

1 Article

2 Damping ratio in carbon fiber reinforced epoxy 3 filament-wound composites using Hilbert transform

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15 **Abstract:** The aim of this work is to evaluate damping ratio (ζ) of unidirectional carbon fiber-
16 reinforced epoxy laminates using an experimental approach, which is then input into a finite
17 element numerical model. The logarithmic decrement and the envelope techniques were calculated
18 using Hilbert transform. Flat unidirectional laminates were manufactured by dry-filament winding
19 and evaluated under dynamic free vibration tests (longitudinally and transverse fiber-oriented
20 specimens). Post-processing and data analysis were carried out with a dedicated code which used
21 a band-pass filter to isolate the first fundamental frequency from the other modes of vibration and
22 from noise in the acquired signal, and then applied the Hilbert transform to estimate the vibration
23 envelope and adjust an exponential curve to it to evaluate structural damping ratio. The
24 experimental results for modal frequencies were compared to numerical predictions from a fitted
25 finite element model.

26 **Keywords:** Hilbert Transform; damping ratio; vibration; finite element modeling; filament winding
27

28 1. Introduction

29 Damping is a key feature to characterize the dynamic behavior of fiber-reinforced composites
30 regarding minimization of resonant vibrations. Different sources of energy dissipation include
31 viscoelastic character of the matrix, fiber arrangement, damping due to fiber/matrix interface,
32 damage, viscoelastic and thermoelastic damping [1]. The control of vibrations and noise in structures
33 can be achieved by using active or passive damping systems, for instance [2,3]. Anyway, for dynamic
34 loadings, damping ratio measurements are essential for performance control and structural safety.

35 The dynamic nature of damping in orthotropic materials is more complex than in isotropic ones
36 due to the combined influence of several parameters, such as degree of orthotropy, fiber orientation,
37 manufacturing method, boundary conditions as well as the vibrating mode on damping and stiffness
38 distributions [4]. There are many methods available for structural damping evaluation related to
39 modal analysis in mechanical structures, namely half-power bandwidth method, logarithmic
40 decrement, rational fraction polynomial, random decrement, subspace stochastic identification (SSI)
41 and fractional energy loss per cycle [5]. Many of them are also applied to evaluate elastic properties
42 and those based on structural vibrations or acoustic measurement are commonly used in composite
43 materials [6].

44 Damping of a composite system can be evaluated, for instance, based on the damping ratio (ζ)
45 or the loss factor (η), which is used in *hysteretic* damping models. In those, the vibration decay of an

46 excited specimen is followed to characterize the damping behavior. The frequency amplitudes are
47 measured by accelerometers and calculated using a digital method, for instance [7].

48 Hanselka and Hoffman [8] evaluated unidirectional glass/epoxy composites applying the
49 resonance technique and found maximum damping when the fibers were oriented at 15° and 45°.
50 Dubenets and Yakovenko [9] determined the effective complex moduli and vibration decrements of
51 fiber-reinforced viscoelastic composites based on the finite element method. Wei and Kukureka [10]
52 used acoustic energy emission and resonance techniques to measure the elastic properties of
53 glass/epoxy composites and evaluated damping in terms of internal friction energy using either
54 logarithmic decrement in free vibration or half power bandwidth, obtaining similar damping values
55 from both techniques. Fenza et al. [11] minimized damping ratio data dispersion, and the associated
56 uncertainties, by applying logarithmic decrement and Hilbert transform to obtain the free vibration
57 decaying envelopes in the time domain for composites at different temperatures. Lavanya et al. [12]
58 evaluated damping behavior of glass/epoxy composites using the half-power bandwidth technique
59 and concluded that the highest damping was found for fibers oriented at 60°.

60 In this context, the aim of this paper is to investigate damping ratio of carbon fiber reinforced
61 epoxy filament wound laminates, in the longitudinal and transverse fiber direction, by applying the
62 logarithmic decrement technique and Hilbert transform. Experiments were carried out and post-
63 processing of the obtained data is performed by a finite element (FE) model.
64

65 2. Logarithmic decrement applied to damping evaluation

66 Materials and Methods should be described with sufficient details to allow others to replicate
67 and build on published results. Please note that publication of your manuscript implicates that you
68 must make all materials, data, computer code, and protocols associated with the publication available
69 to readers. Please disclose at the submission stage any restrictions on the availability of materials or
70 information. New methods and protocols should be described in detail while well-established
71 methods can be briefly described and appropriately cited.

