Cost Models for Vehicle Routing Problems

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Abstract

Most algorithms for solving vehicle routing problems use the criterion of minimizing total travel time in forming their routes. As illustrated in the paper, even in the case of a homogeneous fleet, solutions using other criteria may be preferred to the solution generated by minimizing total travel time. This situation becomes more pronounced when considering a vehicle routing problem where the fleet is non-homogeneous. In this paper, we explore the uses of more complex cost models in deriving solutions to vehicle routing problems, especially in the case of a non-homogeneous fleet. We develop a function called the Measure of Goodness that can be used as the criterion for solving these problems. We show how the Measure of Goodness criterion or special cases of this criterion can be used to compare solutions, generate an initial fleet mix or used as the criterion in a mixed integer program for carrying out between route swaps of the entities to be serviced. The vehicle routing problem emphasized in this paper is the Capacitated Arc Routing Problem with Vehicle/Site Dependencies (CARP-VSD).

1.0 A Dilemma

Consider the point-to-point vehicle routing problem (VRP) whose locations to be serviced are displayed in Figure 1. In this problem, we use Euclidean Distances to measure the distance between the stops and the stops and the depot. We assume there are no service times at the stops, the maximum length of any route is 35 units and all vehicles are identical; i.e., we have a homogeneous fleet of vehicles. We want to develop a set of routes to service these stops. The criterion ordinarily used to form these routes is to minimize total travel time.

The following two solutions (Solution 1 and Solution 2) can be generated to this example using well-known algorithms:

- Solution 1, which is displayed in Figure 2, requires three vehicles. The length of each route is 22.1 units so that the total length of the three routes is 66.3 units. This three-route solution can be generated using the Clark and Wright heuristic algorithm.

- Solution 2, which is displayed in Figure 3, requires two vehicles. The length of each route is 34.83 units so that the total length of the two routes is 69.66 units. This two-route solution can be generated using a ‘route first-cluster second’ or ‘giant tour’ heuristic approach.

Which solution is preferred? How can we find a solution to this dilemma? The three route solution is shorter (66.3 units rather than 69.66 units) but requires an extra vehicle (3 vehicles versus 2 vehicles). Since the algorithms were formed using heuristic algorithms, neither solution is guaranteed to be optimal – whatever optimal means. Traditional algorithms that do not require the number of vehicles to be specified as input can find the three route solution whereas other algorithms that do not specify the number of routes in advance can find either the two route solution or the three route solution. Some algorithms require the number of routes to be specified a-priori. If
an algorithm needs to know the number of vehicles to use, then the question arises as to how to estimate the number of vehicles to use in the solution.

In the above example, just minimizing total travel time may not be adequate to decide upon the preferred solution. One way to determine the preferred solution is to assign costs to the various metrics that can be computed from a solution. By assigning costs to the metrics of the problem, the user can decide if the capital cost associated with an additional vehicle offsets the operating cost of 3.36 additional units of route length in the solution.

The previous example describes one possible use of a cost model - to assist the user in determining the preferred solution to a VRP in the case of a homogeneous fleet. To find the preferred solution, the user determines a Measure of Goodness of a solution by forming a weighted linear combination of many of the factors that can be computed from a solution to a VRP. The user can then compare the Measure of Goodness for the various solutions generated in order to rank the solutions or select the solution that is preferred.
2.0 Comparing Solutions for a Non-Homogeneous Fleet

Comparing solutions to a VRP for a non-homogeneous fleet can become a more complex situation than the same situation for a homogeneous fleet (as described in the previous section). Solutions 3 and 4 illustrate the ambiguities that can arise in solving a non-homogeneous fleet VRP.

- **Solution 3**: The fleet consists of 5 large vehicles and 38 hours of travel time.
- **Solution 4**: The fleet is comprised of 2 large vehicles, 2 medium size vehicles and 2 small vehicles and 44 hours of travel time.

The question as to what solution is best becomes more difficult to answer. Non-homogeneous fleet vehicle routing problems need a cost model that can compare solutions similar to the above two solutions in order to rank the solutions or determine the preferred solution. In some situations, the 5 large vehicle solution may be preferred to the 6 vehicle solution; in other situations, the 6 vehicle solution may be preferred to the 5 vehicle solution. The decision as to the preferred solution can be based on the capital and operating costs of both solutions as well as other factors such as the amount of overtime and the compactness of the routes. All of these factors can be combined into a Measure of Goodness of a solution. It is possible for a feasible solution with a more vehicles and more deadheading to be preferred to a feasible solution with less vehicles and less deadheading if its capital cost is significantly smaller.

In this paper, we want to discuss the use of cost models to generate and evaluate solutions to vehicle routing problems. Our analysis will focus on the use of cost models to generate and evaluate solution to the Capacitated Arc Routing Problem with Vehicle-Site Dependencies (CARP-VSD). The analysis presented in this paper is based on the PhD dissertation of Sniezek (Sniezek, 2001). Although this paper concentrates on analyzing the CARP-VSD, we believe that the ideas presented in this paper can be adapted for analyzing other classes of vehicle routing problems.

