Consider the Initial Value Problem (IVP)

\[
\begin{align*}
    x'(t) &= f(x(t)) \\
    x(0) &= x_0.
\end{align*}
\]  

(1)

Suppose that the initial condition and/or coefficients are uncertain in (1) and given by fuzzy subsets. We have the following Fuzzy Initial Value Problem (FIVP):

\[
\begin{align*}
    x'(t) &\in \tilde{f}(x,K) \\
    x(0) &\in X_0,
\end{align*}
\]  

(2)

where \(K\) and \(X_0\) are fuzzy subsets in \(\mathcal{F}(\mathbb{R}^n)\).

According with H"ullermeier[2], (2) can be interpreted as the following family of differential inclusions:

\[
\begin{align*}
    x'(t) &\in [\tilde{f}(x,K)]^\alpha \\
    x_0 &\in [X_0]^\alpha,
\end{align*}
\]  

(3)

where \([\tilde{f}(x,K)]^\alpha\) and \([X_0]^\alpha\) are the \(\alpha\)-levels of the fuzzy subsets \(\tilde{f}(x,K)\) and \(X_0\), respectively.

As in [1] we saw that for each \(t > 0\), the attainable set obtained under the H"ullermeier’s interpretation for (2), or rather, through a family of differential inclusions of the type gives in (3) coincides with the solution obtained applying the principle extension of Zadeh in deterministic solution. We will define fuzzy flow and a fuzzy equilibrium point for (2) for the deterministic flow extended, in other words, applying the principle extension of Zadeh in deterministic flow. Since then, the theory of stability for (2) will also be established through the extended solution.

We verify that there is an equivalence of stability for the equilibrium points \(\bar{x}\) of (1) and fuzzy of type \(\chi(\bar{x})\) (characteristic of \(\bar{x}\)) of (2). Also, we present some results that relate attractors of (1) and the fuzzy asymptotically stable equilibrium of (2). We explored the malthusian and logistic models with the initial condition being a fuzzy subset, with the fuzzy coefficient and finally both, coefficient and initial condition.

Referências