72 Research manuscripts reporting large datasets that are deposited in a publicly available database
73 should specify where the data have been deposited and provide the relevant accession numbers. If
74 the accession numbers have not yet been obtained at the time of submission, please state that they
75 will be provided during review. They must be provided prior to publication.

76 Logarithmic decrement method enables evaluation of damping ratio for a particular composite
77 system based on free vibration responses. For a theoretical SDOF (single degree of freedom) mass-
78 spring-damper system, the logarithmic decrement (δ) can be calculated as the natural logarithm of
79 the ratio between any two successive positive peak amplitudes of the free vibration response [13].
80 The free vibration response of an underdamped SDOF system may be described by:

$$81 \quad x(t) = X_0 e^{-\zeta \omega_d t} \sin(\omega_d t + \theta) \quad (1)$$

82 where X_0 is the initial magnitude, θ is the phase shift and ω_d the damped natural frequency.

83 Considering any two successive peaks separated by n oscillation cycles, the ratio between these
84 amplitudes is:

$$85 \quad \frac{x_n}{x_0} = e^{\frac{-2\pi n \zeta}{\sqrt{1-\zeta^2}}} \quad (2)$$

86 and the logarithmic decrement is calculated as:

$$87 \quad \delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad (3)$$

88 Expanding the expression above for n cycles between the peaks:

$$89 \quad \delta = \frac{1}{n} \ln\left(\frac{x_0}{x_{0+n}}\right) \quad (4)$$

90 Therefore, damping ratio may be obtained from the logarithmic decrement by:

$$91 \quad \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} \quad (5)$$

92 And, for small values of ζ (<0.1), the following approximation is used:

$$93 \quad \delta \cong 2\pi\zeta \quad (6)$$

94 3. Application of Hilbert transform for envelope and damping evaluations

95 This section may be divided by subheadings. It should provide a concise and precise description
96 of the experimental results, their interpretation as well as the experimental conclusions that can be
97 drawn.

98 Free response vibration signals generally present one (or more) resonance frequencies, also
99 called carrier signal, and another signal corresponding to damping, that modulates (decreasingly)
100 the carrier signal. Thus, the information about damping comes merged with the carrier wave, making
101 the analysis more difficult.

102 The determination of an envelope is an efficient way to work around this problem [14], which is
103 usually done via hardware, with special purpose circuit boards called envelope detectors, or digitally,
104 applying the Hilbert transform [15]. This research focuses on the latter approach, and consists in
105 applying a convolution operation (\otimes) which keeps function mappings at the same domains (i.e. time
106 domain). The Hilbert transform pair is defined as:

$$107 \quad x_h(t) = H\{x(t)\} = \frac{1}{n} x(t) \otimes \frac{1}{t} \quad (7)$$

$$108 \quad x(t) = H^{-1}\{x_h(t)\} = -\frac{1}{\pi} x_h(t) \otimes \frac{1}{t} \quad (8)$$

109 Since both functions are described in the time domain, it is possible to combine them as:

$$110 \quad z(t) = x(t) + jx_h(t) \quad (9)$$

111 The $z(t)$ function is called the analytical signal of $x(t)$, and is a complex signal in which its
112 imaginary part is the Hilbert transform of $x(t)$. Equation (9) can then be rewritten as:

$$113 \quad z(t) = A(t)e^{j\theta(t)} \quad (10)$$

114 where:

$$115 \quad A(t) = \sqrt{[x^2(t) + x_h^2(t)]} \quad (11)$$

$$116 \quad \theta(t) = \tan^{-1} \left[\frac{x_h(t)}{x(t)} \right] = 2\pi f_0 t \quad (12)$$

117 In Equations (11-12), $A(t)$ is the envelope of the response signal $x(t)$, and $\theta(t)$ is the
118 instantaneous phase of $x(t)$. The Hilbert transform, $A(t)$, can be used as a tool for demodulating a
119 constant amplitude and frequency sinusoidal signal, using an inverse exponential decay function to
120 yield the modulated signal (the envelope), where the vibration decrement is due to damping.
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122 4. Experimental procedure

123 Carbon fiber reinforced epoxy towpreg (Toray T700-12K-50C and UF3369 resin) from TCR
124 Composites was used to produce flat laminates using a rectangular stainless steel mandrel (327×228
125 $\times 12$ mm³) and a filament winding system from MFTech coupled with a KUKA robot KR 140 L100.
126 The towpregs were wound onto the mandrel and a shrink tape was used to help consolidation. The
127 laminates were cured by hot compression under 6 ton for 4 h at 130 °C, reaching a final fiber volume
128 fraction of $\approx 72\%$. Unidirectional 4- and 8-layer laminates were produced and cut following
129 longitudinal and transverse directions, yielding [0]4, [90]4, [0]8 and [90]8 specimens. However, the
130 [90]4 laminates could not be used because they were not stiff enough to produce reliable results.