3.0 Capacitated Arc Routing Problems

The CARP–VSD is a variant of the Capacitated Arc Routing problem (CARP). In most CARPs, the following holds:

- The total travel time in any solution is the sum of the service times on the streets and the total deadhead time in the solution.
- The total service time on the streets is assumed to be a constant and is generally much larger than total deadhead time in the solution which is a variable.
Since, in most CARPs, the total deadhead time (a variable) is generally small when compared to the total service time (a constant), basing an algorithm strictly on total deadhead time may give misleading results.

A vehicle class is defined to be a set of vehicles with identical characteristics. In a CARP with a non-homogeneous fleet, the fleet of vehicles used in a solution can come from more than one vehicle class. A homogeneous fleet CARP has one vehicle class.

The CARP-VSD is a CARP where a non-homogeneous vehicle fleet and at least one street (arc or edge) in the underlying network has a vehicle/site dependency. A street has a vehicle/site dependency if vehicles from certain vehicle classes cannot service or traverse the street. The vehicle/site dependency on a street determines the vehicle classes that can service the street, the vehicle classes that can travel along the street but not service the street, and the vehicle classes that can neither travel along the street nor service the street.

The Composite Approach presented in Sniezek (2001) is a procedure for finding a feasible solution to the CARP-VSD. To determine this feasible solution, the Composite Approach breaks the network of streets to be serviced into partitions, assigns a vehicle from a vehicle class to each partition and exchanges streets between partitions. The Composite Approach is integrated with well-known travel path generation procedures to derive an overall algorithm for solving the CARP-VSD. The uses of the cost model described in this paper is key to the successful use of the Composite Approach. Residential solid waste collection serves as the motivating example for solving the CARP-VSD.

4.0 Uses of Cost Models for Solving the CARP-VSD

The Composite Approach uses cost models for the following purposes.

a. To compare alternative solutions to the CARP-VSD.
b. To determine an initial fleet mix for the CARP-VSD.
c. To be the objective function of a mixed integer program that breaks a network into feasible partitions for the CARP-VSD.

Critical to the use of these cost models is the Measure of Goodness of a solution to the CARP-VSD mentioned in section 1 of this paper. In the next section, we want to quantify the notion of the Measure of Goodness for the CARP-VSD.

5. Measure of Goodness for the CARP-VSD

The principal cost factors to any solution to the CARP-VSD are now defined. The Measure of Goodness for any solution to the CARP-VSD is a linear combination of these principal cost factors along with a measure of the compactness of the partitions.

5.1 Principal Cost Factors to a Solution to the CARP-VSD


Each partition (and route) requires a vehicle and the vehicles in a solution can vary in size. Associated with each vehicle is an amortized daily capital cost that can be determined from the purchase cost of the vehicle, maintenance cost, insurance cost, etc, along with the expected lifetime of the vehicle. This daily capital cost is vehicle class dependent since larger vehicles generally have a larger daily capital cost than smaller vehicles. Let $\text{CAP}_k$ represent the daily capital cost of a vehicle from vehicle class $k$.

b. Salary Cost of the Crew Operating the Vehicle.

Associated with each partition is a crew that operates the vehicle. A fixed daily salary including benefits and overhead can be established for a crew. Since the number of persons in a crew can be vehicle class dependent, the salary cost can be vehicle class dependent. In sanitation routing, some vehicle classes require a one person crew while other vehicle classes require a two or three person crew. (As a note, in some countries, sanitation crews have as many as 10 persons in the crew.) Let $\text{SAL}_k$ represent
the daily salary cost of a crew associated with vehicle class k.

c. **Overtime**

The salary cost paid to a crew is for a specified workday (generally 8 hours or 10 hours). Let $\text{TARGET}_k$ represent the target length of the workday of a crew from vehicle class k. If a partition requires a crew longer than $\text{TARGET}_k$, overtime must be paid to the crew operating the vehicle. Let $\text{SHOUR}_k$ represent the hourly cost of overtime for a crew from vehicle class k.

d. **Mileage Cost of Operating the Vehicle.**

The operating cost associated with the vehicle covering the streets on its route can be vehicle class dependent. Let $\text{SMILE}_k$ represent this cost per mile traveled by a vehicle from vehicle class k. $\text{SMILE}_k$ is assumed to be the same whether the vehicle is performing a service or deadhead.

e. **Tipping Cost of a Sanitation Vehicle at the Disposal Facility**

The tipping cost of a vehicle at the disposal facility is a cost that is specific to sanitation VRPs and, hence, becomes a cost factor for vehicle routing problems involving sanitation vehicles. Since a vehicle has a fixed capacity (let $M_k$ be the capacity of a vehicle from vehicle class k), when the vehicle capacity is reached, the vehicle must be emptied at a disposal facility. A vehicle can make several trips to a disposal facility over a day and the vehicle must pay a tipping cost each time it visits the disposal facility. The tipping cost is typically measured as a fixed cost plus a variable cost measured in $/unit of weight. Since the total weight collected over an entire solution is constant regardless of the fleet mix, the weight component of the tipping cost is a constant for any fleet mix. Let $\text{STIP}_k$ be the fixed cost portion of the tipping cost for a vehicle from vehicle class k. In addition, every trip to the disposal facility adds both time and distance to the statistics for a partition.

