS

The channel model, especially the SINR distribution, has significant impact on performance results. Equation (4) provides for all combinations of channel conditions and serving BS, after integration and normalisation to the scenario area, the
downlink SINR distribution, Figure 4. The graph also shows results gained from the openWNS simulator.

The analytic results match the simulator results very well. This way the channel model implemented in the simulator is validated. The results match the calibration results presented in [2].

Next, simulation results gained to calibrate the LTE system model are compared to results of the analytic model. According to [1] UTs are served RoundRobin assigning each station the full bandwidth (100 RBs) in each frame. In this comparison small-scale channel fading is neglected. Figure 5 shows the distribution of the used MCSs. Simulation results for 50 and 500 drops together with 95% confidence intervals are shown. In the analytic model we used a step width of 1 m to numerically integrate over the area.

![Fig. 4. SINR distribution for the whole scenario area.](image)

Analytic and simulation results match very well. Increasing the number of drops does not improve the results significantly.

Figure 6 shows the distribution of the number of nodes per BS for 50 and 500 simulation drops and compares it to a Bernoulli distribution. It is visible that simulation result confidence intervals are significantly reduced when increasing the number of drops. Analysis and simulation match very well.

The user throughput distribution is shown in Figure 7. With 50 drops small differences between the analytic and simulation results around 5 Mbps to 6 Mbps are visible. Minor differences to the analytic results for this throughput range are also visible for 500 drops. With 1000 drops no significant difference between simulated and analytic results are visible.

The CSE $\eta$ is found from Eq. (8) to be 2.269 Bit/s/Hz/Cell. This is very close to the simulator output result of 2.265 Bit/s/Hz/Cell for 1000 drops. The cell edge user spectral efficiency is defined as the 5-percentile of the throughput distribution. It is 0.057 Bit/s/Hz for both, the analytic model and the simulator with 1000 drops. Figure 8 shows how the relative error of the CSE and cell edge user spectral efficiency decreases as the number of drops is increased. Less than 30 drops are required to assure an error below 1 % for the CSE. More than 80 drops are needed to assure an error below 1 % for the cell edge user spectral efficiency.

The influence of the IMT-A small-scale fading model has not been examined in this work. Figure 9 shows simulator results including small-scale fading. It shows that small-scale fading in this model has only little impact on the SINR distribution function since data is transmitted on all 100 RBs.
causing the effective SINR of a transmission to be calculated as the average over many channel realisations. Due to the low speed of 3 km/h assumed in the InH scenario, channel coherence time is long enough to assure accurate channel state information available for scheduling and a negligible error probability due to channel estimation errors.

The authors of [10] show that the Shannon capacity of a flat Rayleigh fading channel can be calculated by introducing a constant SINR shift. Figure 9 shows analytic results if the SINR in Eq. (4) is reduced by 1 dB. This results in an throughput distribution very similar to the simulation result. Still the exact impact of small-scale fading needs to be modelled analytically to obtain the exact shifting factor and to evaluate the assumptions under which an SINR shift can be used to model small-scale fading.

Fig. 8. Relative error of simulation result compared to analytic result.

Fig. 9. Results with small-scale fading.

IV. SUMMARY, CONCLUSION, AND OUTLOOK

An analytic model to verify system level calibration results for spectral efficiency and cell edge user spectral efficiency in the IMT-Advanced Indoor Hotspot scenario is presented. Model parameters like transmission power and the share of radio resources for each User Terminal can be adjusted to evaluate their influence on system performance.

Presented results proof the correct implementation of the openWNS simulator used for LTE performance evaluation. The model empowers other simulator developers and users to verify their LTE-Advanced simulator implementations. We have found that intermediate results like the modulation and coding scheme distribution and especially the number of associated User Terminals show significant differences to the analytical model if the simulation experiment is repeated only few times. Still the key performance indicator results for cell spectral efficiency and cell edge user spectral efficiency show high confidence after only a few (less than 100) simulation drops.

The model can be extended to obtain results for more than two cells. Interference power has then the distribution of the sum of multiple log-normally distributed random variables. The sum can be estimated by a normal distribution as shown in [11]. Antenna patterns can be included as an additional angle dependant factor in the fixed component of the path-loss.

REFERENCES