131 Table 1 shows the material properties of the carbon/epoxy composite system and Table 2
132 presents the four specimens of each family fixed into a clamped-free condition and tested in free
133 vibration response. The data acquisition system was composed of two types of sensors: an Analog
134 Devices ADXL 213 accelerometer, weighing 0.3 g, with 970 mV/g nominal sensitivity, nonlinearity of
135 0.5% of the full scale and ± 1.2 g measurement range; and a Micro-Epsilon OptoNCDT 1300 laser
136 vibrometer sensor, with 10 μ m resolution, ± 10 mm dynamic measurement range and 250 mV/mm
137 sensitivity. The latter one is considered a nonintrusive method since no mass is added to the clamped-
138 free structure. The experimental set-up is shown in Figure 1.

139 The measurements were programmed in Keysight Agilent VEE 7.5 using a Measurement
140 Computing 1208FS A/D board, with 12 bits, 8 input analog channels and a measurement range of ± 10
141 V. The test was repeated 10 times for each specimen and post-processing of acquired data was
142 performed in Matlab 2012, which consisted in selecting the free vibrating part of the signal, filtering
143 its frequency contents around fundamental frequency of the beam followed by evaluating the Hilbert

144 transform to detect the envelope of dynamic response, fitting the exponential curve into that
 145 envelope, and then finally calculating the damping ratio. Some options available in the Matlab Signal
 146 Processing Toolbox for curve fitting were tested and the 'peaks' option was found to result in lower
 147 uncertainty.

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Table 1. Representative properties of the carbon/epoxy 1-D laminate [16].

Property	Description	Value
$E1$ (GPa)	Longitudinal elastic modulus	129.3
$E2 = E3$ (GPa)	Transverse elastic modulus	9.11
$\nu12 = \nu13$	Poisson's ratio in planes 1-2/1-3	0.32
$\nu23$	Poisson's ratio in plane 2-3	0.35
$G12 = G13$ (GPa)	Shear moduli	5.44
$G23$ (GPa)	Transverse shear modulus in plane 2-3	2.10
Xt (MPa)	Longitudinal tensile strength	1409.9
Yt (MPa)	Transverse tensile strength	42.5
Xc (MPa)	Longitudinal compressive strength	-740.0
Yc (MPa)	Transverse compressive strength	-140.3
Spl (MPa)	In-plane shear strength	68.9

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Table 2. Specimens used in the free vibration tests.

Configuration	Length [mm]	Width [mm]	Thickness [mm]
$[0]_8$	234	28.0	3.00
$[0]_8$	203	28.0	3.00
$[0]_8$	195	29.0	3.00
$[0]_8$	240	29.0	3.00
$[90]_8$	199	29.0	3.00
$[90]_8$	170	29.0	3.00
$[90]_8$	185	29.0	3.00
$[90]_8$	158	29.0	3.00
$[0]_4$	141	38.6	0.85
$[0]_4$	143	38.6	0.85
$[0]_4$	205	47.5	0.85
$[0]_4$	211	47.5	0.85

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153 Figure 2 shows a typical response of the original signal measured by the accelerometer and the
 154 corresponding filtered response for the first natural frequency (bending mode). Figure 3 displays the
 155 original frequency spectrum (Fourier Transform) where the fundamental natural frequency can be
 156 identified at 23.23 Hz. This spectrum also shows another resonance around 145 Hz which
 157 corresponds to the 1st torsional mode, as later identified via FE analysis.

158 Figure 4 depicts the post-processing results of the filtered signal (red curve), where the Hilbert
 159 transform defined the envelope (black curve), and the exponential curve fitting (green curve) was
 160 used to define the damping ratio by logarithm decrement.