5.2 **Statistics for a Partition in the CARP-VSD**

In a solution to the CARP-VSD, assume that there are arcs and edges assigned to partition p and that a vehicle from vehicle class k is assigned to partition p. Further, assume that a travel path has been formed through the streets assigned to partition p. Then, the following statistics can be computed for partition p.

a. **Total Distance** ($\text{DIST}_p$)

$\text{DIST}_p$ is the total distance that the vehicle travels in the travel path associated in covering all of the arcs and edges in partition p. This distance includes deadhead distance and the distance to/from the disposal facility and the distance to/from the depot.

b. **Total Time** ($\text{TIME}_p$)

$\text{TIME}_p$ is the total time, including deadhead time, that the vehicle that services partition p requires to traverse the travel path that covers all of the arcs and edges in partition p. This time includes the time to/from the disposal facility and the time to/from the depot.

c. **Overtime** ($\text{OT}_p$)

$\text{OT}_p = \text{MAX} \left[ (\text{TIME}_p - \text{TARGET}_k), 0 \right]$ is the overtime associated with partition p. Since a vehicle from vehicle class k is assigned to partition p, $\text{TARGET}_k$ is known.

d. **Total Trips** ($\text{Z}_p$)

$\text{Z}_p$ is the total number of trips to the disposal facility needed to service all arcs in partition p. We assume that $\text{Z}_p$ includes one trip to the disposal facility at the end of the workday to unload any residual volume on the vehicle. In some sanitation applications, the vehicles can return to the depot at the end of the day partially loaded and not emptied at the disposal facility.

5.3 **Total Cost of a Partition**

Given the above statistics for partition p where a vehicle from vehicle class k is assigned to service partition p, the total cost of this partition is the sum of the fixed costs ($\text{FC}_p$), variable labor costs ($\text{VLC}_p$) and variable routing costs ($\text{VRC}_p$) for the partition where:

$$\text{FC}_p = \text{SCAP}_k + \text{SSAL}_k \quad (1)$$
\[ VLC_p = \text{HOUR}_k \times \text{MAX} \left( \text{TIME}_p - \text{TARGET}_k, 0 \right) \] (2)

\[ VRC_p = \text{MILE}_k \times \text{DIST}_p + \text{TIP}_k \times Z_p \] (3)

### 5.4 Total Cost of a Solution

The total cost of a solution is the sum of the costs of each individual partition; i.e.

\[ \text{Total Solution Cost} = \sum_p (\text{FC}_p + VLC_p + VRC_p) \] (4)

If we assume that there is only one vehicle class, no overtime and one trip per route, then, for any partition \( p \), \( \text{CAP}_k, \text{SAL}_k, \text{MILE}_k \) and \( \text{TIP}_k \) do not depend upon vehicle class \( k \), \( \text{HOUR}_k = 0 \) and \( Z_p = 1 \). Thus, the only variable in the cost of any partition \( p \) is \( \text{DIST}_p \) and the total variable cost of a solution is the total travel time over all of the routes, \( \sum_p \text{DIST}_p \). Minimizing \( \sum_p \text{DIST}_p \) is the typical objective of most VRPs.

### 5.5 Partition Compactness

In many arc routing applications, the partitions must be compact and the interlacing of the routes must be avoided. Let \( C_p \) be the measure of partition compactness for partition \( p \). Assume arc \( a \) is assigned to partition \( p \). There are two nodes that define the endpoints of arc \( a \). \( D_{ap} \) is defined as the distance from the closer of the two nodes making up the endpoints of arc \( a \) to the seed point of partition \( p \). Given this measure of \( D_{ap} \), the measure of compactness for partition \( p \) is

\[ C_p = \sum_a (D_{ap})^2 \] where the sum is over all arcs \( a \) assigned to partition \( p \) (5)

Let \( \text{GEOSPREAD} \) be the measure of compactness for a solution. Then, \( \text{GEOSPREAD} \) is computed as follows:

\[ \text{GEOSPREAD} = \sum_p C_p \] (6)

### 5.6 Measure of Goodness for a Solution

\( \text{MG} \), the Measure of Goodness for a solution can then be computed as follows:

\[ \text{MG} = A \times \text{Total Solution Cost} + B \times \text{GEOSPREAD} \] (7)

In (7), \( A \) and \( B \) are parameters that allow for a tradeoff between the total cost of a solution and the route compactness of a solution to be considered.

### 5.7 Discussion

The evaluation of a solution in Sections 5.1-5.6 allows the user to estimate the total solution cost, measure the compactness of each route in a solution and use \( \text{MG} \) to measure the goodness of the solution. The parameters \( A \) and \( B \) in (7) and the parameters used to compute (4) give the user great flexibility in trading off between the cost of a solution and the route compactness of a solution. The above functions are developed specifically for the CARP-VSD; however, similar functions can be developed for other vehicle routing applications.