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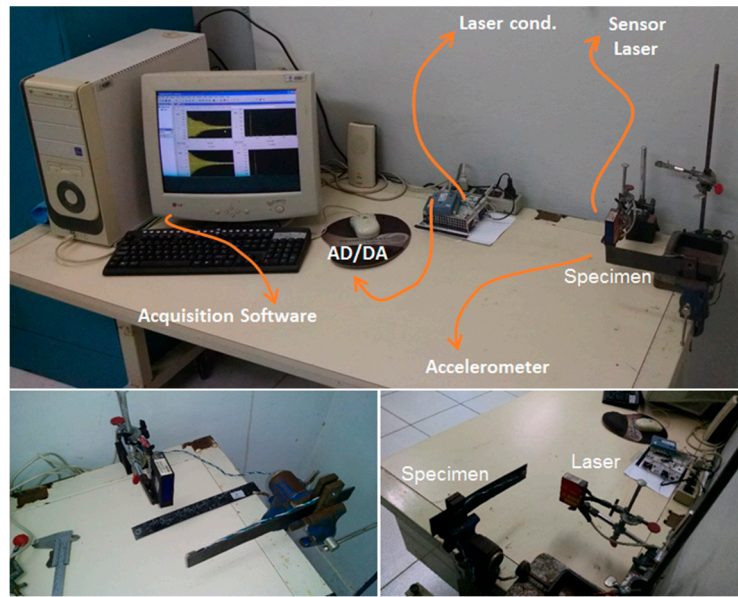


Figure 1. Experimental set-up used for damping ratio evaluation.

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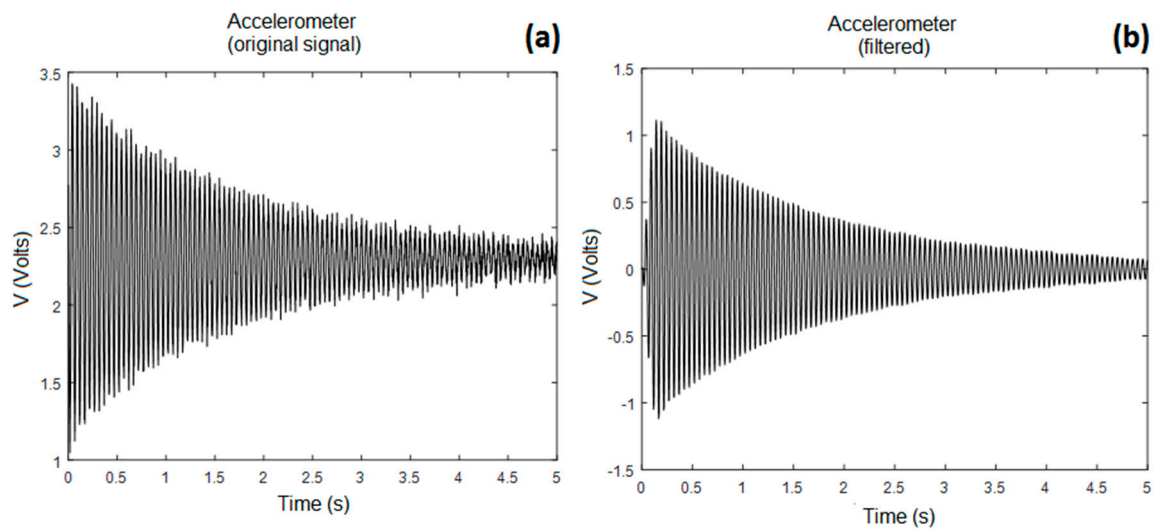


Figure 2. Typical accelerometer output (4th sample, 4th replication): Original (a) and filtered (b) time responses.