### 6. Comparison of Alternative Solutions to a CARP-VSD

The Total Solution Cost (4) or the Measure of Goodness of a solution (7) allows us to compare alternative solutions to a CARP-VSD. Thus, any solution can be measured in terms of distance, time, vehicle configuration, fixed vehicle costs and route compactness. Using (4) to quantify a solution can help answer questions such as the following: Which solution is preferred: A four vehicle solution composed of large vehicles or a five vehicle solution composed of small vehicles? In the above cost model, most of the quantities can be determined from a solution and only a few quantities need to be estimated. Thus, this cost model can serve as a good base for comparing the quality of solution derived by different algorithms to a given vehicle routing problem.

### 7. Use of the Cost Model to Determine an Initial Fleet Mix for the CARP-VSD

The algorithm developed in Sniezek (2001) for determining an initial solution to the CARP-VSD needs an initial fleet mix. We have used the cost model concept described in Section 5 to formulate a mixed integer programming problem in order to find an initial fleet mix. Input to the mixed integer program are the costs identified with each vehicle class and the value of the Vehicle Class Workload Matrix \( V \). The generation of the \( V \) matrix is described in detail in Chapter
3 of Sniezek (2001) and briefly described below in Section 7.2.

7.1 Estimating the Daily Cost of Each Vehicle Class

As described in Section 5, the total cost associated with a partition includes fixed costs (Capital Costs + Crew Salary Costs), variable labor costs (Overtime Cost), and variable routing costs (Mileage Costs + Disposal Costs). Each of these costs is a function of the vehicle class servicing a partition. An example that illustrates the computation of these costs is given in Tables 1-3. Table 1 shows the computation of fixed costs by vehicle class, Table 2 shows how to estimate the variable routing costs by vehicle class, and Table 3 gives an estimated total cost comparison by vehicle class.

The following was assumed in generating these tables:

- Crew size by vehicle class is specified and each member of the crew costs $150/day, including benefits.
- Daily capital cost of a vehicle from any vehicle class is amortized over 1000 workdays (approximately 4 years). The daily capital cost times 1000 gives the total capital cost of the vehicle, by vehicle class.
- The tipping fee is $100. The typical number of trips to the disposal facility is assumed known by vehicle class.
- The mileage on each route is estimated and includes the mileage between the routes and the disposal facility and the mileage to and from the depot. Each mile is assumed to cost $2.
- The overtime associated with each route is zero. This is not a requirement of the solution. However, for estimating costs, the initial assumption is that no overtime will be required.

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Daily Capital Cost</th>
<th>Crew Size</th>
<th>Daily Crew Cost</th>
<th>Daily Fixed Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>2</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>137</td>
<td>2</td>
<td>300</td>
<td>437</td>
</tr>
<tr>
<td>3</td>
<td>156</td>
<td>2</td>
<td>300</td>
<td>456</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
<td>3</td>
<td>450</td>
<td>700</td>
</tr>
</tbody>
</table>

Table 1 – Daily fixed cost comparison among vehicle classes.

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Disposal Trips</th>
<th>Disposal Cost</th>
<th>Route Mileage</th>
<th>Mileage Cost</th>
<th>Daily Routing Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>400</td>
<td>60</td>
<td>120</td>
<td>520</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>300</td>
<td>50</td>
<td>100</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>300</td>
<td>50</td>
<td>100</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>200</td>
<td>40</td>
<td>80</td>
<td>280</td>
</tr>
</tbody>
</table>

Table 2 – Estimated daily routing cost comparison among vehicle classes.

<table>
<thead>
<tr>
<th>Vehicle Class</th>
<th>Fixed Cost</th>
<th>Variable Routing Costs</th>
<th>Variable Labor (OT) Costs</th>
<th>Total Vehicle Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>520</td>
<td>0</td>
<td>920</td>
</tr>
<tr>
<td>2</td>
<td>437</td>
<td>400</td>
<td>0</td>
<td>837</td>
</tr>
<tr>
<td>3</td>
<td>456</td>
<td>400</td>
<td>0</td>
<td>856</td>
</tr>
<tr>
<td>4</td>
<td>700</td>
<td>280</td>
<td>0</td>
<td>980</td>
</tr>
</tbody>
</table>

Table 3 – Estimated daily total cost comparison among vehicle classes.
7.2 Determining the V Matrix

A detailed discussion of determining the $V = (V(i,j))$ matrix is given in Chapter 3 of Sniezek (2001). The entry, $V(i,j)$, in the $V$ matrix represents the estimated number of vehicles from vehicle class $j$ required to service all arcs whose largest vehicle class that can service these arcs is vehicle class $i$.

A sample $V$ matrix for a four vehicle class problem is given in Figure 4. This $V$ matrix is based on an actual 1800 arc and edge street network from Philadelphia, PA. This network contains the actual site dependencies that exist in Philadelphia.

Thus, the element $V(3,1) = 3.604$ says that we estimate that 3.604 vehicles from vehicle class 1 are needed to service all of the streets whose largest vehicle class that can service these streets is vehicle class 3. Many of our computational tests are based on this network.

\[
\begin{array}{cccc}
1.987 & \text{------} & \text{------} & \text{------} \\
0.958 & 0.756 & \text{------} & \text{------} \\
3.604 & 2.807 & 2.807 & \text{------} \\
8.456 & 15.380 & 13.445 & 11.530 \\
\end{array}
\]

Figure 4: $V$ matrix for example

7.3 The Mixed Integer Program to Determine the Initial Fleet Mix

The following mixed integer program uses the estimated daily cost of a vehicle class and the $V$ Matrix to determine an initial fleet mix. The solution to this mixed integer program allocates enough vehicles to satisfy the workload requirements of each row of the $V$ matrix while minimizing the estimated cost of the fleet mix. The formulation of this mixed integer program is the following.

7.3.1 Variables in the Mixed Integer Program

The variables in the mixed integer program are as follows:

- $X_{ij}$ = The number of vehicles used from vehicle class $j$ for allocating (satisfying) the work in row $i$ of the $V$ matrix.
- $S_i$ = The excess work (slack) available after satisfying the requirements for row $i$ of the $V$ matrix. This slack is present because the required number of vehicles needed to satisfy a row of the $V$ matrix may be non-integer while the number of vehicles allocated must be integer. This slack can be used toward meeting the requirements of subsequent rows of the $V$ matrix.

7.3.2 Data Elements Input to the Mixed Integer Program

The data elements input to the mixed integer program are as follows:

- $C_j$ = Estimated total daily cost of a vehicle from vehicle class $j$ as determined in Table 3.
- The matrix $V$ with data elements $V_{ij}$ (see section 7.2 for a description of the $V$ matrix)
- $QMAX_j$ = The maximum number of vehicles available from vehicle class $j$.
- $QMIN_j$ = The minimum number of vehicles from vehicle class $j$ that must be used in the solution. This data element will typically be zero but is included in case there is a contractual obligation to use a certain number of vehicles from a specific vehicle class or some other condition specific to the organization.
- $VCLASS$ = The total number of unique vehicle classes that can be used in the fleet mix. The $V$ Matrix has $VCLASS$ rows and $VCLASS$ columns.

7.3.3 Formulation of the Mixed Integer Program

The mixed integer program (IFM) given in (8) to (15) determines an initial fleet mix for the CARP-VSD. The formulation of the IFM is as follows:
(IFM) \[ \text{Min } Z = \sum_{i} \sum_{j} C_{j} X_{ji} \quad (8) \]

subject to the following conditions:

\[ (\sum_{j=1}^{j_{\text{CLASS}}} \left( \frac{V_{ii}}{V_{ij}} \right) X_{ji}) + S_{i-1} - S_i = V_{i1} \quad \forall \ i \quad (9) \]

\[ \sum_{i=j}^{v_{\text{CLASS}}} X_{ji} = X_j \quad \forall \ j \quad (10) \]

\[ X_j \leq Q_{\text{MAX}j} \quad \forall \ j \quad (11) \]

\[ X_j \geq Q_{\text{MIN}j} \quad \forall \ j \quad (12) \]

\[ S_0 = 0 \quad (13) \]

All \( X_{ji} \) and \( X_j \) variables are integer and nonnegative \quad (14)

All \( S \) variables are nonnegative \quad (15)

The constraints of the mixed integer program represent the following.

- (8) minimizes the estimated total cost of the fleet mix chosen.
- (9) ensures that enough vehicles from the vehicle classes are used to satisfy the workload for each row of the \( V \) matrix. \( V_{ii}/V_{ij} \) is a straightforward proration formula on workload. If \( V_{i1} = 12 \) and \( V_{ij} = 3 \), then it is assumed that a vehicle from vehicle class \( j \) can cover the workload of 4 vehicles from vehicle class 1. As such, only one vehicle from vehicle class \( j \) is needed to cover the work of 4 vehicles from vehicle class 1.
- (10) tallies the number of vehicles used from each vehicle class.
- (11) and (12) are the upper and lower bounds on the total number of vehicles from each vehicle class that can be used in the fleet mix.
- (13) states that there is no slack available toward satisfying row 1 of the \( V \) matrix. Recall that the slack variables are introduced to account for an integer number of vehicles being used to satisfy a non-integer demand in a row of the \( V \) matrix. Since we start with row 1, there can be no slack yet realized from a previous row of the \( V \) matrix and as such, \( S_0 \) is set to zero.
- (14) ensures that the number of vehicles used in the fleet mix are both nonnegative and integer.
- (15) ensures the slack variables are nonnegative.

### 7.3.4 Example

Using the cost estimates and \( V \) matrix defined above and assuming that all vehicles have to be purchased, the IFM becomes the following mixed integer program (IFM2):

\[
\text{MIN } Z = 920X_{11} + 920X_{12} + 920X_{13} + 920X_{14} +
837X_{22} + 837X_{23} + 837X_{24} + 856X_{33} + 856X_{34} +
980X_{44}
\]

SUBJECT TO

\[
X_{11} + S_0 - S_1 = 1.987
\]

\[
X_{12} + 1.27X_{22} + S_1 - S_2 = 0.958
\]

\[
X_{13} + 1.28X_{33} + 1.28X_{33} + S_2 - S_3 = 3.604
\]

\[
X_{14} + 1.20X_{44} + 1.37X_{44} + 1.60X_{44} + S_3 - S_4 = 18.456
\]

\[
X_1 - X_{11} - X_{12} - X_{13} - X_{14} = 0
\]

\[
X_2 - X_{22} - X_{23} - X_{24} = 0
\]

\[
X_3 - X_{33} - X_{34} = 0
\]

\[
X_4 - X_{44} = 0
\]

\[ S_0 = 0 \]

All variables nonnegative

All \( X_{ij} \) and \( X \) variables are integer.