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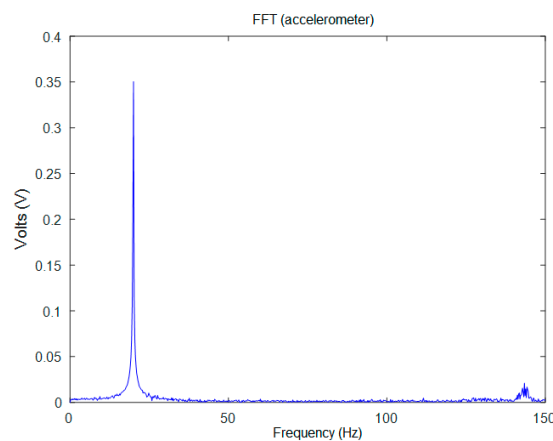
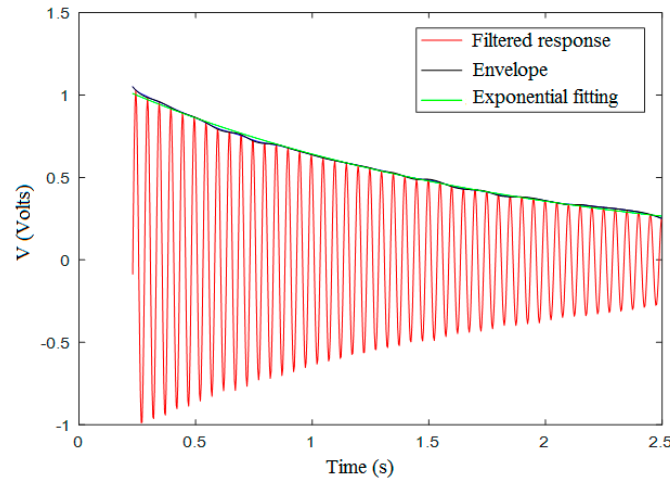


Figure 3. Original free vibration response measured by accelerometer (4th sample, 4th replication).

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Figure 4. Filtered signal acquired from the accelerometer (red curve), envelope defined by Hilbert transform (black curve) and exponential fitting (green curve) used to define damping ratio.

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175 5. Finite element model

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This section is not mandatory, but can be added to the manuscript if the discussion is unusually long or complex.

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For the finite element (FE) model herein built, a degenerated 4-node element with bending and torsion capability was adopted. The first-order shear deformation theory (FSDT) for laminated plates (Mindlin plate theory) was followed, where the displacement field is given by:

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$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z\theta_x(x, y) \\ v(x, y, z) &= v_0(x, y) + z\theta_y(x, y) \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (13)$$

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where u , v , and w are the displacement components of the vector field, and θ_i , with $i = x, y$, are the rotations on these axes.

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The strain tensor for each layer is obtained from the displacement field using:

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$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{Bmatrix} \quad (14)$$

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The stiffness matrix is then obtained by usual Gauss integration points throughout each lamina and from the definition of element strain energy. Thus, for instance, the membrane-bending coupling matrix is calculated by:

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$$\mathbf{K}_{mb}^{(e)} = \sum_{k=1}^{nl} \int_A \mathbf{B}_m^T \mathbf{D} \mathbf{B}_b (z_{k+1} - z_k) dA \quad (15)$$

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where nl is the number of layers along the total thickness of the laminate, \mathbf{D} is the constitutive matrix, already rotated to each layer orientation, \mathbf{B}_m and \mathbf{B}_b are the matrices of shape function derivatives for the membrane and bending components, respectively, and z_k and z_{k+1} are the top and bottom coordinates of k^{th} layer, respectively.

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Superposing the matrices related to each effect, the element matrix can be expressed as:

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$$\mathbf{K}^{(e)} = \mathbf{K}_{mm}^{(e)} + \mathbf{K}_{mb}^{(e)} + \mathbf{K}_{bm}^{(e)} + \mathbf{K}_{bb}^{(e)} + \mathbf{K}_{ss}^{(e)} \quad (16)$$

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where the indexes mm , mb , bm , bb , and ss stand for membrane, membrane-bending, bending-membrane, bending, and shear effects, respectively. The global matrix is then assembled by superposing each correspondent degree of freedom.

201 For the time-domain analysis, the equation of motion is commonly given for a general multi-
 202 degree of freedom (MDOF) system by:

$$203 \quad \mathbf{M}\{\dot{\mathbf{y}}(t)\} + \mathbf{C}\{\dot{\mathbf{y}}(t)\} + \mathbf{K}\{\mathbf{y}(t)\} = \{f(t)\} \quad (17)$$

204 and the corresponding generalized eigenvector-eigenvalue problem is defined as:

$$205 \quad (\lambda_i^2 \mathbf{M} + \lambda_i \mathbf{C} + \mathbf{K})\{\boldsymbol{\varphi}_i\} = \{0\} \quad (18)$$

206 where \mathbf{M} is the global mass matrix, \mathbf{K} is the global stiffness matrix, \mathbf{C} is the global damping matrix,
 207 λ_i is the i^{th} eigenvalue and $\{\boldsymbol{\varphi}_i\}$ the i^{th} eigenvector (i^{th} column of the $\boldsymbol{\phi}$ generalized eigenvector
 208 matrix). So, once the structural matrices are defined and Equation (18) is solved, the damped
 209 frequencies can be evaluated by $f_i = \text{Im}(\lambda_i)/2\pi$ and the damping ratios by $\zeta_i = -\text{Re}(\lambda_i)/(2\pi|\lambda_i|)$.
 210 For time domain simulations, the global damping matrix \mathbf{C} is selected to be proportional to stiffness
 211 and mass, in such a way that it will represent the contribution of the modes with defined damping
 212 ratios (nm measured modes): $\mathbf{C} = \mathbf{M} \sum_{i=1}^{nm} \{\boldsymbol{\varphi}_i\} \zeta_i \{\boldsymbol{\varphi}_i\}^T \mathbf{M}$.

213 A 120-element FE mesh was generated for the finite element analysis (see Figure 5). Prior
 214 convergence mesh analysis was carried out in order to validate the procedure to obtain representative
 215 natural frequencies. The accelerometer mass was added near the free tip of the cantilever to better
 216 represent the experimental set-up (also represented in Figure 5).

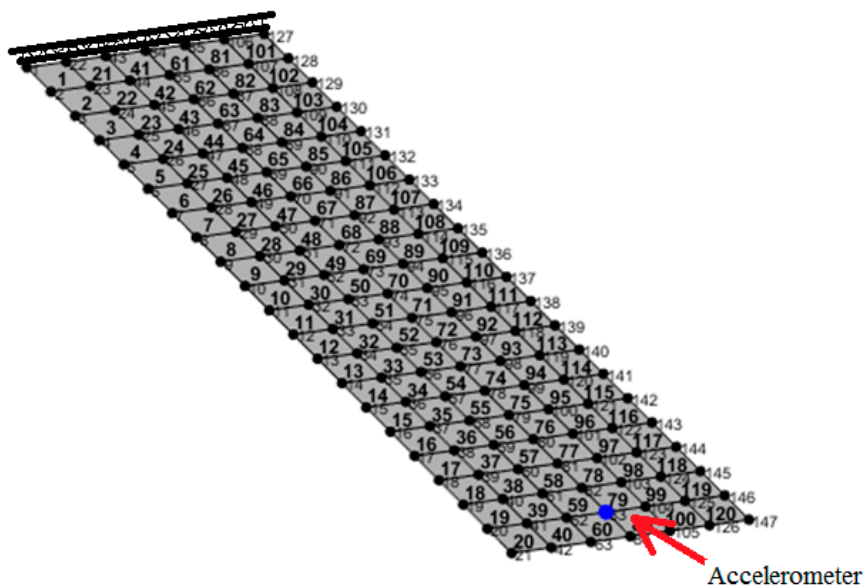


Figure 5. FE mesh illustration with accelerometer position embedded into the FE model.

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220 6. Results and discussion

221 This section is not mandatory, but may be added if there are patents resulting from the work
 222 reported in this manuscript.

223 The experimental results of natural frequencies and damping ratio for each sample and its
 224 replications are shown in Table 3-5. Damping was found to vary according to the number of layers,
 225 ply orientation and with specimen length, as expected. The observed damping variability between
 226 samples of the same family was considered low, being attributed to changes in overall viscous
 227 damping (air drag around oscillating structure), which is more significant for small and simple
 228 structures as those herewith tested. It was also observed low variability between replications in the
 229 same sample and composite specimen. For instance, in case of specimen [0]4 (Table 3), the samples
 230 have standard deviation lower than 0.4% in damping ratio. In the sample specimen, for instance,
 231 samples 1 and 2 of Table 3, for an increase in the specimen length of about 3% this resulted in
 232 decreasing damping ratio about 37%. Experimental data using accelerometer or the laser transducer
 233 resulted in very similar results, so the optical sensor data was preferred, since this acquisition method
 234 is non-intrusive.

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Table 3. Natural frequencies and damping ratio results for the [0]₄ carbon/epoxy specimens.