The solution to IFM2 is \( X_1 = 2 \), \( X_2 = 6 \), \( X_3 = 3 \), and \( X_4 = 7 \) and the value of the objective function of IFM2 is 16290. Thus, the initial fleet for this example is the following: two vehicles from vehicle class 1, six vehicles from vehicle class 2, three vehicles from vehicle class 3, and seven vehicles from vehicle class 4 and the estimated total cost of the solution is $16,290/day.

### 7.3.5 Company Owned Fleet Example

If we assume that the company already owns 25 Class 1 vehicles and only Class 1 vehicles can be used (a homogeneous fleet), then there is no capital cost associated with these 25 vehicles and the daily cost of the Class 1 vehicles would drop from $920 to $820. This fleet would have an estimated cost of $20,500/day ($820*25). The 25 Class 1 vehicle fleet is the minimum number of Class 1 vehicles needed to cover total demand.
Assuming the daily cost of a vehicle from vehicle class 1 is $820, then the fleet mix determined by the IFM is two vehicles from vehicle class 1, six vehicles from vehicle class 2, three vehicles from vehicle class 3, and seven vehicles from vehicle class 4. The total cost of the IFM is $16,090. This fleet mix is $4,410/day less expensive than the company owned fleet of 25 Class 1 vehicles. In other words, the company saves $4,410/day by purchasing vehicles and incurring additional capital costs rather than using the vehicles that they already own. The salvage value of the company owned vehicles is not considered in the IFM.

7.3.6 Using the IFM as a Cost Estimation Tool

We then analyzed the IFM to determine if the IFM could be used as a tool to estimate the cost of a CARP-VSD. We evaluated 42 different fleet mixes by the objective function of the IFM. Then, each of the 42 fleet mixes was processed by a partition generation heuristic called the Vehicle Decomposition Algorithm (VDA). The VDA is described in Sniezek (2001). The VDA is initiated with the fleet mix found from the IFM.

The VDA partitions the area to be serviced into partitions as dictated by the fleet mix found from in the IFM. To complete a VDA solution, the travel paths are generated, deadhead travel distance and time as well as actual trips to the disposal facility are determined and overtime is computed. The determination of the VDA solution requires considerably more data preparation and software development that the determination of the IFM solution. We used the estimated vehicle costs computed in Table 3; that is to say, we assumed that the company had to purchase the entire fleet. The results from our 42 scenarios are listed in Table 4. These problems are described in Chapter 4, Section 3 of Sniezek (2001).

The results were very encouraging. We observed the following.

- In 8 of these 42 trials, the estimated cost found by the MIP was within 1% of the cost found from the VDA solution.
- In 20 of these 42 trials, the estimated cost found by the MIP was within 2% of the cost found from the VDA solution.
- In 28 of these 42 trials, the estimated cost found by the MIP was within 3% of the cost found from the VDA solution.
- In total, 41 of the 42 trials, the estimated cost found by the MIP was within 6% of the cost found from the VDA solution.
- The worst estimated cost found by the MIP was 6.33% from cost found the VDA solution.
- The average deviation of the total estimate cost found by the MIP from the cost found by the VDA solution over all 42 scenarios was 2.31%.
- In 38 of the 42 scenarios, the cost found by the VDA solution was lower than the estimated cost found by solving the IFM.

It appears that the estimated cost of a solution found from the IFM with the assumed fleet mix appears to be close to the actual cost of the VDA solution. Thus, we concluded that the IFM could be a very useful tool for determining a fleet mix and for estimating the cost of the fleet mix without having to generate the partitions and travel paths, thereby avoiding considerable data preparation. Therefore, the MIP may be very useful in a sketch planning environment where we are only trying to estimate the cost of a fleet mix. In this case, we want to get an idea of what the best fleet mix might be without having to generate a feasible solution for each case. As Sniezek (2001) shows, generating a complete solution to the CARP-VSD for large problems can be complex and lengthy computationally.
8.0 Use of the Cost Model in the Objective of an Mathematical Program for Partitioning a Street Network

The Composite Approach proposed by Sniezek (2001) for generating a solution to the CARP-VSD involves solving a mixed integer program (MPP) as a between-route route improvement procedure. A description of the MPP can be found in Sniezek (2001).

Input to the MPP is a subset of the routes that contain no more than 250 arcs and edges. The MPP has the capability of breaking the arcs and edges in the subset of the routes into partitions and assigning a vehicle from the proposed fleet mix to each of the partitions. The objective of the MPP is the Measure of Goodness (see equation (7)). In the objective function, we can vary the weights A and B in (7) to try to find a better solution that emphasizes either total cost or compactness or, in some cases, try to improve both total cost and compactness by assigning them equal weight in the objective function.