Replication	Sample 1		Sample 2		Sample 3		Sample 4	
	<i>f</i> 1	ξ	<i>f</i> 1	ξ	<i>f</i> 1	ξ	<i>f</i> 1	ξ
1	41.66	0.0042	20.23	0.0062	40.45	0.0187	19.63	0.0137
2	41.66	0.0066	20.43	0.0056	40.05	0.0271	19.43	0.0136
3	41.66	0.0043	20.23	0.0066	39.45	0.0162	19.63	0.0141
4	41.86	0.0083	20.23	0.0062	39.25	0.0181	19.43	0.0141
5	41.86	0.0114	20.23	0.0060	39.25	0.0187	19.43	0.0134
6	41.86	0.0117	20.23	0.0057	39.25	0.0190	19.43	0.0131
7	41.86	0.0087	20.03	0.0050	39.05	0.0183	19.43	0.0125
8	41.86	0.0081	20.03	0.0045	39.25	0.0154	19.43	0.0135
9	41.86	0.0097	20.03	0.0045	40.25	0.0140	19.63	0.0129
10	41.86	0.0090	20.03	0.0042	40.25	0.0110	19.43	0.0131
Mean	41.8	0.008	20.2	0.005	39.7	0.018	19.5	0.013
Standard deviation	0.097	0.003	0.135	0.001	0.534	0.004	0.097	0.001
CV	0.002	0.311	0.007	0.153	0.013	0.238	0.005	0.039

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Good agreement for modal frequency between experiments and updated numerical model, in the frequency range of the measured data was found, as will be presented in the following, indicating that this updated numerical model can be used to evaluate the dynamic behavior of composite structures.

Table 4. Natural frequencies and damping ratio results for the [0]₈ carbon/epoxy specimens.

Replication	Sample 1		Sample 2		Sample 3		Sample 4	
	<i>f</i> 1	ξ	<i>f</i> 1	ξ	<i>f</i> 1	ξ	<i>f</i> 1	ξ
1	57.65	0.0076	74.49	0.0048	87.30	0.0086	59.47	0.015
2	57.65	0.0094	74.49	0.0048	87.10	0.0054	59.47	0.018
3	57.25	0.0084	74.49	0.0051	87.10	0.0073	59.27	0.022
4	57.25	0.0061	74.49	0.0051	87.10	0.0054	59.27	0.012
5	57.25	0.0066	74.08	0.0054	87.10	0.0064	59.47	0.018
6	57.25	0.0031	74.49	0.0058	87.09	0.0069	59.27	0.021
7	57.65	0.0102	74.49	0.0063	87.10	0.0092	59.27	0.011
8	57.45	0.0112	74.69	0.0053	87.10	0.0069	59.27	0.019
9	57.25	0.0074	74.49	0.0062	87.30	0.0074	59.27	0.012
10	57.65	0.0105	74.49	0.0051	87.10	0.0083	59.27	0.013
Mean	57.4	0.008	74.5	0.005	87.1	0.007	59.3	0.016
Standard deviation	0.199	0.002	0.148	0.001	0.085	0.001	0.097	0.004
CV	0.003	0.299	0.002	0.102	0.001	0.180	0.002	0.249

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The results indicate a mean damping ratio (ζ) of 9×10^{-3} (coefficient of variation ca. 15%) for [0]₈ sample (Table 4), 4.5×10^{-3} (coefficient of variation ca. 8%) for [90]₈ sample (Table 5) and 1.1×10^{-2} (coefficient of variation ca. 18%) for the [0]₄ laminate (Table 3). It is worth mentioning that these values lie in the same order of magnitude expected for carbon/epoxy composite damping properties [17]. It can also be said that, the longer the sample, the greater the damping.

Using the results presented in Table 5, for Sample 4 (and 4th replication), for instance, the finite element model was updated with the experimentally identified damping ratio. Numerical mode shapes associated with 1st, 2nd and 4th frequencies are shown in Figure 6 and simulated results for transverse displacement are presented in Figure 7. Analysis of displacement and power spectral density (PSD) of a node at the center of the free tip (node 84) can be used to identify structural modal frequencies. The following values for the first four damped natural frequencies were obtained:

256 $f_1=37.41$ Hz, $f_2=261.90$ Hz, $f_3=361.88$ Hz and $f_4=550.51$ Hz. One can see that the first natural frequency
 257 is very similar to the experimental one indicated in Table 5.

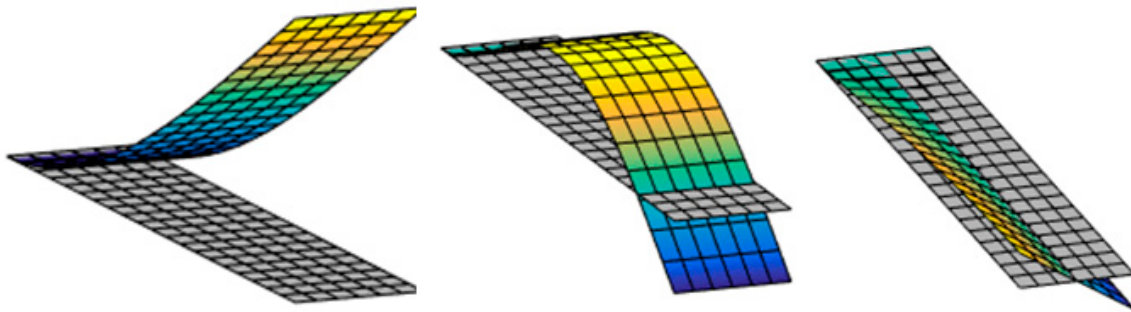
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Table 5. Natural frequencies and damping ratio results for the [90]8 carbon/epoxy specimens.

Replication	Sample 1		Sample 2		Sample 3		Sample 4	
	f_1	ξ	f_1	ξ	f_1	ξ	f_1	ξ
1	23.23	0.0034	33.04	0.0041	26.63	0.0055	37.44	0.0044
2	23.23	0.0048	33.04	0.0047	26.63	0.0061	37.44	0.0045
3	23.23	0.0029	33.04	0.0047	26.63	0.0059	37.44	0.0043
4	23.23	0.0048	33.04	0.0044	26.63	0.0054	37.44	0.0042
5	23.23	0.0053	33.04	0.0045	26.63	0.0051	37.44	0.0045
6	23.23	0.0046	33.04	0.0046	26.63	0.0056	37.44	0.0042
7	23.23	0.0048	33.04	0.0048	26.63	0.0051	37.44	0.0043
8	23.23	0.0048	33.04	0.0039	26.63	0.0057	37.44	0.0044
9	23.23	0.0042	33.04	0.0048	26.63	0.0053	37.44	0.0041
10	23.23	0.0046	33.04	0.0043	26.63	0.0048	37.44	0.0044
Mean	23.2	0.004	33.0	0.004	26.6	0.005	37.4	0.004
Standard deviation	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
CV	0.000	0.163	0.000	0.068	0.000	0.070	0.000	0.031

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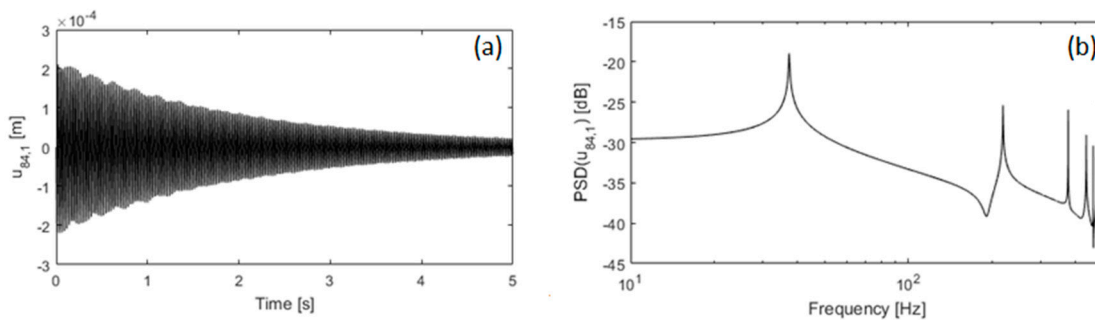


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Figure 6. Mode shapes corresponding to 1st (bending), 2nd (bending) and 4th (torsion) modes.



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Figure 7. Simulated time response for the transverse displacement of node 84 with damping ratios extracted from experimental data (a) and PSD for the displacement (b).

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7. Conclusions

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In this study, Hilbert transform was successfully applied for the identification of damping ratio in carbon/epoxy composite laminates. This method was readily implemented and considered a reliable alternative for damping ratio parameter calculations. Stability of this method was verified by the low dispersion of the obtained data (low coefficient of variation), considering the whole set of test repetitions for replicates. Experimental results revealed that damping varied according to the number of layers, ply orientation and specimen length. The damping variability between samples of the same family was attributed to changes in overall viscous damping (air drag around oscillating structures),

276 which is more pronounced in small and simple structures as those samples tested. Also, the proposed
277 method using the Hilbert transform technique allowed quick and systematic evaluation of the
278 variability between samples.
279

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