Further, we add the following two constraints to the MPP.

Total Solution Cost \( \leq \) Value of Total Solution Cost from Previous Solution

GEOSPREAD \( \leq \) Value of GEOSPREAD from Previous Solution

Constraints (16) and (17) insure that the solution found by the MPP is no worse than the previous solution.

Using the Measure of Goodness as the objective function along with constraints (16) and (17) gives the user tremendous flexibility in trying to find an improved solution. The user has complete control on the values of A and B and the values of the right hand sides of (16) and (17). If desired, the user can choose to improve GEOSPREAD while allowing for a slight increase in cost. Conversely, the user can choose to decrease cost by allowing for a little less compactness. Several scenarios of using the composite procedure in this context are described in Sniezek (2001).

9.0 Conclusions

In this paper, we have shown that the use of most complex cost models for the analysis of vehicle routing problems can lead to finding good and implementable solutions to vehicle routing problems. Further, these cost models can be used to find an initial fleet mix or as the objective function in a mathematical programming problem that performs between-route swaps. The motivating example in this analysis is the CARP/VSD.

We further have shown that the cost model function called the Measure of Goodness, can generate a reasonably accurate solution to the CARP-VSD without having to generate a complete of routes and schedules. This allows us to estimate the cost of alternative non-homogeneous fleets in a very rapid manner. The notion of a cost model, although not unique in transportation applications, is relatively unique when comparing vehicle routing solutions to the same problem. In the 1970s, transit systems generally used these cost models to estimate the costs of bus systems. In these unit cost models, the estimated cost of a proposed timetable for a transit system was simply the following:

\[
\text{Cost} = A \times (\text{Number of Buses}) + B \times (\text{Number of Miles Traversed})
\]

These cost models for analyzing bus systems were extended in Bodin, Rosenfield and Kydes (1978) and Bodin, Rosenfield and Kydes (1981) to include factors such as the number of bus drivers needed, overtime, maintenance costs, etc.

Further, these cost models allows for the use of overtime in generating routes and analyzing solutions. These models confirm a long-term conjecture of the authors that using overtime can help to generate less expensive solutions to vehicle routing problems because of the savings in capital cost that results. These cost models allow for the tradeoff between overtime and increased capital cost to be fully explored.
<table>
<thead>
<tr>
<th>Problem Name</th>
<th>Fleet Mix Used</th>
<th>IFM Cost</th>
<th>VDA Cost</th>
<th>Net Difference</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSD1-ST1</td>
<td>2-1-3-11</td>
<td>$16,025</td>
<td>$15,858</td>
<td>($167)</td>
<td>-1.04%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
<td>3-17-0-0</td>
<td>$16,989</td>
<td>$16,735</td>
<td>($254)</td>
<td>-1.50%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
<td>7-0-0-12</td>
<td>$18,200</td>
<td>$17,048</td>
<td>($1,152)</td>
<td>-6.33%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
<td>4-5-5-5</td>
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<td>$16,409</td>
<td>($636)</td>
<td>-3.73%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
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<td>$16,022</td>
<td>($268)</td>
<td>-1.65%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
<td>6-2-5-6</td>
<td>$17,354</td>
<td>$16,737</td>
<td>($617)</td>
<td>-3.56%</td>
</tr>
<tr>
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<td>4-2-6-22</td>
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<td>$31,205</td>
<td>($845)</td>
<td>-2.64%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
<td>6-0-32-0</td>
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<td>-2.39%</td>
</tr>
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<td>$31,793</td>
<td>$31,985</td>
<td>$192</td>
<td>0.60%</td>
</tr>
<tr>
<td>VSD1-ST1</td>
<td>8-10-10-10</td>
<td>$34,090</td>
<td>$32,709</td>
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</tr>
<tr>
<td>VSD1-ST2</td>
<td>4-7-3-20</td>
<td>$31,707</td>
<td>$32,072</td>
<td>$365</td>
<td>1.15%</td>
</tr>
<tr>
<td>VSD1-ST3</td>
<td>6-3-8-34</td>
<td>$48,199</td>
<td>$47,640</td>
<td>($559)</td>
<td>-1.16%</td>
</tr>
<tr>
<td>VSD1-ST3</td>
<td>9-0-48-0</td>
<td>$49,368</td>
<td>$47,146</td>
<td>($2,222)</td>
<td>-4.50%</td>
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<tr>
<td>VSD1-ST3</td>
<td>6-14-0-32</td>
<td>$48,598</td>
<td>$47,741</td>
<td>($857)</td>
<td>-1.76%</td>
</tr>
<tr>
<td>VSD1-ST3</td>
<td>13-15-15-15</td>
<td>$52,055</td>
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<td>($2,397)</td>
<td>-4.60%</td>
</tr>
<tr>
<td>VSD1-ST3</td>
<td>6-11-4-31</td>
<td>$48,531</td>
<td>$48,143</td>
<td>($388)</td>
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</tr>
<tr>
<td>VSD1-ST3</td>
<td>8-10-5-30</td>
<td>$49,410</td>
<td>$48,013</td>
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<td>-2.83%</td>
</tr>
<tr>
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<td>7-4-3-6</td>
<td>$18,236</td>
<td>$18,006</td>
<td>($230)</td>
<td>-1.26%</td>
</tr>
<tr>
<td>VSD2-ST1</td>
<td>12-3-3-3</td>
<td>$19,059</td>
<td>$18,366</td>
<td>($693)</td>
<td>-3.64%</td>
</tr>
<tr>
<td>VSD2-ST1</td>
<td>12-8-1-1</td>
<td>$19,572</td>
<td>$19,174</td>
<td>($398)</td>
<td>-2.03%</td>
</tr>
<tr>
<td>VSD2-ST1</td>
<td>8-7-5-1</td>
<td>$18,479</td>
<td>$18,029</td>
<td>($450)</td>
<td>-2.44%</td>
</tr>
<tr>
<td>VSD2-ST2</td>
<td>14-8-6-11</td>
<td>$35,492</td>
<td>$35,690</td>
<td>$198</td>
<td>0.56%</td>
</tr>
<tr>
<td>VSD2-ST2</td>
<td>29-5-5-5</td>
<td>$40,045</td>
<td>$38,197</td>
<td>($1,848)</td>
<td>-4.61%</td>
</tr>
<tr>
<td>VSD2-ST2</td>
<td>23-20-1-1</td>
<td>$39,736</td>
<td>$38,644</td>
<td>($1,092)</td>
<td>-2.75%</td>
</tr>
<tr>
<td>VSD2-ST2</td>
<td>19-14-10-1</td>
<td>$38,738</td>
<td>$36,482</td>
<td>($2,256)</td>
<td>-5.82%</td>
</tr>
<tr>
<td>VSD2-ST3</td>
<td>20-12-10-16</td>
<td>$52,684</td>
<td>$53,141</td>
<td>$457</td>
<td>0.87%</td>
</tr>
<tr>
<td>VSD2-ST3</td>
<td>41-8-8-8</td>
<td>$59,104</td>
<td>$57,294</td>
<td>($1,810)</td>
<td>-3.06%</td>
</tr>
<tr>
<td>VSD2-ST3</td>
<td>40-26-1-1</td>
<td>$60,398</td>
<td>$59,163</td>
<td>($2,235)</td>
<td>-3.04%</td>
</tr>
<tr>
<td>VSD2-ST3</td>
<td>23-20-19-1</td>
<td>$55,144</td>
<td>$54,499</td>
<td>($645)</td>
<td>-1.17%</td>
</tr>
<tr>
<td>VSD3-ST1</td>
<td>6-5-5-5</td>
<td>$18,885</td>
<td>$17,809</td>
<td>($1,076)</td>
<td>-5.70%</td>
</tr>
<tr>
<td>VSD3-ST1</td>
<td>12-5-2-3</td>
<td>$19,877</td>
<td>$19,121</td>
<td>($756)</td>
<td>-3.80%</td>
</tr>
<tr>
<td>VSD3-ST1</td>
<td>6-10-2-3</td>
<td>$18,542</td>
<td>$18,405</td>
<td>($137)</td>
<td>-0.74%</td>
</tr>
<tr>
<td>VSD3-ST1</td>
<td>6-5-8-2</td>
<td>$18,513</td>
<td>$18,418</td>
<td>($95)</td>
<td>-0.51%</td>
</tr>
<tr>
<td>VSD3-ST1</td>
<td>12-10-9-9</td>
<td>$35,934</td>
<td>$35,365</td>
<td>($569)</td>
<td>-1.58%</td>
</tr>
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<td>VSD3-ST1</td>
<td>24-10-4-6</td>
<td>$39,754</td>
<td>$37,614</td>
<td>($2,140)</td>
<td>-5.38%</td>
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<td>VSD3-ST1</td>
<td>12-20-4-6</td>
<td>$37,084</td>
<td>$36,929</td>
<td>($155)</td>
<td>-0.42%</td>
</tr>
<tr>
<td>VSD3-ST1</td>
<td>12-10-16-4</td>
<td>$37,026</td>
<td>$36,666</td>
<td>($360)</td>
<td>-0.97%</td>
</tr>
<tr>
<td>VSD3-ST3</td>
<td>18-15-13-14</td>
<td>$53,963</td>
<td>$53,337</td>
<td>($626)</td>
<td>-1.16%</td>
</tr>
<tr>
<td>VSD3-ST3</td>
<td>36-15-6-9</td>
<td>$59,631</td>
<td>$56,633</td>
<td>($2,998)</td>
<td>-5.03%</td>
</tr>
<tr>
<td>VSD3-ST3</td>
<td>18-30-6-9</td>
<td>$55,626</td>
<td>$54,509</td>
<td>($1,117)</td>
<td>-2.01%</td>
</tr>
<tr>
<td>VSD3-ST3</td>
<td>18-15-24-6</td>
<td>$55,539</td>
<td>$54,889</td>
<td>($650)</td>
<td>-1.17%</td>
</tr>
</tbody>
</table>

Total: $1,500,078 $1,465,384 ($34,694) -2.31%

Table 4 - Estimated solution costs versus actual solution costs.